

**Ph. D. Preliminary examination in Industrial Organization, September 2005**

**Answers to questions 1 and 2**

**1.**

(a) Let  $c$  be the price charged by the monopolist to the retailers. The Cournot equilibrium in the game between retailers is given by:

$$q_1^* = q_2^* = \frac{a-c}{3b}, \quad Q^* = \frac{2(a-c)}{3b}, \quad P^* = \frac{a+2c}{3}, \quad \pi_1^* = \pi_2^* = \frac{(a-c)^2}{9b} > 0.$$

Thus the monopolist's profit function is given by:

$$\pi_M = \frac{2(c-k)(a-c)}{3b}$$

which is maximized at  $c^* = \frac{a+k}{2}$  with corresponding profits  $\pi_M^* = \frac{(a-k)^2}{6b}$ .

(b) Let  $F$  be the fixed fee.  $F$  is a fixed cost, hence it does not affect the choice of output by the retailers. Thus the Cournot equilibrium in the game between retailers is the same as in part (a). The

monopolist can set  $F = \pi_1^* = \pi_2^* = \frac{(a-c)^2}{9b}$  and extract the entire profit of each retailer. Hence the monopolist's profit function in this case is

$$\pi_M = \frac{2(c-k)(a-c)}{3b} + 2 \frac{(a-c)^2}{9b}$$

which is maximized at  $\hat{c} = \frac{a+3k}{4}$  with corresponding profits  $\hat{\pi}_M = \frac{(a-k)^2}{4b}$ .

(c) The two-part tariff is obviously better.

(d) With Bertrand competition, since the good is homogeneous, we have  $p_1^* = p_2^* = c$ . Hence  $\pi_1^* = \pi_2^* = 0$  and with a two-part tariff it must be  $F = 0$ . Hence in this case there is no difference between linear pricing and a two-part tariff. The monopolist's profit function in this case is

$$\pi_M = (c - k) \frac{(a - c)}{b}$$

which is maximized at  $c^* = \frac{a + k}{2}$  [same as in part (a)] with corresponding profits  $\hat{\pi}_M = \frac{(a - k)^2}{4b}$

[same as in part (b)]. Thus linear pricing with Bertrand competition gives the monopolist the same profits as a two-part tariff under Cournot competition.

## 2.

Given the price  $P$  set by the cartel, each fringe firm chooses that level of output at which marginal cost ( $= q$ ) is equal to price. Thus total output of the fringe is  $(5 - k)p$ . Hence the residual demand function for the cartel is  $Q_R = 5 - 5P - (5 - k)P$ . Now the cartel chooses  $P$  to maximize

$$\pi_C(P, k) = \left( \frac{P[5 - (10 - k)P]}{k} - \frac{\left( \frac{5 - (10 - k)P}{k} \right)^2}{2} \right)$$

The solution is  $P^*(k) = \frac{2}{4 - \left(\frac{k}{5}\right)^2}$ . The profits of a firm in the fringe are:

$$\pi_F(k) = \frac{2}{\left(4 - \left(\frac{k}{5}\right)^2\right)^2} \text{ while the profits of a firm in the cartel are } \pi_C(k) = \frac{1}{\left(4 - \left(\frac{k}{5}\right)^2\right)} . \text{ Thus we}$$

have:

$k$	0	1	2	3	4	5
$\pi_C$	–	0.126	0.130	0.137	0.149	0.167
$\pi_F$	0.125	0.128	0.136	0.151	0.177	–

Hence there is only one stable cartel with  $k = 3$ :  $\pi_C(3) > \pi_F(2)$  and  $\pi_F(3) > \pi_C(4)$ .