University of California, Davis
Department of Economics

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE,

MACROECONOMICS

Answer Key: 200E Questions

July, 2023

Question 3 (*20 points*)

This question examines two different potential drivers of the business cycle — shocks to productivity and shocks to the supply of labor. Consider the following decentralized business cycle model. There are a continuum of identical households and the representative household maximizes lifetime expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln(C_t) - \frac{1}{\gamma_t} N_t \right) \tag{1}$$

subject to their budget constraint:

$$C_t + I_t = w_t N_t + r_t^k K_t + \Pi_t \tag{2}$$

where w_t is the real wage, N_t is hours worked, I_t is investment, r_t^k is the rental price of capital and Π_t are profits from firms. As usual, K_t is the capital stock available at time t (i.e. the closing stock of capital at the end of $t-1$). γ_t affects the dis-utility of supplying labor and will be discussed below. Assume $0 < \beta < 1$.

Capital is related to investment I as follows:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{3}$$

but assume full depreciation so $\delta = 1$.

There are a continuum of identical firms who produce under perfect competition. Firms produce output, Y_t, using capital and labor. The production function is given by:

$$Y_t = A_t K_t^{\alpha} (N_t)^{1-\alpha} \tag{4}$$

where $0 < \alpha < 1$. A_t is total factor productivity.

This model features two exogenous shocks. The first is a shock that affects the dis-utility of supplying labor, γ_t. The second is a shock to productivity, A_t. Both γ_t and A_t are stochastic and follow a stationary Markov process. There is no trend growth.

a) Write down the household's problem in recursive form and write down the firm's maximization problem. Derive the household's first order conditions and the firm's optimal hiring rules.

Sketch answer

Denoting k_t^S as capital chosen by the representative household and K_t as the aggregate capital stock: The recursive formulation is:

$$V(k_t^S, K_t, A_t, \gamma_t) = \max_{C_t, N_t, k_{t+1}^S} \left(\ln C_t - \frac{1}{\gamma_t} N_t^s + \beta E_t V(k_{t+1}^S, K_{t+1}, A_{t+1}, \gamma_{t+1}) \right)$$

subject to

$$C_t + I_t = w_t N_t^s + r_t^k k_t^s + \Pi_t$$
$$k_{t+1}^s = I_t + (1-\delta) k_t^s$$

and setting this up as a constrained problem:

$$\begin{aligned} V(k_t^S, K_t, A_t, \gamma_t) &= \max_{C_t, N_t, k_{t+1}^S} (\ln C_t - \frac{1}{\gamma_t} N_t^s + \beta E_t V(k_{t+1}^S, K_{t+1}, A_{t+1}, \gamma_{t+1}) && (5) \\ &+ \lambda_t (w_t N_t^s + r_t^k k_t^s + \Pi_t - C_t - k_{t+1}^s)) && (6) \end{aligned}$$

where we have already used the restriction that $\delta = 1$.

FOCs:

$$[C_t] \qquad \frac{1}{C_t} = \lambda_t$$
$$[K_{t+1}] \qquad \lambda_t = \beta E \partial V(k_{t+1}^S, K_{t+1}, A_{t+1}, \gamma_{t+1}) / \partial k_{t+1}^S$$
$$[N_t] \qquad \frac{1}{\gamma_t} = \lambda_t w_t$$

Using the Envelope theorem:

$$\partial V(k_t^S, K_t, A_t, \gamma_t) / \partial k_t^S = \lambda_t r_t^k$$

we can then write the Euler equation as follows:

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \left(r_{t+1}^k \right) \right]$$

Firms:

Denote k_t^d and N_t^d as the capital stock and labor input demanded by the representative firm:

$$\max_{N_t^d, k_t^d} A_t k_{d,t}^{\alpha} N_{d,t}^{1-\alpha} - w_t N_t^d - r_t^k k_t^d \tag{7}$$

$$r_t^k = \alpha A_t k_{d,t}^{\alpha-1} N_{d,t}^{1-\alpha} = \alpha Y_t / k_t^d$$
$$w_t = (1-\alpha) A_t k_{d,t}^{\alpha} N_{d,t}^{-\alpha} = (1-\alpha) Y_t / N_t^d$$

b) Carefully define a recursive competitive equilibrium.

Sketch answer:

A recursive competitive equilibrium is a value function $V(A_t, \gamma_t, k_t^s, K_t)$ decision rules $k_{t+1} = k(A_t, \gamma_t, k_t^s, K_t)$, $c_t = c(A_t, \gamma_t, k_t^s, K_t)$, $N_t = n(A_t, \gamma_t, k_t^s, K_t)$, a law of motion for the aggregate capital stock $K_{t+1} = g(A_t, \gamma_t, K_t)$, stochastic processes for A_t and γ_t, and prices $\left\{w(A_t, \gamma_t, K_t), r^k(A_t, \gamma_t, K_t)\right\}$

Such that

- Given the pricing functions and the law of motion for capital, the value function and decision rules solve the household's problem (the allocation satisfies all the first order conditions)
- The firm's optimality conditions are satisfied
- All markets clear:

$$k_t^d = k_t^s = K_t$$
$$N_t^d = N_t^s = N_t$$
$$Y_t = C_t + I_t$$

  and the law of motion for the individual state is consistent with the law of motion for the aggregate state (rational expectations):

$$g(A_t, \gamma_t, K_t) = k^s(A_t, \gamma_t, k_t^s, K_t)$$

c) Using your answers to parts (a) and (b) show that the first order conditions for this decentralized economy coincide with the equilibrium conditions from the social planner's problem. Explain why this is the case. In the course of your answer, make sure you show that firms make zero profit and that the aggregate resource constraint is satisfied. (**Hints:** you do not need to set up the social planner's problem for this question. The goal is to combine your results from parts (a) and (b) to find the equivalent equilibrium conditions).

Sketch answer

First, impose the market clearing conditions from part (b) and the rational expectations assumption. Next, we can combine the Euler Equation with the firm's FOC. This yields

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \left(\alpha \frac{Y_{t+1}}{K_{t+1}} \right) \right]$$

Expected consumption growth is related to the expected marginal product of capital.[1] This is equivalent to the planner's Euler equation.

[1] This is easier to see from the linearized version of this expression, as we discussed in class.

Next consider the labor market. Imposing equilibrium and combining the labor supply and labor demand conditions:

$$[N_t] \qquad \frac{C_t}{\gamma_t} = (1-\alpha)\frac{Y_t}{N_t}$$

The marginal rate of substitution between consumption and leisure equals the marginal product of labor. This is also the planner's optimality condition for labor.

Substituting the optimal hiring rules for capital and labor into the profit function shows that the firm makes zero profit (which should be shown step by step).

Substituting these optimal hiring rules into the household budget constraint, together with the zero profit result implies the aggregate resource constraint is satisfied:

$$Y_t = C_t + I_t$$

(again this should be shown step by step).

Together with the law of motion for capital and the stochastic processes, these are the equilibrium conditions from the planner's problem.

Why is this the case? This model is perfectly competitive and does not feature any distortions or market failures. Firms are price takers and make zero profit. All households are identical and, under rational expectations, the individual capital stock coincides with the average (aggregate) capital stock in the economy. As a result, the welfare theorems hold and the equilibrium allocation for the decentralized competitive equilibrium coincides with the equilibrium allocation for the social planner's problem. Models with distortionary taxes, externalities or monopolistic competition would not satisfy this condition (and we saw models like this later in the quarter).

d) Using guess and verify, find the solved policy functions for output, consumption, investment and hours worked. Guess that investment is a constant share of output, and that hours worked are a constant share of γ_t.

Sketch answer

Let's guess $K_{t+1} = I_t = BY_t$. The resource constraint then implies $C_t = (1-B)Y_t$. Let's also guess that $N_t = Q\gamma_t$.

Substitute the consumption and investment guesses into the Euler equation:

$$\frac{1}{(1-B)Y_t} = \alpha\beta E_t\left[\frac{1}{(1-B)Y_{t+1}}\frac{Y_{t+1}}{BY_t}\right]$$

Cancelling terms:

$$1 = \alpha\beta E_t\left[\frac{1}{B}\right]$$

At this point, the expectations term now only depends on the constant B, so we can drop the expectations operator.

As a result, $B = \alpha\beta$. $K_{t+1} = I_t = \alpha\beta Y_t$ and $C_t = (1-\alpha\beta)Y_t$.

Now substitute the guess for consumption into the planner's labor condition:

$$[N_t] \qquad \frac{(1-B)Y_t}{\gamma_t} = (1-\alpha)\frac{Y_t}{N_t}$$

Cancel the Y_t terms, rearrange and use the solution for B:

$$N_t = \frac{1-\alpha}{(1-\alpha\beta)}\gamma_t \tag{8}$$

As a result, $Q = \frac{1-\alpha}{(1-\alpha\beta)}$

Naturally, the policy functions for output, investment and consumption are expressed as

$$y_t = A_t K_t^{\alpha}\left(\frac{1-\alpha}{(1-\alpha\beta)}\gamma_t\right)^{1-\alpha} \tag{9}$$

$$K_{t+1} = \alpha\beta A_t K_t^{\alpha}\left(\frac{1-\alpha}{(1-\alpha\beta)}\gamma_t\right)^{1-\alpha} \tag{10}$$

$$C_t = (1-\alpha\beta)A_t K_t^{\alpha}\left(\frac{1-\alpha}{(1-\alpha\beta)}\gamma_t\right)^{1-\alpha} \tag{11}$$

$$N_t = \frac{1-\alpha}{(1-\alpha\beta)}\gamma_t \tag{12}$$

e) Compare and contrast the business cycle properties implied by these two shocks. How well does each type of shock account for the business cycle dynamics we see in the data? Explain. In the course of your answer, make sure you discuss how, and why, these two shocks affect output, consumption, investment and hours worked in this model. (The model is very stylized. You can therefore focus on the general predictions rather than precise quantitative magnitudes.).

Sketch answer

In the data consumption, investment, (total) hours worked and GDP all co-move strongly. In the model however, TFP shocks generate co-movement of output, consumption and investment, but no response of hours worked.

Let's first discuss the response of hours worked to a TFP shock. Because $\delta = 1$, the wealth and substitution effects of a rise in TFP offset each other. Higher TFP implies a higher marginal product of labor, implying a greater incentive to supply labor, i.e. the substitution effect. On the other hand, higher TFP raises income and wealth, encouraging more leisure, i.e. the wealth effect. In this case, these mechanisms offset each other and hours worked do not move in equilibrium.

Now let's discuss the response of consumption, investment and output. The increase in TFP raises production possibilities. GDP increases directly. Households value consumption smoothing in this model and know that TFP is higher today and will be lower in the future (from the Markov process). In general, this provides an incentive to consume more, but also to invest today. Higher TFP raises the marginal product of capital today, which also generates the incentive to accumulate capital.

Labor supply shocks also generate a co-movement of consumption, investment and GDP. A higher value of γ_t implies a lower dis-utility from working and, as a result, labor supply will increase. An increase in hours worked leads to an increase in GDP. This also increases the marginal product of capital, encouraging capital accumulation. Notice that A_t and $\gamma_t^{(1-\alpha)}$ enter the solutions for output, investment and consumption the same way. In this sense, they are both shocks that can "account" for the movements we seen in the data.

In this model, however, only the labor supply shock can generate a movement in hours worked. Still, one question is whether N_t should be seen as average hours per worker or total hours worked. While total hours moves in the data, hours per worker tend to be much less responsive over the business cycle. If we interpret N_t as hours per worker the labor supply shock, arguably, generates movements in hours worked for the "wrong" reason. That said, this utility function is linear in hours worked. Hansen's indivisible labor model would provide one rationalization for why changes in N_t could be seen as movements in employment. Another difference with the TFP shock is that the real wage falls when γ_t rises. In contrast, TFP shocks will increase the real wage on impact.

Question 4 (*20 points*)

This question uses the New Keynesian model to examine whether higher labor income taxes could be used to reduce inflation, as sometimes suggested by policymakers. For simplicity, let's assume the economy starts in steady state and then consider a surprise, temporary, increase in taxes.

There are a continuum of identical households and the representative household's (period) utility function is:

$$\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} \tag{13}$$

In nominal terms, the household's budget constraint is:

$$Q_t B_{t+1} + P_t C_t = B_t + (1-\tau_t) P_t w_t N_t + \Pi_t + T_t \tag{14}$$

C_t is consumption, N_t is hours worked, w_t is the real wage, τ_t is the distortionary labor income tax rate, Π_t are firm profits distributed lump sum, T_t are lump-sum transfers from the government. B_t are bonds in period t (i.e. the closing stock of bonds in $t-1$). P_t is the aggregate price level. Q_t is the bond price. $\sigma > 0$ and $\psi > 0$.

Aside from the introduction of taxes, this model is the same as we saw in class.

In linearized form, the household's Euler equation and labor supply conditions are:

$$E_t \hat{c}_{t+1} - \hat{c}_t = \frac{1}{\sigma}(\hat{i}_t - E_t \hat{\pi}_{t+1}) \tag{15}$$

$$\hat{w}_t = \sigma \hat{c}_t + \psi \hat{n}_t + \hat{\tau}_t \tag{16}$$

The linearized equilibrium conditions for firms are:

$$\hat{y}_t = \hat{n}_t \tag{17}$$

$$\hat{w}_t = \hat{mc}_t \tag{18}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \lambda \hat{mc}_t \tag{19}$$

The resource constraint is:

$$\hat{y}_t = \hat{c}_t \tag{20}$$

Monetary policy follows a simple Taylor Rule:

$$\hat{i}_t = \phi_\pi \hat{\pi}_t \tag{21}$$

The (linearized) labor income tax rate follows a Markov process:

$$\hat{\tau}_t = \rho \hat{\tau}_{t-1} + e_t \tag{22}$$

where $0 < \rho < 1$ and e_t is an i.i.d. disturbance. Tax revenues generated by the income tax are redistributed lump-sum to households via T. The government does not issue any debt and there is no government spending in this model.

In percentage deviations from steady state: $\hat{mc}_t$ is real marginal cost, $\hat{c}_t$ is consumption, $\hat{w}_t$ is the real wage, $\hat{n}_t$ is hours worked, $\hat{y}_t$ is output. In deviations from steady state: $\hat{i}_t$ is the nominal interest rate, $\hat{\pi}_t$ is inflation and $\hat{\tau}_t$ is the labor income tax rate. λ is a function of model parameters, including the degree of price stickiness.[2] Assume that $\phi_\pi > 1$ and β is the household's discount factor, where $0 < \beta < 1$.

a) Using the relevant linearized equations above, show that, under flexible prices, the natural rate of output $\hat{y}_t^n$ (in % deviations from steady state) in this model depends on the labor income tax rate. In particular, show that:

$$\hat{y}_t^n = -\frac{1}{\sigma + \psi}\hat{\tau}_t \tag{23}$$

Provide some economic intuition for this result.

Sketch answer

The key is to note that the natural rate of output occurs under flexible prices, so $\hat{mc}_t = 0$. Use equations 17 and 20 in equation 16, and set $\hat{mc}_t = \hat{w}_t = 0$. Rearrange this expression for $\hat{y}_t$ and put a superscript on the variable to denote the result under flexible prices. This yields the expression requested. Each step in this process should be shown in the answer.

The income tax discourages work by lowering the after tax real wage. Each unit of labor supplied now earns less. As a result, labor supply will contract (note this is a negative shock to the labor supply curve, equation 16). Since hours are used to produce output, output will fall as well.

b) Now assume prices are sticky. Show that the Phillips Curve can be written as:

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \lambda(\sigma + \psi)\hat{x}_t + \lambda\hat{\tau}_t \tag{24}$$

where $\hat{x}_t$ is a measure of the output gap. In this question, define the output gap as $\hat{x}_t = \hat{y}_t - \hat{y}_t^e$, where $\hat{y}_t^e$ is the *efficient* level of output (in % deviations from steady state). The efficient level of output is defined as the level of output that would be obtained under flexible prices and in an economy without distortions. Note that, under flexible prices, the labor income tax is the only time-varying distortion in this model. You can therefore assume that $\hat{y}_t^e = 0$, $\forall t$.

[2] $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ where θ is the probability that a firm cannot adjust its price and $0 \leq \theta < 1$.

Sketch answer

Combining equations 16, 18, 17 and 20 yields an expression for marginal cost in terms of output:

$$\hat{mc}_t = (\psi + \sigma)\hat{y}_t + \hat{\tau}_t$$

Next note that $\hat{x}_t = \hat{y}_t$ given that $\hat{y}_t^e = 0$. Substitute this result into equation 19 yields the Phillips Curve in the question. Each step in this process should be shown in the answer.

c) Using the method of undetermined coefficients, find the response of the output gap, $\hat{x}_t$, and inflation, $\hat{\pi}_t$, to an exogenous increase in the labor income tax rate when prices are sticky and monetary policy follows the Taylor Rule above. To do this, guess that the solution for each variable is a linear function of the shock $\hat{\tau}_t$. (**Hint:** you will need to rewrite the consumption Euler equation in terms of the output gap $\hat{x}_t$).

Sketch answer

Let's guess that the solution is of the form:

$$\hat{x}_t = \Lambda_x \hat{\tau}_t$$
$$\hat{\pi}_t = \Lambda_\pi \hat{\tau}_t$$

Let's also define the slope of the Phillips Curve as $\kappa = \lambda(\sigma + \psi)$ to keep the algebra a bit tidier (as we did in class).

The steps are the same as in the technology shock example in problem set 8. We substitute the Taylor Rule into the Euler (although here we replace $\hat{c}_t = \hat{x}_t$) and substitute the guesses into Euler and the Phillips Curve. We also make use of the stochastic process for $\hat{\tau}_t$. All steps in your working should be shown.

Eventually you should arrive at the following solution for the unknown coefficients:

$$\Lambda_\pi = \lambda \frac{(1-\rho)\sigma}{\sigma(1-\rho)(1-\beta\rho) + \kappa(\phi_\pi - \rho)} > 0 \tag{25}$$
$$\Lambda_x = \lambda \frac{\rho - \phi_\pi}{\sigma(1-\rho)(1-\beta\rho) + \kappa(\phi_\pi - \rho)} < 0 \tag{26}$$

The denominator is positive in both expressions, this is because $\sigma > 0$, $0 < \rho < 1$, $0 < \beta < 1$, $\kappa > 0$ and $\phi_\pi > 1$. The positive sign in the first expression then comes from noting that $0 < \rho < 1$ and $\sigma > 0$. The negative sign in the second expression come from the fact that $\rho < \phi_\pi$.

d) If policymakers increase the tax rate $\hat{\tau}_t$, will this reduce inflation? Explain how and why an increase in the labor income tax affects the output gap $\hat{x}_t$ and inflation $\hat{\pi}_t$ in this model.

Sketch answer

Labor income tax shocks look like "cost" shocks in this model (what we called u_t in Topic 9). The output gap, $\hat{x}_t$ falls, whereas inflation rises. Because this is a distortionary labor tax, the natural rate of output falls as discussed in part (a): the fall in the natural rate (and output) reflects the fact that a rise in income taxes lowers the after tax real wage and thus the incentive to supply labor. To produce the same level of output as before, firms will now have to offer a higher wage. This shock affects firm costs, which is what generates the inflationary pressure.

If prices were flexible, all firms would raise prices and reduce production. With sticky prices, some firms can't adjust prices. Goods produced by these firms are therefore relatively cheaper. Demand for these goods is therefore relatively higher and output does not fall as much as under flexible prices. The gap between output and the natural rate of output is therefore positive: $\hat{y}_t - \hat{y}_t^n > 0$, consistent with the rise in inflation over time.

What may be surprising is that a *contractionary* fiscal policy of this kind *raises* inflation. This is because the shock is actually more like a cost-push shock and increases distortions in the economy. The effect works through labor supply substitution effects (note that tax revenues are then redistributed back to households lump sum). Higher taxes will therefore not reduce inflation. In fact, higher taxes will *raise* inflation via their effects on costs.

e) Find the response of the nominal interest rate. How does monetary policy respond to a tax increase in this model? Do you find this surprising? Explain.

Sketch answer

The policy rule is:

$$\hat{i}_t = \phi_\pi \hat{\pi}_t$$

Plugging in the result for inflation, the solved policy function for the nominal interest rate is:

$$\hat{i}_t = \phi_\pi \lambda \frac{(1-\rho)\sigma}{\sigma(1-\rho)(1-\beta\rho) + \kappa(\phi_\pi - \rho)} > 0$$

Higher inflation leads to higher nominal interest rates. A unique stable solution requires that interest rates are increased by more than inflation (the Taylor Principle). Because this shock is inflationary, when taxes are increased, the monetary policymaker *raises* interest rates. In this model, this occurs because a tax hike is a cost shock. This makes sense in the context of the model but, in the policy debate, you will sometimes hear commentators argue that contractionary fiscal policy might have to be offset with a more expansionary monetary policy. The logic here would be that higher taxes would lower aggregate demand, leading to lower inflation and output. Monetary policy would then lower interest rates. In the model above, however, the distortionary labor income tax increase is not a negative aggregate demand shock. Instead, it is a shock that raises firms' costs.