University of California, Davis
Department of Economics
Microeconomics

Date: June 21, 2021
Time: 3 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

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Please answer **two** of the following three questions.
If you answer more than two questions please indicate which two you want to be graded.

QUESTION 1

Consider the following game. A worker proposes a wage $w \in \{1,2,3\}$ to a potential employer and the employer then accepts or rejects the proposal. If the proposal is accepted, the worker's payoff is w and the employer's payoff is $\theta - w$, where $\theta \geq 1$ is the worker's contribution to the employer's profit (the worker's productivity). If the proposal is rejected, each player's payoff is 0.

(a) Assume first that the value of θ is common knowledge.

(a.1) Draw the extensive-form game.

(a.2) Find all the subgame-perfect equilibria of this game. (Clearly, your answer should be conditional on the value of θ.)

(a.3) For a value of θ of your choice, find a Nash equilibrium which is not subgame perfect.

(b) Continue to assume that $w \in \{1,2,3\}$ and add the assumption that $\theta \in \{2,3\}$. Modify the game so that the worker still knows the value of θ, but the employer only knows that either $\theta = 2$ or $\theta = 3$ and considers them equally likely. Furthermore, all of this is common knowledge.

(b.1) Draw the extensive-form game that results from applying the Harsanyi transformation to this situation of incomplete information.

(b.2) Find two pure-strategy **pooling** weak sequential equilibria (where the worker makes the same wage request at her information sets).

(b.3) Are there any pure-strategy **separating** weak sequential equilibria (where the worker makes different wage requests at her information sets)?

(c) Further modify the game of Part (b) by supposing that (1) $w \in \{2,3\}$ and (2) *before she makes her proposal*, the worker has the opportunity to acquire a signal *s* at a cost to her of $\dfrac{2}{\theta}$ (thus her choice is: either acquire *s* by paying $\frac{2}{\theta}$ or not acquire *s*). Whether the signal is acquired or not has no effect on the payoffs, except - of course - for the the cost to the worker. The employer observes whether or not the worker has acquired the signal.

(c.1) Draw the new extensive-form game.

(c.2) Find a pure-strategy separating weak sequential equilibrium and state explicitly the payoffs of both players at the equilibrium.

QUESTION 2: Risk Apportionment

Considering only lotteries that pay in non-negative units of money, suppose that W, X, Y and Z are independent lotteries with bounded support. Assume that $W >_1 X$ and $Y >_1 Z$.[1] Consider the following compound lotteries:

- $\mathcal{K}$ pays $W + Y$ or $X + Z$ with equal probabilities, $1/2$;
- $\mathcal{L}$ pays $W + Z$ or $X + Y$ with equal probabilities, $1/2$.

In what follows, you will compare these two compound lotteries in terms of stochastic dominance.

(a) Argue that if a Bernoulli index $u : \mathbb{R}_+ \to \mathbb{R} \in \mathbb{C}^2$ is strictly increasing and strictly concave, then the function

$$v(r) = \mathrm{E}[u(X + r)] - \mathrm{E}[u(W + r)]$$

is strictly increasing.

(b) Argue now that, under the same assumptions about u,

$$\mathrm{E}[v(Y)] > \mathrm{E}[v(Z)].$$

(c) Argue that, still under the assumptions on u,

$$\tfrac{1}{2}\left\{\mathrm{E}[u(W + Y)] + \mathrm{E}[u(X + Z)]\right\} < \tfrac{1}{2}\left\{\mathrm{E}[u(W + Z)] + \mathrm{E}[u(X + Y)]\right\}.$$

(d) Interpret the previous result in terms of an expected utility maximizer's preferences and attitudes towards wealth and risk. How does she rank the compound lotteries $\mathcal{K}$ and $\mathcal{L}$? Provide intuition for this result.

(e) How do lotteries $\mathcal{K}$ and $\mathcal{L}$ rank in terms of stochastic dominance?

(f) Using a virtually identical argument, one can prove the following: As before, suppose that W, X, Y and Z are independent lotteries, but assume that $W >_1 X$ and $Y >_2 Z$ now.[2] If $u : \mathbb{R}_+ \to \mathbb{R} \in \mathbb{C}^3$ is strictly increasing, strictly concave, and has strictly positive third derivative everywhere, then

$$\mathrm{E}[u(W + Y)] + \mathrm{E}[u(X + Z)] < \mathrm{E}[u(W + Z)] + \mathrm{E}[u(X + Y)]. \qquad (*)$$

Interpret this result in terms of the preferences of an expected utility maximizer and her attitudes towards wealth, risk and prudence: how does she rank the following compound lotteries:

- $\mathcal{M}$ pays $W + Y$ or $X + Z$ with equal probabilities, $1/2$;
- $\mathcal{N}$ pays $W + Z$ or $X + Y$ with equal probabilities, $1/2$?

[1] Recall the notation used in class: $W >_1 X$ means that random variable W first-order stochastically dominates random variable X.

[2] Recall again the notation from class: $>_2$ denotes second-order stochastic dominance.

QUESTION 3: Dynamic General Equilibrium

Consider an exchange economy with society $\mathcal{I} = \{1, \ldots, I\}$. There are $L + K$ commodities and trade takes place in two periods:

1. In the morning, L commodities are traded. Individual i is endowed with ω_ℓ^i units of commodity $\ell = 1, \ldots, L$, and her consumption is x_ℓ^i. The price per unit of commodity ℓ is p_ℓ.
2. In the afternoon, the other K commodities are traded. The endowment and consumption of commodity $k = 1, \ldots, K$ by individual i are ψ_k^i and y_k^i, respectively. The price per unit of commodity k is q_k.
3. In the evening, each agent consumes. If agent i has purchased the bundle $x = (x_1, \ldots, x_L)$ in the morning and the bundle $y = (y_1, \ldots, y_K)$ in the afternoon, her utility in the evening is $u^i(x) + v^i(y)$.

In the morning, besides trading the corresponding commodities, individual i chooses an amount m^i of nominal savings. If positive, m^i becomes nominal wealth in the afternoon; if negative, it is nominal debt that the agent must honor.

In the afternoon, given the prices $q = (q_1, \ldots, q_K)$ and her savings m^i, agent i solves the problem

$$\max_{y \in \mathbb{R}_+^K} \left\{ v^i(y) \,:\, q \cdot y \leq q \cdot \psi^i + m^i \right\}, \tag{1}$$

where $\psi^i = (\psi_1^i, \ldots, \psi_K^i)$. In the morning, given $p = (p_1, \ldots, p_L)$ and anticipating q, she solves

$$\max_{(x,m) \in \mathbb{R}_+^L \times \mathbb{R}} \left\{ u^i(x) + V^i(m, q) \,:\, p \cdot x + m \leq p \cdot \omega^i \right\}, \tag{2}$$

where $\omega^i = (\omega_1^i, \ldots, \omega_L^i)$ and

$$V^i(m, q) = \max_{y \in \mathbb{R}_+^K} \left\{ v^i(y) \,:\, q \cdot y \leq q \cdot \psi^i + m \right\}.$$

The minister of finance of this economy is worried that the agents may be acting silly. She would prefer it if, instead of solving the two problems (2) and (1), agent i solves the intertemporal problem

$$\max_{(x,y,m) \in \mathbb{R}_+^L \times \mathbb{R}_+^K \times \mathbb{R}} \left\{ u^i(x) + v^i(y) \,:\, p \cdot x + m \leq p \cdot \omega^i \text{ and } q \cdot y \leq q \cdot \psi^i + m \right\}. \tag{3}$$

Not having taken the second-year GE course in his Ph.D., the dean of the most prominent economics department in the economy is worried that in the model he learned in the first-year course, people were assumed to solve the static problem

$$\max_{(x,y) \in \mathbb{R}_+^L \times \mathbb{R}_+^K} \left\{ u^i(x) + v^i(y) \,:\, p \cdot x + q \cdot y \leq p \cdot \omega^i + q \cdot \psi^i \right\}, \tag{4}$$

as if all trade took place at the same time.

There are three definitions of equilibrium for this economy.

1. The definition that the dean understands is:[1] a tuple $(p, q, \mathbf{x}, \mathbf{y})$, where $\mathbf{x} = (x^1, \ldots, x^I)$ and $\mathbf{y} = (y^1, \ldots, y^I)$, is a *static equilibrium* if
   - for each individual, the pair (x^i, y^i) solves the static problem (4);
   - $\sum_i x^i = \sum_i \omega^i$ and $\sum_i y^i = \sum_i \psi^i$.
2. The minister would wish that the following was the definition of equilibrium: a tuple $(p, q, \mathbf{x}, \mathbf{y}, \mathbf{m})$, where $\mathbf{m} = (m^1, \ldots, m^I)$, is an *intertemporal equilibrium* if
   - for each individual the triple (x^i, y^i, m^i) solves the intertemporal problem (3);
   - $\sum_i x^i = \sum_i \omega^i$, $\sum_i y^i = \sum_i \psi^i$, and $\sum_i m^i = 0$.
3. The actual definition of equilibrium is: a tuple $(p, q, \mathbf{x}, \mathbf{y}, \mathbf{m})$ is a *dynamic equilibrium* if
   - for each individual the pair (x^i, m^i) solves the morning problem (2), and the bundle y^i solves the afternoon problem (1);
   - $\sum_i x^i = \sum_i \omega^i$, $\sum_i y^i = \sum_i \psi^i$, and $\sum_i m^i = 0$.

The definition of efficiency, on the other hand, does not change: allocation $(\mathbf{x}, \mathbf{y})$ is efficient if there does not exist another allocation $(\tilde{\mathbf{x}}, \tilde{\mathbf{y}})$ such that $u^i(\tilde{x}^i) + v^i(\tilde{y}^i) \geq u^i(x^i) + v^i(y^i)$ for all i, with strict inequality for some.

The point of this question is to argue that the three definitions of equilibrium are not that different.

(a) Argue that if pair (x^i, m^i) solves problem (2) and bundle y^i solves problem (1), then triple (x^i, y^i, m^i) is feasible in problem (3).

(b) Argue that if $(p, q, \mathbf{x}, \mathbf{y}, \mathbf{m})$ is a dynamic equilibrium, then it is also an intertemporal equilibrium.

(c) Argue that if $(p, q, \mathbf{x}, \mathbf{y}, \mathbf{m})$ is an intertemporal equilibrium, then $(p, q, \mathbf{x}, \mathbf{y})$ is a static equilibrium.

(d) State minimal conditions under which if tuple $(p, q, \mathbf{x}, \mathbf{y}, \mathbf{m})$ is a dynamic equilibrium, then allocation $(\mathbf{x}, \mathbf{y})$ is efficient.

(e) Suppose that function $v^i(y)$ is locally non-satiated for all i. Argue that if $(p, q, \mathbf{x}, \mathbf{y}, \mathbf{m})$ is a dynamic equilibrium, then there do not exist a coalition $\mathcal{H} \subseteq \mathcal{I}$ and a sub-allocation of afternoon bundles $(\tilde{y}^i)_{i \in \mathcal{H}}$ such that

   (i) $\sum_{i \in \mathcal{H}} m^i \geq 0$,

   (ii) $\sum_{i \in \mathcal{H}} \tilde{y}^i = \sum_{i \in \mathcal{H}} \psi^i$,

   (iii) $v^i(\tilde{y}^i) \geq v^i(y^i)$ for all $i \in \mathcal{H}$, and

   (iv) $v^i(\tilde{y}^i) > v^i(y^i)$ for some $i \in \mathcal{H}$.

   Interpret this result, in particular for the case when $\mathcal{H} = \mathcal{I}$.

[1] This is the definition that we learned in the course.