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PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

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Directions: Answer four questions, at least one from each part.

**Part I**

1. Consider a monetary economy populated by identical agents with preferences given by:

$$\sum_{t=0}^{\infty} \beta^t [\ln c_t + \ln d_t]$$

where  $c_t$  denotes a “cash” good whose purchases are subject to a cash-in-advance constraint while  $d_t$  is a “credit” good that can be acquired via barter. Agents produce output via the Cobb-Douglas production function  $y_t = Ak_t^\alpha$ . Revenue from the sale of this output is combined with money chosen in the previous period and the lump sum monetary transfer from the government in order to purchase both cash and credit goods, capital, and acquire new money. As stated above, purchases of the cash good are also subject to the CIA constraint:

$$M_{t-1} + \tau_t \geq P_t c_t$$

where  $\tau_t$  denotes the lump sum monetary transfer. The aggregate money stock in this economy grows at the constant (gross) rate,  $\mu$  implying the law of motion:  $\bar{M}_t = \mu \bar{M}_{t-1}$ . Note that there is no uncertainty in this economy. Given this environment, do the following

- (a) Define a steady-state equilibrium in this economy.
  - (b) Does the model exhibit superneutrality?
  - (c) What is the rate of inflation that maximizes steady-state utility? Compare this to the “Friedman Rule”.
2. Consider the following representative agent monetary model: In each period agents choose labor,  $h_t$ , consumption,  $c_t$ , money,  $M_t$ , and one-period nominal bonds,  $B_t$ , in order to maximize lifetime expected utility given by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \ln c_t + \ln \left( \frac{M_t}{P_t} \right) + \phi \ln (1 - h_t) \right] \right\}$$

Agents have access to a linear technology that transforms labor into output,  $y_t$ . This technology is:

$$y_t = \xi_t h_t$$

The term  $\xi_t$  represents a stochastic technology shock with constant trend. That is:

$$\xi_t = \lambda^t z_t$$

Where  $\lambda > 1$ , and  $z_t \sim i.i.d.$  with  $E(z_t) = 1$ . The money stock is also growing randomly with  $M_t = M_{t-1}\mu_t$  where  $\mu_t \sim i.i.d.$  with  $E(\mu_t) > 1$ . Each period agents use the nominal proceeds from selling output together with their nominal assets to buy consumption and new assets. Given this environment, do the following:

- (a) Express the agents' maximization as a dynamic programming problem.
- (b) Define a recursive monetary equilibrium.
- (c) Characterize the behavior of equilibrium real balances, labor, and nominal interest rates.

## Part II

3. A researcher is interested in estimating the hybrid Phillips curve

$$\pi_t = \mu E_t \pi_{t+1} + (1 - \mu)\pi_{t-1} + \gamma x_t + \epsilon_t \quad (1)$$

where  $\pi_t$  is the inflation rate;  $x_t$  is the output gap; and  $\epsilon_t$  is a supply shock. Assume that inflation and the output gap are directly observable variables for which a sample of  $T$  observations is available for estimation. This Phillips curve has been derived from first principles so that the parameters  $\theta = (\mu \ \gamma)'$  are a known function of a vector of deep parameters  $\phi$ , which may include such parameters as the discount rate, the per period probability of price adjustment, the elasticity of the labor supply with respect to the real wage, etc. Hence,  $\theta = h(\phi)$ . Answer the following questions (*please be very brief: I am looking for specific, but short answers*):

- (a) Set up the sample moment conditions for this problem that would allow one to estimate the parameters  $\theta$  by GMM (you do not need to write the quadratic function, nor derive the first order conditions, nor anything fancy, but you need to be specific about what instruments you think should be used).
- (b) What is the optimal weighting matrix for this GMM problem (just state the theoretical matrix)? How would you then set the GMM estimation routine to accomplish estimation with the optimal weighting matrix (just words, but be sure to mention what you would do to obtain a sample estimate of the theoretical weighting matrix)?
- (c) What is the minimum number of moment conditions needed for the model to be identified in terms of the parameters  $\theta$ ?

- (d) Given estimates  $\hat{\theta}_T$  from the GMM procedures outlined above, set up the minimum distance problem that would allow you to obtain estimates of the deep parameters  $\phi$  (all I need is an expression for the appropriate quadratic form). Be sure to provide the form of the optimal weighting matrix (to do this, you do not need to solve the first order conditions of the GMM step, just indicate with notation what it is).
- (e) What conditions are necessary to identify the parameters  $\phi$  with this procedure? Hint: there are two sets of conditions: one has to do with the dimensions of the first stage and second stage parameter vectors. What is the other?
- (f) Which procedure produces more efficient estimates of  $\phi$ , this two step (GMM-MD) procedure or applying GMM directly to expression (1) with the parameters  $\theta$  replaced directly by the parameters  $\phi$ ? Would this second estimator affect your answer in (e)?
- (g) What are the advantages/disadvantages of GMM with respect to maximum likelihood estimation of expression (1). Please be brief?

4. Suppose a researcher is interested in estimating the following IS equation

$$x_t = \gamma E_t x_{t+1} - \frac{1}{\sigma} r_t + u_t \quad (2)$$

where  $x_t$  is the output gap, and  $r_t = i_t - E_t \pi_{t+1}$ , that is, the real interest rate expressed as the difference between the nominal interest rate  $i_t$  and next period's expected inflation  $E_t \pi_{t+1}$ . Data is available on GDP, potential GDP, nominal interest rates and the rate of inflation for a sample of  $T$  observations. Answer the following questions (*please be very brief: I am looking for specific, but short answers*):

- (a) Set up the state-space representation for expression (2) in terms of the state variables (be explicit in pointing out how these relate to the variables in expression (2)) and the available observable variables (remember the ultimate objective is to estimate the parameters  $\gamma$  and  $\sigma$ ). You may assume that real interest rates are first order Markov. Notice that you do not need to derive the Kalman recursions or anything of that sort.
- (b) Suppose instead that you believe real interest rates follow an  $AR(p)$  process. Recast your state-space appropriately.
- (c) Suppose that you wanted to estimate the output gap yourself instead of relying on the potential GDP measure provided to you. Set up a separate state-space model for the purpose of computing this output gap measure only and explain (in words, I do not need a bunch of equations) how you could recover a measure of the output gap if you had estimated values of this state-space.

- (d) Data on the GDP variables are available quarterly but data on interest rates and inflation are available monthly. Suppose expression (2) reflects the dynamics of the output gap at a monthly frequency given by that  $IS$  relation. Explain how you would use the state-space representation to estimate the monthly model. Specifically, what would you need to do (in words, no equations needed) to estimate such model by maximum likelihood (“use the EM algorithm” is not the answer I am looking for here. I need a few more specifics)?
- (e) Suppose a sample of quarterly data is available for  $x_t, i_t,$  and  $\pi_t$ . Hence you construct a  $VAR(p)$  with these three variables. Set-up a minimum distance procedure (a la Sbordone) that would allow you to estimate the parameters  $\gamma$  and  $\sigma$  (you do not need to derive the first order conditions). What weighting matrix should the researcher use if he were concerned with obtaining consistent (rather than efficient) parameter estimates (just words, no formulas needed)?
- (f) If instead the researcher is interested in efficiency, what weighting matrix should he use (just words, no formulas needed)?

### Part III

5. Consider a purely forward-looking version of the Calvo model. A firm that reoptimizes in period  $t$  sets its price  $P_t^*$  to maximize the present value of profits generated while that price remains effective,

$$\max_{P_t^*} E_t \sum_{j=0}^{\infty} \alpha^j \Lambda_{t,t+j} [P_t^* Y_{t+j|t} - \psi_{t+j}(Y_{t+j|t})],$$

where  $1 - \alpha$  is the probability that a firm can reoptimize,  $\Lambda_{t,t+j}$  is a stochastic discount factor at date  $t$  for payments received at  $t+j$ ,  $Y_{t+j|t}$  is output produced for a firm that last reset its price in period  $t$ , and  $\psi_{t+j}(\cdot)$  is marginal cost. Explain how this expression can be derived from a dynamic program.

6. Researchers like Bilal and Klenow and Nakamura and Steinsson study microeconomic data on prices. Briefly summarize the lessons of these micro studies for macroeconomic models of sticky prices.