University of California, Davis Department of Economics

# PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

## MACROECONOMICS

### June 29, 2020

**Directions:** The exam consists of three questions. Question 1 concerns ECN 200D (Geromichalos), question 2 concerns ECN 200E (Cloyne), and question 3 concerns ECN 200F (Caramp). You only need to answer two out of the three questions. If you prefer (and have time), you can answer all three questions and your grade will be based upon the best two scores. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. You have 3 hours to complete the exam and an additional 20 minutes of reading time.

#### Question 1 (50 points)

Consider the Mortensen-Pissarides model in continuous time. Labor force is normalized to 1 and there are 2 types of workers. A worker of type  $i = \{L, H\}$  enjoys a benefit  $z_i$  while unemployed, and  $z_L < z_H$ . Let  $\pi \in (0, 1)$  denote the measure of type-*H* workers. With random matching the probability with which a firm matches with a certain type of worker depends only on the relative measure of this type in the pool of unemployed.

The rest of the model is standard. Unemployed workers and firms (with one vacancy each) search for each other. Let the measure of unemployed workers be denoted by  $u = u_L + u_H$ , where  $u_i$  is the measure of unemployed workers of type *i*. Also, let the measure of vacant firms be denoted by  $\nu$ , which will be determined endogenously by free entry. A CRS and increasing (in both arguments) matching function,  $m(u, \nu)$ , brings together unemployed workers and vacant firms. It will be useful to define the market tightness as  $\theta \equiv \nu/u$ . Once a match has been formed, the wage is determined through Nash bargaining, with  $\beta \in (0, 1)$  representing the worker's power.

The output of *all* jobs is p > 0 per unit of time, i.e., p does not depend on the worker's type and  $p > z_i$ , for all i. Also, while a firm is searching for a worker it has to pay a search (or recruiting) cost, pc > 0, per unit of time. All jobs are exogenously destroyed at rate  $\lambda > 0$ . All agents discount future at the rate r > 0. Throughout this question focus on steady state equilibria.

a) Write down the value functions for a firm and a worker of each type in all possible states.

b) Exploiting the free entry condition of firms, derive the analogue of the job creation (JC) curve for this economy.

c) Using the same methodology as in the lectures (adjusted to accommodate the differences in the new environment), derive the wage curve (WC) for this economy.

d) Combine the JC curve and the WC curve determined in the previous parts in order to provide an equation that (implicitly) determines the equilibrium  $\theta$ . Compare with the analogous equilibrium condition from the lectures.

For the remainder of this question, we will make an important change in the environment. We will assume that the total production of a filled job is given by px, where x is the match-specific (idiosyncratic) productivity. (But we will **maintain the assumption** that there are two types of workers and they enjoy a different unemployment benefit.) As in the theory of "endogenous job destruction" (Chapter 2 of Pissarides' book), we will assume that existing matches get hit by a productivity shock at rate  $\lambda$ ; when that happens the random variable x attains a new value drawn from the cdf G(x). This distribution is *iid* over time and has support in the interval [0, 1].

New jobs are always created at x = 1. Assume that every time a match obtains a new value x, the firm and the worker decide whether it is worth keeping the match alive and, if yes, they renegotiate over the wage. To answer the following questions make a conjecture about the equilibrium form, similar to the one we made in class in the endogenous job destruction theory.

e) In this new environment what is the unemployment *rate* of workers of type i = L and H? Is it the same?

f) Write down the value functions for a firm in all possible states.

g) Write down the value functions for a worker of each type in all possible states.

h) Without characterizing equilibrium (just using the economic intuition you have developed by studying these types of models) answer the following question:

In the model with exogenous job destruction L-type workers are worse off compared to H-type workers because they get paid a lower wage. How is the well being of L-types compared to that of H-types affected as we switch from the model of exogenous to the model of endogenous job destruction? (**Hint:** Compare the wage the two types receive but also the length of time for which they get to keep this wage, since now job destruction is endogenous.)

### Question 2 (50 points)

This question considers the macroeconomic effects of a time-varying sales tax in the New Keynesian model.

There are a continuum of identical households. The representative household makes consumption (C) and labor supply (N) decisions to maximize lifetime expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln C_t - \chi \frac{N_t^{1+\psi}}{1+\psi} \right) \tag{1}$$

subject to their budget constraint:

$$(1+\tau_t^s)C_t + B_t = w_t N_t + (1+i_{t-1})\frac{P_{t-1}}{P_t}B_{t-1} + D_t + T_t$$
(2)

where  $w_t$  is the real wage,  $N_t$  is hours worked,  $B_t$  are real bond holdings at the end of period t,  $i_{t-1}$  is the nominal interest rate paid between t-1 and t,  $P_t$  is the price of the final consumption good and  $D_t$  are real profits from firms that are distributed lump sum.  $T_t$  are real lump sum transfers from the government. As usual,  $0 < \beta < 1$ and  $\psi > 0$ .  $\tau_t^s$  is a sales tax charged on the purchases of consumption goods.

The production side of the model is the standard New Keynesian environment. Monopolistically competitive intermediate goods firms produce an intermediate good using labor. Intermediate goods firms face a probability that they cannot adjust their price each period (the Calvo pricing mechanism). Intermediate goods are aggregated into a final (homogenous) consumption good by final goods firms. The production side of the economy, when aggregated and linearized, can be described by the following set of linearized equilibrium conditions (the production function, the optimal hiring condition for labor and the dynamic evolution of prices):

$$\hat{y}_t = \hat{n}_t \tag{3}$$

$$\hat{w}_t = \hat{m}c_t \tag{4}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \lambda \hat{mc}_t \tag{5}$$

The resource constraint is: 
$$\hat{y}_t = \hat{c}_t$$
 (6)

Monetary policy follows a simple Taylor Rule:  $\hat{i}_t = \phi_\pi \hat{\pi}_t$  (7)

The (linearized) sales tax rate follows an AR(1) process:  $\hat{\tau}_t^s = \rho \hat{\tau}_{t-1}^s + e_t$  (8)

 $e_t$  is i.i.d. and tax revenues are redistributed lump-sum to households. In percentage deviations from steady state:  $\hat{m}c_t$  is real marginal cost,  $\hat{c}_t$  is consumption,  $\hat{w}_t$  is the

real wage,  $\hat{n}_t$  is hours worked,  $\hat{y}_t$  is output. In deviations from steady state:  $\hat{i}_t$  is the nominal interest rate,  $\hat{\pi}_t$  is inflation and  $\hat{\tau}_t^s$  is the sales tax rate.  $\lambda$  is a function of model parameters, including the degree of price stickiness.<sup>1</sup> Assume that  $\phi_{\pi} > 1, 0 < \rho < 1$ .

a) First consider the representative household's problem. Write down the household's problem in *recursive form* and derive the household's first order conditions.

b) Show that the linearized first order condition for labor supply from part (a) is:

$$\hat{w}_t = \hat{c}_t + \psi \hat{n}_t + \hat{\tau}_t^s \tag{9}$$

and that the flexible price natural rate of output (in linearized form) is given by:

$$\hat{y}_t^n = -\frac{1}{1+\psi}\hat{\tau}_t^s \tag{10}$$

**Hints:** you will need to use equations (3), (4), (6) and (9). To simplify the algebra, define  $\hat{\tau}_t^s = \ln\left(\frac{1+\tau_t^s}{1+\tau_{ss}^s}\right)$  and assume a zero steady state tax rate,  $\tau_{ss}^s = 0.2$ 

c) This model can be reduced to two equations:

$$E_t \tilde{y}_{t+1} - \tilde{y}_t = (\phi_\pi \hat{\pi}_t - E_t \hat{\pi}_{t+1}) + \frac{\psi}{1+\psi} (1-\rho) \hat{\tau}_t^s$$
(11)

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \kappa \tilde{y}_t \tag{12}$$

plus the stochastic process for the sales tax.  $\tilde{y}_t = \hat{y}_t - \hat{y}_t^n$  is the output gap.  $\kappa = (1+\psi)\lambda$ .

Using the method of undetermined coefficients, find the response of the output gap and inflation to an exogenous cut in sales taxes when prices are sticky and monetary policy follows the Taylor Rule above. To do this, guess that the solution for each variable is a linear function of the tax shock  $\hat{\tau}_t^s$ .

d) Discuss how, and why, a sales tax cut affects the natural rate of output, the output gap and inflation in this model.

e) Now suppose the monetary policymaker attempts to target the *natural* real interest rate. Is this policy optimal from a welfare perspective in this model? Explain. You do not need to derive anything, answer using your knowledge of this model.

(Hints: To answer this question, think about what the first best allocation,  $\hat{y}_t^e$ , would look like and whether the policymaker can close the welfare relevant output gap  $\hat{x}_t =$  $\hat{y}_t - \hat{y}_t^e$ . Assume the steady state is efficient).

 $<sup>\</sup>overline{ {}^{1}\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} }$ where  $\theta$  is the probability that a firm cannot adjust its price. <sup>2</sup>If the tax rate is sufficiently small, this implies  $\hat{\tau}_{t}^{s} = \tau_{t}^{s}$  (approximately). Another way to think about this is that the policy choice variable is  $(1+\tau_{t}^{s})$ , so  $\hat{\tau}_{t}^{s} = \frac{(1+\tau_{t}^{s})-(1+\tau_{ss}^{s})}{(1+\tau_{ss}^{s})}$  but with a steady state tax rate of zero  $\tau_{ss}^s = 0$ .

#### Question 3 (50 points)

Consider the following economy. There are two periods, 0 and 1, and a single consumption good. The economy is populated by two types of agents: financiers (F) and entrepreneurs (E). There is a mass one of each type. All agents in the economy have preferences given by  $U = C_1 + \beta C_2$ , where  $C_t$  denotes consumption in period t and  $\beta$  is a parameter between 0 and 1.

Financiers have deep pockets (i.e., they have a large endowment every period). Entrepreneurs have access to a variable scale project: investing I units in period 0 yields RI units of the consumption good in period 1 in case of success and 0 in case of failure. If the entrepreneur puts effort, then the probability of success is  $p_H$ . If entrepreneur i shirks, then the probability of success is  $p_L < p_H$  but she gets a private benefit  $B_iI$ . Assume that  $B_i$  is observable by Es and Fs. Entrepreneurs differ in their private benefit of shirking  $B_i$ , which is distributed according to the cumulative distribution function  $G(B_i)$  in the compact set  $[\underline{B}, \overline{B}]$ , with  $E[B_i] = B$ .

E's have limited liability at both dates  $(C_1 \ge 0, C_2 \ge 0)$  and access to initial funds  $N_i$ , distributed in the population according to cumulative distribution function  $F(N_i)$  in the compact set  $[\underline{N}, \overline{N}]$  with  $E[N_i] = N$ . Assume that the distribution of private benefit  $B_i$  is independent of the distribution of net worth  $N_i$ .

Denote the interest rate of the economy by r. Assume that

$$\frac{p_H R}{1+r} > 1 > \frac{p_L R + B}{1+r}$$

a) State the conditions an optimal contract must satisfy. Let  $\rho_i \equiv R - \frac{B_i}{p_H - p_L}$ . Show that  $\rho_i I$  is the maximum amount an entrepreneur of type *i* can commit to repaying at t = 1 (for this reason we will call  $\rho_i$  as "pledgeability").

b) State the entrepreneurs' problem. Show that Es invest up to the maximum possible scale, i.e.,

$$I = \frac{N_i}{1 - \frac{p_H \rho_i}{1 + r}}.$$

How does the project scale depend on  $B_i$ ? Explain. **Hint:** Since Fs have a large endowment, the Es keep all the surplus from the project.

c) Let  $I \equiv \int_{\underline{B}}^{\overline{B}} \int_{\underline{N}}^{\overline{N}} I(N_i, B_i) dF(N_i) dG(B_i)$  be the aggregate investment in this economy. Argue that if the financiers' endowment is sufficiently large, then  $1 + r = \frac{1}{\beta}$ . Let  $e_0$  denote the Fs' endowment in period 0. What is the minimum value of  $e_0$  such that  $1 + r = \frac{1}{\beta}$  in equilibrium?

d) Start from a situation in which there is no heterogeneity so that  $\underline{B} = \overline{B} = B$ and  $\underline{N} = \overline{N} = N$ . Does an increase in heterogeneity in  $N_i$  (but keeping the average constant) increase, decrease or not change aggregate investment I? How about an increase in heterogeneity in  $B_i$  (keeping the average constant)? **Hint:** If X is a random variable and h(x) is a convex function of x, then E[h(X)] > h(E[X]).

Now suppose that entrepreneurs can hire a monitoring service (at a cost), which reduces the private return of shirking to  $b_i I$  where  $b_i = \phi B_i$  with  $\phi \in (0, 1)$ . We will study whether entrepreneurs have an incentive to hire the monitoring service.

e) Suppose that the cost of monitoring is cI and an entrepreneur *i* hires the service. How does the contract with Fs change? Show that the scale of the project is larger with monitoring if and only if

$$c \le (1-\phi)p_H \frac{R-\rho_i}{1+r}$$

**Hint:** Carefully state the conditions a contract must satisfy.

f) What is the maximum cost such that E chooses monitoring? How does the maximum cost depend on i? Explain the intuition of why E chooses to pay to be monitored.

g) Suppose now that the cost of monitoring a project is c, independent of the scale. Calculate the maximum cost that an entrepreneur i is willing to pay. How does the maximum cost depend on  $N_i$  and  $B_i$ ?

h) Given your answer to g), under what conditions are entrepreneurs more likely to pay for monitoring? Explain the intuition.