

Ph. D. Preliminary examination in Industrial Organization, June 2004

Answers to questions 1 and 2

1.

$$N := 81 \quad c_1 := 0 \quad c_2 := \frac{1}{3}$$

The consumer who is indifferent between buying and not buying is given by the solution, w.r.t. θ , of the following equation: $E = E - p + \theta k$. The solution is $\theta = p/k$. Consumers with θ greater than this value will buy. Thus demand is given by:

$$D(p, k) := N \cdot \left(1 - \frac{p}{k}\right) \quad \text{demand for a composite good of quality } k$$

SECOND STAGE: choice of prices The profit functions are

$$\Pi_1(p_1, p_2, k_1, k_2) := p_1 \cdot D(p_1 + p_2, \min(k_1, k_2)) - c_1 \cdot k_1 \cdot D(p_1 + p_2, \min(k_1, k_2))$$

$$\Pi_2(p_1, p_2, k_1, k_2) := p_2 \cdot D(p_1 + p_2, \min(k_1, k_2)) - c_2 \cdot k_2 \cdot D(p_1 + p_2, \min(k_1, k_2))$$

$$\text{Given } \frac{\partial}{\partial p_1} \Pi_1(p_1, p_2, k_1, k_2) = 0 \quad \frac{\partial}{\partial p_2} \Pi_2(p_1, p_2, k_1, k_2) = 0$$

$$P(k_1, k_2) := \text{Find}(p_1, p_2) \rightarrow \begin{pmatrix} \frac{1}{3} \cdot \min(k_1, k_2) - \frac{1}{9} \cdot k_2 \\ \frac{1}{3} \cdot \min(k_1, k_2) + \frac{2}{9} \cdot k_2 \end{pmatrix}$$

Nash equilibrium prices

$$P(k_1, k_2)_1 + P(k_1, k_2)_2 \text{ simplify } \rightarrow \frac{2}{3} \cdot \min(k_1, k_2) + \frac{1}{9} \cdot k_2$$

Price paid by consumers

$$Q(k_1, k_2) := D(P(k_1, k_2)_1 + P(k_1, k_2)_2, \min(k_1, k_2))$$

$$Q(k_1, k_2) \text{ simplify } \rightarrow 9 \cdot \frac{3 \cdot \min(k_1, k_2) - k_2}{\min(k_1, k_2)}$$

equilibrium quantity

Thus the first-stage profit functions are:

$$F\Pi_1(k_1, k_2) := \Pi_1(P(k_1, k_2)_1, P(k_1, k_2)_2, k_1, k_2)$$

$$F\Pi_2(k_1, k_2) := \Pi_2(P(k_1, k_2)_1, P(k_1, k_2)_2, k_1, k_2)$$

$$F\Pi_1(k_1, k_2) \text{ simplify } \rightarrow \frac{(-3 \cdot \min(k_1, k_2) + k_2)^2}{\min(k_1, k_2)}$$

$$F\Pi_2(k_1, k_2) \text{ simplify } \rightarrow \frac{(-3 \cdot \min(k_1, k_2) + k_2)^2}{\min(k_1, k_2)}$$

$$A := \begin{pmatrix} F\Pi_1(3,3) & F\Pi_1(3,6) & F\Pi_1(3,9) \\ F\Pi_1(6,3) & F\Pi_1(6,6) & F\Pi_1(6,9) \\ F\Pi_1(9,3) & F\Pi_1(9,6) & F\Pi_1(9,9) \end{pmatrix} \quad A = \begin{pmatrix} 12 & 3 & 0 \\ 12 & 24 & 13.5 \\ 12 & 24 & 36 \end{pmatrix}$$

$$B := \begin{pmatrix} F\Pi_2(3,3) & F\Pi_2(3,6) & F\Pi_2(3,9) \\ F\Pi_2(6,3) & F\Pi_2(6,6) & F\Pi_2(6,9) \\ F\Pi_2(9,3) & F\Pi_2(9,6) & F\Pi_2(9,9) \end{pmatrix} \quad B = \begin{pmatrix} 12 & 3 & 0 \\ 12 & 24 & 13.5 \\ 12 & 24 & 36 \end{pmatrix}$$

For a Nash equilibrium we are looking for an element which is a maximum in its column in matrix A and a maximum in its row in matrix B. Since $A = B$, we are looking for an element in A which is both a maximum in its column and a maximum in its row. All and only the diagonal elements satisfy both requirements. Thus there are three subgame-perfect equilibria given by:

- (1) $k_1 = k_2 = 3$, $p_1 = 2/3 = 0.67$, $p_2 = 5/3 = 1.67$
- (2) $k_1 = k_2 = 6$, $p_1 = 4/3 = 1.33$, $p_2 = 10/3 = 3.33$
- (3) $k_1 = k_2 = 9$, $p_1 = 2$, $p_2 = 5$.

Each firm prefers (3) to (2) to (1).

To check the ranking from the point of view of consumers, first notice that the indifferent consumer is the same at all equilibria: it is given by $\frac{p_1 + p_2}{k} = \frac{7}{9}$. Consumers with

$\theta < \frac{7}{9}$ do not buy at any of the equilibria and are therefore indifferent among all three. Each consumer

with $\theta \geq \frac{7}{9}$ will buy at every equilibrium and get a utility of:

$$\text{At equilibrium (1): } U = E + (3\theta - \frac{7}{3})$$

at equilibrium (2): $U = E + 2\left(3\theta - \frac{7}{3}\right)$

at equilibrium (3): $U = E + 3\left(3\theta - \frac{7}{3}\right)$

Thus all consumers who buy prefer (3) to (2) to (1).

A less precise method is to calculate consumer using the demand function:

$$CS(k) := \frac{1}{2} \cdot [k - (P(k, k)_1 + P(k, k)_2)] \cdot Q(k, k)$$

$$CS(k) \text{ simplify } \rightarrow -(4 \cdot k - 3 \cdot \min(k, k)) \cdot \frac{-3 \cdot \min(k, k) + k}{\min(k, k)}$$

$$CS(3) = 6 \quad CS(6) = 12$$

$$CS(9) = 18$$

Thus consumers also prefer (3) to (2) to (1). Hence the same is true of society.

2.

(a) Let A be advertising expenditure and p be the price. Let $a = \frac{A}{c}$. Then, if $p + a \leq E$, the profit function is

$$\begin{aligned} \pi(a, p) &= 2a(p - k) - c \cdot a & \text{if } a \leq \frac{L}{2} \\ &= L(p - k) - c \cdot a & \text{if } a > \frac{L}{2} \end{aligned}$$

Having $p + a > E$ cannot be profit maximizing, because in that case the consumers at points $L+a$ and $L-a$ prefer not to buy. Hence the effective market is a proper subset of $\left[\frac{L}{2} - a, \frac{L}{2} + a\right]$ and the firm can increase its profits by keeping p constant and reducing a (revenue and production costs remain the same, while advertising costs decrease). From the above expression it is clear that if a maximizes π then it must be that

$$a \leq \frac{L}{2}.$$

A second requirement for profit maximization is that the consumers who live at points $L+a$ and $L-a$ pay the reservation price, that is,

$$p + a = E.$$

In fact, if $p + a < E$, then profits can be increased by keeping a constant and increasing p (revenue increases, production and advertising costs do not change).

Using the constraints $a \leq \frac{L}{2}$ and $p = E - a$, the profit function becomes

$$\pi(a) = 2a(E - a - k) - c \cdot a \quad \text{with } a \leq \frac{L}{2}$$

Now, $\frac{\partial \pi}{\partial a} = 2E - 4a - 2k - c$.

- If $\left. \frac{\partial \pi}{\partial a} \right|_{a=\frac{L}{2}} \geq 0$, i.e. if $E \geq L + k + \frac{c}{2}$ then the profit-maximizing solution is $p = E - a$ and $a = \frac{L}{2}$ (or $A = c \frac{L}{2}$).
- If $\left. \frac{\partial \pi}{\partial a} \right|_{a=\frac{L}{2}} < 0$, i.e. if $E < L + k + \frac{c}{2}$ then the profit-maximizing solution is $p = E - a$ and a the

solution to $2E - 4a - 2k - c = 0$, that is, $a = \frac{2E - 2k - c}{4}$.

Thus if $E = 16, L = 10, k = 2$ and $c = 4$, then we are in the first case and the profit-maximizing solution is $a = \frac{L}{2} = 5$ and $p = E - a = 16 - 5 = 11$. Profits are $\pi = 70$.

If $E = 10, L = 17, k = 6, c = 4$, then we are in the second case and the profit-maximizing solution is $a = 1$ and $p = E - a = 9$. Profits are $\pi = 2$.

(b) Let firm 1 be the one located at point 0 and firm 2 the one located at point L . Let p_i be the price of firm i and A_i the advertising expenditure of firm i ($i = 1, 2$). Given p_1 and p_2 , let z be the location of the consumers who (if aware of the two products) is indifferent between the two firms. Then z is the solution to $p_1 + z = p_2 + (L - z)$, that is,

$$z = \frac{p_2 - p_1 + L}{2}.$$

Then in the **second stage** (where p_1 and p_2 – and thus z – are fixed) the demand functions are:

$$D_1 = \begin{cases} A_1 & \text{if } A_1 \leq z \\ z & \text{if } A_1 > z \text{ and } A_2 \geq (L - z) \\ \min(A_1, L - A_2) & \text{if } A_1 > z \text{ and } A_2 < (L - z) \end{cases}$$

$$D_2 = \begin{cases} A_2 & \text{if } A_2 \leq (L - z) \\ L - z & \text{if } A_2 > (L - z) \text{ and } A_1 \geq z \\ \min(A_2, L - A_1) & \text{if } A_2 > (L - z) \text{ and } A_1 < z \end{cases}$$

Thus the second-stage profit functions are:

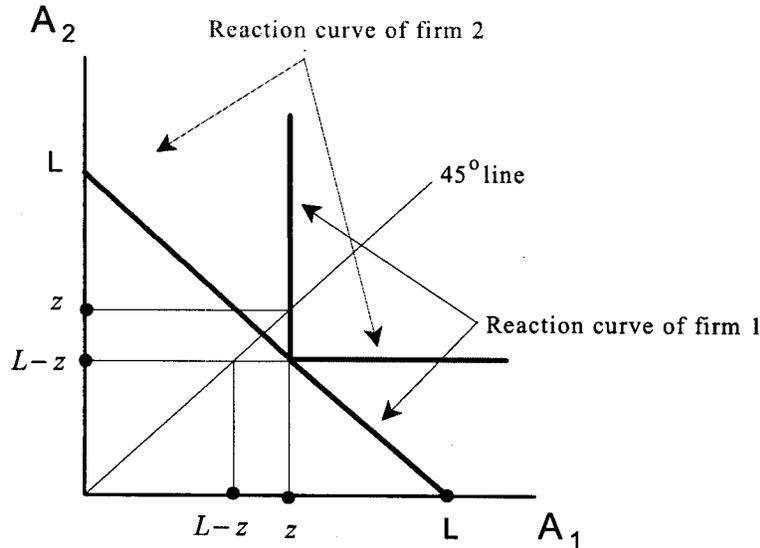
$$\pi_1 = \begin{cases} p_1 A_1 - A_1 & \text{if } A_1 \leq z \\ p_1 z - A_1 & \text{if } A_1 > z \text{ and } A_2 \geq (L - z) \\ p_1 \min(A_1, L - A_2) - A_1 & \text{if } A_1 > z \text{ and } A_2 < (L - z) \end{cases}$$

$$\pi_2 = \begin{cases} p_2 A_2 - A_2 & \text{if } A_2 \leq (L - z) \\ p_2 (L - z) - A_2 & \text{if } A_2 > (L - z) \text{ and } A_1 \geq z \\ p_2 \min(A_2, L - A_1) - A_2 & \text{if } A_2 > (L - z) \text{ and } A_1 < z \end{cases}$$

Thus firm 1 will only choose either $A_1 = 0$ (if $p_1 \leq 1$) or $A_1 = z$ (if $p_1 > 1$ and $A_2 \geq L - z$) or $A_1 = L - A_2$ (if $p_1 > 1$ and $A_2 < L - z$) and similarly for firm 2. Thus the reaction functions are (assuming $p_1 > 1$ and $p_2 > 1$):

$$R_1(A_2) = \begin{cases} L - A_2 & \text{if } A_2 \leq L - z \\ z & \text{if } A_2 \geq L - z \end{cases} \quad \text{and} \quad R_2(A_1) = \begin{cases} L - A_1 & \text{if } A_1 \leq z \\ L - z & \text{if } A_1 \geq z \end{cases}$$

The reaction curves look as follows (the picture below shows the case where $z > L - z$):



Thus the unique equilibrium of the second stage (assuming $p_1 > 1$ and $p_2 > 1$) is $(A_1 = z, A_2 = L - z)$.

Working backwards to the first stage we get the following payoff functions (recall that

$$z = \frac{p_2 - p_1 + L}{2}):$$

$$\pi_1 = \begin{cases} 0 & \text{if } p_1 \leq 1 \\ (p_1 - 1) \frac{p_2 - p_1 + L}{2} & \text{if } p_1 > 1 \end{cases} \quad \text{and}$$

$$\pi_2 = \begin{cases} 0 & \text{if } p_2 \leq 1 \\ (p_2 - 1) \left(L - \frac{p_2 - p_1 + L}{2} \right) & \text{if } p_2 > 1 \end{cases}$$

(assuming that $0 \leq z = \frac{p_2 - p_1 + L}{2} \leq L$). To find the Nash equilibrium solve

$$\frac{\partial \Pi_1}{\partial p_1} = 0, \quad \frac{\partial \Pi_2}{\partial p_2} = 0$$

The solution is: $p_1 = p_2 = L + 1$ with corresponding profits of $\frac{L^2}{2}$.

Answer to question 6

This problem turned out to be more non-standard than intended, because the second term of A_t should have been $x_{t-1}(p_{t-1} - p_t)$. The answers here refer to the question as written, not as intended.

- a) The incentives for the manager are horrible, from the consumer viewpoint. Notice that the 2nd term penalizes the firm for dropping prices on inframarginal units. Since the usual profit function already incorporates this sort of inframarginal loss, this scheme doubly penalizes the firm for dropping prices, compared to a monopolist. So you should expect that you'd end up with a higher than monopoly price.

To formalize this, set up the Bellman d.p. problem as:

$$\max V_t = \pi_t - \pi_{t-1} + x_{t-1}(p_t - p_{t-1}) + \delta(\pi_{t+1} - \pi_t + x_t(p_{t+1} - p_t))$$

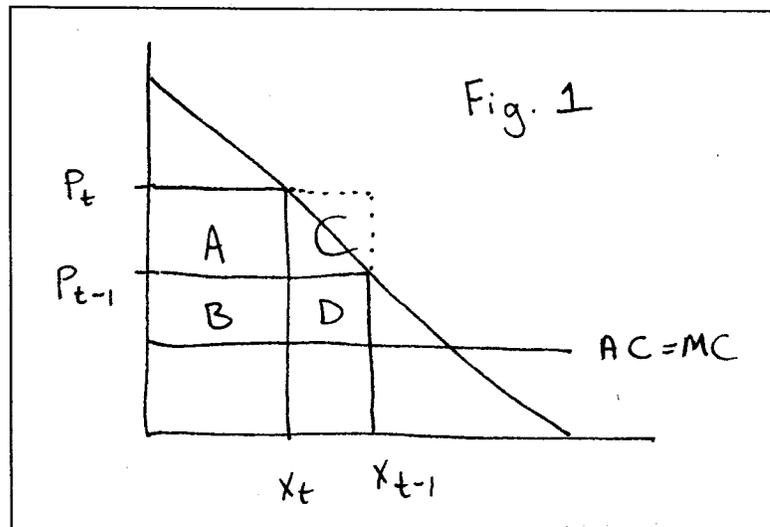
Note you can leave out the Y terms, because they aren't sensitive to the manager's actions. The FOCs lead to

$$L_t |\varepsilon_t| = 1 + (1/(1-\delta))(x_{t-1}/x_t - \delta) \quad (*)$$

where L is the Lerner index and ε is elasticity of demand. In the steady state, $x_{t-1} = x_t$ and the RHS simplifies to 2. So the Ramsey number is twice as high as it would be for an unregulated monopolist. See how the "double penalty" on inframarginal loss nicely translates into a doubling of the Ramsey number? Note this equation does not imply that the firm keeps raising prices forever; you can show that if the firm set an infinite price (i.e. a price leading to zero x sold) last period it will get zero profit if it does that again but positive profit if it *lowers* price.

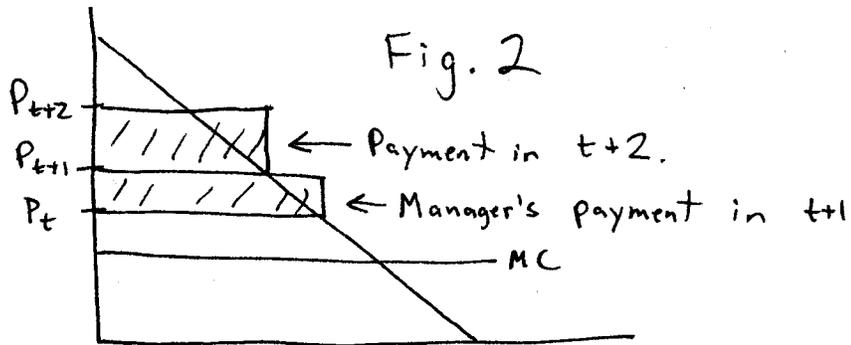
Graphically, look at Fig. 1.

The $\pi_t - \pi_{t-1}$ term gives the manager $(A + B) - (B + D) = A + D$. The incentive term $x_{t-1}(p_t - p_{t-1})$ gives an additional $(A + C)$. So the total increment (ignoring discounting) is $2A + C - D$ when the firm raises price. It is the "-D" term that keeps the firm from wanting to raise price forever. It forces the firm to care at least a bit about the loss of CS.



- b) The discount factor's effect is the same as in the AISS: the more patient the firm is, the smaller the steps the firm will take. The RHS of (*) is increasing in δ (remember $\delta \leq 1$) if quantity is decreasing (price is rising). So the larger is δ (the more patient the firm is) the lower the Lerner index (and price) will be at any given period. You could also argue pictorially.

- c) This looks closest to the AISS (the F-V mechanism). It would be that exactly if the 2nd term were $x_{t-1}(p_{t-1} - p_t)$.
- d) See Fig 2, drawn for a sequence of increasing prices. Now the manager has no term like “-D” in (a) to keep it from raising price to infinity. In fact, the manager’s incentive now is an award for approximate *loss* in consumer surplus. The zero-profit part of B will never matter in this case; the firm won’t be in the zero profit region.



- e) See (d).
- f) The regulator needs to observe realized current profit and price and last period’s output. The regulator does NOT need to know the cost or demand functions.