

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Instructions: *Answer all four questions, each question on a separate sheet of paper.*

1.
 - a. Write down a stochastic monetary model in which the Fisher relationship holds and one in which it does not.
 - b. The U.S. now issues inflation-indexed bonds. It has been stated that by comparing the yields on inflation indexed bonds and equivalent maturity nominally denominated bonds one can uncover investors' forecast of inflation. Do you agree? Use your answer in part (a) to support your argument.

2. Consider the following variant of Stockman's cash-in-advance model. The primary distinction is that asset and goods markets are assumed to meet sequentially with the asset market preceding the goods market. The assets in the economy are: capital (k_t), one-period nominal bonds B_t , one-period real bonds (b_t), and money (M_t). Nominal bonds costs 1\$ in period t and return N_t in the following period while real bonds cost one unit of consumption in period t and return R_t units of consumption in period $t+1$. In the asset market, agents receive the returns from capital (i.e. the revenue from the sale of output in last period's goods market), the returns from bonds, money not spent in last period's goods market, and the lump-sum monetary transfer and use this to buy new bonds, money, and purchase investment (investment is not subject to the CIA constraint). Next agents visit the goods market where money is used to finance consumption (i.e. consumption is subject to the cash-in-advance constraint). The aggregate money stock grows at the constant rate $\mu > 0$. (Note that, as in Stockman, agents' use capital to produce output via the production function $f(k_t)$. This output is sold in the goods market. The law of motion for capital is standard with depreciation of $\delta \in (0,1)$.) Households have standard time separable preferences as in Stockman.
 - (a) Set up the agent's maximization problem as a dynamic programming problem. Identify the state and control variables.
 - (b) Prove that in this economy, the Lagrange multiplier associated with the budget constraint is equal to agent's marginal utility of consumption. Explain.
 - (c) Define a steady-state equilibrium. Is money superneutral in this economy?
 - (d) Show that the Fisher relationship holds.

3. Suppose the economy is described by the following new-Keynesian model

$$\begin{aligned} y_t &= E_t y_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}) + \varepsilon_t \\ \pi_t &= \beta E_t \pi_{t+1} + k y_t + v_t \\ i_t &= \phi_\pi (\pi_t - \pi^*) + \phi_y y_t \end{aligned}$$

where y is the output gap, π is the inflation rate and ε , and v are i.i.d. random disturbances. We also assume $1 > \beta > 0$, $k > 0$, $\sigma > 0$.

- (a) Briefly comment on the micro-foundations behind the new-Keynesian model just presented.
- (b) If $\phi_y = 0$, explain intuitively why $\phi_\pi > 1$ is needed to ensure that the equilibrium will be unique.
- (c) If both ϕ_π and ϕ_y are nonnegative, the condition

$$k(\phi_\pi - 1) + (1 - \beta)\phi_y > 0 \quad (1)$$

implies that the economy can still have a unique, stable equilibrium even when

$$1 - \frac{(1 - \beta)\phi_y}{k} < \phi_\pi < 1$$

Explain intuitively why some values of $\phi_\pi < 1$ are still consistent with stability when $\phi_y > 0$.

- (d) Now suppose that the economy is backward-looking a la Rudebusch and Svensson (1999) and in particular takes on the following form:

$$\begin{aligned} y_t &= y_{t-1} - \sigma^{-1}(i_{t-1} - \pi_{t-1}) + \varepsilon_t \\ \pi_t &= \beta \pi_{t-1} + k y_{t-1} + u_t \\ i_t &= \phi_\pi \pi_t + \phi_y y_t \end{aligned}$$

but that the central bank has decided on its optimal policy based on the original forward-looking model. In fact, it has decided on a policy that sets $\phi_\pi = 1$ and sets ϕ_y so that it meets condition (1). Under what conditions will this choice of policy also be stable for the backward-looking economy? What lesson do you extract from this result?

4. Consider a model of agency costs. Assume there are two types of agents, entrepreneurs and lenders, both risk neutral. Entrepreneurs have access to a stochastic technology that contemporaneously transforms i units of consumption goods into ωi units of capital, with i distributed *i.i.d.*, $i > 0$, $E(i) = 1$, with distribution Φ and density ϕ . ω is privately observed by entrepreneurs but can only be observed by lenders by incurring a monitoring cost μi capital units. The lending contract stipulates an interest rate r^k . Thus, an entrepreneur borrows $(i - n)$ and repays $(1 + r^k)(i - n)$. The entrepreneur defaults if $\omega < (1 + r^k)(i - n) / i \equiv \bar{\omega}$. The lender only monitors in case of default, in which case, he can confiscate all the returns on the project. The optimal contract is completely defined by the pair $(i, \bar{\omega})$ since the implied rate of interest can be found from the condition $(1 + r^k) \equiv \bar{\omega} i / (i - n)$.
- Calculate expected entrepreneurial income. *Hint:* notice that $i\bar{\omega} = (1 + r^k)(i - n)$ so that you can write income as a function of $i, \bar{\omega}$
 - Calculate expected lender's income. Use the same hint as above.
 - Design the optimal contract as the pair $(i, \bar{\omega})$ that maximizes entrepreneur's expected return subject to the lender being indifferent between lending or retaining funds. Give necessary conditions for the entrepreneur's participation.
 - Derive the first order conditions.
 - Carlstrom and Fuerst use this model of agency costs into a monetary business cycle model in order to study the *amplification* of monetary shocks due to the Bernanke, Gertler, and Gilchrist financial accelerator effect. Briefly comment on the results obtained by both approaches and comment on the sources for their difference.