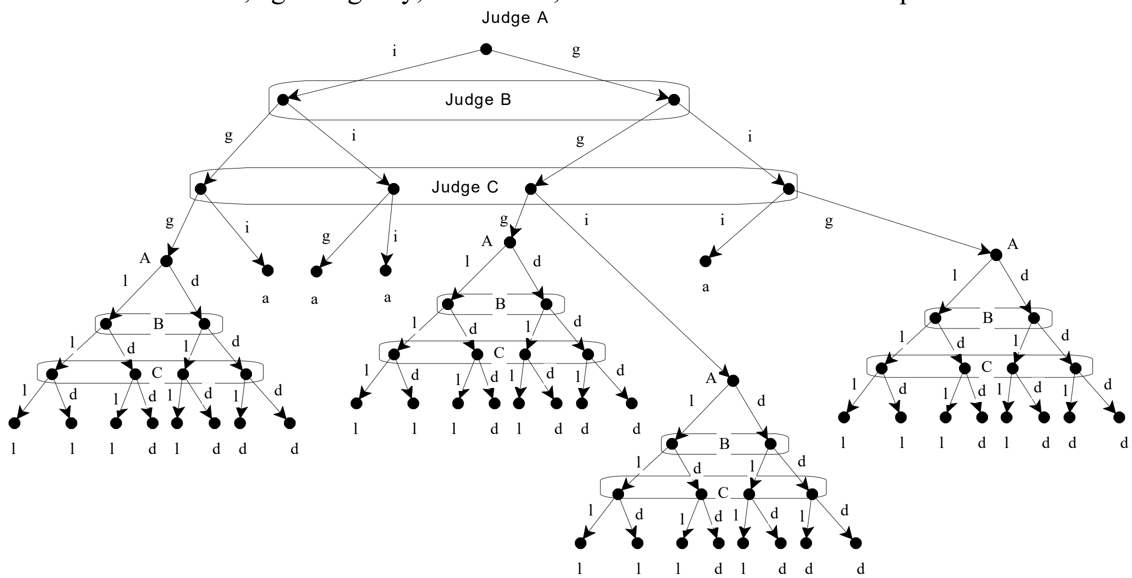
ANSWER KEYS Micro Prelim June 24, 2019

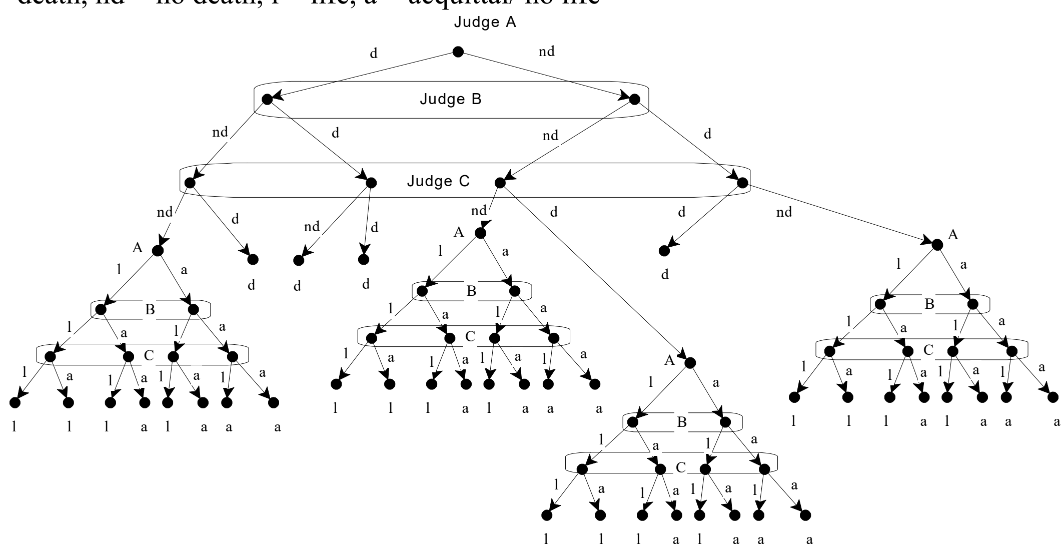
QUESTION 1 ANSWER KEYS

[The key observation is that, under majority voting, when there are only two choices voting sincerely (that is, according to one's true preferences) is a weakly dominant strategy.]

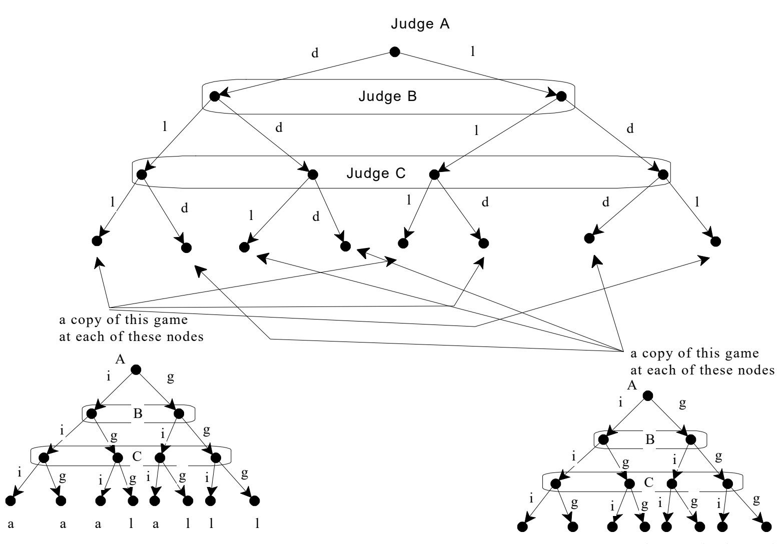
(a) "i" stands for innocent, "g" for guilty, "l" for life, "d" for death and "a" for acquittal.



(b) d = death, nd = no death, l = life, a = acquittal/ no life



(c)



(d) In the game of part (c), Judge A has **nine** information sets. Hence judge A has $\mathbf{2^9 = 512}$ possible strategies. One of these 512 strategies is the following: vote "d" initially and then if the defendant is found guilty vote "i" if the first-stage vote is a majority for death and vote "g" otherwise.

Note for (e)-(g): in these games (2nd stage and reduced 1st stage) there is no difference between dominant-strategy equilibrium and the outcome of the IDWDS procedure (deletion has to be done in one step).

(e) In the **IG procedure** the first vote is between 'guilty' and 'innocent'. If you are found innocent, then you are acquitted. If, instead, you are found guilty, then there is a second vote where each judge has to vote either for 'life' (L) or 'death' (D). Thus the second-stage vote (if the outcome of the first stage was 'guilty') is:

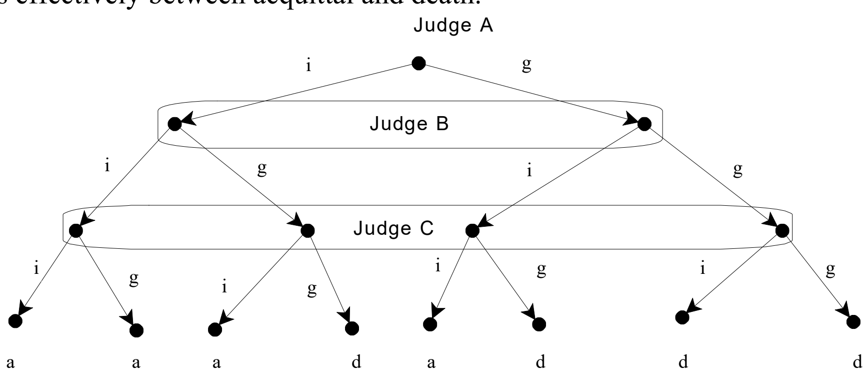
Judge C chooses L

Judge A \ Judge B	L	D
L	life	life
D	life	death

Judge C chooses D

Judge A \ Judge B	L	D
L	life	death
D	death	death

At this stage, for judge A voting 'D' is a dominant choice, for judge B voting 'L' is a dominant choice and for judge C voting 'D' is a dominant choice. That is, voting sincerely (i.e. according to one's true ranking) is a dominant strategy for every judge. Hence everybody can predict that the outcome of the second-stage vote will be 'death'. Thus the game reduces to the following ('i' stands for 'innocent' and 'g' for 'guilty'), where the choice is effectively between acquittal and death.



The corresponding strategic form is:

Judge C chooses i

Judge A \ Judge B	i	g
i	acquitted	acquitted
g	acquitted	death

Judge C chooses g

Judge A \ Judge B	i	g
i	acquitted	death
D	death	death

In this game for judge A voting 'g' is a dominant strategy, for judge B voting 'i' is a dominant strategy and for judge C voting 'i' is a dominant strategy. Thus the outcome of the IG procedure is that you are acquitted.

(f) In the **SP procedure** the first vote is between 'death' and 'no death'. If the majority voted for 'death' then you are executed. If, instead, the outcome of the first vote was 'no death', then there is a second vote where each judge has to vote either for 'life' (L) or 'acquittal' (A). Thus the second-stage vote (if the outcome of the first stage was 'no death') is as follows:

Judge C chooses L

Judge A \ Judge B	L	A
L	life	life
A	life	acquitted

Judge C chooses A

Judge A \ Judge B	L	A
L	life	acquitted
A	acquitted	acquitted

At this stage, for judge A voting 'L' is a dominant choice, for judge B voting 'L' is a dominant choice and for judge C voting 'A' is a dominant choice (once again, truthful voting is a dominant strategy). Thus everybody can predict that the outcome of the second-stage vote will be 'life'. On the basis of this prediction, the first-stage vote reduces to one where d means death and "no death" means life, that is, the game reduces to the following ('D' stands for 'death' and 'N' for 'no death'):

Judge C chooses D (rows: Judge A; columns: Judge B)

Judge A \ Judge B	D	N
D	death	death
N	death	life

Judge C chooses N

Judge A \ Judge B	D	N
D	death	life
N	life	life

In this game for judge A voting 'D' is a dominant strategy, for judge B voting 'L' is a dominant strategy and for judge C voting 'D' is a dominant strategy. Thus the outcome of the SP procedure is that you are sentenced to death.

(g) In the **PA procedure** the first vote is between 'death' and 'life'. If the majority voted for 'death' then the next vote will be between 'acquittal' (a) and 'guilty' (g), where 'guilty' will mean 'death sentence'. If, instead, the outcome of the first vote was 'life', then the next vote will be between 'acquittal' (a) and 'guilty' (g), where 'guilty' will mean 'life sentence'. Thus there are two different second-stage games, depending on the outcome of the first-stage vote.

CASE 1: the outcome of the first-stage vote was **death.** In this case the second-stage game becomes:

Judge C chooses a

Judge A \ Judge B	a	g
a	acquitted	acquitted
g	acquitted	death

Judge C chooses g

Judge A \ Judge B	a	g
a	acquitted	death
g	death	death

In this game, for judge A voting 'g' is a dominant choice while for judges B and C voting 'a' is a dominant choice. Thus everybody can predict that the outcome of this second-stage vote will be 'acquittal'.

CASE 2: the outcome of the first-stage vote was **life.** In this case the second -stage game becomes:

Judge C chooses a

Judge A \ Judge B	a	g
a	acquitted	acquitted
g	acquitted	life

Judge C chooses g

Judge A \ Judge B	a	g
a	acquitted	life
g	life	life

In this game, for judges A and B voting 'g' is a dominant choice while for judge C voting 'a' is a dominant choice. Thus everybody can predict that the outcome of this second-stage vote will be 'guilty'. Thus there are two second-stage games. If life was chosen as the applicable penalty, then in the second stage there will be a majority for the life sentence. If death is the predetermined penalty, then in the second stage there will be a majority in favor of acquittal. Thus the first-stage vote reduces to one where "life" means life and "death" means acquittal. The game thus reduces to one where the judges have to choose between 'death', D, and 'life', L:

Judge C chooses D

Judge A \ Judge B	D	L
D	acquittal	acquittal
L	acquittal	guilty/life

Judge C chooses L

Judge A \ Judge B	D	L
D	acquittal	guilty/life
L	guilty/life	guilty/life

In this game for judges A and B voting 'L' is a dominant strategy, while for judge C voting 'D' is a dominant strategy. Thus the outcome of the PA procedure is that you are sentenced to life in prison.

(h) Obviously, the IG procedure.

Question 2

a. President Tony Dumb of the United States of Absurdistan is worried about his reelection prospects. He believes that his reelection prospects are strongly positively correlated with the stock market. That's why he announced on Twitter that he wants to *subsidize* returns on the stock market. When you went to heat your lunch in the microwave of the Stevens lounge across Professor Schipper's office, you overheard him mumbling to himself that this was a really good idea. You express surprise to Professor Schipper that you find him in agreement with President Dumb. Professor Schipper answers smilingly that you should check out the effect of the subsidy yourself.

Let w denote the initial wealth of the voter. Consider an asset that yields a return r_g in the good state and a return r_b in the bad state, with $r_g > 0 > r_b$. That is, when the voter invests $x \geq 0$ into the asset, her wealth becomes $(w - x) + x(1 + r_g)$ in the good state and $(w - x) + x(1 + r_b)$ in the bad state. She assigns probability $\pi \in (0, 1)$ to the good state and the remaining probability to the bad state. We assume that r_g, r_b, and π are such that the expected return is strictly positive. We also assume that her twice continuously differentiable Bernoulli utility function $u(\cdot)$ is strictly increasing in wealth. Finally, assume that she is risk averse (and not risk neutral).

aa. Show that her optimal investment x^0 in the absence of subsidies is strictly positive.

`This is essentially the subsidy-analogue of the tax-version in Varian, H. ‘‘Intermediate microeconomics: A modern approach, Appendix to Chapter 12, Norton, 9th edition, 2014.`

`First write wealth in the good and bad state, respectively, as`

$$\begin{aligned}(w - x) + x(1 + r_g) &= w + xr_g \\ (w - x) + x(1 + r_b) &= w + xr_b\end{aligned}$$

`The expected utility is`

$$\mathbb{E}[u(x)] = \pi u(w + xr_g) + (1 - \pi)u(w + xr_b)$$

`We maximize expected utility over` x. `The first-order condition is`

$$\frac{d\mathbb{E}[u(x^0)]}{dx} = \pi u'(w + x^0 r_g)r_g + (1 - \pi)u'(w + x^0 r_b)r_b \equiv 0$$

`We verify the second-order condition:`

$$\frac{d^2\mathbb{E}[u(x)]}{dx^2} = \pi u''(w + xr_g)r_g^2 + (1 - \pi)u''(w + xr_b)r_b^2 < 0$$

`because investors are risk averse, i.e.,` $u''(w) < 0$ `for all` w.

To see that the optimal investment x^0 is strictly positive, assume to the contrary that it is zero. Consider the first derivative at zero,

$$\begin{aligned}\frac{d\mathbb{E}[u(0)]}{dx} &= \pi u'(w)r_g + (1-\pi)u'(w)r_b \\ &= u'(w)(\pi r_g + (1-\pi)r_b) > 0\end{aligned}$$

because the Bernoulli utility function is strictly monotone increasing in wealth and the expected return is positive. I.e., the expected utility is increasing in the first dollar invested in the asset. Thus, $x^0 > 0$.

ab. Assume now that the subsidy is $s > 0$ per unit of return. That is, in the good state the return after subsidy is $(1+s)r_g$ and in the bad state the return after subsidy is $(1+s)r_b$. Show how her optimal investment after installation of the subsidy differs from her optimal investment without the subsidy.

The expected utility with subsidy is

$$\mathbb{E}[u(x)] = \pi u(w + x(1+s)r_g) + (1-\pi)u(w + x(1+s)r_b).$$

Consider the first-order condition

$$\frac{d\mathbb{E}[u(x)]}{dx} = \pi u'(w+x^s(1+s)r_g)(1+s)r_g + (1-\pi)u'(w+x^s(1+s)r_b)(1+s)r_b \equiv 0.$$

Simplify

$$\pi u'(w + x^s(1+s)r_g)r_g + (1-\pi)u'(w + x^s(1+s)r_b)r_b \equiv 0.$$

Recall from aa.

$$\pi u'(w + x^0 r_g)r_g + (1-\pi)u'(w + x^0 r_b)r_b \equiv 0.$$

Since u is strictly monotone, we must have

$$x^0 = x^s(1+s)$$

or equivalently

$$x^s = \frac{x^0}{1+s}.$$

But this implies $x^s < x^0$!

ac. Discuss/interpret your results.

At a first glance the result is counter-intuitive. Usually we believe that a consumer buys more of what is subsidized. At a second glance, the result is not surprising. When the subsidy is imposed, the investor has more of a gain in the good state but also more of a loss in the bad state. The subsidy increases the expected return but also the risk. By scaling the investment down by $\frac{1}{1+s}$, the investor can reproduce the same expected return as before the subsidy.

b. Suppose an agent faces two distributions of payoffs, F and G. Suppose that for any payoff x, the probability of x given that some payoff not below x is drawn is lower under F than G. Does there exist an expected utility maximizer with monotone increasing Bernoulli utility function who strictly prefers G over F?

Hint: Assume that F and G have densities f and g, respectively. The probability of x conditional on a payoff being drawn not below x under F is $\frac{f(x)}{1-F(x)}$. We can now formalize the hypothesis as $\frac{f(x)}{1-F(x)} \leq \frac{g(x)}{1-G(x)}$ for all x. This is called the monotone hazard rate condition and used a lot in asymmetric information economics.

`The answer is ``No.'' We need to show that the monotone hazard rate condition implies first-order stochastic dominance. I.e.,`

$$\frac{f(x)}{1-F(x)} \leq \frac{g(x)}{1-G(x)} \text{ for all } x$$

implies

$$\int u(x)dF(x) \geq \int u(x)dG(x) \text{ for all increasing } u.$$

`Thus, there is no little` u `with which` G `is preferred to` F `by an expected utility maximizer.`

`The proof of`

$$\int u(x)dF(x) \geq \int u(x)dG(x) \text{ for all increasing } u$$

`if and only if`

$$F(x) \leq G(x) \text{ for all } x$$

`you can find in my slides.`

`Consider`

$$\frac{f(x)}{1-F(x)} \leq \frac{g(x)}{1-G(x)} \text{ for all } x.$$

`Integrate both sides`

$$\int_0^x \frac{f(t)}{1-F(t)}dt \leq \int_0^x \frac{g(t)}{1-G(t)}dt \text{ for all } x.$$

`This is equivalent to`

$$\ln(1-F(x)) \geq \ln(1-G(x)) \text{ for all } x$$

`and therefore`

$$F(x) \leq G(x) \text{ for all } x.$$

`To see the claimed equivalence, note that`

$$\frac{d(1-F(x))}{dx} = -\frac{F(x)}{dx} = -f(x).$$

Thus,

$$-\frac{d\ln(1-F(x))}{dx} = \frac{\frac{-d(1-F(x))}{dx}}{1-F(x)} = \frac{f(x)}{1-F(x)}.$$

Taking integrals on both sides yields

$$-\ln(1-F(x)) = \int_0^x \frac{f(t)}{1-F(t)}dt.$$

Question 3: ANSWER KEYS

(a) A *Nash-Walras equilibrium* is an array $(p, q, \vec{x}, \vec{y}, X, Y)$where $\vec{x} = (x^1, \dots, x^I)$ and $\vec{y} = (y^1, \dots, y^I)$ are, respectively, profiles of consumption bundles of the first L goods and of the last good, such that:

i. for each i, (x^i, y^i) solves the problem

$$\max_{x,y} \left\{ u^i \left(x, y + \textstyle\sum_{j \neq i} y^j, y \right) : p \cdot x + qy \leqslant p \cdot w^i + s^i(qY - p \cdot X) \right\};$$

ii. for the firm, (X, Y) solves the problem

$$\max_{\hat{X}, \hat{Y}} \left\{ q\hat{Y} - p \cdot \hat{X} : \hat{Y} = f(\hat{x}) \right\};$$

iii. markets clear: $\sum_i x^i + X = \sum_i w^i$ and $\sum_i y^i = Y$.

(b) For each consumer, if we define the Lagrangean

$$\mathcal{L}^i = u^i \left(x, y + \textstyle\sum_{j \neq i} y^j, y \right) + \lambda^i [p \cdot w^i + s^i(qY - p \cdot X) - p \cdot x - qy],$$

where $\lambda^i > 0$, we can derive the first-order conditions

$$\begin{aligned} D_x u^i(x^i, Y, y^i) &= \lambda^i p \\ \partial_Y u^i(x^i, Y, y^i) + \partial_{y^i} u^i(x^i, Y, y^i) &= \lambda^i q, \end{aligned} \tag{1}$$

which must hold in addition to the budget constraint with equality.

For the firm, if we simply re-write its problem as

$$\max_{\hat{X}} \left\{ qf(\hat{X}) - p \cdot \hat{X} \right\},$$

we get the first-order condition

$$qDf(X) = p, \tag{2}$$

which must hold in addition to the technological constraint with equality.

(c) An *allocation* is a tuple $(\vec{x}, \vec{y}, X, Y)$ such that $f(X) \leqslant Y$, $\sum_i x^i + X \leqslant \sum_i w^i$ and $\sum_i y_i \leqslant Y$. Allocation $(\vec{x}, \vec{y}, X, Y)$ is *Pareto efficient* if there does not exist an alternative allocation

$$((\hat{x}^i, \hat{y}^i)_{i=1}^I, \hat{X}, \hat{Y})$$

such that $u^i(\hat{x}^i, \hat{Y}, \hat{y}^i) \leqslant u^i(x^i, X, y^i)$ for all i, with strict inequality for some. (For simplicity, note that if an allocation is efficient, the conditions that define its feasibility must all hold with equality.)

(d) Allocation $(\vec{x}, \vec{y}, X, Y)$ is Pareto efficient only if it maximizes, over

$$((\hat{x}^i, \hat{y}^i)_{i=1}^I, \hat{X}, \hat{Y}),$$

the function

$$u^1(\hat{x}^1, \hat{Y}, \hat{y}^1)$$

subject to the conditions that

$$u^i(\hat{x}^i, \hat{Y}, \hat{y}^i) = u^i(x^i, Y, y^i)$$

for all $i \geqslant 2$, $\sum_i \hat{y}^i = \hat{Y} = f(\hat{X})$, and $\sum_i \hat{x}^i + \hat{X} = \sum_i w^i$.

(e) Denoting by $\bar{u}^i = u^i(x^i, \sum_j y^j, y^i)$ for all $i \geqslant 2$, the Lagrangean of the previous problem is

$$u^1(\hat{x}^1, \textstyle\sum_j \hat{y}^j, \hat{y}^1) + \displaystyle\sum_{i \geqslant 2} [u^i(\hat{x}^i, \textstyle\sum_j \hat{y}^j, \hat{y}^i) - \bar{u}^i] + \gamma[f(\hat{X}) - \sum_i \hat{y}^i] + \delta \cdot [\sum_i (w^i - \hat{x}^i) - \hat{X}],$$

with all $\mu^i > 0$, $\gamma > 0$ and $\delta \gg 0$. The first-order conditions for efficiency are, thus, that for each consumer

$$\begin{aligned} &\mu^i D_x u^i(x^i, Y, y^i) = \delta \\ &\mu^i [\partial_Y u^i(x^i, Y, y^i) + \partial_{y^i} u^i(x^i, Y, y^i)] + \textstyle\sum_{j \neq i} \mu^j \partial_Y u^j(x^j, Y, y^j) = \gamma, \end{aligned} \tag{3}$$

while

$$\gamma D f(X) = \delta. \tag{4}$$

These conditions must hold in addition to the constraints of the problem.

As in class, let us now assume that some equilibrium allocation is Pareto efficient. Then, conditions (1) to (4) must hold simultaneously. Conditions (1) and (3) imply that

$$\mu^i \lambda^i p = \delta \tag{5}$$

and

$$\mu^i \lambda^i q + \textstyle\sum_{j \neq i} \mu^j \partial_Y u^j(x^j, Y, y^j) = \gamma \tag{6}$$

for all i, while, from (2) and (4)

$$\frac{1}{\gamma} \delta = \frac{1}{q} p. \tag{7}$$

From (5) and (7), since $p \gg 0$ under our assumptions,

$$\mu^i \lambda^i = \frac{\gamma}{q},$$

in which case Eq. (6) becomes

$$\frac{\gamma}{q} \cdot q + \textstyle\sum_{j \neq i} \mu^j \partial_Y u^j(x^j, Y, y^j) = \gamma.$$

This implies that, for all i,

$$\textstyle\sum_{j \neq i} \mu^j \partial_Y u^j(x^j, Y, y^j) = 0.$$

Since $\mu^j > 0$ and $\partial_Y u^j$, the latter is possible only if the sum contains in fact no summands, namely if there is no $j \neq i$ in the population.

(f) Suppose not: fix a Lindahl equilibrium

$$(p, \vec{q}, r, \vec{x}, y, \vec{z}, X, Y, Z),$$

and suppose that alternative allocation

$$((\hat{x}^i, \hat{y}^i)_{i=1}^I, \hat{X}, \hat{Y})$$

is such that $u^i(\hat{x}^i, \hat{Y}, \hat{y}^i) \leqslant u^i(x^i, X, y^i)$ for all i, with strict inequality for some.

Since each u^i is strictly monotone, by condition (i) it must be that

$$p \cdot \hat{x}^i + q^i \textstyle\sum_j \hat{y}^j + r\hat{y}^i \geqslant p \cdot w^i + s^i(\sum_j q^j Y + rZ - p \cdot X)$$

for all i, with strict inequality for some. Adding up, this implies that

$$p \cdot \textstyle\sum_i \hat{x}^i + \sum_i q^i \sum_i \hat{y}^i + r\sum_i \hat{y}^i \geqslant p \cdot \sum_i w^i + \sum_i q^i Y + rZ - p \cdot X \qquad (8)$$

By (ii), on the other hand,

$$\textstyle\sum_i q^i \sum_i \hat{y}^i + r\sum_i \hat{y}^i - p \cdot \hat{X} \leqslant \sum_i q^i Y + rZ - p \cdot X. \qquad (9)$$

Together, Eqs. (8) and (9) imply that

$$p \cdot \textstyle\sum_i \hat{x}^i > p \cdot (\sum_i w^i - \hat{X}),$$

which is impossible since $p \gg 0$ and $\sum_i \hat{x}^i - \hat{X} \leqslant \sum_i w^i$.

(g) In the Lindahl problem, the Lagrangean of each consumer is

$$u^i\,(x, y, z) + \lambda^i[p \cdot w^i + s^i(\textstyle\sum_j q^j Y + rZ - p \cdot X) - p \cdot x - qy - rz],$$

where $\lambda^i > 0$, we can derive the first-order conditions

$$\begin{aligned} D_x u^i(x^i, Y, z^i) &= \lambda^i p \\ \partial_Y u^i(x^i, Y, z^i) &= \lambda^i q^i \\ \partial_{y^i} u^i(x^i, Y, z^i) &= \lambda^i r \end{aligned} \qquad (10)$$

which must hold in addition to the budget constraint with equality. For the firm, if we simply re-write its problem as

$$\max_{\hat{X}} \left\{ (\textstyle\sum_i q^i + r) f(\hat{X}) - p \cdot \hat{X} \right\},$$

we get the first-order condition

$$(\textstyle\sum_i q^i + r) Df(X) = p, \qquad (11)$$

which must hold in addition to the technological constraint with equality.

If we now let $\mu^i = 1/\lambda^i$, $\delta = p$ and $\gamma = \sum_i q^i + r$, then Eqs. (10) and (11) restore Eqs. (3) and (4). This confirms the previous result, that Lindahl equilibrium allocations are Pareto efficient.

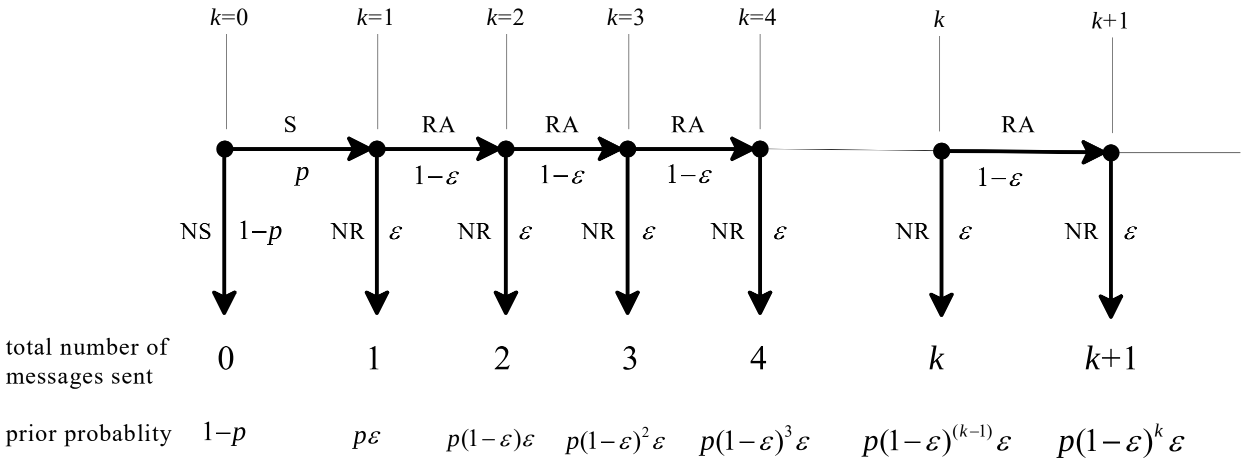
The intuition of this result is not very complicated. As in class, the use of personalized prices whose sum is paid to the producer implies that the external effect of the mixed good is in fact internalized in the compensation to the firm. Unlike in class, however, the market for rights over the mixed good is necessary to allocate it correctly to the consumers.

QUESTION 4 ANSWER KEYS

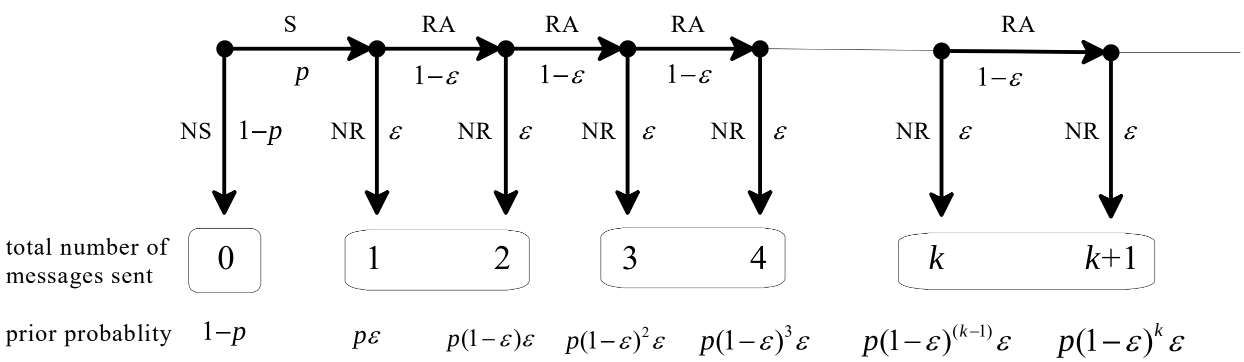
(a) The tree is as follows

Legend: S = initial message sent, NS = initial message non sent, NR = last message not received
RA = last message received and automatically acknowledged

k is the number of edges preceding the node and thus also the total number of messages sent at the terminal node following the node in question

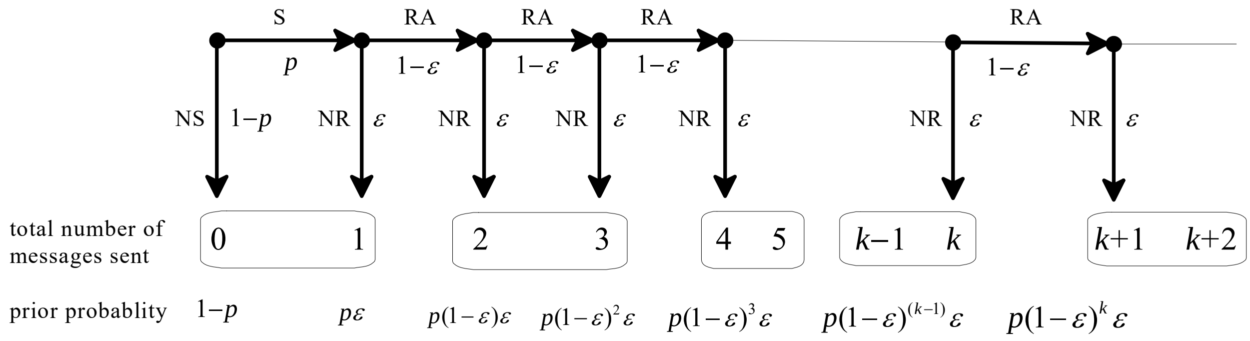


(b) If the total number of messages sent is 0 (because Player 1 ascertained that the enemy position is H) , then Player 1 knows; thus one cell of Player 1's partition is $\{0\}$. Suppose that Player 1 has sent the initial message but did not receive an acknowledgment; then Player 1 will consider it possible that her initial message was lost or that it was received and acknowledged (thus increasing the total number of messages sent to 2) but the acknowledgment was lost; thus another cell of her information partition is $\{1,2\}$, etc. Thus the information partition of Player 1 is as follows:



The information partition of Player 1

(c) If by 6:01am Player 2 has not received any communication then he will consider it possible that Player 1 did not send any messages or that Player 1 sent a message but it got lost; thus one cell of Player 2's information partition is $\{0,1\}$. If Player 2 receives the initial e-mail of Player 1 but does not receive Player 1's acknowledgment of his automatically generated acknowledgment, then Player 2 considers it possible that his own acknowledgment was lost or that it was received but Player 1's acknowledgment of the acknowledgment was lost; thus another call of Player 2's information partition is $\{2,3\}$, etc. Thus the information partition of Player 2 is as follows:



The information partition of Player 2

(d) The set of states is the set of natural numbers $\mathbb{N} = \{0,1,2,3,\ldots\}$. The proposition that the enemy position is *H* is represented by the event $\{0\}$ and the proposition that the enemy position is *L* is represented by the event $\{1,2,3,\ldots\} = \mathbb{N} \setminus \{0\}$. The common knowledge partition consists of only one set, namely the entire set $\mathbb{N}$. Thus no matter how many messages are successfully exchanged it is never common knowledge that the enemy position is *L*.

(e) **Suppose that *k* is odd.** Then Player 1 knows that at least *k* messages were sent at each cell of the form $\{k+j, k+j+1\}$, for every $j \in \{0,2,4,...\}$. Let us focus on the case where $j = 0$, that is, on the cell $\{k, k+1\}$. The prior probability of state k is $p(1-\varepsilon)^{(k-1)}\varepsilon$ and the prior probability of state $k+1$ is $p(1-\varepsilon)^k\varepsilon$. Thus the posterior probability of state k is $\frac{p(1-\varepsilon)^{k-1}\varepsilon}{p(1-\varepsilon)^{k-1}\varepsilon + p(1-\varepsilon)^k\varepsilon} = \frac{1}{2-\varepsilon}$ and the posterior probability of state $k+1$ is $\frac{1-\varepsilon}{2-\varepsilon}$. If the state is $k+1$, then cell of the partition of Player 2 is $\{k+1, k+2\}$ so that Player 2 knows that at least *k* messages were sent and, given the agreed-upon strategy, he will attack; if the state is k, then cell of the partition of Player 2 is $\{k-1, k\}$ so that Player 2 does not know that at least *k* messages were sent and, given the agreed-upon strategy, he will not attack.

state	k	$k+1$
probability	$\frac{1}{2-\varepsilon}$	$\frac{1-\varepsilon}{2-\varepsilon}$
Player 2's decision	N	A

Hence if Player 1 attacks her expected payoff is $0\frac{1}{2-\varepsilon} + (1+c)\frac{1-\varepsilon}{2-\varepsilon} = 3\frac{1-\varepsilon}{2-\varepsilon}$ and if she does not attack her payoff is $c = 2$. Thus she will be willing to attack if and only if $3\frac{1-\varepsilon}{2-\varepsilon} \geq 2$ which is equivalent to $-\varepsilon \geq 1$ which cannot be true. Hence there is at least one situation (namely state *k*) where Player 1 knows that at least *k* messages were sent and yet she is not willing to implement strategy $\hat{s}_k$ even if she trusts that Player 2 will follow strategy $\hat{s}_k$.

Suppose that *k* > 0 is even and focus on state *k*. Player 2's information set is $\{k, k+1\}$ and thus he knows that at least *k* messages were sent. The conditional probabilities are the same as above. If the state is $k + 1$ then Player 1's information set is $\{k+1, k+2\}$ and, given the agreed-upon strategy, she will attack; if the state is k, then cell of the partition of Player 1 is $\{k-1, k\}$ so that Player 1 does not know that at least *k* messages were sent and, given the agreed-upon strategy, she will not attack.

state	k	$k+1$
probability	$\frac{1}{2-\varepsilon}$	$\frac{1-\varepsilon}{2-\varepsilon}$
Player 1's decision	N	A

The calculations are now the same as above: if Player 2 attacks his expected payoff is $0\frac{1}{2-\varepsilon} + (1+c)\frac{1-\varepsilon}{2-\varepsilon} = 3\frac{1-\varepsilon}{2-\varepsilon}$ and if he does not attack her payoff is $c = 2$; thus he will be willing to attack if and only if $3\frac{1-\varepsilon}{2-\varepsilon} \geq 2$ which is equivalent to $-\varepsilon \geq 1$ which cannot be true. Hence there is at least one situation (namely state *k*) where Player 2 knows that at least *k* messages were sent and yet he is not willing to implement strategy $\hat{s}_k$ even if she trusts that Player 1 will follow strategy $\hat{s}_k$.

Thus, no matter whether *k* is odd or even, one of the two players would deviate from strategy $\hat{s}_k$ (in at least one situation where he/she knows that at least *k* messages were sent) even if the player trusts that the other player will follow strategy $\hat{s}_k$.

(f) From the above calculations it is clear that the necessary and sufficient condition is $(1+c)\frac{1-\varepsilon}{2-\varepsilon} \geq c$, that is, $c \leq 1-\varepsilon$.

(g) First of all, note that when $c = 0.3$ and $\varepsilon = 0.2$ the inequality of part (f) is indeed satisfied. Any $k \geq 1$ will work (that is, both players will rationally follow strategy $\hat{s}_k$). Given *k*, the probability that at least one of them will **not** attack is $P(N;k) = 1 - p + p\sum_{j=0}^{k-1}(1-\varepsilon)^j \varepsilon$. This is minimized when $k = 1$. Player 1 will attack after sending the first e-mail to Player 2, but Player 2 will only attack if he gets Player 1's e-mail. Thus the probability that one of them will **not** attack is the probability of *H* (which is $1-p$) plus the probability of *L* (which is *p*) multiplied by the probability that the first e-mail will be lost (which is ε): $(1-p)+p\varepsilon+=1-0.8p$, so that the probability that they both will attack is $0.8p$. (If they agreed on $k = 2$, both would attack if at least three messages were successfully sent, hence a lower probability and even more so for larger values of *k*).