University of California, Davis
Department of Economics
Microeconomics

Date: August 14, 2019
Time: 5 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Please answer **any three** of the following four questions
[If you answer all four questions please indicate which three you want to be graded]

QUESTION 1

Having seen the success of Uber, you have decided to start a similar company in Davis, which you will call Unter. Unter will only provide rides within the city of Davis. The new and revolutionary feature of Unter is that, instead of charging per ride, it is based on a yearly membership. If you become a member of Unter then, within the year covered by your membership, you can take as many rides as you like for free. Each Davis resident is characterized by a pair (r,κ), where r is the reservation price for membership (he/she is willing to become a member as long as the membership charge is **less than or equal to** r) and κ is the number of rides that he/she would take per year (note that κ is a constant, independent of the cost of becoming a member: riding a car around Davis is not particularly enjoyable, thus residents would make use of Unter only if needed; note also that each resident, if indifferent between applying and not applying for membership, he/she will apply). Unter's cost of providing each ride is constant and equal to $c > 0$. You find yourself in a situation of asymmetric information: each Davis resident knows his/her own (r,κ), but you (the owner of Unter) only know the distribution P over the set of possible pairs (r,κ) [$P(r,\kappa)$ is the fraction of Davis residents who are characterized by the pair (r,κ)]. **You are risk neutral.** Let N be the number of Davis residents. Because of tax reasons, you need to run a small company by enrolling as members **not more than K residents.** From now on by "price" we will mean the membership fee.

(a) Let us start with a simple example, where $N = 24{,}000$ and $K = 50$; furthermore, the possible pairs (r,κ) and, for each pair, the number of residents who are characterized by that pair, are given in the following table.

(r,κ):	(100,8)	(120,12)	(100,16)	(100,24)	(140,24)	(120,36)	(140,36)	(120,40)	(140,72)
residents:	2,000	1,000	3,000	2,000	1,000	2,000	4,000	4,000	5,000

What price will you charge? [You preferences reflect the capitalist society in which you grew up: you want to make as much money as possible! Clearly, your answer will have to be conditional on the value of c, which is the cost of providing each ride.]

(b) For this question assume that $K < \frac{N}{10}$, $r \in \{1,2,\ldots,10\}$ and $\kappa \in \{1,2,\ldots,10\}$. Let n_{ij} be the number of residents for whom $r = i$ and $\kappa = j$ (thus $\sum_{i=1}^{10}\sum_{j=1}^{10} n_{ij} = N$). Assume a uniform distribution, that is, $n_{ij} = n_{st}$ for every $i,j,s,t \in \{1,2,\ldots,10\}$.

(b.1) Find the profit-maximizing price for Unter.

(b.2) Suppose that $K = 1{,}000$ and you have decided to start your business only if you expect to make a profit of at least \$3,900. For what values of c will you decide to start your business?

(c) For this question assume that $K < \frac{N}{15}$, $r \in \{16,17,18,19,20\}$ and $\kappa \in \{1,2,3,4,5\}$. Let n_{ij} be the number of residents for whom $r = i$ and $\kappa = j$. For every i and j, let $n_{ij} = \begin{cases} 0 & \text{if } i > j+15 \\ \frac{N}{15} & \text{if } i \le j+15 \end{cases}$.

(c.1) Find the profit-maximizing price for Unter.

(c.2) Suppose that $K = 1{,}000$ and you have decided to start your business only if you expect to make a profit of at least \$3,900. For what values of c will you decide to start your business?

Question 2

When we studied consumer theory in ECN200A, we introduced various functions like utility functions, indirect utility functions, expenditure functions, Walrasian demand functions, Hicksian demand functions etc. There are other functions that are sometimes useful in the context of consumer theory. Let us use our knowledge of consumer theory and the techniques we learned to study one such function that is not discussed in Mas-Colell, Whinston, and Green (1995).

Fix a consumption bundle $g \in X \subseteq \mathbb{R}^L_+$ with $g \neq 0$. We will use this consumption bundle as a reference point. We want to define a function that measures *how many units of this reference consumption bundle g a consumer is willing to give up in order to move from some utility level $\underline{u}$ to some consumption bundle $x \in X$.* Such a function may be useful in the context of development economics of societies in which one commodity (e.g., rice) is a natural reference commodity already. It is also of conceptual significance as it helps us to understand the consumer problem as a problem of maximizing the difference between benefits and costs.

To this end, for reference consumption bundle $g \in X$, $g \neq 0$, and utility level $\underline{u}$, define the *benefit function* by

$$b(x, \underline{u}) = \begin{cases} \max\{\beta \in \mathbb{R} : u(x - \beta g) \geq \underline{u}, x - \beta g \in X\} & \text{if } x - \beta g \in X, u(x - \beta g) \geq \underline{u} \text{ for some } \beta \\ -\infty & \text{otherwise} \end{cases}$$

a. Let's try first to understand this function graphically by assuming $L = 2$. Consider first Figure 1 (a). It depicts an indifference curve representing a utility level u and a reference consumption bundle g. Further, it depicts b, the number of units of g the consumer is willing to give up to move from the indifference curve representing u to the consumption bundle x. Explain now what happens in Figure 1 (b).

b. Explain what happens in Figure 1 (c).

c. Let's derive the benefit function for the case of a Cobb-Douglas utility function $u(x) = \prod_{\ell=1}^{L} x_\ell^{\alpha_\ell}$ for $\alpha_\ell > 0$, $\ell = 1, ..., L$, $x \in \mathbb{R}^L_{++}$. Set $g = (1, 0, ..., 0)$. Then

$$b(x, \underline{u}) = \max \beta \text{ s.t. } (x_1 - \beta)^{\alpha_1} \prod_{\ell \geq 2} x_\ell^{\alpha_\ell} \geq \underline{u}.$$

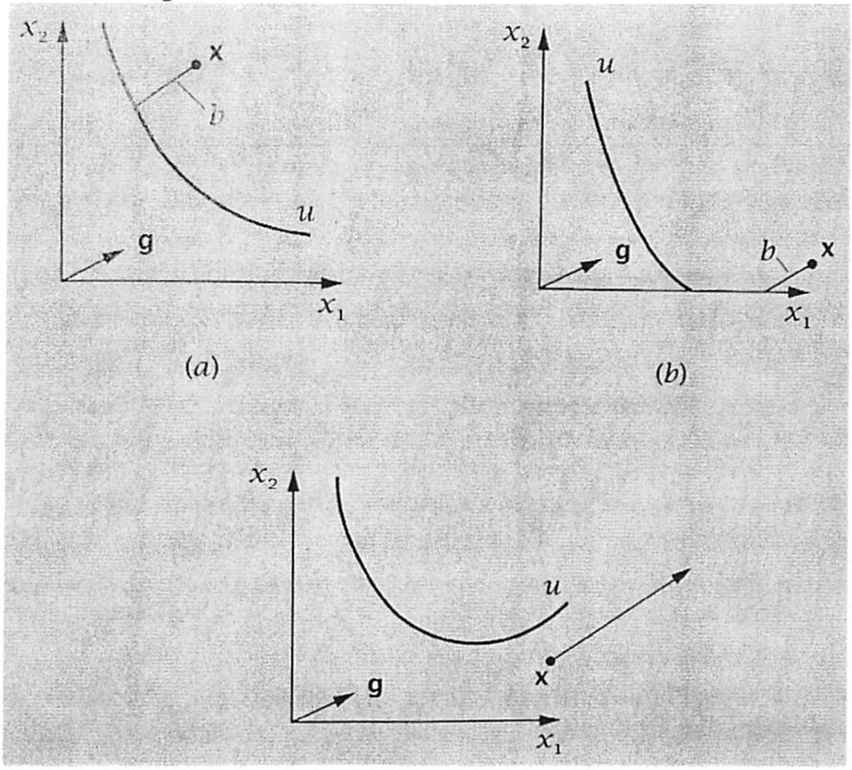
Derive $b(x, \underline{u})$ (I.e., solve for the β that corresponds to $b(x, \underline{u})$.)

d. Consider now again the general definition of the benefit function defined above. Argue that $b(x, \underline{u})$ is nonincreasing in $\underline{u}$.

e. Argue that if $x \in \mathbb{R}^L_+$ and $x + \alpha g \in \mathbb{R}^L_+$, then $b(x + \alpha g, \underline{u}) = \alpha + b(x, \underline{u})$.

f. Show that if the utility function u is quasiconcave with respect to x, then $b(x, \underline{u})$ is concave with respect to the x.

Figure 1: Three Cases of the Benefit Function



g. Assume that the utility function u is continuous, $g \geq 0$, $g \neq 0$, and $X = \mathbb{R}^L_+$. Let $p \in \mathbb{R}^L_{++}$. Assume further that $p \cdot x^* > 0$ and $b(x^*, u^*) = 0$ with $u^* = u(x^*)$. Show that if $x^* \in X$ solves the problem

$$\max_{x \in X} b(x, u^*) - p \cdot x,$$

then x^* also solves the problem

$$\max_{x \in X} u(x) \text{ s.t. } p \cdot x \leq w,$$

where as in class w represents the consumer's wealth.

h. Assume that the utility function u is continuous, locally nonsatiated, and that $g \geq 0$, $g \neq 0$, and $X = \mathbb{R}^L_+$. As before, let $p \in \mathbb{R}^L_{++}$ and $p \cdot g = 1$. Show that if $x^* \in X$ solves the problem

$$\max_{x \in X} u(x) \text{ s.t. } p \cdot x \leq w,$$

then x^* also solves the problem

$$\max_{x \in X} b(x, u^*) - p \cdot x,$$

with $u^* = u(x^*)$.

Question 3

In this exercise you will argue that two economies with different endowments are very unlikely to have the same equilibrium prices, even if they have the same preferences.

Consider an exchange economy with L commodities and 2 individuals. Individual preferences are represented by functions of the form $u^i : \mathbb{R}^L_{++} \to \mathbb{R}$. Both of these functions are assumed to be of class C^2, differentiably strictly monotone, and differentiably strictly quasi-concave, and to satisfy the interiority property.[1]

(a) Write the "extended approach" function

$$F(p, x^1, x^2, \lambda^1, \lambda^2, w^1, w^2)$$

that can be used to characterize competitive equilibrium.[2]

(b) Now define the function

$$G(p, x^1, x^2, \lambda^1, \lambda^2, w^1, w^2, \hat{p}, \hat{x}^1, \hat{x}^2, \hat{\lambda}^1, \hat{\lambda}^2, \hat{w}^1, \hat{w}^2) = \begin{pmatrix} F(p, x^1, x^2, \lambda^1, \lambda^2, w^1, w^2) \\ F(\hat{p}, \hat{x}^1, \hat{x}^2, \hat{\lambda}^1, \hat{\lambda}^2, \hat{w}^1, \hat{w}^2) \end{pmatrix}.$$

Taking for granted that F is transverse to 0, argue that so is G.

(c) Consider next the function

$$\begin{pmatrix} G(p, x^1, x^2, \lambda^1, \lambda^2, w^1, w^2, \hat{p}, \hat{x}^1, \hat{x}^2, \hat{\lambda}^1, \hat{\lambda}^2, \hat{w}^1, \hat{w}^2) \\ p_2 - \hat{p}_2 \end{pmatrix}$$

and argue that its Jacobean consists of the matrix

$$\begin{pmatrix} D_{x^1,x^2,\lambda^1,\lambda^2,w^1,w^2,\hat{x}^1,\hat{x}^2,\hat{\lambda}^1,\hat{\lambda}^2,\hat{w}^1,\hat{w}^2} G & \partial_{p_2} G & \partial_{\hat{p}_2} G \\ 0 & 1 & -1 \end{pmatrix}$$

in addition *only* to some more columns.

(d) Taking for granted that matrix

$$D_{x^1,x^2,\lambda^1,\lambda^2,w^1,w^2,\hat{x}^1,\hat{x}^2,\hat{\lambda}^1,\hat{\lambda}^2,\hat{w}^1,\hat{w}^2} G$$

has full row rank, argue that the function defined in part (c) is transverse to 0 too.

(e) Conclude that, generically on $(w^1, w^2, \hat{w}^1, \hat{w}^2)$, if p is a vector of equilibrium prices for economy $\{(u^1, w^1), (u^2, w^2)\}$ and $\hat{p}$ is one for economy $\{(u^1, \hat{w}^1), (u^2, \hat{w}^2)\}$, then $p \neq \hat{p}$.

(f) Using what we learned in class about generic determinacy of competitive equilibrium, provide intuition for this result.

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[1] Recall that these assumptions imply that if the individual endowments are interior, then so are the individual demands, and that the latter can be characterized by the standard first-order conditions.

[2] Recall that the roots of these function satisfy all the conditions that characterize equilibrium: both agents' first-order conditions and budget constraints, and market clearing for all non-numéraire commodities.

QUESTION 4

Consider a market for a homogeneous good, which is produced at **zero cost**. Market inverse demand is given by $P = 1 - 2Q$ (where Q is industry output). Let π_M be monopoly profits and π_d be Cournot duopoly profits in this market. A firm, call it *M*, is currently a monopolist in this market but faces a potential entrant, call it *PE*. They play the following game. *M* chooses a level of investment, *k*, where *k* can be **any nonnegative number.** This decision is observed by *PE*. Next, *PE* decides whether or not to enter the market. If *PE* does not enter, *M* remains a monopolist and earns profits equal to $(2k+1)\pi_M - k$, while *PE* earns zero profits and the game ends. If *PE* enters, *M* observes *PE's* decision and decides whether or not to exit the market. If it exits, it earns $\theta - k$, while *PE* earns profits equal to $\pi_M - F$, where *F* is the cost of entry. If *M* does not exit, then we have a Cournot duopoly, with corresponding equilibrium profits of $(2k+1)\pi_d - k$ for *M* and $\pi_d - F$ for *PE*. Assume throughout that $\pi_M > F > \pi_d$ and that, if indifferent between exiting and staying, *M* chooses to stay and this is common knowledge between *M* and *PE*.

(a) Calculate π_M and π_d.

(b) Assume that the value of θ is common knowledge. Show the structure of the game by sketching the extensive form.

(c) Still assuming that the value of θ is common knowledge, find the subgame-perfect equilibrium of the game (clearly, your answer should be conditional on the value of θ).

For the remaining questions, assume that there are only two possible values of *k*: 0 and $\hat{k}$, that is, $k \in \{0, \hat{k}\}$. The value of θ is private information to *M*. However, it is commonly known that there are only two possible values: θ_H and θ_L, with $\theta_H > \theta_L > 0$. Let $p \in (0,1)$ be the probability that *PE* assigns to θ_H [and $(1-p)$ the probability that *PE* assigns to θ_L]. *PE*'s beliefs are common knowledge between *M* and *PE* as is the fact that *M* knows the true value of θ. Thus we have a situation of incomplete information.

(d) Using the Harsanyi transformation sketch the extensive form of the corresponding imperfect-information game. Make sure that information sets are clearly drawn. [You can simplify the sketch by replacing the Cournot duopoly interactions with terminal nodes and associating with them the corresponding equilibrium profits.]

(e) For the game of part (*d*) show that under the following parameter restrictions there is no pure-strategy separating weak sequential equilibrium (that is, there is no pure-strategy equilibrium where the two types of *M* make different investment choices):

$$\hat{k} = \frac{1}{40}, \quad \theta_L = \frac{1}{20}, \quad \theta_H = \frac{1}{15}.$$

(f) With the parameter values of part (*e*), and assuming that the players are risk neutral, for what values of p is there a pooling weak sequential equilibrium where both types of *M* choose $\hat{k}$ and, observing this, *PE* stays out?