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Date: July 1, 2009
Time: 3 hours
Reading Time: 20 minutes

PRELIMINARY EXAM FOR THE Ph.D. DEGREE

Answer 4 questions total, at least one from each part. This is a closed book exam.

PART I

Question 1. Point allocation: (a) 30%; (b) 20%; (c) 10%; (d) 20%; (d) 20%.

Consider the estimator $\widehat{\boldsymbol{\beta}}$ that minimizes

$$Q_N(\boldsymbol{\beta}) = \frac{1}{N}\sum\nolimits_{i=1}^{N}(y_i - \exp(\mathbf{x}_i'\boldsymbol{\beta}))^2,$$

where $\boldsymbol{\beta}$ is a $k \times 1$ parameter vector and $\mathbf{x}_i$ is a $k \times 1$ stochastic regressor vector and it is assumed that $(y_i, \mathbf{x}_i)$ are iid over i.
In the true model

$$\begin{aligned} \mathrm{E}[y_i|\mathbf{x}_i] &= \exp(\mathbf{x}_i'\boldsymbol{\beta}_0) \\ \mathrm{V}[y_i|\mathbf{x}_i] &= \sigma_{i0}^2. \end{aligned}$$

State clearly any additional assumptions needed below.

(a) Obtain plim $Q_N(\boldsymbol{\beta})$ and hence prove the consistency of $\widehat{\boldsymbol{\beta}}$.
Hint: It is helpful in the objective function to re-write $y_i - \exp(\mathbf{x}_i'\boldsymbol{\beta}) = [y_i - \exp(\mathbf{x}_i'\boldsymbol{\beta}_0)] + [\exp(\mathbf{x}_i'\boldsymbol{\beta}_0) - \exp(\mathbf{x}_i'\boldsymbol{\beta})]$
You need not formally verify any LLN and CLT used here.

(b) Obtain the limit distribution of $\sqrt{N}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0)$.
You need not formally verify any LLN and CLT used here.

(c) Given your answer in part (b), provide complete details on a method to obtain consistent standard errors.

(d) Present in detail an iterative method that permits computation of $\widehat{\boldsymbol{\beta}}$.

(e) Suppose it is additionally assumed that $\sigma_{i0}^2 = \exp(\mathbf{x}_i'\boldsymbol{\beta}_0)$. Present a method to obtain an estimate of $\boldsymbol{\beta}$ that is more efficient than $\widehat{\boldsymbol{\beta}}$ given above.

Question 2. Point allocation: (a) 10%; (b) 15%; (c) 20%; (d) 15%; (e) 20% (f) 20%.

For the random variable y^* with exponential distribution the density function, distribution function, mean, variance and truncated mean are given by

$$\begin{aligned} f(y^*) &= \gamma \exp(-\gamma y^*), \qquad y^* > 0, \qquad \gamma > 0 \\ F(y^*) &= 1 - \exp(-\gamma y^*), \\ \mathrm{E}[y^*] &= 1/\gamma \\ \mathrm{Var}[y^*] &= 1/\gamma^2 \\ \mathrm{E}[y^*|y^* > c] &= c + 1/\gamma \end{aligned}$$

We incorporate regressors by specifying

$$\gamma_i = \exp(\mathbf{x}_i'\boldsymbol{\beta}),$$

where $\mathbf{x}_i$ and $\boldsymbol{\beta}$ are $k \times 1$ vectors. Throughout we assume a sample of size N with independence over i.

(a) Suppose $\mathbf{x}'\boldsymbol{\beta} = \alpha + \delta z$ and we obtain estimates $\widehat{\alpha} = 0.3$ and $\widehat{\delta} = 0.4$. Give a meaningful interpretation of how the conditional mean of y^* changes as z changes.

(b) Suppose we fully observe $\mathbf{x}_i$ but do not fully observe y_i^*. Instead we observe

$$\begin{aligned} d_i &= 1 \qquad \text{if } y_i^* \geq 2 \\ d_i &= 0 \qquad \text{if } y_i^* < 2. \end{aligned}$$

Give the objective function for the MLE of $\boldsymbol{\beta}$ given data $(d_i, \mathbf{x}_i)$, $i = 1, ..., N$.

(c) Suppose we fully observe $\mathbf{x}_i$ but do not fully observe y_i^*. Instead we observe

$$\begin{aligned} y_i &= y_i^* \qquad \text{if } y_i^* \geq 2 \\ y_i &= 0 \qquad \text{if } y_i^* < 2. \end{aligned}$$

Give the objective function for the MLE of $\boldsymbol{\beta}$ given data $(y_i, \mathbf{x}_i)$, $i = 1, ..., N$.

(d) Suppose we observe only

$$y_i = y_i^* \qquad \text{if } y_i > 2,$$

and the associated $\mathbf{x}_i$ when $y_i > 2$. i.e. there is no record of either $\mathbf{x}_i$ or y_i when $y_i^* < 2$. State how you would consistently estimate $\boldsymbol{\beta}$ using a nonlinear least squares package.

(e) Suppose we have panel data when y_{it} is exponentially distributed with parameter γ_{it} where

$$\gamma_{it} = \exp(\alpha_i + \mathbf{x}_{it}'\boldsymbol{\beta}), \quad i = 1, ..., N, \quad t = 1, ..., T,$$

where $\alpha_1, ..., \alpha_n$ and $\boldsymbol{\beta}$ are parameters to estimate. For this part there is no censoring or truncation. State whether or not it is possible to consistently estimate $\boldsymbol{\beta}$ using a program for exponential regression if N is small and $T \to \infty$. If it is possible then say how you would do so.

(f) Consider the same setup as part (c), but suppose T is small and $N \to \infty$. Consider the estimator $\widehat{\boldsymbol{\beta}}$ that solves

$$\sum_{i=1}^{N} [y_{it} - (\bar{y}_i/\bar{\lambda}_i)\lambda_{it}]\mathbf{x}_{it} = \mathbf{0},$$

where $\lambda_{it} = \exp(\mathbf{x}_{it}'\boldsymbol{\beta})$ so $\gamma_{it} = \alpha_i \lambda_{it}$, $\bar{y}_i = T^{-1}\sum_{t=1}^{T} y_{it}$, and $\bar{\lambda}_i = T^{-1}\sum_{t=1}^{T} \lambda_{it}$.
Will $\widehat{\boldsymbol{\beta}}$ be consistent for $\boldsymbol{\beta}$? Explain.

PART II FOLLOWS ON NEXT PAGE

PART II

Question 3. Point allocation: each part equally weighted.

Assume that $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})' \in \mathbb{R}^2$ is iid$N(0, \Omega)$ where Ω is a positive definite, symmetric 2×2 covariance matrix with

$$\Omega = \begin{bmatrix} \omega_{11} & 0 \\ 0 & \omega_{22} \end{bmatrix}.$$

Assume that $z_t' = (y_t, x_t)$ is the solution to the structural model

$$B_0 z_t = B_1 z_{t-1} + \varepsilon_t$$

with initial condition $z_0 = 0$. The coefficient matrices B_0 and B_1 are given by

$$B_0 = \begin{bmatrix} 1 & 0 \\ b_2 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} b_1 + 1 & b_1 b_3 \\ b_2 + b_2 b_1 + 1 & b_3 (b_2 b_1 + 1) + 1 \end{bmatrix}.$$

(a) Find the reduced form for z_t. Express the variance Σ of the reduced form innovations in terms of Ω, b_1, b_2 and b_3 and find the reduced form coefficients Π in terms of b_1, b_2 and b_3.

(b) Find the error correction representation of z_t.

(c) Show that z_t is cointegrated.

(d) Write down the likelihood for the parameters of the reduced form.

(e) Write down expressions for the estimators of the reduced form parameters Π and Σ.

Question 4. Point allocation: each part equally weighted.

Assume that u_t is iid$N(0,1)$. Assume that z_t is the solution to

$$z_t = \pi z_{t-1} + u_t.$$

with initial condition $z_0 = 0$. You have strong prior beliefs that $\pi = 1$.

(a) Choose a Minnesota style prior to reflect your believes.

(b) Find the posterior distribution of π.

(c) Find the Bayesian Estimator for π when the Loss function is quadratic.

(d) Compare the Bayesian Estimator to the Maximum Likelihood Estimator.

(e) Assume that the true value of π is $\pi_0 = .95$. Compare the forecast mean squared error (MSE) of a one step ahead forecast based on $\pi = .95$ with the forecast MSE of a forecast based on $\pi = 1$.

PART III FOLLOWS ON NEXT PAGE

PART III

Question 5. Point allocation: equal weights for each question.

Consider the within-group estimator of the model

$$y_{it} = \alpha y_{i,t-1} + \delta_i + v_{it} \qquad |\alpha| < 1$$

where $\{(y_{i0}, ..., y_{iT}),\ i = 1, ..., N\}$ is a random sample with finite first- and second-order moments and

$$E(v_{it}|y_{i,t-1}, ..., y_{i0}; \delta_i) = 0$$

and the δ_i are individual specific intercepts.

(a) Explain (in words if you prefer) what is the Nickell bias and its source (I just need the intuition). Practically speaking, when is this bias likely to be important and when is it likely to be irrelevant.

(b) Provide an alternative method of estimation to the within-group estimator that will avoid the Nickell bias. Provide intuition as to why this alternative method works. In addition, point out some of the possible practical short-comings of this method in practical applications.

(c) Consider now the problem of testing the null $H_0 : \alpha = 1$ (assuming that the condition $|\alpha| < 1$ is not known to be true or false a priori) against a homogenous alternative in which the number of available cross-sections is small but each cross-section is relatively large. Describe a way to set-up a regression that would allow you to test this null and comment on the asymptotic properties of your test. Discuss some of the properties of v_{it} that one must consider when configuring such a test.

Question 6. Point allocation: equal weights for each question.
Suppose you want to estimate the model

$$y_i = x_i\beta + u_i$$

where $\{y_i, x_i\}_{i=1}^N$ are an i.i.d. sample with finite first- and second-order moments; $E(u_i|x_i) = 0$ and the variance of u_i is known and equal to one.

(a) Briefly describe a Bayesian computational method that would allow you to estimate this model with the constraint $\beta > 0$. Make sure you detail your choice of prior, likelihood, and posterior simulation method.

(b) Suppose that in your answer to part (b) you decide on an accept/reject method based on an importance density given by an exponential density with mean of 1. Explain the advantages/disadvantages of this choice and how the Metropolis and Metropolis-Hastings algorithms will differ.

(c) Instead, suppose estimate the unconstrained model. Describe a way to assess the constraint $\beta > 0$.

END OF EXAM.