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Time: 3 hours
Reading Time: 20 minutes

PRELIMINARY EXAM FOR THE Ph.D. DEGREE

Answer 4 questions total, at least one from each of parts I and II, and at least one from either parts III or IV. Closed book exam.

PART I

Question 1. Point allocation: equal weights for each part.

Consider the regression model:

$$Y^* = X'\beta + \varepsilon$$

with ε distributed $N(0, \sigma^2)$ conditional on X, Y^* denoting log-income and X including covariates that predict income such as age. Suppose that low income wages below some threshold L are excluded from the sample. Answer the following questions:

(a) Will the least squares estimator of β for the observed values (Y^*, X) be a consistent estimator of β? Why or why not? Show analytically.

(b) What is the expectation of Y^* conditional on X in the observed sample? Could you use this conditional expectation to give an estimator of β? If so, analytically describe an estimator. If not, describe (in words or analytically) why, and provide an alternate method of estimating β.

(c) What is the conditional likelihood function of Y^* given the observed X in this sample?

(d) Suppose now that you are informed that, in addition to excluding low income wages, the sample you have also excluded high income wages above some threshold H. In addition, assume that $H = 2L$. Answer parts (b)-(c) again.

(e) With the new information given in part (d), suggest **two** alternate estimators of the parameters than the one you suggested previously.

Question 2. Point allocation: each part equally weighted.

Consider the two equation structural model:

$$\begin{aligned} Y_1 &= \beta_{11} Y_2 + \gamma_{11} Z_1 + \gamma_{12} Z_2 + u_1 \\ Y_2 &= \beta_{12} Y_1 + \gamma_{23} Z_3 + \gamma_{24} Z_4 + u_2 \end{aligned}$$

Answer the following questions:

(a) Determine the identification of the two equations in terms of the rank and order conditions.

(b) Derived the reduced form and determine the effect of the overidentifying restrictions on the coefficients of the reduced form. Derive a test of the overidentifying restrictions based on estimates of the reduced form coefficients.

(c) Specify an efficient full information estimator for the structural model. Derive a test of the overidentifying restrictions.

(d) Suppose you believe that $E(u_1, u_2) = 0$. Derive an efficient full information estimator in this situation. How could you test whether the covariance restriction is true?

PART II FOLLOWS ON NEXT PAGE

PART II

Question 3. Point Allocation: (a) 20%, (b) 10%, (c) 10%, (d) 20%, (e) 20%. (f) 10%

Assume that $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})' \in \mathbb{R}^2$ is iid$N(0, \Omega)$ where Ω is a positive definite, symmetric 2×2 covariance matrix with

$$\Omega = \begin{bmatrix} \omega_{11} & 0 \\ 0 & \omega_{22} \end{bmatrix}.$$

Assume that $z_t' = (y_t, x_t)$ is the stationary solution to the structural model

$$B_0 z_t = B_1 z_t + \varepsilon_t.$$

The coefficient matrices B_0 and B_1 are given by

$$B_0 = \begin{bmatrix} 1 & 0 \\ \beta & 1 \end{bmatrix}, \ B_1 = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}.$$

Assume that enough restrictions are placed on β and B_1 such that z_t is stationary.

(a) Find the reduced form for z_t. Express the variance Σ of the reduced form innovations in terms of Ω, B_0 and B_1 and find the reduced form coefficients Π in terms of B_0 and B_1.

(b) Assume you have a sample $z_1, ..., z_T$. Write down the approximate likelihood function of the reduced form parameters, ignoring initial conditions.

(c) Find the maximum likelihood estimators of the reduced form parameters.

(d) How would you test the hypothesis that $\beta = 0$.

(e) Write down the impulse response function of the structural innovations in terms of B_0 and B_1.

(f) Provide an interpretation of the zero restriction imposed on B_0.

Question 4. Point Allocation: (a) 20%, (b) 20%, (c) 20%, (d) 20%, (e) 20%.

Let ε_t be an iid sequence with $\varepsilon_t \sim N(0,1)$ and let u_t be an iid sequence with $Eu_t = 0$ and $Eu_t^2 = 1$. Assume that

$$\begin{aligned} y_t &= \alpha x_t + u_t \\ x_t &= \varepsilon_t + \theta \varepsilon_{t-1} \end{aligned}$$

with $|\alpha| < 1$, $\operatorname{cov}(\varepsilon_t, u_t) = \sigma_{\varepsilon u} \neq 0$.

(a) Is x_t, as determined by the above difference equations, stationary? Weakly stationary? Prove your answer.

(b) Find the spectral density of x_t.

(c) You are interested in estimating the parameter α. You use OLS

$$\hat{\alpha} = \frac{\sum_{t=1}^{T} x_t y_t}{\sum_{t=1}^{T} x_t^2}.$$

Describe the asymptotic properties of the OLS estimator.

(d) Is there a better estimator than $\hat{\alpha}$ for α? If so, write down a formula for a better procedure. What are the asymptotic properties of the proposed alternative procedure.

(e) Now assume that you want to estimate θ. What are the conditions for identification of this parameter?

PART III FOLLOWS ON NEXT PAGE

PART III

Question 5. Point allocation: (a) 20%; (b) 20%; (c) 10%; (d) 10%; (e) 10%; (f) 30%.

Consider the regression model

$$y_i = \exp(\mathbf{x}_i'\boldsymbol{\beta}) + u_i,$$

where $\boldsymbol{\beta}$ is a $K \times 1$ vector and u_i are independent over i with $\mathrm{E}[u_i|\mathbf{x}_i] = 0$ and $\mathrm{V}[u_i|\mathbf{x}_i] = \sigma_i^2$. Consider the estimator $\widehat{\boldsymbol{\beta}}$ that minimizes

$$Q(\boldsymbol{\beta}) = \sum\nolimits_{i=1}^{N} (y_i - \exp(\mathbf{x}_i'\boldsymbol{\beta}))^2.$$

(a) Give the Newton-Raphson iterations for computing the estimator, with the simplification that the expected value of the Hessian is used in place of the Hessian.

(b) Give a formula to compute a consistent estimate of the variance matrix of $\widehat{\boldsymbol{\beta}}$.

(c) Provide a direct interpretation of the coefficient $\widehat{\beta}_j$ in this model.

(d) Give a complete formula that allows computation of the average marginal effect in this model of a change in an indicator regressor variable using the finite-difference method.

(e) Define the residual to be $\widehat{u}_i = y_i - \exp(\mathbf{x}_i'\widehat{\boldsymbol{\beta}})$. Will $\sum_{i=1}^{N} \widehat{u}_i = 0$? Explain.

(f) Suppose that $\mathrm{E}[u_i|\mathbf{x}_i] \neq 0$. Instead there exist $K + 2$ variables $\mathbf{z}_i$ that satisfy $\mathrm{E}[u_i|\mathbf{z}_i] = 0$. Explain in detail how to calculate a consistent and efficient estimator of $\boldsymbol{\beta}$.

Question 6. Point allocation: (a) 10%; (b) 10%; (c) 20%; (d) 10%; (e) 10%; (f) 20%; (g) 20%.

Consider the panel data model

$$y_{it} = \alpha_i + \beta x_{it} + u_{it},\ i = 1, ..., N,\ t = 1, ..., T,$$

where u_{it} are independently distributed over i and t, $\mathrm{E}[u_{it}|\alpha_i, x_{it}] = 0$, and x_{it} is a scalar regressor. **If you need to make any additional assumptions in answering the following, state them clearly.**

(a) State how to obtain a consistent estimator of β using only an OLS program under the assumption that x_{it} is uncorrelated with α_i.

(b) State how to obtain a consistent estimator of β using only an OLS program under the assumption that x_{it} is correlated with α_i.

(c) State in some detail how to determine whether or not x_{it} is correlated with α_i.

(d) You are told that x_{it} has small within variation and large between variation. What does this mean and how, if at all, will it effect your estimators in part (a) and part (b)?

(e) Suppose x_{it} is correlated with both α_i and u_{it}. State how to obtain a consistent estimator of β.

(f) Suppose $x_{it} = y_{i,t-1}$. State how to obtain a consistent and efficient estimator of β.

(g) Suppose we write the above model in matrix notation as

$$\mathbf{y}_i = \mathbf{e}\alpha_i + \beta\mathbf{x}_i + \mathbf{u}_i,$$

where $\mathbf{y}_i$, $\mathbf{x}_i$, and $\mathbf{u}_i$ are $T \times 1$ vectors with t^{th} entries y_{it}, x_{it} and u_{it}, and $\mathbf{e}$ is an $N \times 1$ vector of ones. Consider the estimator

$$\widetilde{\boldsymbol{\beta}} = \left[\sum\nolimits_i \mathbf{x}_i'\mathbf{Q}\mathbf{x}_i\right]^{-1} \sum\nolimits_i \mathbf{x}_i'\mathbf{Q}\mathbf{y}_i,$$

where $\mathbf{Q} = (\mathbf{I}_T - \frac{1}{T}\mathbf{e}\mathbf{e}')$. Obtain the variance of $\widetilde{\boldsymbol{\beta}}$ under the assumption that u_{it} is i.i.d. $(0, \sigma^2)$.

PART IV FOLLOWS ON NEXT PAGE

PART IV

Question 7. Point allocation: equal weights for each question.

This question contains 6 short questions. I am looking for concise and specific answers and many do not even require that you use any math notation. Do not ramble, you do not have the time.

(a) Briefly explain (in words) the concept of uniform convergence and its usefulness (i.e., in what way is it used in asymptotic theory).

(b) In a regime-switching model with two states, say 1 and 2, you may remember that the auxiliary vector ξ_t is a 2×1 vector that is equal to the first column of the matrix I_2 when the state is $s_t = 1$, and is the second column of I_2 when the state is $s_t = 2$. This vector is used to formalize the conditional transition probabilities. Explain, in words, the differences and usefulness of each of the following entities: $\widehat{\xi}_t$; $\widehat{\xi}_{t+1|t}$; $\widehat{\xi}_{t|T}$. Be sure to discuss how an empirical practitioner may use each quantity in practice.

(c) What are the advantages and disadvantages of using the equal-weights matrix over the optimal weighting matrix in a minimum distance (e.g. GMM) type problem?

(d) Consider the following state-space model

$$\begin{aligned} y^*_{t+1} &= \rho y^*_t + \varepsilon_{t+1} \\ y_t &= y^*_t + u_t \end{aligned}$$

where $E(\varepsilon_t^2) = \sigma^2_\varepsilon$; $E\left(u_t^2\right) = \sigma^2_u$. Identify the state variable, the observed variable and hence the state equation and the observation equation. What is the mean-squared error of the one-period ahead forecast (i.e. $P_{t+1|t}$ in the notation used in the Kalman filter) when there is no measurement error? Suppose you need to provide your best guess about the unobserved variable $\{y^*_t\}_{t=1}^T$ to a policy maker (he may then use this information for some other analysis with other data). Assuming $\sigma^2_u \neq 0$, what quantity (or quantities) from the Kalman filter should you provide him with (and why)?

(e) Suppose you are interested in constructing a 95% simultaneous confidence region for the parameters θ_1 and θ_2. These parameters must meet the following restrictions: $0 > \theta_1 > 1$ and $\theta_2 > 0$. Provide an algorithmic method based on simulation techniques to construct such a region. You may assume that the posterior density for each parameter individually is available and is easy to take draws from.

(f) Briefly explain what is a particle filter, when and what is it used for, and give the intuition as to how it works.

Question 8. Point allocation: equal weights for each question.
This question contains 6 short questions. I am looking for concise and specific answers and many do not even require that you use any math notation. Do not ramble, you do not have the time.

(a) Briefly explain (in words) the concept of stochastic equicontinuity and in what way is it useful in asymptotic derivations.

(b) Suppose a consistent vector of parameter estimates $\widehat{\gamma}_T$ of dimension $k \times 1$ is available out of which a researcher can construct sample moment conditions $h(\mathbf{z}_t; \widehat{\gamma}_T; \theta) = 0$ to estimate the structural parameters of interest θ by some minimum distance method. Assume also that an estimate of the optimal weighting matrix, $\widehat{W}$, is available. Under what conditions will the parameters θ be identified (make sure that you consider *all* the conditions needed)? Would your answer change if there existed a set of moment conditions $g(\mathbf{z}_t; \theta) = 0$ that did not require the first stage estimates $\widehat{\gamma}_T$? Explain your answer.

(c) Explain (in words) in what way is the mean value theorem useful when deriving the asymptotic distribution of a minimum distance estimator.

(d) Propose a simple test (and explain what type of a test it is, Wald, LM or LR) and the way to compute it for the null hypothesis $H_0 : \theta_2 = 0$ in the model

$$y_t = \theta_1 x_{1t} + \theta_2 x_{2t} + \varepsilon_t$$

but without actually estimating that model. Instead, you can use the estimates from the model

$$y_t = \theta_1 x_{1t} + u_t$$

and any other auxiliary regression that you like. You may make any assumptions that you want as long as you are explicit about them.

(e) You are told a state variable ξ_t follows a Gaussian first order autoregression. The observed variable y_t depends linearly on this state variable and on a Gaussian white noise. Further, from the properties of the multivariate normal, you know that if $\mathbf{z}_1$ and $\mathbf{z}_2$ are two vectors of random variables such that

$$\begin{pmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{pmatrix} \sim N\left(\begin{bmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{bmatrix}; \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} \right)$$

then $\mathbf{z}_2 | \mathbf{z}_1 \sim N(\mathbf{m}, \Sigma)$ where

$$\begin{aligned} \mathbf{m} &= \boldsymbol{\mu}_2 + \Omega_{21}\Omega_{11}^{-1}(\mathbf{z}_1 - \boldsymbol{\mu}_1) \\ \Sigma &= \Omega_{22} - \Omega_{21}\Omega_{11}^{-1}\Omega_{12} \end{aligned}$$

Compute $\widehat{\xi}_{t+1|t}$ and $\widehat{\xi}_{t|t}$.

(f) Suppose T_1 observations from an experiment are available to estimate the parameter vector β in a linear regression where the residual variance is known. Call these estimates $\widehat{\beta}_1$. Later on, an additional T_2 observations are made available. Provide a method to estimate $\widehat{\beta}$ using $\widehat{\beta}_1$ and the new T_2 observations only, that would be equivalent to the estimate you would obtain if you used instead the sample of $T_1 + T_2$ observations. Be specific about the formula for this estimator.

END OF EXAM.