University of California, Davis
Department of Economics
Microeconomics

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Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

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Please answer **all three** questions

QUESTION 1

Consider the following game played by two firms. Firm 1 moves first and chooses the quality of its product: either H (high) or L (low). Firm 2 moves second, **after observing Firm 1's choice**, and chooses the quality of its own product: H or L. After this second choice, the two qualities become common knowledge between the two firms and the two firms play a **simultaneous** game where Firm 1 chooses the price $p_1 \in [0,\infty)$ of its product and Firm 2 chooses the price $p_2 \in [0,\infty)$ of its product. A high-quality good is produced at a constant marginal cost of 4, while a low-quality good is produced at a constant marginal cost of 2. There are no fixed costs. When both firms choose quality H the demand function is given by $Q = 16 - P$ and when both firms choose quality L the demand function is given by $Q = 6 - P$; in both cases P is the lowest price: since the products are identical, consumers buy only from the firm with the lower price and, if both firms charge the same price, then consumers split themselves equally between the two firms. When one firm chooses H and the other chooses L then the demand functions are given as follows (p_H is the price charged by firm H and p_L is the price charged by firm L):

for firm H: $q_H = \begin{cases} 10 - \frac{p_H}{4} + \frac{p_L}{4} & \text{if } p_H > p_L \\ 16 - \frac{p_H}{4} & \text{if } p_H \le p_L \end{cases}$, for firm L: $q_L = \begin{cases} 0 & \text{if } p_H \le p_L \\ 3 + \frac{p_H}{4} - \frac{p_L}{4} & \text{if } p_H > p_L \end{cases}$.

The objective of each firm is to maximize its own profits.

(a) Find the pure-strategy subgame-perfect equilibrium (SPE) of this game. [Hint: first look for a solution in the range $p_H > p_L$ and then verify that it is a solution for the original problem.]

Suppose now that the industry does not exist yet and the government runs an auction with two bidders and what is being auctioned is the right to be Firm 1 in the above game. Thus, whoever wins the auction will be Firm 1 in the above game and whoever loses the auction will be Firm 2. Call the participants in the auction Players A and B. The two players are "selfish and greedy", that is, their objective is to maximize their own wealth. In all of the auctions of Parts (b)-(d) the following applies: (1) the auction is a simultaneous sealed-bid auction, (2) the winner is the player who submits the higher bid (the other player is called the loser), (3) if the two bids are the same, then Player A will be declared the winner, (4) bids can be **any non-negative** real numbers.

(b) Case 1. The auction is a **first-price** auction (the winner pays her own bid and the loser pays nothing). Find all the pure-strategy SPEs of the two-stage game just described (the first stage is the auction and the second stage is the game described at the beginning). Prove that what you claim to be SPEs are indeed SPEs and that there are no other SPEs. If your claim is that there are no SPEs, then prove it.

(c) Case 2. The auction is a **second-price** auction (the winner pays the bid of the loser and the loser pays nothing). Find all the pure-strategy SPEs of the two-stage game. Justify your answer.

(d) Case 3. The auction is an **all-pay first-price** auction (each player pays her own bid, **including the loser**). Find all the pure-strategy SPEs of the two-stage game. Prove that what you claim to be SPEs are indeed SPEs and that there are no other SPEs. If your claim is that there are no SPEs, then prove it.

Question 2

When you work late in the office, you have a chance to meet quiet members of our department. One of them, inconspicuous but ever present is Manfredi di Notte, a modest unassuming fellow who roams the halls at night, especially close to the kitchen area across my office. Although cross-species communication is not easy, I happened to chat with him the other night about his interesting consumption problem. It turns out that he is not just a consumer à la Mas-Colell, Whinston, and Green (1995, Chapter 3), maximizing utility over burgers and coke subject to his usual budget constraint. His personal physician, Dottore Pierfrancesco Scarabaeus, also asked him to satisfy a dietary constraint limiting his daily intake of calories. Obesity is a real problem for a cockroach when trying to slip beneath office doors. That is, he is a consumer with *two* constraints, one budget constraint and one dietary constraint.

a. Suppose Manfredi maximizes a strictly quasi-concave and monotone utility function $u(x, y)$ in the amount of burgers, $x \geq 0$, and coke, $y \geq 0$, subject to the two constraints,

$$p_x x + p_y y \leq w$$

$$k_x x + k_y y \leq d,$$

where $p_x > 0$ denotes the price of a unit of burgers, $p_y > 0$ the price of a unit of coke, $k_x > 0$ the calories per unit of burgers, $k_y > 0$ the calories per unit of coke, $w > 0$ the wealth, and $d > 0$ his maximal allowed calories.

Consider an economic analyst, Anna Lyst, who is unaware of the dietary constraint. She observes two consumption bundles of Manfredi, denoted by (x', y') and (x'', y'') at differences prices and wealth situations $(p'_x, p'_y, w') > 0$ and $(p''_x, p''_y, w'') > 0$, respectively.

Is it possible that Manfredi violates WARP for these observations because of the dietary constraint that Anna Lyst is unaware of? Argue why not or provide a graphical counterexample.

b. Assume further that Manfredi's utility function is given by

$$u(x, y) = x^{\alpha} y^{1-\alpha}$$

for $\alpha \in (0, 1)$. Derive the set of first-order conditions for his constrained utility maximization problem. You can neglect the non-negativity constraints if you can argue why they are satisfied in this problem anyway.

c. Solve for/derive step-by-step Manfredi's demand function as function of prices, wealth, calories, and maximal allowed calories. Illustrate the solutions with figures.

d. Suppose that Manfredi's consumption bundle is determined by the tangency point w.r.t. his dietary constraint and that it is not the case that $m = \gamma d$, $p_x = \gamma k_x$, and $p_y = \gamma k_y$ for some $\gamma > 0$. (Call it the d-tangency solution.) What is his marginal utility of wealth at this point?

e. UC Davis Custodial Services put in exemplary effort in not disturbing the habitat of roaches.[1] In fact, they are so concerned about the well-being of roaches that they created the position of "Vice-Chancellor for Blattodeo Affairs & Welfare". They engaged a renowned head hunting firm to carefully vet qualified candidates. After a thorough nationwide search and with the approval of the Office of the President, Anna Lyst was appointed to the position. She already made a splash with her latest interview revealing that she is learning about dietary constraints in town hall meetings with her numerous staff and working really hard to estimate Manfredi's loss of welfare due to his dietary constraint. She employs you as Assistant Vice-Chancellor for Blattodeo Affairs & Welfare. You desperately need to come up with a dollar-number of Manfredi's loss of welfare due to his dietary constraint.

   Continue to assume that Manfredi's consumption bundle is determined by the tangency point w.r.t. his dietary constraint and that it is not the case that $m = \gamma d$, $p_x = \gamma k_x$, and $p_y = \gamma k_y$ for some $\gamma > 0$. For simplicity, assume that $p_x = 1$. How can you measure in dollar values Manfredi's loss of welfare due to the dietary constraint? In particular, how much would be Manfredi willing to pay for not being subjected to the dietary constraint?

f. The Athletic Roach Center (ARC) offers a fitness program tailor-made and free-of-charge for members aspiring to get into every crevice. Let $z \geq 0$ denote the units of fitness. Fitness causes a disutility to Manfredi. His utility function is now $u(x, y, z) = \alpha \ln(x) + (1 - \alpha) \ln(y) - z$. At the same time, fitness allows Manfredi to loosen up his dietary constraint, which is now $k_x x + k_y y \leq d + z$. That is, when he exercises, he is allowed to eat more.

   Continue to assume that Manfredi is at the d-tangency solution. State a condition that determines how much Manfredi likes to exercise and derive his optimal amount of exercises. (I.e., think of a condition derived from a first-order condition.)

[1] All persons, events, and institutions mentioned herein are fictitious except for Professor Schipper, the existence of roaches, and the aforementioned sentence related to the custodial service. (E.g., Professor Schipper's office hasn't been swept for the entire spring quarter.) Any other resemblance to actual persons, events, and institutions is entirely coincidental.

Question 3

In the model of production economies we studied in class, we assumed the ownership structure of the firms. In this question, you will see a way to endogenize that variable.

Consider a two-person economy with L commodities. The individuals are $i = 1, 2$. Each has an endowment $w^i \in \mathbb{R}^L_+$ of commodities and preferences represented by the utility function $u^i : \mathbb{R}^L_+ \to \mathbb{R}$. A technology $Y \subseteq \mathbb{R}^L$ represents the production plans that can be achieved by the two individuals *working together*. In addition, each individual can choose to work individually, in which case her technology is $Y^i \subseteq \mathbb{R}^L$. All these technologies satisfy possibility of inaction,[1] and there are benefits to cooperation in the sense that

$$Y^1 + Y^2 = \{y \in \mathbb{R}^L \mid \exists (y^1, y^2) \in Y^1 \times Y^2 : y^1 + y^2 = y\} \subseteq Y.$$

There are competitive markets for all the commodities, and the prices are denoted by $p \in \mathbb{R}^L$.

Define a *price-taking cooperative equilibrium* to be a tuple $(p, \bar{x}^1, \bar{x}^2, \bar{y}, m^1, m^2)$ such that:

(a) for both i, $\bar{x}^i \in \arg\max_x \{u^i(x) : p \cdot x \leq p \cdot w^i + m^i\}$;

(b) for the firm, $(\bar{y}, m^1, m^2)$ satisfies:

   (i) $\bar{y} \in \arg\max_y \{p \cdot y : y \in Y\}$

   (ii) $m^1 + m^2 = p \cdot \bar{y}$,

   (iii) $m^1 \geq \max_y \{p \cdot y : y \in Y^1\}$, and

   (iv) $m^2 \geq \max_y \{p \cdot y : y \in Y^2\}$; and

(c) $\bar{x}^1 + \bar{x}^2 = w^1 + w^2 + \bar{y}$.

For this definition

1. Interpret the concept of price-taking cooperative equilibrium in economic terms, emphasizing condition (b).

2. Argue that if $(p, \bar{x}^1, \bar{x}^2, \bar{y}, m^1, m^2)$ is a price-taking cooperative equilibrium, then

$$u^1(\bar{x}^1) \geq \max_y \{u^1(w^1 + y) : y \in Y^1\}$$

[1] That is, $0 \in Y^1 \cap Y^2 \cap Y$.

and

$$u^2(\bar{x}^2) \geq \max_y \left\{u^2(w^2 + y) : y \in Y^2\right\}$$

3. Determine conditions under which if $(p, \bar{x}^1, \bar{x}^2, \bar{y}, m^1, m^2)$ is a price-taking cooperative equilibrium, then allocation $(\bar{x}^1, \bar{x}^2, \bar{y})$ is efficient in the production economy with firm Y, and prove the resulting theorem.

4. Define the numbers
$$s^1 = \frac{m^1}{m^1 + m^2} \text{ and } s^2 = \frac{m^2}{m^1 + m^2}.$$
Argue that if $(p, \bar{x}^1, \bar{x}^2, \bar{y}, m^1, m^2)$ is a price-taking cooperative equilibrium, then $(p, \bar{x}^1, \bar{x}^2, \bar{y})$ is a competitive equilibrium of the production economy with firm Y, where each i owns a proportion s^i of it.

5. Interpret these results in terms of economics. In particular, what do they mean regarding the definition of competitive equilibrium and its welfare properties?

6. Generalize the definition of price-taking cooperative equilibrium to the case of an arbitrary number I of individuals.