

Ph. D. Preliminary examination in Industrial Organization, July 2008

Answers to questions 1 and 2

1.

(a) The monopolist uses a two-part tariff and serves only the high demand consumer

In this case it is best to serve the high-demand consumer.

The monopolist sets $p = c$ inducing the consumer to purchase $\frac{\theta_2 - c}{\theta_2} \rightarrow \frac{5}{6} = 0.833$ units

and sets a tariff of

$$T := \frac{1}{2} \cdot (\theta_2 - c) \cdot \frac{\theta_2 - c}{\theta_2} \quad T \rightarrow \frac{25}{12} = 2.083$$

The monopolist's profits are thus equal to T.

(b) The monopolist uses bundling and serves only the high demand consumer

This case replicates the outcome of part A. The bundle consists of $\frac{\theta_2 - c}{\theta_2} \rightarrow 4 - 4q = 0.833$ units

and it is sold for $V := T + c \cdot \frac{\theta_2 - c}{\theta_2} \quad V \rightarrow \frac{35}{12} = 2.917$

The monopolist's profit is equal to T, as in case A: $T \rightarrow \frac{25}{12} = 2.083$

(c) The monopolist uses a menu of two-part tariffs and serves both consumers

In this case it is optimal to charge the high-demand consumers $p = c$ thereby inducing efficient consumption of

$$\frac{\theta_2 - c}{\theta_2} \rightarrow \frac{5}{6} = 0.833 \quad \text{units.} \quad A(p) := \frac{(\theta_1 - p)^2}{2 \cdot \theta_1} \quad B(p) := (p - c) \cdot \frac{\theta_1 - p}{\theta_1}$$

$$C(p) := (p - c) \cdot \left(\frac{\theta_2 - p}{\theta_2} - \frac{\theta_1 - p}{\theta_1} \right) \quad D(p) := \frac{1}{2} \cdot (p - c) \cdot \left(\frac{\theta_2 - c}{\theta_2} - \frac{\theta_2 - p}{\theta_2} \right)$$

Furthermore, if the monopolist charges a price p to low-demand consumer, then it will set his tariff so as to extract his entire surplus, that is, it will set a tariff of

$$T_1(p) := A(p) \quad T_1(p) \rightarrow \frac{1}{8} \cdot (4 - p)^2$$

Given such a two-part tariff targeted to the low-demand consumer, the high-demand consumer can also choose that tariff and realize a surplus of

$$S_2(p) := \frac{1}{2} \cdot (\theta_2 - p) \cdot \left(\frac{\theta_2 - p}{\theta_2} \right) - T_1(p) \quad S_2(p) \text{ simplify } \rightarrow 1 - \frac{1}{24} \cdot p^2$$

Thus the tariff for the high-demand consumer has to be set so that he gets at least the above surplus if he chooses the two-part tariff targeted to him, that is,

$$T_2(p) := \frac{1}{2} \cdot \left[(\theta_2 - c) \cdot \left(\frac{\theta_2 - c}{\theta_2} \right) \right] - S_2(p) \quad T_2(p) \text{ simplify } \rightarrow \frac{13}{12} + \frac{1}{24} \cdot p^2$$

Thus the monopolist's profits are

$$\Pi(p) := 2 \cdot A(p) + 2 \cdot B(p) + C(p) + D(p) \quad \Pi(p) \text{ simplify } \rightarrow \frac{25}{12} + \frac{1}{4} \cdot p - \frac{1}{12} \cdot p^2$$

$$T_1(p) + T_2(p) + (p - c) \cdot \left(\frac{\theta_1 - p}{\theta_1} \right) \text{ simplify } \rightarrow \frac{25}{12} + \frac{1}{4} \cdot p - \frac{1}{12} \cdot p^2 \quad [\text{alternative computation of } \Pi(p)]$$

$$\text{Given } \frac{d}{dp} \Pi(p) = 0 \quad p_1 := \text{Find}(p) \rightarrow \frac{3}{2} \quad \frac{\theta_2}{\theta_1} \cdot c = 1.5$$

Thus the optimal menu of two-part tariffs is

$$\text{for low demand: } p_1 \rightarrow \frac{3}{2} = 1.5 \quad T_1(p_1) \rightarrow \frac{25}{32} = 0.781$$

$$\text{for high demand } c \rightarrow 1 \quad T_2(p_1) \rightarrow \frac{113}{96} = 1.177$$

$$\text{corresponding profits: } \Pi(p_1) \rightarrow \frac{109}{48} = 2.271 \quad \text{Thus this option is better than option A (serving only the high-demand consumer).}$$

(d) The monopolist uses a menu of bundles and serves both consumers

In this case it is optimal to include the efficient quantity in the bundle for the high-demand consumer

$$\frac{\theta_2 - c}{\theta_2} \rightarrow \frac{5}{6} = 0.833 \quad \text{units.}$$

Furthermore, if the bundle for the low-demand consumer includes q units, then the optimal price for the bundle is

$$V_1(q) := \int_0^q [\theta_1 \cdot (1 - x)] dx \quad V_1(q) \text{ simplify } \rightarrow 4 \cdot q - 2 \cdot q^2$$

Such a bundle is available also to the high-demand consumer and would yield him a surplus of

$$s_2(q) := \int_0^q [\theta_2 \cdot (1 - x)] dx - V_1(q) \quad s_2(q) \rightarrow 2 \cdot q - q^2$$

Thus the bundle targeted to the high-demand consumer must give him at least this surplus

$$V_2(q) := \int_0^{\frac{\theta_2 - c}{\theta_2}} [\theta_2 \cdot (1 - x)] dx - s_2(q) \qquad V_2(q) \text{ simplify } \rightarrow \frac{35}{12} - 2 \cdot q + q^2$$

Thus the monopolist's profit is

$$\text{Pr}(q) := V_1(q) + V_2(q) - c \cdot \left(q + \frac{\theta_2 - c}{\theta_2} \right) \qquad \text{Pr}(q) \text{ simplify } \rightarrow q - q^2 + \frac{25}{12}$$

Given $\frac{d}{dq} \text{Pr}(q) = 0$ $q_1 := \text{Find}(q) \rightarrow \frac{1}{2}$ $1 - \frac{c}{2 \cdot \theta_1 - \theta_2} = 0.5$

Thus the optimal menu of bundles is:

for the low-demand $q_1 \rightarrow \frac{1}{2} = 0.5$ $V_1(q_1) \rightarrow \frac{3}{2} = 1.5$

for the high-demand $\frac{\theta_2 - c}{\theta_2} \rightarrow \frac{5}{6} = 0.833$ $V_2(q_1) \rightarrow \frac{13}{6} = 2.167$

The corresponding profits are $\text{Pr}(q_1) \rightarrow \frac{7}{3} = 2.333$

(e) The monopolist prefers d to c, c to b and is indifferent between b and a

(f) In cases a and b consumer surplus is zero. In cases c and d the surplus of the low-demand consumer is zero. In case c the high-demand consumer has a surplus of

$$s_2(p_1) \rightarrow \frac{29}{32} = 0.906$$

In case d the high-demand consumer has a surplus of $s_2(q_1) \rightarrow \frac{3}{4} = 0.75$

Thus the ranking in terms of consumer surplus is: c better than d, d better than b, b the same as a

(g) Now we look at total surplus

$$\text{TS} := \begin{pmatrix} \text{"Case a"} & T \\ \text{"Case b"} & T \\ \text{"Case c"} & \Pi(p_1) + S_2(p_1) \\ \text{"Case d"} & \Pi(q_1) + s_2(q_1) \end{pmatrix} \quad \text{TS} \rightarrow \begin{pmatrix} \text{"Case a"} & \frac{25}{12} \\ \text{"Case b"} & \frac{25}{12} \\ \text{"Case c"} & \frac{305}{96} \\ \text{"Case d"} & \frac{37}{12} \end{pmatrix} = \begin{pmatrix} \text{"Case a"} & 2.083 \\ \text{"Case b"} & 2.083 \\ \text{"Case c"} & 3.177 \\ \text{"Case d"} & 3.083 \end{pmatrix}$$

Thus from the point of view of total surplus, c is better than d, d is better than b and b is the same as a

2.

$$p(q) \rightarrow 46 - 2 \cdot q \quad C(q) \rightarrow 32 + 2 \cdot q$$

(a) Scenario 1. Since there is no possibility of entry and there is no discounting, the incumbent chooses, in each period, the output that maximizes

$$\Pi(q) := q \cdot p(q) - C(q) \quad \Pi(q) \text{ simplify} \rightarrow 44 \cdot q - 2 \cdot q^2 - 32$$

$$\text{Given } \frac{d}{dq} \Pi(q) = 0$$

$$q_M := \text{Find}(q) \rightarrow 11$$

$$p_M := p(q_M) \quad p_M = 24$$

$$\Pi_M := \Pi(q_M) \quad \Pi_M = 210$$

Thus the firm produces $q_M = 11$ units and makes a profit of $\Pi_M = 210$ in each period for a total of

$$2 \cdot \Pi_M = 420$$

(b) Scenario 2. If the incumbent has committed to output q for period 2 then the entrant's residual dem:

$$\text{res}(q_e, q) := p(q_e + q) \quad \text{res}(q_e, q) \text{ simplify} \rightarrow 46 - 2 \cdot q_e - 2 \cdot q$$

The potential entrant's profit function is:

$$\Pi_e(q_e, q) := q_e \cdot \text{res}(q_e, q) - C(q_e)$$

$$\Pi_e(q_e, q) \text{ simplify} \rightarrow 44 \cdot q_e - 2 \cdot q_e^2 - 2 \cdot q_e \cdot q - 32$$

$$\text{Given } \frac{d}{dq_e} \Pi_e(q_e, q) = 0$$

$$q_E(q) := \text{Find}(q_e) \rightarrow 11 - \frac{1}{2} \cdot q$$

reaction function of entrant

$$\Pi_E(q) := \Pi_e(q_E(q), q)$$

$$\Pi_E(q) \text{ simplify } \rightarrow 210 - 22 \cdot q + \frac{1}{2} \cdot q^2 \quad \text{entrant's maximum profit}$$

Given

$$\Pi_E(q) = 0 \quad q_{\text{limit}} := \text{Find}(q)^T \rightarrow \begin{pmatrix} 14 \\ 30 \end{pmatrix} \quad q_E(14) = 4$$

$$q_E(30) = -4$$

$$q_{\text{lim}} := q_{\text{limit}_1}$$

$$q_{\text{lim}} = 14$$

limit output: if the incumbent commits to an output of 14 per period then the entrant cannot make positive profits and will therefore stay out. **Thus entry deterrence is possible.**

(c,d,e) Then the incumbent's profits in each period are: $\Pi(14) = 192$

for a total of $2 \cdot \Pi(14) = 384$

The incumbent's profit function is thus

$$\Pi_i(q) := \begin{cases} (2 \cdot \Pi(q)) & \text{if } q \geq q_{\text{lim}} \\ (\Pi(q) + \Pi(q + q_E(q))) & \text{otherwise} \end{cases}$$

$$f(q) := \Pi(q) + \Pi(q + q_E(q)) \quad f(q) \text{ simplify } \rightarrow 44 \cdot q - \frac{5}{2} \cdot q^2 + 178$$

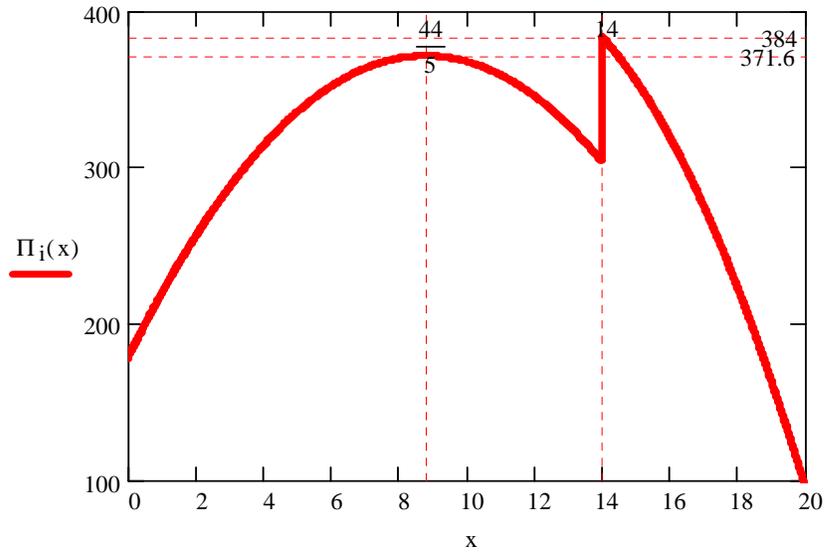
given $\frac{d}{dq} f(q) = 0 \quad q_{\text{emax}} := \text{find}(q) \rightarrow \frac{44}{5}$

$$f(q_{\text{emax}}) = 371.6$$

The best alternative is to commit to 44/5 units but in this case the incumbent's total profits would only be 371.6

Graphically, the incumbent's profit function in Scenario 2 is as follows

$$x := 0, 0.01 \dots 20$$



Thus the incumbent chooses to commit to 14 units of output in each period, entry does not take place and the incumbent's total profits are 384.

(f,g,h) Scenario 3. Since the incumbent cannot commit to a level of output for period 2, in period 2 there will be a standard Cournot game. If profits at the Cournot equilibrium are positive then the incumbent cannot deter entry.

$$pr_1(q_1, q_2) := q_1 \cdot p(q_1 + q_2) - C(q_1)$$

$$pr_2(q_1, q_2) := q_2 \cdot p(q_1 + q_2) - C(q_2)$$

Given $\frac{d}{dq_1} pr_1(q_1, q_2) = 0$

$\frac{d}{dq_2} pr_2(q_1, q_2) = 0$

$$\text{CNE} := \text{Find}(q_1, q_2) \rightarrow \begin{pmatrix} \frac{22}{3} \\ \frac{22}{3} \end{pmatrix}$$

$$\text{CNE } \Pi := pr_2(\text{CNE}_1, \text{CNE}_2) \rightarrow \frac{680}{9}$$

positive, so entry will take place

Then the incumbent will choose monopoly output in period 1 ($q_M = 11$) and Cournot equilibrium output in period 2 ($\text{CNE}_1 \rightarrow \frac{22}{3}$) and its total profits are $\Pi_M + \text{CNE } \Pi = 285.55556$