Question 1 Answer Key

a) Define an ADE for this economy.

- *Agents* are $i \in \{1, 2\}$.

- *Endowments* are

$$e_t^1 = \begin{cases} e_t & \text{if } t \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

$$e_t^2 = \begin{cases} e_t & \text{if } t \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

- An *allocation* is $\{\{c_t^i\}_{t=0}^\infty\}_{i=1}^2$ with $c_t^i \geq 0 \, \forall t, i$.

- An *ADE* is a sequence $\{\hat{p}_t\}_{t=0}^\infty$ and allocations $\{\{\hat{c}_t^i\}_{t=0}^\infty\}_{i=1}^2$ such that

  1. Given $\{\hat{p}_t\}_{t=0}^\infty$, for $i = 1, 2$,

  $$\{\hat{c}_t^i\}_{t=0}^\infty \in \underset{\{c_t^i\}_{t=0}^\infty}{\arg\max} \left\{ \sum_{t=0}^\infty \beta^t ln(c_t^i) \; : \; \forall t \geq 0 : c_t^i \geq 0, \text{ and } \sum_{t=0}^\infty \hat{p}_t(c_t^i - e_t^i) \leq 0 \right\}$$

  2. $\sum_{i=1}^2 \hat{c}_t^i = \sum_{i=1}^2 e_t^i \; \forall t \geq 0$.

a) Define a SME for this economy.

- *Agents* are $i \in \{1, 2\}$.

- *Endowments* are

$$e_t^1 = \begin{cases} e_t & \text{if } t \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

$$e_t^2 = \begin{cases} e_t & \text{if } t \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

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- An *SME* is a sequence of Arrow-Debreu Security (ADS) prices $\{\hat{q}_t\}_{t=0}^{\infty}$ and allocations for consumption and ADS $\{\{\hat{c}_t^i, \hat{a}_t^i\}_{t=0}^{\infty}\}_{i=1}^2$ such that

1. Given $\{\hat{q}_t\}_{t=0}^{\infty}$, for $i = 1, 2$,

$$\{\hat{c}_t^i, \hat{a}_t^i\}_{t=0}^{\infty} \in \underset{\{c_t^i, a_t^i\}_{t=0}^{\infty}}{\arg\max} \left\{ \sum_{t=0}^{\infty} \beta^t ln(c_t^i) \right\}$$

$$s.t. \quad 1. \quad \hat{c}_t^i \geq 0 \ \forall i, t$$
$$2. \quad \hat{c}_t^i + \hat{q}_t^i \hat{a}_{t+1}^i = e_t^i + \hat{a}_t^i \ \forall i, t$$
$$3. \quad \hat{a}_{t+1}^i \geq -\bar{A}^i \ \forall i, t$$

2. $\sum_{i=1}^2 \hat{c}_t^i = \sum_{i=1}^2 e_t^i \ \forall t \geq 0$.

3. $\sum_{i=1}^2 \hat{a}_t^i = 0 \ \forall t \geq 0$.

b) You can use any method you like here. You should find that ADE prices satisfy

$$p_t = \frac{\beta^t}{e_t}$$

Then, after normalizing for $p_0 = 2$, we have

$$p_t = \frac{\beta^t}{2 - e^{-t}}, \forall t$$

Prices are decreasing in t for two reasons:

(1) people discount future consumption and endowments

(2) e_t increases with time so the good becomes more plentiful (i.e. supply goes up), which leads the fall in prices.

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Question 2 Answer Key

a) The matching rate of a vacant firm with a worker of type i $(i = 1, 2)$ is

$$a_F^i = \frac{m(u, v)}{v} \frac{u_i}{u} = q(\theta) \frac{u_i}{u}$$

The matching rate of a unemployed worker of type i with a vacant firm is

$$a_w^i = \frac{m(u, v)}{u} = \theta q(\theta)$$

which is not a function of i.

The flows in and out of unemployment of **type 1's** worker is given by

$$\dot{u}_1 = \theta q(\theta) u_1 - \lambda e_1 \qquad (1)$$
$$u_1 + e_1 = \pi \qquad (2)$$

Hence,

$$\theta q(\theta) u_1 = \lambda(\pi - u_1)$$

The beveridge curve of the type 1's worker is

$$u_1 = \frac{\lambda \pi}{\lambda + \theta q(\theta)} \qquad (3)$$

The flows in and out of unemployment of **type 2's** worker is given by

$$\dot{u}_2 = \theta q(\theta) u_2 - a e_2 \qquad (4)$$
$$u_2 + e_2 = 1 - \pi \qquad (5)$$

Hence,

$$\theta q(\theta) u_2 = a(1 - \pi - u_2)$$

The beveridge curve of the type 2's worker is

$$u_2 = \frac{a(1 - \pi)}{a + \theta q(\theta)} \qquad (6)$$

b) The unemployment rate within Type 1 workers' population is

$$\gamma_1 = \frac{\lambda}{\lambda + \theta q(\theta)}$$

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The unemployment rate within Type 2 workers' population is

$$\gamma_2 = \frac{a}{a + \theta q(\theta)}$$

Since $a > \lambda$, it is easy to see that

$$\gamma_1 < \gamma_2$$

.

c) For future reference, it's good to know the fraction of total unemployed workers who are type 1:

$$x(\theta) = \frac{u_1}{u} = \frac{u_1}{u_1 + u_2}$$
$$= \frac{\lambda\pi(a + \theta q(\theta))}{a\lambda + \theta q(\theta)\big[\lambda\pi + a(1 - \pi)\big]} \tag{7}$$

The value functions for firms are:

$$rV = -pc + q(\theta)\Big[xJ_1 + (1 - x)J_2 - V\Big] \tag{8}$$
$$rJ_1 = p - w_1 - \lambda J_1 \tag{9}$$
$$rJ_2 = -w_m - aJ_2 \tag{10}$$

The value functions for workers are:

$$rU_i = z + \theta q(\theta)\Big(W_i - U_i\Big) \tag{11}$$
$$rW_1 = w_1 + \lambda\Big(U_1 - W_1\Big) \tag{12}$$
$$rW_2 = w_m + a\Big(U_2 - W_2\Big) \tag{13}$$

d) Due to free entry, $V = 0$. Therefore,

$$xJ_1 + (1 - x)J_2 = \frac{pc}{q(\theta)}$$

We can derive the JC curve as below:

$$x(\theta)\frac{p - w_1}{r + \lambda} - (1 - x(\theta))\frac{w_m}{r + a} = \frac{pc}{q(\theta)} \tag{14}$$

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e) The bargaining problem only happens between Type 1's worker and firms:

$$\max_{w_1} \quad (W_1 - U_1)^\beta J_1^{1-\beta}$$

As usual, we can derive that $(1 - \beta)(W_1 - U_1) = \beta J_1$. Therefore,

$$\Rightarrow (1 - \beta)\left[\frac{w_1 + \lambda U_1 - rU_1 - \lambda U_1}{r + \lambda}\right] = \beta\frac{p - w_1}{r + \lambda}$$

$$\Rightarrow w_1 = \beta p + (1 - \beta)rU_1$$

$$\Rightarrow w_1 = \beta p + (1 - \beta)\Big(z + \theta q(\theta)(W_1 - U_1)\Big)$$

$$\Rightarrow w_1 = \beta p + (1 - \beta)z + \theta q(\theta)\beta J_1$$

Hence, the WC curve is

$$w_1 = \beta p + (1 - \beta)z + \theta q(\theta)\beta\frac{p - w_1}{r + \lambda} \tag{15}$$

f) What follows is a very detailed discussion of existence and uniqueness (copies from the Ak for the Final in 2016). In the prelim I specifically asked students to not go into detail. Hence, a short discussion of existence and uniqueness was sufficient for obtaining full credit.

Firstly, we want to check whether the following condition satisfy:

$$w_{JC}(\theta = 0) > w_{WC}(\theta = 0)$$

When $\theta = 0$, $x(\theta) = \pi$ and JC becomes

$$\pi\frac{p - w_1}{r + \lambda} - (1 - \pi)\frac{w_m}{r + a} = 0$$

$$\Rightarrow w_1 = p - \underbrace{\frac{1 - \pi}{\pi}w_m\frac{r + \lambda}{r + a}}_{\text{wedge, we want it not to be huge}}$$

When $\theta = 0$, WC becomes

$$w_1 = \beta p + (1 - p)z + \underbrace{\theta q(\theta)}_{=0}\beta\left(\frac{p - w_1}{r + \lambda}\right)$$

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$$\Rightarrow w = \beta p + (1 - \beta)z$$

Here, the existence requires

$$p - \frac{1 - \pi}{\pi} w_m \frac{r + \lambda}{r + a} > \beta p + (1 - p)z$$

$$\Leftrightarrow (p - z)(1 - \beta) > \frac{1 - \pi}{\pi} w_m \frac{r + \lambda}{r + a} \tag{16}$$

To be clear, we want p, π, a to be big enough while w_m, λ to small enough to ensure the existence of equilibrium.

To show the monotonicity of JC and WC curve, we want to solve for WC and JC:

$$JC: \quad w_1 = p - \frac{pc(r + \lambda)}{q(\theta)x(\theta)} - \frac{1 - x(\theta)}{x(\theta)} \frac{(r + \lambda)w_m}{r + a}$$

$$WC: \quad w_1 = \frac{(1 - \beta)z(r + \lambda)}{r + \lambda + \beta\theta q(\theta)} + \beta p \frac{r + \lambda + \theta q(\theta)}{r + \lambda + \beta\theta q(\theta)}$$

Let $\theta q(\theta) \equiv f(\theta)$ and $f'(\theta) > 0$ because

$$\frac{d(\theta q(\theta))}{d\theta} = q(\theta) + \theta q'(\theta) = q(\theta)\left[1 + q'(\theta)\frac{\theta}{q(\theta)}\right] = q(\theta)\left[1 - \eta(\theta)\right] > 0$$

Taking the derivative of the WC with respect to θ, we can show that

$$w_1'(\theta) = \frac{\beta(1 - \beta)(r + \lambda)(p - z)f'(\theta)}{(r + \lambda + \beta f(\theta))^2} > 0$$

Hence, we have shown that WC is monotonically increasing with θ. And before we take derivative of the JC, we firstly want to show the derivative of $x(\theta)$. We can derive that

$$x(\theta) = \frac{\lambda\pi(a + f(\theta))}{a\lambda + f(\theta)\left[\lambda\pi + a(1 - \pi)\right]}$$

$$x'(\theta) = \frac{a\lambda\pi f'(\theta)(1 - \pi)(\lambda - a)}{D^2} < 0$$

because of $a > \lambda$. Now take the derivative of JC:

$$w_1'(\theta) = \frac{pc(r + \lambda)}{[q(\theta)x(\theta)]^2}\big[\underbrace{q'(\theta)x(\theta)}_{\ominus} + \underbrace{q(\theta)x'(\theta)}_{\ominus}\big] + \frac{(r + \lambda)w_m}{(r + a)x^2(\theta)}\underbrace{x'(\theta)}_{\ominus} < 0$$

Hence, JC is monotonically decreasing with θ. Combined with the fact we have shown above, we can derive that a unique equilibrium exists if and only if 16 inequality holds.

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Question 3 Answer Key

a) Consider first System A. Under this taxation system, the typical households problem is:

$$V(k, K) = \max_{c,k'} \left\{ u(c) + \beta V(k', K') \right\}$$

$$s.t. \quad c + k' = w + rk(1 - \tau) + (1 - \delta)k, \tag{17}$$

$$K' = H(K), \tag{18}$$

$$w = w(K) = F_2(K, 1), \tag{19}$$

$$r = r(K) = F_1(K, 1), \tag{20}$$

where k is the individual capital, and K is the aggregate capital. Moreover, (17) is the household's budget constraint, (18) is the aggregate law of motion of capital, (19) and (20) follow directly from market clearing.

Now cosinder System B. Under this taxation system, the typical households problem is:

$$V(k, K) = \max_{c,k'} \left\{ u(c) + \beta V(k', K') \right\}$$

$$s.t. \quad c = w + [r + (1 - \delta)(1 + \tau)]k - (1 + \tau)k', \tag{21}$$

$$K' = H(K), \tag{22}$$

$$w = w(K) = F_2(K, 1), \tag{23}$$

$$r = r(K) = F_1(K, 1), \tag{24}$$

which admits a similar interpretation as the problem under System A (although the budget constraint is very different, as I hinted in the question).

b) Consider first System A. Taking the FOC on the agent's problem yields the following Euler equation:

$$u'(c) = \beta[(1 - \tau)F_1(K', 1) + 1 - \delta] u'(c'),$$

which in steady state implies:

$$1 = \beta[(1 - \tau)F_1(K, 1) + 1 - \delta].$$

This gives us the answer to our question:

$$K_A^*(\tau) \equiv \left\{ K : 1 = \beta[(1 - \tau)F_1(K, 1) + 1 - \delta] \right\}. \tag{25}$$

c) Now consider System B. Taking the FOC on the agent's problem yields the following Euler equation:

$$u'(c)(1 + \tau) = \beta[F_1(K', 1) + (1 - \delta)(1 + \tau)] u'(c').$$

which in steady state implies:

$$1 + \tau = \beta[F_1(K, 1) + (1 - \delta)(1 + \tau)].$$

Therefore,

$$K_B^*(\tau) \equiv \left\{ K : 1 + \tau = \beta[F_1(K, 1) + (1 - \delta)(1 + \tau)] \right\}. \tag{26}$$

d) With $F(K, N) = K^a N^{1-a}$, we have $F_1(K, 1) = aK^{a-1}$. Then, all we need to do is substitute this expression into the definitions (25) and (26) and solve with respect to K. It is easy to show that:

$$K_A^*(\tau) = \left[\frac{a\beta(1 - \tau)}{1 - \beta(1 - \delta)} \right]^{\frac{1}{1-a}}, \tag{27}$$

$$K_B^*(\tau) = \left[\frac{a\beta}{(1 + \tau)[1 - \beta(1 - \delta)]} \right]^{\frac{1}{1-a}}. \tag{28}$$

e) At $\tau = 0$, we have $K_A^* = K_B^* = \left[\frac{a\beta}{1 - \beta(1-\delta)} \right]^{\frac{1}{1-a}}$, which is a familiar expression, since this is the steady state capital stock chosen by the social planner (or by a society without distortionary taxes). Both K_A^* and K_B^* are decreasing in τ, but it is easy to prove that $K_A^* < K_B^*$ for any $\tau > 0$. This is because System A is more severe than B. More precisely, the tax on capital income is a tax on all the household's capital. The tax on investment is just a tax on newly created capital. It is also easy to verify that $K_A^*(1) = 0$, while $K_B^*(1) > 0$.

f) The total tax revenue under system B is

$$T_B = \tau i = \tau \delta K_B^*(\tau) = \tau \delta \left[\frac{a\beta}{(1 + \tau)[1 - \beta(1 - \delta)]} \right]^{\frac{1}{1-a}}.$$

This is not surprising, since in the steady state investment is just enough to replace the depreciated capital. Notice that we can re-write

$$T_B = B \tau (1 + \tau)^{\frac{1}{a-1}},$$

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where $B \equiv \delta \left[\frac{a\beta}{1-\beta(1-\delta)} \right]^{\frac{1}{1-a}}$ is just a positive parameter. This makes differentiation of T_B with respect to τ very easy. We have:

$$\frac{\partial T_B}{\partial \tau} = B(1+\tau)^{\frac{1}{a-1}} \left[1 + \frac{\tau}{(a-1)(1+\tau)} \right].$$

The sign of $\partial T_B / \partial \tau$ depends only on the sign of the term inside the square bracket. More specifically, $\partial T_B / \partial \tau$ will be positive if and only if

$$\tau < \frac{1-a}{a}.$$

Notice that if $a \leq 1/2$, then $(1-a)/a \geq 1$. In this case, the inequality above will be satisfied for all a, τ, and, hence, the Laffer curve will be upward sloping for all a, τ.

But if $a > 1/2$, then $(1-a)/a < 1$. This means that for low τ (i.e., lower than $(1-a)/a$) the Laffer curve will be upward sloping, but for high τ (i.e., higher than $(1-a)/a$) it will be downward sloping.

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Macroeconomics Prelim Answer Key, August 2017

Question 4 (*20 points*)

Consider the following decentralized real business cycle model with no trend growth. There is a continuum of households and the representative household's preferences are given by:

$$\ln\left(c_t - hC_{t-1}\right) - \frac{1}{2}n_t^2 \tag{1}$$

Where c is individual consumption and C is aggregate consumption (which the household takes as given but, in equilibrium, individual and aggregate consumption will be equal). The household is infinitely lived and maximizes utility subject to their budget constraint. The household budget constraint can be written as:

$$c_t + k_{t+1} - (1-\delta)k_t = w_t n_t + r_t^k k_t + \pi_t \tag{2}$$

where w is the real wage, n is hours worked, k is capital, r^k is the rental price of capital and π are profits from firms.

The representative firm produces output using capital k and labor n_t:

$$y_t = A_t k_t^\alpha n_t^{1-\alpha} \tag{3}$$

Total factor productivity A follows a Markov chain over the set $A = \{a_1, \ldots a_N\}$ with transition probabilities given by p_{ij}. The resource constraint is

$$Y_t = C_t + I_t$$

where upper case letters denote aggregate variables.

a) Write down the household's problem in recursive form and write down the firm's maximization problem. Derive the household's first order conditions and the firm's optimal hiring rules.

Answer:

Households:

$$V(A_t, k_t, K_t, C_{t-1}) = \max_{c_t, n_t, k_{t+1}} \ln(c_t - hC_{t-1}) - \frac{1}{2}n_t^2 + \beta E_t V(A_{t+1}, k_{t+1}, K_{t+1}, C_t)$$

$$+ \lambda_t (w_t n_t + r_t^k k_t + \pi_t - c_t - k_{t+1} + (1-\delta)k_t)$$

Firms:

$$\max_{k_t, n_t} \pi_t = \max A_t k_t^\alpha n_t^{1-\alpha} - w_t n_t - r_t^k k_t$$

Household first order conditions:

1

$$\frac{1}{c_t - hC_{t-1}} - \lambda_t = 0$$

$$n_t - \lambda_t w_t = 0$$

$$-\lambda_t + \beta E_t V_{k_{t+1}}(A_{t+1}, k_{t+1}, K_{t+1}, C_t) = 0$$

Envelope condition:

$$V_{k_t} = \lambda_t (r_t^k + 1 - \delta)$$

Which implies:

$$\lambda_t = \beta E_t \left(\lambda_{t+1}(r_{t+1}^k + 1 - \delta)\right)$$

Firm's first condition conditions:

$$w_t = (1 - \alpha) A_t k^\alpha n_t^{-\alpha}$$

$$r_t^k = \alpha A_t k_t^{\alpha-1} n_t^\alpha$$

b) Carefully define a recursive competitive equilibrium. Take care to distinguish between the aggregate and individual state variables and explain any market clearing conditions.

Answer:

A recursive competitive equilibrium is a value function $V(k_t, A_t, K_t, C_{t-1})$, decision rules $k_{t+1} = g_k(k_t, K_t, A_t, C_{t-1})$, $c_t = g_c(k_t, K_t, A_t, C_{t-1})$, $n_t = g_n(k_t, K_t, A_t, C_{t-1})$, a law of motion for the aggregate capital stock $K_{t+1} = G(K_t, A_t, C_{t-1})$ and prices $\{w(K_t, A_t, C_{t-1}), r^k(K_t, A_t, C_{t-1})$ Such that:

1. Given the pricing functions and the law of motion for K, the value function and decision rules solve the household's problem (the allocation satisfies all the first order conditions)

2. The firm's optimality conditions are satisfied.

3. All markets clear:

$$k_t^d = k_t^s = K$$

$$G(K_t, A_t, C_{t-1}) = g_k(k_t, K_t, A_t, C_{t-1})$$

$$n_t^s = n_t^d = N_t$$

$$c_t = C, y_t = Y_t, i_t = I_t$$

and

$$Y_t = C_t + I_t$$

Firms make zero profits. To see this substitute the equilibrium conditions for the wage and the rental price of capital back into the profit function, this produces:

$$\pi_t = 0$$

c) Linearize the consumption Euler equation you found in part (a) around the deterministic steady state.

Let's linearize the marginal utility of consumption and the Euler equation in terms of λ separately. Note that in equilibrium $c = C$.

$$\hat{\lambda}_t = \frac{1}{1-h}\hat{c}_t - \frac{h}{1-h}\hat{c}_{t-1}$$

For simplicity, let's define the real interest rate as

$$r_t = r_t^k + 1 - \delta$$

The linearized Euler in terms of λ is:

$$\hat{\lambda}_t = E_t\left(\hat{\lambda}_{t+1} + \hat{r}_{t+1}\right)$$

Combining these two linearized conditions yields the linearized Euler in terms of consumption.

$$\frac{1+h}{1-h}\hat{c}_t - \frac{h}{1-h}\hat{c}_{t-1} = E_t\frac{1}{1-h}\hat{c}_{t+1} + E_t\hat{r}_{t+1}$$

When $h = 0$ this collapses to the standard consumption Euler without habits.

d) With reference to your answer in part (c) (if you can), discuss how a TFP shock affects consumption in this model. Would the dynamics of consumption be different if consumption preferences were given by $\ln(c_t - hc_{t-1})$? (Hint: note the second term is now household consumption at $t - 1$, not aggregate consumption. You also do not need to derive anything for this question).

Answer:

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In the absence of habits, consumption increases because the TFP shock makes households richer. To the extent that households also supply more labor, this will also increase consumption. Consumption therefore jumps on impact. Some have questioned whether consumption is really this sensitive in response to changes in permanent income. With habits the consumption jump is ameliorated. This is because an extra unit of consumption does not provide the same units of utility — utility is now partly provided by past consumption (note that because everyone is the same aggregate consumption equals individual consumption, assuming households are distributed on the unit interval).

If the utility function included lagged individual level consumption there would be extra dynamics in the consumption response. To see this, re-derive the marginal utility of consumption. The condition now includes an extra term. Households internalize the effect of the consumption choice today on utility tomorrow.

e) What is the labor supply elasticity in this model? Given this calibration, how well will the model match the data? Explain.

Answer:

Linearizing the labor supply conditions yields:

$$\hat{n}_t = \hat{w}_t + \hat{\lambda}_t$$

Holding the marginal utility of wealth constant ($\hat{\lambda}_t = 0$), a 1% rise in wages therefore leads to a 1% rise in hours worked. The labor supply elasticity is 1.

The model we saw in class had a labor supply elasticity of 4. Even this did not produce enough amplification relative to the data. In class we saw the Hansen model which produces an infinite labor supply elasticity and includes an extensive margin — this did much better matching the data. In general, the model requires a high labor supply elasticity to match the data. In this model the labor supply elasticity is on the low side.

Question 5 (*20 points*)

The question asks you to consider the effects of shocks to household preferences in the New Keynesian model.

Households

The household's per period utility function is:

$$\left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} \right) Z_t \qquad (4)$$

where Z is a household preference shock, N is hours worked and C is real consumption. The linearized equilibrium conditions are:

$$E_t \hat{c}_{t+1} - \hat{c}_t = \frac{1}{\sigma}(\hat{i}_t - E_t \hat{\pi}_{t+1}) - \frac{1}{\sigma}(1-\rho)\hat{z}_t \qquad (5)$$

$$\hat{w}_t = \sigma \hat{c}_t + \psi \hat{n}_t \qquad (6)$$

And for firms

$$\hat{y}_t = \hat{n}_t \qquad (7)$$

$$\hat{w}_t = \hat{\phi}_t \qquad (8)$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \kappa \tilde{y}_t \qquad (9)$$

κ is inversely related to the degree of degree of price stickiness and $\tilde{y}_t$ is the output gap (relative to the model with flexible prices)

$$\tilde{y}_t = \hat{y}_t - \hat{y}_t^n \qquad (10)$$

Resource constraint

$$\hat{y}_t = \hat{c}_t \qquad (11)$$

Policy:

$$\hat{i}_t = \phi_\pi \hat{\pi}_t \qquad (12)$$

where $\phi_\pi > 1$. Preference shocks follow an AR(1) process (in percentage deviations from steady state)

$$\hat{z}_t = \rho \hat{z}_{t-1} + e_t \qquad (13)$$

In percentage deviations from steady state: $\hat{z}_t$ is the preference shock, $\hat{\phi}_t$ is real marginal cost, $\hat{c}_t$ is consumption, $\hat{w}_t$ is the real wage, $\hat{n}_t$ is hours worked, $\hat{y}_t$ is output. In deviations from steady state: $\hat{i}_t$ is the nominal interest rate, $\hat{\pi}_t$ is inflation.

a) Using the equilibrium conditions above, show that this model can be represented by the standard 3 equations

$$E_t \tilde{y}_{t+1} - \tilde{y}_t = \frac{1}{\sigma}(\hat{i}_t - E_t \hat{\pi}_{t+1} - \hat{r}_t^n) \qquad (14)$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \kappa \tilde{y}_t \qquad (15)$$

$$\hat{i}_t = \phi_\pi \hat{\pi}_t \qquad (16)$$

Where the natural real rate of interest is:

$$\hat{r}_t^n = (1 - \rho)\hat{z}_t \qquad (17)$$

and where $\hat{z}_t$ follows the process in equation 13. (Hint: start by showing that the natural rate of output is constant in this model).

Answer:

Let's start by examining the natural rate of output. Flexible prices implies $\hat{w}_t = 0$. From the labor supply condition:

$$0 = \sigma \hat{y}_t^n + \psi \hat{y}_t^n$$

where this also makes use of the production function and the resource constraint (linearized). From this expression we can solve for the natural rate of output, but we can see that it will be zero (in deviations from steady state). Preference shocks do not affect the natural rate of output in this model. This means:

$$\tilde{y}_t = \hat{y}_t$$

Next, use this result, and the resource constraint, in the consumption Euler:

$$E_t \tilde{y}_{t+1} - \tilde{y}_t = \frac{1}{\sigma}(\hat{i}_t - E_t \hat{\pi}_{t+1} - (1 - \rho)\hat{z}_t)$$

Finally, let's find the natural rate of interest. With flexible prices the output gap is zero. Output is equal to the natural rate. The LHS is therefore 0. Note that the real rate is defined as

$$\hat{r}_t = \hat{i}_t - E_t \pi_{t+1}$$

and after imposing flexible prices the real rate we're solving for is therefore the natural rate of interest.

$$0 = \frac{1}{\sigma}(\hat{r}_t^n - (1 - \rho)\hat{z}_t)$$

Rearranging this last expression yields equation 17.

The Phillips curve and monetary policy rule were already given in the question, so all that was require for part (a) was to find equations (14) and (17).

b) Using the method of undetermined coefficients, find the response of the output gap and inflation to an exogenous decrease in $\hat{z}_t$ when prices are sticky and monetary policy follows the Taylor Rule above. To do this, guess that the solution for each variable is a linear function of the shock $\hat{z}_t$:

$$\tilde{y}_t = \Lambda_y \hat{z}_t$$

$$\hat{\pi}_t = \Lambda_\pi \hat{z}_t$$

Answer:

Substitute the guesses into equation (15) and make use of equation (13) to substitute for the $E_t \hat{z}_{t+1}$ term. Solving for Λ_y in terms of Λ_π yields:

$$\Lambda_y = \frac{\Lambda_\pi (1 - \beta\rho)}{\kappa}$$

Next, substitute the guesses and equation (17) into equation (14) and make use of equation (13) to substitute for $E_t \hat{z}_{t+1}$ terms. Also make use of the expression for Λ_y that we just derived. Solve for Λ_π:

$$\Lambda_\pi = \kappa \left(\sigma(1 - \beta\rho) + \frac{\kappa(\phi_\pi - \rho)}{1 - \rho} \right)^{-1} > 0$$

Combining this with the solution for Λ_y we found above:

$$\Lambda_y = (1 - \beta\rho) \left(\sigma(1 - \beta\rho) + \frac{\kappa(\phi_\pi - \rho)}{1 - \rho} \right)^{-1} > 0$$

If $\hat{z}_t$ falls both output and inflation fall.

c) Interpret your results. In particular, explain how, and why, preference shocks affect the output gap and inflation. Briefly comment on how a decrease in $\hat{z}_t$ relates to typical recessions we see in the data.

Answer:

A negative shock to consumer preferences is like a shock to demand. It leads to a fall in output and inflation. Why? As consumer demand falls some firms cannot adjust their price (sticky prices come from the Calvo pricing mechanism). Some firms cut price and some firms cut output. As a result both prices and output fall. If prices were not sticky there would be a fast adjustment of prices by all firms and output would remain unchanged. This was shown in part (a) where the natural rate of output was unaffected by the demand shock. This accords with common views of recessions — a collapse in demand that leads to a fall in output and inflation.

The Λ terms also make sense. A higher κ — more flexible prices — raises the effect on inflation and lowers the effect on output. As ϕ_π rises the response of output and inflation gets smaller. This makes sense because ϕ_π is the coefficient in the monetary policy rule, a higher coefficient implies more aggressive policy.

d) Instead of following the Taylor Rule above, policy is now set optimally. Derive the optimal monetary policy rule under discretionary policy. (Hint: As in class, assume that the loss function has quadratic terms for the output gap and inflation, with a relative weight ϑ on the output gap. For simplicity, assume the steady state is efficient). What is

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the optimal path for the output gap and inflation in response to preference shocks under this policy?

Answer:

Optimal policy under discretion means the policy maker resets policy choices each period. We are told to assume an efficient steady state, so $\tilde{y}$ appears in the loss function. The policymaker therefore solves a static problem:

$$\min_{\tilde{y}, \pi} \frac{1}{2} \left(\hat{\pi}_t + \vartheta \tilde{y}_t \right)$$

subject to the Phillips Curve. If we denote the Lagrange multiplier on the constraint as ξ_t, the first order conditions for inflation and the output gap are:

$$\hat{\pi}_t + \xi_t = 0$$

$$\vartheta \tilde{y}_t - \kappa \xi_t = 0$$

Combining these equations leads to a targeting rule that keeps the output gap proportional to inflation:

$$\hat{\pi}_t = -\frac{\vartheta}{\kappa} \tilde{y}_t$$

There are no trade-off shocks in the Phillips Curve so it is possible to close the output gap and the inflation gap at the same time. 0 is a solution to this policy rule, it minimizes the loss function and is consistent with the IS and PC equations. Optimal policy therefore completely offset the recession in part (c).

e) Now suppose the central bank has access to a credible commitment technology. How would your answer to (d) change if the central bank followed the optimal policy rule under commitment? You do not need to derive anything, but explain how commitment policy differs from discretionary policy and whether this would change the optimal path for the output gap and inflation.

Answer:

Commitment policy is able to shape expectations of future inflation. When there is a trade-off, making promises about the future can improve outcomes. Because the Phillips Curve and the IS curve are forward looking, expectations of the future affect outcomes today. In this model, however, there is no trade off. Discretionary policy is able completely stabilize the output gap and inflation. There is no additional benefit to commitment policy in this model (as long as there is an efficient steady state and no cost push shocks).

Question 6 (*10 points*)

This question is about fiscal policy in the real business cycle model. You do not need to derive anything and keep your answers clear and concise.

a) TFP shocks can explain the positive correlation of GDP, consumption and investment in the data. Government consumption shocks cannot explain these facts. Is this statement true or false? Explain. (Note: in this model, government consumption shocks are defined as the purchase of consumption goods by the government. This spending is not productive and does not enter the household's utility function.)

Answer:

The statement is correct. TFP shocks raise output directly and households consume and save more. Investment rises, as does consumption. This comes from the wealth effect, although there will also be substitution effects and further amplification as households increase hours worked. The RBC model does relatively well matching the co-movement of output, consumption and investment. Government spending shocks, however, raise output but lower consumption and investment. They generate *negative* wealth effects as the lifetime tax burden rises. Output increases because people work harder to offset the fall in consumption. There is a negative correlation between output and consumption and investment.

b) The government wants to stimulate private consumption and GDP. They propose a temporary, debt-financed, tax cut. In the RBC model, will a tax cut have the desired effect on the economy? Start by considering lump sum taxes, and then discuss how this result might change for other types of taxes.

Answer:

A lump sum tax cut will have no effect on consumption or GDP in this model. This is due to Ricardian equivalence. Households understand any tax cut must be repaid later and save it. The timing of taxes is therefore irrelevant. The size of the tax cut is also irrelevant.

If taxes are distortionary the tax cut will have an effect. Labor tax cuts will stimulate labor supply, and hence raise GDP and consumption. This is because they raise the take-home hourly wage. For capital taxes, the tax cut will stimulate investment, and therefore GDP (although in principle an aggressive capital tax cut could make it better to save than consume). A consumption tax cut will stimulate consumption, although investment could fall as savings decline. The timing of the tax cut matters — this is because the taxes are distortionary. It matters at what point in time the distortions are relaxed.