University of California, Davis
Department of Economics
**Microeconomics**

Date: August 18, 2022
Time: 5 hours
Reading Time: 20 minutes

**PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE**

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Please answer **all three** questions

## QUESTION 1

Consider a game-theoretic version of Michael Spence's job market signaling model. There are two firms and one worker. The worker's productivity can be either $\theta_L$ or $\theta_H$ with $0 < \theta_L < \theta_H$. The worker knows her own productivity, while the firms only know that the fraction $p$ of the population of workers has low productivity $\theta_L$ and the fraction $(1-p)$ has high productivity $\theta_H$ (with $0 < p < 1$). The worker can acquire education at any level $e \in [0, \bar{e}]$ (with $0 < \bar{e}$). Education does not affect the worker's productivity. Let $c(e,\theta)$ be the cost of acquiring education level $e$ for a worker of productivity $\theta$ and assume that, for every $e \in (0, \bar{e}]$, $c(e,\theta_L) > c(e,\theta_H)$ and for every $e_1, e_2 \in [0, \bar{e}]$ with $e_1 < e_2$, $c(e_1,\theta_L) < c(e_2,\theta_L)$ and $c(e_1,\theta_H) < c(e_2,\theta_H)$ and, finally, $c(0,\theta_L) = c(0,\theta_H) = 0$. The "incomplete-information signaling game" is played as follows. First the worker observes the value of $\theta \in \{\theta_L, \theta_H\}$ and chooses the amount of education $e \in [0, \bar{e}]$. Then the two firms observe the chosen value of $e$ and simultaneously make a wage offer to the worker; denote by $s_i \in [0, \infty)$ the wage offer by firm *i*. Finally, the worker chooses the higher of the two wage offers or randomizes with equal probability between the two offers if they are equal. The worker's payoff is $s - c(e,\theta)$, where $s$ is the wage she accepted, and the payoff of firm $i \in \{1,2\}$ is $\theta - s_i$ if the worker accepted the wage $s_i$ offered by firm *i* or 0 if the worker accepted the offer of the other firm. All the players are risk neutral.

**(a)** Draw the extensive-form game for the case where there are only two possible levels of education: $e_1$ and $e_2$ and two possible wage offers $w_1$ and $w_2$. **No need to write the payoffs** and no need to represent the acceptance decision of the worker.

For questions (b) and (c) refer to the full game (not the simplified version of Part (a)).

**(b)** For each player, describe the set of pure strategies.

**(c)** Restricting attention to "symmetric" weak sequential equilibria where the two firms use the same strategy, describe a pooling weak sequential equilibrium of the game (where "pooling" means that the worker makes the same choice irrespective of her type). Give enough details to support the claim that what you are proposing is indeed a weak sequential equilibrium.

For the next question go back to the simplified version of the game of Part (a) assuming the following values:

$$\theta_L = 1.5, \quad \theta_H = 3.5, \quad e_1 = 0, \quad e_2 = 2, \quad w_1 = 1, \quad w_2 = 3$$
$$c(0,\theta_L) = c(0,\theta_H) = 0, \quad c(2,\theta_L) = 4, \quad c(2,\theta_H) = 2$$

**(d)** Describe **in full detail** a separating weak sequential equilibrium of the game (where "separating" means that the worker makes different choices depending on her type).

## QUESTION 2

a. Let $\boldsymbol{p} = (p_1, p_2, p_3, p_4)$ be the price vector consisting of prices for commodities 1 to 4. Further, denote by $w$ the wealth level of a consumer. Assume $(\boldsymbol{p}, w) >> \mathbf{0}$. Derive the Walrasian demand functions for the following utility functions:

(i) $u(x_1, x_2, x_3, x_4) = \min\left\{\sqrt{x_1 x_2}, \sqrt{x_3 x_4}\right\}$

(ii) $u(x_1, x_2, x_3, x_4) = \sqrt{x_1 x_2} + \sqrt{x_3 x_4}$

b. Consider a consumer whose utility function over bundles $(x_1, x_2) \in \mathbb{R}_+ \times (1, \infty)$ is given by

$$u(x_1, x_2) = \ln(x_1 + 1) + \ln(x_2 - 1).$$

We denote by $p_1 > 0$ and $p_2 > 0$ the prices of commodities 1 and 2, respectively, and by $w > 0$ the wealth of the consumer.

(i) For which vectors $(p_1, p_2, w) >> 0$ does the consumer consume strict positive amounts of both commodities?

(ii) Derive the Walrasian demand function.

(iii) Derive the indirect utility function.

(iv) Consider now $n$ consumers. Consumer $i \in \{1, ..., n\}$ has utility function

$$u^i(x_1^i, x_2^i) = a^i \ln(x_1^i + b^i) + \ln(x_2^i - 1)$$

with $a^i, b^i > 0$. Which restrictions do we need to place on $a^i$ and $b^i$ such that aggregate demands for commodities 1 and 2 are determined by prices $p_1$ and $p_2$, the sum $\sum_{i=1}^n w^i$, and does not depend on the distribution of wealth? How is this answer related to the Gorman form?

Question 3: The Core of an Economy

(a) Define the *weak core* of exchange economy $\{\mathcal{I}, \mathbf{u}, \mathbf{w}\} = \{\mathcal{I}, (u^i, w^i)_{i \in \mathcal{I}}\}$ as the set of its allocations $\mathbf{x}$ such that there do not exist $\mathcal{H} \subseteq \mathcal{I}$ and $(\hat{x}^i)_{i \in \mathcal{H}}$ for which $\sum_{i \in \mathcal{H}} \hat{x}^i = \sum_{i \in \mathcal{H}} w^i$ and $u^i(\hat{x}^i) > u^i(x^i)$ for all $i \in \mathcal{H}$. Argue that:

  i. the core is a subset of the weak core; and

  ii. if all preferences are continuous and strictly monotone, the core and the weak core are the same set.

(b) Given an exchange economy $\{\mathcal{I}, \mathbf{u}, \mathbf{w}\}$, prove the following:

  i. If $\mathbf{w}$ is efficient, then it is a core allocation.

  ii. If each $u^i$ is strongly quasiconcave and $\mathbf{w}$ is efficient, then $\mathbf{w}$ is the only core allocation.

(c) Consider a two-person exchange economy

$$\{\mathcal{I} = \{1, 2\}, \mathbf{u} = (u^1, u^2), \mathbf{w} = (w^1, w^2)\},$$

and suppose that $(p, x^1, x^2)$ is a competitive equilibrium. Argue that if $(x^1, x^2)$ is *not* in the core of the economy, then it must be Pareto inefficient.