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PRELIMINARY EXAM FOR THE Ph.D. DEGREE

Answer 4 questions, at least one from each part. Closed Book exam.

PART I

Question 1. Point allocation: (a) 10%; (b) 20%; (c) 20%; (d) 20%; (e) 10%; (f) 20%.

Consider the following density for the continuous positive random variable y

$$f(y) = \exp(-\theta/y)\theta^2(1/y)^3/2; \quad y > 0,\ \theta > 0,$$

where it can be shown that

$$\begin{aligned} \mathsf{E}[y] &= \theta \\ \mathsf{V}[y] &= \infty \\ \mathsf{E}[1/y] &= 2/\theta \\ \mathsf{V}[1/y] &= 2/\theta^2 \end{aligned}$$

Suppose we have a random sample $(y_i, \mathbf{x}_i)$, $i = 1, ...N$, where $\mathbf{x}_i$ is a $k \times 1$ nonstochastic regressor vector and y_i has the above density with

$$\theta_i = \exp(\mathbf{x}_i'\boldsymbol{\beta}),$$

where $\boldsymbol{\beta} = \boldsymbol{\beta}_0$ in the data generating process.

For much of this question we are concerned with the properties of the MLE of $\boldsymbol{\beta}$ under conditions weaker than correct specification of the density.

You can apply any laws of large numbers and central limit theorems without formally verifying the necessary assumptions.

(a) Give the formula for the objective function $Q_N(\boldsymbol{\beta})$ equal to N^{-1} times the log-likelihood function.

(b) Obtain $\operatorname{plim} Q_N(\boldsymbol{\beta})$.

(c) Given your answer in (b), what assumptions are the essential assumptions to ensure consistency of $\widehat{\boldsymbol{\beta}}$ that maximizes $Q_N(\boldsymbol{\beta})$.

(d) Assuming that the density is correctly specified, give the limit distribution of $\widehat{\boldsymbol{\beta}}$. [Your derivation can be as brief as possible].

(e) Hence give a method to test at the 5% significance level $H_0 : \beta_j = 1$ against $H_1 : \beta_j \neq 1$, where β_j is the coefficient of the j^{th} regressor.

(f) Now suppose that the conditional density is misspecified, though in such a way that the MLE $\widehat{\boldsymbol{\beta}}$ retains its consistency. Give the formula for a consistent estimate of the asymptotic variance-covariance matrix of $\widehat{\boldsymbol{\beta}}$. [Your answer and derivation can be as brief as possible].

Question 2. Point allocation: (a) 20%; (b) 20%; (c) 15%; (d) 15%; (e) 30%.

2. Consider a linear regression model based on the **discrete** latent variable y_i^* which is Poisson distributed with density

$$f^*(y_t^*) = e^{-\mu_i}\mu_i^{y_i^*}/y_i^*!, \qquad y_i^* = 0, 1, 2, ...$$

where $\mu_i = \exp(\mathbf{x}_i'\boldsymbol{\beta})$ and the data are independent over i. Note that for the Poisson

$$\begin{aligned} \mathrm{E}[y^*] &= \mu \\ \mathrm{E}[y^*|y^* &> 0] = \mu/(1-e^{-\mu}) \\ \mathrm{V}[y^*] &= \mu \\ \mathrm{V}[y^*|y^* &> 0] = \mu e^{-\mu}/(1-e^{-\mu})^2. \end{aligned}$$

In this question the variable y_i^* is not completely observed.

(a) Suppose we observe only

$$\begin{aligned} y_i = 1 \quad &\text{if } y_i^* \geq 1 \\ y_i = 0 \quad &\text{if } y_i^* = 0 \end{aligned}$$

Give with justification the objective function for a consistent estimator of $\boldsymbol{\beta}$.

(b) Suppose we observe only

$$y_i = y_i^* \quad \text{if } y_i^* \geq 1.$$

Give with justification the objective function for the MLE of $\boldsymbol{\beta}$.

(c) In the same situation as in part (b), give an alternative consistent estimator for $\boldsymbol{\beta}$ that uses nonlinear least squares.

(d) In the same situation as in part (b), give an alternative consistent estimator for $\boldsymbol{\beta}$ that uses GMM.

(e) Suppose that now we completely observe y^*, but the data are panel data. We suppose that y_{it} is Poisson distributed with density

$$f(y_{it}) = e^{-\mu_{it}}\mu_i^{y_{it}}/y_{it}!, \qquad y_{it} = 0, 1, 2, ...$$

Suppose we obtain the Poisson MLE of $\boldsymbol{\beta}$ by Poisson regression with mean $\mu_{it} = \exp(\mathbf{x}_{it}'\boldsymbol{\beta})$. What are the properties of this estimator (consistency and efficiency) if in fact $\mu_{it} = \alpha_i \exp(\mathbf{x}_{it}'\boldsymbol{\beta})$ where the additional component α_i is assumed to be iid $[1, \sigma_\alpha^2]$? Give a solid explanation.

PART II

Question 3. Point allocation: (a) 30%; (b) 45%; (c) 25%

Consider the model

$$y_t = \beta t + u_t \tag{1}$$

where t is a time trend, u_t is an i.i.d. sequence with mean zero and variance σ^2. Suppose we have a sample of size T, where the first R observations are used to estimate the model, and $P = T\text{-}R$ observations are reserved to test the out-of-sample forecasting performance of the model. Specifically, we are interested in evaluating the one period-ahead forecasts generated by

$$\hat{y}_t = \hat{\beta}_R t$$

where $\hat{\beta}_R$ refers to the least-squares estimate of expression (1) with the first R observations, and $t = R+1,\ R+2,\ \ldots,\ T.$

Define the P one period-ahead forecast errors for $t = R+1,\ \ldots,\ T$ as

$$\hat{u}_t = y_t - \hat{y}_t$$

A test of forecasting bias can be constructed by estimating the following regression with these forecast errors

$$\hat{u}_t = \alpha + \varepsilon_t$$

and then testing the null hypothesis $H_0 : \alpha = 0$ with a simple t-statistic.

The following results are provided for convenience:

$$\frac{1}{T^{v+1}}\sum^{T} t^{v} \to \frac{1}{v+1}; \quad \frac{1}{T^{1/2}}\sum^{T} \frac{t}{T} v_t \to N(0, \sigma^2/3) \text{ for } v_t \text{ a M.D.S.}$$

Answer the following questions:

(a) Suppose $R \to \infty$, derive the asymptotic distribution of $\hat{\beta}_R$.

(b) Suppose $T, R, P \to \infty; P/R \to \pi < \infty$. Derive the asymptotic distribution of the t-statistic of the null $H_0 : \alpha = 0$.

(c) Suppose that instead of the distribution you derived in part (b) you incorrectly assume $\pi = 0$. In what way would the results of the test of forecasting bias be affected? Draw a graph to help with the interpretation.

Question 4. Point allocation: (a) 15%; (b) 25%; (c) 25%; (d) 35%

Suppose the data are generated by the following AR(2)

$$y_t = (1+\rho)y_{t-1} - \rho y_{t-2} + \varepsilon_t;\ \ \varepsilon_t \overset{i.i.d.}{\sim} (0,\sigma^2);\ \ |\rho|<1$$

(a) Show that this AR(2) is not stationary.
(b) Calculate the impulse response for this process.
Hint: $y_{t+s} - (y_{t-1} + \varepsilon_t) = \Delta y_{t+s} + \ldots + \Delta y_t + \varepsilon_t$

Suppose now that you incorrectly specify the process as

$$y_t = \beta y_{t-1} + u_t$$

(c) Calculate the limit in probability of the least squares estimator for β.
(d) Derive the asymptotic distribution for $\hat{\beta}$.

You may sue the following results without proof:

The Functional Central Limit Theorem:

$$\frac{\sqrt{T}}{\sigma} X_T(.) \overset{d}{\rightarrow} \psi(1)W(.);\ \ X_T(.) = \frac{1}{T}\sum^{[Tr]} u_t;\ \ \psi(L)u_t = \varepsilon_t$$

The Continuous Mapping Theorem

PART III

Question 5. Point allocation: (a) 25%; (b) 25%; (c) 25%; (d) 25%

Suppose that the solution of a representative agent's portfolio allocation problem is given by the following Euler equation

$$1 = \beta E_t\left[M_{t+1}\frac{(P_{t+1}+D_{t+1})}{P_t}\right]$$

where P_t is the price of a financial asset and D_t is the dividend paid out by the financial asset. Suppose that the stochastic discount factor, M_{t+1}, is given by

$$M_{t+1} = C_t / C_{t+1}$$

where C_t is consumption. In addition, suppose you have quarterly time series data on P_t and D_t for one financial asset, and on C_t. Finally, suppose you estimate β using GMM with the moment conditions $E(Z_t u_{t+1}) = 0$, where $Z_t = (1 \quad X_t)'$, $u_{t+1} = 1 - \beta X_{t+1}$, and $X_t = M_t(P_t + D_t)/P_{t-1}$.

(a) Write down Hansen's *J*-statistic and explain how you would use it to make inference about the validity of the moment conditions and the model implied by the above Euler equation. If you were to reject the null hypothesis of Hansen's *J*-test, what would you do?

(b) Suppose you are unable to reject the null hypothesis of Hansen's *J*-test and that your estimate of β is 0.96 with a standard error of 1.5. This standard error is calculated using the usual formula, which is a function of the variance of the sample moment conditions, the first derivative of the moment conditions, and the weighting matrix. Do these results imply that the parameter β is well identified? If so, explain why. If not, explain how you would determine whether this parameter is well identified.

(c) Suppose that you estimate β using the moment condition $E(X_{t+1}u_{t+1}) = 0$ rather than $E(Z_t u_{t+1}) = 0$. Show that your estimate is inconsistent and explain in words why it is inconsistent.

(d) Suppose you are unwilling to assume that stochastic discount factor is given by $M_{t+1} = C_t / C_{t+1}$. Instead, you are reluctant to go beyond the general model $M_{t+1} = f(W_{t+1})$, where W_{t+1} denotes a large vector of macroeconomic variables. Your friend suggests that you take the first *k* principal components of W_{t+1} and assume that *f* is linear, i.e., specify $M_{t+1} = \alpha_0 + \alpha_1 V_{1,t+1} + ... + \alpha_k V_{k,t+1}$, where $V_{i,t+1}$ denotes the i^{th} principal component of W_{t+1}. Write down the sample moment conditions that you would use to obtain the GMM estimates of the unknown parameters ($\alpha_0, \ldots, \alpha_k, \beta$) and describe some advantages and disadvantages of this modeling approach.

Question 6: Point allocation: (a) 25%; (b) 25%; (c) 25%; (d) 25%.

Suppose that as part of your dissertation you are estimating by ordinary least squares a linear regression model $y = X\beta + \varepsilon$. You are concerned about parameter stability in your model and perform the F-tests suggested by Bai and Perron (Econometrica, 1998). Your results are printed below:

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**********************************************************
Output from the Bai and Perron testing procedure
**********************************************************
a) supF tests against a fixed number of breaks
--------------------------------------------------------
The supF test for 0 versus 1 breaks (scaled by q) is: 7.56
The supF test for 0 versus 2 breaks (scaled by q) is: 19.89
The supF test for 0 versus 3 breaks (scaled by q) is: 17.55
-----------------------
The critical values at the 5% level are (for k=1 to 3):
12.89 11.60  10.46
--------------------------------------------------------
b) Dmax tests against an unknown number of breaks
--------------------------------------------------------
The UDmax test is: 19.89
(the critical value at the 5% level is: 13.27 )
**********************************************************
The WDmax test at the 5% level is: 17.95
(The critical value is: 14.19 )
**********************************************************
c) supF(l+1|l) tests using global optimizers under the null
--------------------------------------------------------
The supF( 2 | 1 ) test is : 19.94
The supF( 3 | 2 ) test is : 9.83
**********************************************************
The critical values of supF(i+1|i) at the 5% level are (for i=1 to 3 ) are:
12.89 14.50  15.42
**********************************************************
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(a) Draw conclusions about the number of breaks in your sample.

(b) Suppose you conclude that there are some breaks in the parameters of your model and you want to incorporate these into your analysis. One of your options is to treat the breaks as deterministic as in the Bai/Perron approach. A second option is to model the breaks as random draws from some distribution using, for example, a Markov switching model. Outline the advantages and disadvantages of each of these approaches.

(question 6 continued on next page)

(c) Suppose you want estimate the Markov switching model discussed in part (b). Briefly outline the steps you would take to estimate the parameters using the EM algorithm. What will be the properties of your estimate? (Hint: If you were to observe the state variable S, then the log-likelihood function would be $L(Z;\theta)$, where θ denotes the parameters of the model and Z includes both S and the observed data (y,X). The state variable S indicates which observations reside in each regime. You may express your answer in terms of $L(Z;\theta)$; you do not need to give a formula for $L(Z;\theta)$.)

(d) Suppose you estimate the Markov switching and deterministic breaks models discussed in part (b), but you suspect that neither model adequately fits the data. Explain how you could use nonparametric analysis to gain insight into the stability of your model.