

**MACROECONOMICS PRELIM, JULY 2017**  
**ANSWER KEY FOR QUESTIONS 1-3**

**Question 1**

a) The Planner's problem is indeed much easier, because one only needs to describe the allocations, and not the prices of all commodities (which are infinite sequences). What allows us to use this technique here, is the fact that in this environment both Welfare Theorems hold. So we know that the competitive allocation and the Planners allocation will coincide. After characterizing the Planners allocation, we can construct the whole competitive equilibrium, like we did in class.

b) Using any technique you like, you can arrive at the following Euler condition (which is necessary for the dynamic maximization):

$$\frac{1}{k_t^a - k_{t+1}} = \frac{a\beta k_{t+1}^{a-1}}{k_{t+1}^a - k_{t+2}}.$$

If you impose the guess I gave you as a hint, and after a little bit of algebra, you will find that

$$k_{t+1} = gk_t^a = a\beta k_t^a.$$

Notice that this term is in  $(0, 1)$ . This result simply means that each period agents should invest a part equal to  $g$  of the output and eat the remaining  $1 - g$ .

Also, since we know that in equilibrium  $k_t = x_t$ , and  $x_0$  is given, we can fully characterize the whole capital stock allocation. In particular, for all  $T > 0$ , we have

$$k_T = (a\beta)^{1+a+\dots+a^{T-1}} (x_0^a)^T.$$

c) Regardless of the initial condition, this economy will always converge to a steady state. To find it, take the limit as  $T \rightarrow \infty$  in the equation above. We get

$$k^* = \lim_{T \rightarrow \infty} k_T = (a\beta)^{\frac{1}{1-a}}.$$

Regarding the rental rate of capital in the long run, we know that  $r_t = F_K(k_t, 1)$ . Therefore, as  $t \rightarrow \infty$ , we have  $r^* = 1/\beta$ . Similarly,  $w_t = F_N(k_t, 1)$ . Therefore, as  $t \rightarrow \infty$ , we have  $w^* = (1 - a)(a\beta)^{a/(1-a)}$ .

## Question 2

a) This is of course wrong. We are not in a Walrasian world where wages depend only on (marginal) productivity. We are in a world with (search and) bargaining, and with bargaining the wage will depend on the workers' outside options, which are not equal.

b) As I point out in the question, the probability with which a firm will meet a type  $i$  depends only on the relative measure of this types in the aggregate pool of unemployed. But also, the destruction rate of jobs does not depend on the type of worker employed. These two taken together imply that  $u_i/u$  is just equal to  $\pi_i$ . The following value functions follow naturally:

$$rV = -pc + q(\theta) \sum_{i=1}^N \pi_i (J_i - V), \quad (1)$$

$$rJ_i = p - w_i - \lambda J_i. \quad (2)$$

c) Similarly, for the workers we have

$$rU_i = z_i + \theta q(\theta) (W_i - U_i), \quad (3)$$

$$rW_i = w_i + \lambda (U_i - W_i). \quad (4)$$

d) Since in equilibrium  $V = 0$ , (1) implies that

$$\sum_{i=1}^N \pi_i J_i = \frac{pc}{q(\theta)}. \quad (5)$$

Next, solve (2) with respect to  $J_i$  and plug what you found into (5). After some algebra you should arrive at the following equation, which is the model's **JC curve**:

$$\sum_{i=1}^N \pi_i w_i = p - \frac{pc(r + \lambda)}{q(\theta)}.$$

e) Solving the bargaining problem for a typical match between a firm and a worker of type  $i$  will yield (as usual)

$$w_i = \beta p + (1 - \beta)rU_i = \beta p + (1 - \beta)z_i + (1 - \beta)\theta q(\theta)(W_i - U_i),$$

where the second equality follows from (3). Replacing  $(1 - \beta)(W_i - U_i)$  with  $\beta J_i$  (from the standard solution to the bargaining problem), will yield

$$w_i = \beta p + (1 - \beta)z_i + \theta q(\theta)\beta J_i. \quad (6)$$

Now the trick here is to multiply (6) by  $\pi_i$  and sum over all  $i$ 's to obtain

$$\sum_{i=1}^N \pi_i w_i = \beta p + (1 - \beta) \sum_{i=1}^N \pi_i z_i + \theta q(\theta) \beta \sum_{i=1}^N \pi_i J_i,$$

or, using the definition of the term  $\bar{z}$  and equation (5), we finally obtain

$$\sum_{i=1}^N \pi_i w_i = \beta p + (1 - \beta) \bar{z} + \theta \beta p c,$$

which is our model's **wage curve**.

f) Just replace the LHS of the WC from the JC curve to obtain one equation in one unknown:

$$p - \frac{pc(r + \lambda)}{q(\theta)} = \beta p + (1 - \beta) \bar{z} + \theta \beta p c.$$

Notice that after a little algebra we can write this as:

$$(1 - \beta)(p - \bar{z}) = pc \frac{r + \lambda + \beta \theta q(\theta)}{q(\theta)}.$$

This is identical to the equilibrium condition we saw in class, after one replaces  $z$  with  $\bar{z}$ .

g) To find the equilibrium value  $w_i$ , go back to (6) and this time replace  $J_i$  from (2). The resulting equation will involve only  $w_i$ , parameters, and the term  $\theta$  which is uniquely defined in the previous part. Now, just solve with respect to  $w_i$  to obtain:

$$w_i = p - \frac{(1 - \beta)(p - z_i)(r + \lambda)}{r + \lambda + \theta q(\theta)}.$$

h) Of course, the equilibrium wage  $w_i$  is increasing in  $z_i$  as workers who have a better outside option will be compensated for that through the bargaining process. In the formula above, it looks as if  $w_i$  is linear in  $z_i$ , but don't forget that  $z_i$  also implicitly affects the equilibrium value of  $\theta$  (but you did not have to worry too much about this second order effect- a simple mention is enough). In any case, as the distribution of workers' types becomes more dispersed, so does the distribution of equilibrium wages, due to the obvious positive relationship between the two.

### Question 3

a) The buyer's value function is given by

$$W(m, d) = \max_{X, H, \hat{m}} \left\{ U(X) - H + \beta V(\hat{m}) \right\},$$

s.t.  $X + \phi\hat{m} = \phi m + H - d + T,$

where all variables are explained in the question, and  $T$  is the lump-sum monetary transfer to the buyer. Using the standard procedure described in class, we will find that

$$W(m, d) = \Lambda + \phi m - d,$$

where  $\Lambda$  just summarizes a number of terms that are not related to the state variables.

b) As I hinted in the question, here I wanted you to use your intuition: In a type-0 meeting, the buyer is not constrained. So, of course, she will want to trade the first-best,  $q^*$ . Now, the question is what will she give in return? The answer is, a promise for an amount of numeraire good that will make the seller just happy to produce the first-best (recall that the buyer makes a TIOLI offer). Hence, the payment must be such that the seller's surplus,  $d - q$ , is zero. Summing up, we will have  $q_0 = q^*$  and  $d = q^*$ .<sup>1</sup>

In a type-1 meeting, again the buyer would love to go to the first-best, but now she may be constrained by her cash holdings. Thus, the bargaining solution is as follows:  $q_1 = \min\{\phi m, q^*\}$  and  $x = \min\{m, q^*/\phi\}$ . In words,  $q^*/\phi$  is the amount of money that allows the buyer to buy the first best  $q^*$ . Then, either  $m \geq q^*/\phi$  and the buyer gets  $q_1 = q^*$ , or  $m < q^*/\phi$ , and the buyer gives up all her money just to get  $q_1 = \phi m < q^*$ .

c) As I explained, there is no need to spend time on deriving the objective because it is very intuitive (assuming one has understood the environment well). The objective function here is:<sup>2</sup>

$$J(\hat{m}) = (-\phi + \beta\hat{\phi})\hat{m} + \beta(1 - \sigma)[u(q_1(\hat{m})) - \hat{\phi}x(\hat{m})],$$

where  $q_1(\hat{m})$ ,  $x(\hat{m})$  are the terms of trade in a type-1 meeting, explained in the bargaining solution above. But as we know, in these types of models, if the cost of carrying money is positive (an assumption maintained here), the buyer will never bring more than the amount that would buy her  $q^*$ . Another way of saying this is that "we will be in the binding branch of the bargaining solution" (this is the language we often used in class), i.e., where  $q_1 = \phi m$  and  $x(m) = m$ . So we can re-write this objective as

$$J(\hat{m}) = (-\phi + \beta\hat{\phi})\hat{m} + \beta(1 - \sigma)[u(\hat{\phi}\hat{m}) - \hat{\phi}\hat{m}].$$

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<sup>1</sup> This is a special (and easier) case of what we saw in class, except now the compensation function  $z(g)$  is just  $g$  because the buyer has all the bargaining power. (Thus, she is suppressing the seller to work for a payment equal to her cost.)

<sup>2</sup> Clearly, there is no term that starts with  $\sigma$ , because with that probability the buyer meets a seller who accepts credit and her money is not useful in that meeting.

d) The next step is to obtain the FOC and evaluate it in equilibrium. The FOC yields:

$$\phi = \beta\hat{\phi} + \beta(1 - \sigma)[\hat{\phi}u'(\hat{\phi}\hat{m}) - \hat{\phi}].$$

In the steady state equilibrium, we have  $z = \phi M = \hat{\phi}\hat{M}$ , and, moreover, here due to the TIOLI offer assumption, we will also have  $z = q_1$  (because  $z(q) = q$ ).

Summing up, in the steady state equilibrium, we have  $q_0 = q^*$ , and  $q_1$  is given implicitly by<sup>3</sup>

$$i = (1 - \sigma)[u'(q_1) - 1].$$

e) This part is very easy, especially since I am giving you a lot of information about this functional form. Since  $q^* = \gamma$ , we have immediately that  $q_0 = \gamma$ . And since  $u'(q) - 1 = \gamma - q$ , we have

$$q_1 = \gamma - \frac{i}{1 - \sigma}.$$

f) Again very easy if you have done the work so far. As I tell you in the hint, we want to make sure that  $\sigma$  is not that large that it would discourage agents from carrying money altogether. As we saw  $q_1 = z$  but also  $q_1 = \gamma - i/(1 - \sigma)$ . Thus, the upper bound for which a monetary equilibrium will be sustained is given by

$$\bar{\sigma} \equiv 1 - \frac{i}{\gamma}.$$

g) As I explained in the question, the welfare function here is

$$\mathcal{W} = \sigma[u(q_0) - q_0] + (1 - \sigma)[u(q_1) - q_1],$$

where  $q_0, q_1$  were determined in the previous part. Notice that  $\sigma$  does not change the level of  $q_0$  (because it is equal to  $q^*$  always) but it effects how often agents get it. On the other hand,  $\sigma$  affects both the likelihood of getting  $q_1$  and its value (through the money demand channel). The first derivative of  $\mathcal{W}$  with respect to  $\sigma$  is

$$\frac{\partial \mathcal{W}}{\partial \sigma} = \frac{\gamma^2}{2} - q_1 \left( \gamma - \frac{q_1}{2} \right) - (\gamma - q_1) \frac{i}{(1 - \sigma)}. \quad (7)$$

Showing that this expression is always negative may take a little time, but showing that it is negative at  $\sigma = \bar{\sigma}$  (as I suggested in the hint) is very easy. First, if  $\sigma = \bar{\sigma}$  we know that  $q_1 = 0$  (that was the very definition of  $\bar{\sigma}$ ). So evaluating (7) at  $\sigma = \bar{\sigma}$  will give:

$$\frac{\partial \mathcal{W}}{\partial \sigma} = -\frac{\gamma^2}{2} < 0.$$

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<sup>3</sup> Originally, you should find an expression that has exactly the same RHS, and on the LHS you should have  $\frac{1+\mu-\beta}{\beta}$ . But through the Fisher equation that last expression is simply equal to  $i$ .

If you care about the general result (i.e., for all  $\sigma \in [0, \bar{\sigma}]$ ), a little more algebra in (7) will eventually lead to

$$\frac{\partial \mathcal{W}}{\partial \sigma} = (\gamma - q_1) \left[ \frac{1}{2}(\gamma - q_1) - \frac{i}{1 - \sigma} \right],$$

which after using the definition of  $q_1$  can be re-written as

$$\frac{\partial \mathcal{W}}{\partial \sigma} = -\frac{1}{2} \left( \frac{i}{1 - \sigma} \right)^2,$$

which is undoubtedly negative as long as  $i > 0$  (which is true).

What is going on here and this strange result arises? (Strange, because you would think that more credit is good for the economy.) It should be quite clear by now: A higher  $\sigma$  is good because it implies that *ex post* there will be many meetings where the first best will be traded. But *ex ante* it discourages agents from bringing money. It turns out that with this functional form, the second force is so powerful that welfare end up being decreasing in  $\sigma$  for the whole domain!

# Macroeconomics Prelim AK, Spring 2017

## Question 4 (20 points)

Consider the planner's problem for a real business cycle model with inelastic labor supply (essentially the stochastic growth model) and no trend growth. Preferences are given by:

$$\ln C_t \quad (1)$$

Output is produced using capital  $K$

$$Y_t = A_t K_t^\alpha \quad (2)$$

where  $K_t$  is the capital stock at the start of period  $t$ , and  $A_t$  is a TFP shock and is governed by a discrete state Markov chain. There are adjustment costs to capital, which evolves according to the following production function:

$$K_{t+1} = K_t^\delta I_t^{1-\delta} \quad (3)$$

When  $\delta = 0$ , this becomes the simple model we saw in class with full depreciation (i.e. where  $K_{t+1} = I_t$ ). The resource constraint is

$$Y_t = C_t + I_t$$

a) Write down the recursive formulation of planner's problem. Use two constraints: the typical resource constraint and the capital production function. Denote the Lagrange multiplier on the resource constraint as  $\lambda_t$  and the one on the capital production constraint as  $\lambda_t q_t$ .

**Answer:** The recursive formulation is:

$$V(K_t, A_t) = \max_{C_t, I_t, K_{t+1}} (\ln C_t + \beta E_t V(K_{t+1}, A_{t+1}))$$

subject to

$$A_t K_t^\alpha = C_t + I_t$$

$$K_t^\delta I_t^{1-\delta} = K_{t+1}$$

so we can formulate the planner's problem as follows:

$$V(K_t, A_t) = \max_{C_t, I_t, K_{t+1}} (\ln C_t + \beta E_t V(K_{t+1}, A_{t+1}) + \lambda_t (A_t K_t^\alpha - C_t - I_t) + \lambda_t q_t (K_t^\delta I_t^{1-\delta} - K_{t+1}))$$

b) Derive the first order conditions. Provide an intuitive explanation of  $q_t$ .

**Answer:**

$$\begin{aligned}\frac{1}{C_t} &= \lambda_t \\ \lambda_t q_t &= \beta E \partial V(K_{t+1}, A_{t+1}) / \partial K_{t+1} \\ \frac{1}{q_t} &= (1 - \delta) K_t^\delta I_t^{-\delta}\end{aligned}$$

$q_t$  is the price of capital in terms of consumption goods. Note that the RHS of this expression is the marginal product of investment (for the production of capital tomorrow), which must be equal to the price of consumption relative to capital (the inverse of  $q_t$ ). This is Tobin's  $q$  that we discussed in the lectures.

After using the Envelope theorem and tidying up we get:

$$\begin{aligned}q_t &= \frac{1}{(1 - \delta)} \frac{I_t}{K_{t+1}} \\ q_t &= \beta E_t \left[ \frac{C_t}{C_{t+1}} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{\delta}{1 - \delta} \frac{I_{t+1}}{K_{t+1}} \right) \right]\end{aligned}$$

where the first expression has made use of the capital production function again.

c) Using guess and verify, solve the model and find the policy functions for  $K_{t+1}$ ,  $I_t$  and  $C_t$  (Hint: start by guessing that investment and consumption are a constant share of output).

**Answer:**

Let's guess that the solution for investment is as follows:

$$I_t = i_0 Y_t$$

where  $i_0$  is a constant to be found. From the resource constraint, this implies

$$C_t = (1 - i_0) Y_t$$

Combine the FOCs to eliminate  $q_t$  and substitute in the guesses:

$$\frac{I_t}{K_{t+1}} \frac{1}{1 - \delta} = \beta E_t \left[ \frac{c_0 Y_t}{c_0 Y_{t+1}} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{\delta}{1 - \delta} \frac{i_0 Y_{t+1}}{K_{t+1}} \right) \right]$$

$K_{t+1}$  is predetermined and cancels on the LHS and the RHS.  $Y_{t+1}$  also cancels out on the RHS. Everything that's left only depends on period  $t$ , so the  $E$  term doesn't matter. Using the guess again for  $I_t$  on the LHS implies:

$$i_0 Y_t = \beta(1 - \delta) \left( \alpha + \frac{i_0 \delta}{1 - \delta} \right) Y_t$$

We can therefore solve for  $i_0$

$$i_0 = \frac{\beta \alpha (1 - \delta)}{1 - \beta \delta}$$

This makes sense: if  $\delta = 0$ , the capital production function therefore looks like the standard model we've seen before with full depreciation and the policy function for  $K_{t+1}$  would have the familiar form  $K_{t+1} = I_t = \beta\alpha Y_t$ . The full solution is:

$$I_t = \frac{\beta\alpha(1-\delta)}{1-\beta\delta} A_t K_t^\alpha$$

and

$$K_{t+1} = \left( \frac{\beta\alpha(1-\delta)}{1-\beta\delta} \right)^{1-\delta} A_t^{1-\delta} K_t^{(\delta+\alpha(1-\delta))}$$

$$C_t = \left( 1 - \frac{\beta\alpha(1-\delta)}{1-\beta\delta} \right) A_t K_t^\alpha$$

d) Explain why we can interpret  $\delta$  as a parameter that affects the degree of capital adjustment costs in this model. Briefly discuss how, and why, the responses of consumption and investment to TFP shocks vary with  $\delta$ . Why do some macroeconomic modelers prefer to include capital adjustment costs in their models?

#### Answer

Examining the solution above we can see that when  $\delta > 0$ ,  $I_t$  and  $K_{t+1}$  are less responsive to TFP shocks. Consumption also responds more. As  $\delta$  falls towards 0 we can see that the response of investment gets larger. This means that  $\delta$  controls how strongly investment and new capital respond to TFP shocks. This is why the model is described as featuring adjustment costs. When  $\delta$  is larger, each unit of investment produces fewer units of new capital. When  $\delta = 0$  investment will jump a lot of impact. Some modelers have questioned whether investment is really this responsive to TFP shocks. Adjustment costs allow modelers to produce more persistent movements in investment and the capital stock.

### Question 5 (20 points)

Consider the following set of *linearized* equilibrium conditions for the standard New Keynesian model. The only difference from the model we saw in class is that the government can now purchase a basket of goods  $G_t$ , which is completely funded by lump sum taxes. This is like a pure government demand shock: assume that  $G_t$  is not productive and does not provide utility.

In percentage deviations from steady state:  $\hat{\phi}_t$  is real marginal cost,  $\hat{c}_t$  is consumption,  $\hat{w}_t$  is the real wage,  $\hat{n}_t$  is hours worked,  $\hat{y}_t$  is output. In deviations from steady state:  $\hat{i}_t$  is the nominal interest rate,  $\hat{\pi}_t$  is inflation.  $\sigma$  and  $\psi$  come from household preferences  $\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi}$ .

*Households*

$$E_t \hat{c}_{t+1} - \hat{c}_t = \frac{1}{\sigma} (\hat{i}_t - E_t \hat{\pi}_{t+1}) \quad (4)$$

$$\hat{w}_t = \sigma \hat{c}_t + \psi \hat{n}_t \quad (5)$$

*Firms*

$$\hat{y}_t = \hat{n}_t \quad (6)$$

$$\hat{w}_t = \hat{\phi}_t \quad (7)$$

$$\hat{\pi}_t = \beta E_t (\hat{\pi}_{t+1}) + \kappa \tilde{y}_t \quad (8)$$

$\kappa$  is inversely related to the degree of price stickiness and  $\tilde{y}_t$  is the output gap (relative to the model with flexible prices)

$$\tilde{y}_t = \hat{y}_t - \hat{y}_t^n \quad (9)$$

*Resource constraint*

$$\hat{y}_t = \gamma_c \hat{c}_t + \hat{g}_t \quad (10)$$

$\gamma_c$  is the steady state share of consumption in output. To make the math easier,  $\hat{g}_t$  is the deviation of government spending from steady state relative to output.

*Policy:*

$$\hat{i}_t = \phi_\pi \hat{\pi}_t \quad (11)$$

where  $\phi_\pi > 1$ . Government spending follows an AR(1) process

$$\hat{g}_t = \rho \hat{g}_{t-1} + e_t \quad (12)$$

a) Show that the natural level of output can be written as

$$\hat{y}_t^n = \Gamma \hat{g}_t$$

$$\Gamma \equiv \frac{\sigma}{\sigma + \psi \gamma_c}$$

Explain the mechanism through which an increase in government spending leads to an

increase in output in this model with flexible prices (similar to the RBC model).  
 (Hint: start by combining equations 5, 6, 7 and 10 and note that, under flexible prices  $\hat{\phi}_t = 0$ ).

**Answer**

Since this is a flexible price solution, we want to find the solution for the natural rate of output. Note that since  $Y_t = N_t$  (as can be seen from equation (6)), the marginal product of labor is always 1. In linearized form, the flexible price allocation therefore implies  $\hat{\phi}_t = \hat{w}_t = 0$ . Using this, and the equations given in the hint implies:

$$\hat{y}_t = -\gamma_c \frac{\psi}{\sigma} \hat{n} + \hat{g}_t$$

and, since this is a flexible price solution, the response of output is the response of the natural rate of output:  $\hat{y}_t^n$ . Rearranging yields:

$$\hat{y}_t^n = \frac{\sigma}{\sigma + \gamma_c \psi} \hat{g}_t$$

where the coefficient is defined as  $\Gamma$ , the effect of a rise in government spending on GDP in the flexible price model. A rise in government spending implies an increase in the lifetime tax burden. This creates a negative wealth effect on households. Since the labor supply curve depends on the marginal utility of consumption, lower consumption implies more hours worked (equivalently, leisure and consumption are normal goods, so the wealth effect leads to a fall in both). Consumption falls (government spending crowds out private spending). Since households work more hours, output increases. From the solution, note that this effect is always less than 1.

For the rest of this question note that this model can be simplified to the familiar 3 equations:

$$E_t \tilde{y}_{t+1} - \tilde{y}_t = \frac{\gamma_c}{\sigma} (\hat{i}_t - E_t \hat{\pi}_{t+1} - \hat{r}_t^n) \quad (13)$$

$$\hat{\pi}_t = \beta E_t (\hat{\pi}_{t+1}) + \kappa \tilde{y}_t \quad (14)$$

$$\hat{i}_t = \phi_\pi \hat{\pi}_t \quad (15)$$

Plus the process for government spending

$$\hat{g}_t = \rho \hat{g}_{t-1} + e_t \quad (16)$$

and a definition of the natural real rate of interest

$$\hat{r}_t^n = \frac{\sigma}{\gamma_c} (1 - \Gamma) (1 - \rho) \hat{g}_t \quad (17)$$

b) Using the method of undetermined coefficients, find the response of the output gap and inflation to an exogenous increase in  $\hat{g}_t$  when prices are sticky and monetary policy follows the Taylor Rule above. To do this, guess that the solution for each variable is a linear function of the shock  $\hat{g}_t$ :

$$\tilde{y}_t = \Lambda_y \hat{g}_t$$

$$\hat{\pi}_t = \Lambda_\pi \hat{g}_t$$

(Hint: follow the steps used in the problem set. Start by substituting the guesses, the monetary policy rule and the AR(1) process for  $\hat{g}_t$  into the dynamic equations (13) and (14).)

### Answer

Using the guesses in (14) implies

$$\pi_t = \Lambda_\pi \hat{g}_t = (\beta \Lambda_\pi \rho + \kappa \Lambda_y) \hat{g}_t$$

Comparing coefficients, the solution must satisfy:

$$\Lambda_\pi = \frac{\kappa}{1 - \beta \rho} \Lambda_y$$

Next, using the guesses, the AR(1) process and the policy rule in equation (13):

$$\tilde{y}_t = \Lambda_y \hat{g}_t = \Lambda_y \rho \hat{g}_t - \frac{\gamma_c}{\sigma} (\phi_\pi \Lambda_\pi \hat{g}_t - \Lambda_\pi \rho \hat{g}_t) - (\Gamma - 1)(1 - \rho) \hat{g}_t$$

Comparing coefficients and substituting the result we found for  $\Lambda_\pi$ , the solution (after doing a fair bit of simplification) must satisfy:

$$\Lambda_y = \frac{(1 - \rho)(1 - \Gamma)}{1 - \rho + \Psi}$$

where

$$\Psi = \frac{\kappa \gamma_c (\phi - \rho)}{\sigma (1 - \beta \rho)}$$

and, from above,

$$\Lambda_\pi = \frac{\kappa}{1 - \beta \rho} \Lambda_y$$

c) How, and why, does the response of GDP differ from the model with flexible prices? Do positive government spending shocks increase inflation? Provide economic intuition and (if you can) discuss the solution you found in part (b).

### Answer

Let's inspect the solution to part (b). We know that stickier prices implies a flatter Phillips Curve. This means a lower value for  $\kappa$ .  $\Psi$  is smaller and  $\Lambda_y$  is larger. Since this is the response of the output gap — i.e. output relative to the model with flexible prices — sticker prices implies a larger effect of government spending increases on GDP. This is intuitive. Government spending shocks are demand shocks. When prices are flexible all firms adjust prices and output only increases because households want to supply more labor. But when prices are sticky, firms who don't adjust prices are willing to satisfy the extra demand by raising production (instead of prices). Labor demand therefore

increases. In addition to the wealth effect in part (a), there is also a sticky price demand effect. Inflation also increases with the demand shock (some firms adjust prices), although sticker prices will tend to limit the effect on inflation on impact. This can be seen from the equation for  $\Lambda_\pi$  above. The demand effect that raises  $\Lambda_y$ , also pushes up prices. But some firms don't adjust their price, which dampens the effect on inflation, as can be seen from the  $\kappa$  term.

d) In principle, could monetary policy fully stabilize the output gap and inflation after a government spending shock? (Hint: think about how the 3-equation setup above looks like the cases we studied in class)? Would there be any additional benefit from conducting optimal monetary policy under commitment?

#### Answer

This model is similar to the 3-equation model we saw in class without any cost-push, trade-off inducing, shocks. In class, TFP and preference shocks entered the model via the real natural rate of interest. In this model, government spending enters in the same way. Government spending shocks are demand shocks that affect the real natural rate of interest.

In principle, if we could achieve a real interest rate equal to the real natural rate of interest we could stabilize the output gap and inflation. This can be seen by iterating equation (13) forward, as in the lectures. With a completely stabilized output gap, we can see from equation (14) (iterated forward) that inflation will also be stabilized.

In the current model the Taylor Rule is, in general sub-optimal, but because this model features the divine coincidence, we could replicate full stabilization if we take  $\phi_\pi$  to infinity. Other rules might also work, such as rule that includes the natural rate (in addition to some inflation feedback). The targeting rule for optimal policy under discretion would also work.

Welfare losses in this model come from variations in inflation and the output gap. In the absence of cost-push shocks, optimal policy should seek to stabilize both completely (divine coincidence). Optimal policy set under discretion will be completely able to stabilize the output gap and inflation every period and achieve a zero loss. Furthermore, since there are no trade-offs for the policymaker in this model, there will be no additional benefit to optimal commitment policy.

### Question 6 (10 points)

This question is about the standard decentralized real business cycle model. You do not need to derive anything for this question and keep your answers clear and concise.

a) *Briefly* explain the mechanisms through which TFP shocks affect output, consumption, hours worked and investment in the standard RBC model. How well does the model replicate the business cycle facts seen in the data? How would adding habits in consumption affect the dynamics of consumption and investment?

#### Answer

First consider the response of consumption and investment. The TFP shock directly raises output, so households would like to consume. The Euler equation implies households want to smooth consumption. There is a jump in consumption on impact — the shock is unforeseen — but then consumption is smooth. As a result, investment rises a lot initially, and this is converted back into consumption over time.

Higher wealth implies higher consumption and leisure, which would lead to lower labor supply. For a temporary shocks the wealth effects are small. The substitution effect: the wage and MPK are higher, so it's worthwhile to work harder today. When the shock is very persistent the wealth effect becomes more important so the labor response is smaller. The dynamics depend on the wage profile and the real interest rate (the MPK).

The overall response of output is a combination of the direct effect of TFP, the response of capital and labor. Since they all rise, output rises by more than the TFP shock.

The model replicates the procyclical nature of output, consumption, investment and hours worked in the data. Consumption is smoother than output and investment is more volatile than output, as in the data. But, the model features little internal propagation. Hours worked need to respond a lot, which implies a very high labor supply elasticity, something that has been questioned in the data.

Some have also argued that consumption does not respond so strongly to changes in permanent income (excess smoothness). Adding habits to the model creates a hump-shaped response for consumption, which some modelers prefer. Note that investment then responds *more* strongly.

b) Suppose you want to solve the model using computational methods. Explain one approach, the advantages of this method and the steps you would need to take.

#### Answer

We covered two possible methods for solving the model computationally, you can discuss either.

#### *Value function iteration - key points*

First set up the problem recursively. In the computer we'll need to discretize the state space, possibly centering things around the deterministic steady state. We then calibrate

the parameters we need to. Some deep parameters such as the elasticity of inter-temporal substitution can be matched using external information like micro data.

Next we set up the value function as a vector of optimal utility values for each realization of the state variables (e.g. each  $(k, z)$ ).

Make an initial guess of the value function e.g. all zeros.

Plug this into the Bellman equation and find a new value function, e.g. for the stochastic growth model:

$$V_1(k, z) = \max F(k, k', z) + \beta E(V_0(k', z')|z)$$

We then check if  $|V_1 - V_0| < \varepsilon$ . If it's close enough we stop, otherwise we go again.

Once we have the value function we can find the optimal choices given the states today, this calculation gives us the policy functions for the control variables.

Advantages: global non-linear solution, very reliable (relatively general conditions ensure convergence).

#### *Linearization and Eigenvalue decomposition - key points*

Set-up the problem and derive all the first order conditions. Linearize all the equilibrium equations around the steady state. Calibrate the model's parameters such that our variables (or ratios) match long-run observations in the data. Some deep parameters such as the elasticity of inter-temporal substitution can be matched using external information like micro data. Set up the system:

$$\mathbb{E}_t \mathbf{A} \begin{bmatrix} \hat{x}_{t+1} \\ \hat{w}_{t+1} \end{bmatrix} = \mathbf{B} \begin{bmatrix} \hat{x}_t \\ \hat{w}_t \end{bmatrix} \quad (18)$$

where  $x$  is the vector of state variables.  $w$  is the vector of control variables.

The solution is of the form:

$$\hat{w}_t = \mathbf{F} \hat{x}_t \quad (19)$$

$$\hat{x}_{t+1} = \mathbf{P} \hat{x}_t. \quad (20)$$

Check the Blanchard-Kahn conditions for  $B^{-1}A$ :

- Unstable eigenvalues = number of controls (jumps) and stable = number of states  $\rightarrow$  unique solution.
- Too many unstable: explosive solution. Too few unstable: multiple equilibria.

The Blanchard Kahn algorithm yields matrices  $F$  and  $P$  via eigenvalue decomposition of  $B^{-1}A$  and the imposing TVC condition (if the Blanchard Kahn conditions are met).

We could then use these policy functions to plot impulse response functions: the percentage deviation of a variable from steady state following a particular shock realization.

Advantages: fast, can handle large models, much easier when we face a decentralized model which cannot be represented as a planner's problem.