University of California, Davis
Department of Economics
Microeconomics

Date: June 20, 2023
Time: 5 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

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Please answer **all three** questions

QUESTION 1

Consider a population of n identical individuals who have the same von Neumann-Morgenstern utility-of-money function U (with $U' > 0$, $U'' < 0$), the same initial wealth W and are facing the same potential loss L (with $0 < L \leq W$). The objective (based on long-run historical data) probability of loss is p (with $0 < p < 1$). However, an insurance conspiracy site, called *I-anon*, has convinced all these individuals that the probability of loss is p_L, with $0 < p_L < p$. The insurance industry is a monopoly and the monopolist knows the true probability p and also knows that all of its potential customers believe it to be p_L.

(a) Show that the monopolist will not offer full insurance to these individuals. You can do so graphically or analytically.

(b) For this part only, assume that $W = 4{,}000$, $L = 1{,}800$, $p_L = \frac{1}{7}$, $p = \frac{1}{5}$, $U(\$m) = \ln(m)$.

(b.1) What is the subjective (i.e. based on p_L) expected utility of not insuring? And the objective (i.e. based on p) one? [If you forgot to bring a calculator, just write the expressions that you would input into a calculator.]
(b.2) What contract will the monopolist offer? You don't need to solve for it, just write the relevant equation(s) that must be solved to find it.

(c) For general values of the parameters (that is, do not assume the values given in part b) give a necessary and sufficient condition for the existence of a contract that yields positive profits.

From now on, assume general values of the parameters (that is, do not assume the values given in part b) and assume that the population is divided into two groups. Group R consists of rational individuals who, having informed themselves, know that their probability of loss is the true one, namely p. Group I consists of the I-anon individuals who believe that their probability of loss is p_L. Let $n_R > 0$ be the number of individuals who belong to group R and $n_I > 0$ be the number of individuals who belong to group I (with $n_R + n_I = n$). All of this is known to the monopolist.

(d) Suppose first that, for each customer, the monopolist can tell if s/he belongs to Group R or Group I and can offer different contracts to different individuals. What contracts will the monopolist offer?

(e) Suppose now that the monopolist cannot tell the two types apart.
(e.1) Write the profit-maximization problem for the monopolist (no need to solve it).
(e.2) Describe in words the profit-maximizing menu of contracts (no need to calculate it) and show it in a wealth diagram.

(f) Say that consumers are *exploited* if they end up buying insurance that makes them **objectively** (that is, in terms of the true or objective probability of loss) **worse-off** than they would be without insurance. In each of parts (b.2) [for the general case] and (d) and (e) determine if there are any consumers who are exploited.

Question 2

Consider an exchange economy with two commodities. Suppose that for each individual, the utility function $u^i : \mathbb{R}^2_+ \to \mathbb{R}$ is continuous, strictly quasi-concave and strictly monotone, and her endowment is strictly positive: $w^i \in \mathbb{R}^2_{++}$.

If we fix the price of good 1 at 1 and denote the price of commodity 2 as $p \in \mathbb{R}_{++}$, the individual demands are

$$x^i(p) = \arg\max_{x \in \mathbb{R}^2_+} \left\{ u^i(x) : x_1 + p x_2 \leq w^i_1 + p w^i_2 \right\}.$$

These functions are continuous and you can take for granted that each of them satisfies the following properties:

(i) for all $\Delta > 0$, there exists $\underline{\pi}^i > 0$ such that $x^i_2(p) > \Delta$ if $p \leq \underline{\pi}^i$; and

(ii) for all $\Delta > 0$, there exists $\bar{\pi}^i > 0$ such that $x^i_1(p) > \Delta$ if $p \geq \bar{\pi}^i$.

(These properties simply say that if one commodity becomes arbitrarily cheap while the other remains expensive, the individuals' demand for the cheap commodity becomes unboundedly large.)

Define the function $Z(p) = \sum_i [x^i_2(p) - w^i_2]$.

(a) How do you interpret the function Z?

(b) Argue that if $Z(p^*) = 0$, then $((1, p^*), (x^i(p^*))_{i \in \mathcal{I}})$ is a competitive equilibrium for the economy.

(c) Argue that there exist $\underline{\pi} > 0$ and $\bar{\pi} > 0$ such that $Z(p) > 0$ if $p \leq \underline{\pi}$ and $Z(p) < 0$ if $p \geq \bar{\pi}$.

(d) The intermediate value theorem says the following:

> *Suppose that $X \subseteq \mathbb{R}$ is an interval, function $f : X \to \mathbb{R}$ is continuous, and $\underline{x}, \bar{x} \in X$ are such that $\underline{x} < \bar{x}$, $f(\underline{x}) > 0$, and $f(\bar{x}) < 0$. Then, there exists $x^* \in (\underline{x}, \bar{x})$ for which $f(x^*) = 0$.*

Use this theorem to prove that there exists $p^* \in \mathbb{R}_{++}$ for which $Z(p^*) = 0$.

(e) Argue that this economy has at least one competitive equilibrium.

(f) Consider now the demand of agent $i = 1$ as a function also of her endowments:

$$x^1(p, w^1) = \arg\max_{x \in \mathbb{R}^2_+} \left\{ u^1(x) : x_1 + px_2 \leq w^1_1 + pw^1_2 \right\}.$$

Argue that

$$x^1(p, w^1 + \delta(p, -1)) = x^1(p, w^1).$$

(g) Redefining

$$Z(p, w^1) = [x^1_2(p, w^1) - w^1_2] + \sum_{i \neq 1} [x^i_2(p) - w^i_2],$$

argue that for all δ

$$Z(p, w^1 + \delta(p, -1)) = Z(p, w^1) + \delta.$$

(h) Formalize the following claim, which the previous point proves:

> Suppose that $(1, p)$ is a competitive equilibrium price vector of an exchange economy with continuous and strictly quasi-concave utility functions, and with strictly positive endowments for individual $i = 1$. There exists another economy where:
>
> (i) the only difference is the endowments of agent $i = 1$,
>
> (ii) the magnitude of this difference is arbitrarily small, and
>
> (iii) $(1, p)$ is <u>not</u> a competitive equilibrium price vector of this other economy.

Question 3

The economics faculty is at lunch in the department. Professor Clark mentions that he taught Giffen goods in his principles class today. He laments that it is hard to find examples of Giffen goods. He suggests that wine could be a Giffen good as people sometimes buy it when the price is higher rather than lower. Professor Rapson interjects that when people buy somewhat pricier wine rather than cheap wine, it is most likely due to the fact that they cannot distinguish bad from good wine and take the price as a quality signal. Professor Schipper quips that if Professor Clark had paid attention to consumer theory when studying at Harvard, then he would know that Giffen good implies inferior good. And clearly wine is not an inferior good. There is a moment of silence. It is not clear whether economic logic stifled the conversation or Professor Schipper's arrogant undertone. Realizing latter, Professor Schipper asks (more rhetorically than seriously) how to overcome the argument that Giffen good implies inferior good.

a. Does the proposition that Giffen good implies inferior good depend on the existence of a utility function? Explain.

b. It dawns on Professor Schipper that we have to go beyond standard consumer theory in order to overturn the proposition that Giffen good implies inferior good. Wine is consumed in social settings. Could consumption externalities allow for a Giffen good that is not inferior? Being a slow thinker, Professor Schipper poses this as a prelim problem. Here is the problem description:

   Professor C spends his wealth w on wine and other goods. We denote by $x_1 \geq 0$ and $y_1 \geq 0$ Professor C's spending on other goods and wine, respectively. Professor C also cares about the wine that others drink. Since Professor Schipper only theorizes about alcohol, we enlist the help of Professor T who has non-trivial practical experience with wine. Denote by y_2 the amount of wine consumed by Professor T.[1] The price of wine is $p > 0$. The price of spending on other goods is normalized to 1. Professor C's problem is now

$$\max_{x_1, y_1} u(x_1, y_1, y_2)$$

   subject to the budget constraint

$$x_1 + py_1 \leq w.$$

   As usual, utility functions of economists are well-behaved, that is, the utility function of Professor C is concave and continuously differentiable with strict positive gradient in the interior of its domain.

   Ignoring the non-negativity constraints, write down the Lagrangian and state the first-order conditions. Assuming an interior and unique solution, simplify as much as possible and arrive at a system of equations that does not involve multipliers.

[1] As always, names, characters, and incidents are either the products of the author's imagination or used in a fictitious manner. Any resemblance to actual persons, living or dead, or actual events is purely coincidental.

At the moment, you do *not* need to solve for solutions $x_1(p, w, y_2)$ and $y_1(p, w, y_2)$. Are the first-order conditions also sufficient?

c. From now on, assume that

$$u(x_1, y_1, y_2) := x_1 + \boldsymbol{a} \cdot \boldsymbol{y} - \frac{1}{2} \boldsymbol{y} \cdot B \boldsymbol{y} \tag{1}$$

with

$$\boldsymbol{a} := \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} >> \boldsymbol{0} \tag{2}$$

$$\boldsymbol{y} := \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \tag{3}$$

$$B := \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \tag{4}$$

We assume that B is a positive definite symmetric matrix.

Specialize the system of equations from part a. to the utility function given by equations (1) to (4). We restrict prices, wealth, and parameters such that a satiation point of the utility function is outside any budget sets we consider.

d. Use the system of equations from part c. to solve for Professor C's Walrasian demand functions.

e. Check whether wine is a Giffen good for Professor C. How does his demand for wine change with the price?

f. How is Professor C's consumption of wine affected by Professor T's consumption of wine?

g. Professor C thinks that our earlier answer about how his consumption of wine responds to changes of the price of wine is wrong. He argues that if he cares about Professor T's consumption of wine, then there should also be an effect of the price change via Professor T's change of consumption of wine. Let's analyze this argument. Obviously, we need a model for Professor T's consumption. Assume that his utility function is given by

$$v(y_1, x_2, y_2) = y_1 x_2 y_2$$

That is, Professor T also cares about Professor C's consumption. Here x_2 denotes Professor T's spending on other goods. His budget set is given by

$$x_2 + p y_2 \leq m$$

for $m > 0$. Derive a condition on the parameters under which Professor C's consumption of wine (not a typo; we care about Professor C's consumption) is decreasing in the price of wine and explain. For this, you will have to derive explicitly Professor T's demand for wine.

h. Find a condition on the parameters such that Professor C's consumption of wine increases in the price of wine. Argue that wine is not an inferior good to him. Why do we have a counterexample to the proposition? Explain what's going on.

i. Did we miss something in our analysis? We know that Professor T also cares about Professor C's consumption of wine. Shouldn't Professor T's consumption of wine also depend on Professor C's consumption of wine? In other words, aren't both professors' consumption decisions interdependent so that we would need game theory to solve the problem? Explain why we do not need a fixed-point argument like Nash equilibrium (or an iterated best "wine consumption" response argument) to solve the problem.