University of California, Davis
Department of Economics
Advanced Economic Theory

Date: June 25, 2008
Time: 3 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

You need to answer four questions, at least one from each part.

Part 1

1. (Temptation and Self-Control)

a) Let X be a finite set of alternatives, and $\mathcal{M}$ the set of its non-empty subsets ("menus"). Consider a decision maker with menu preferences described by the functional form of Gul und Pesendorfer's (2000) model of "Temptation and Self-Control" that is based on a committment utility $u : X \to \mathbf{R}$ and temptation utility $t : X \to \mathbf{R}$.

- As in class, think of the $x \in X$ as abstract alternatives, disregarding the fact that GP study specifically menus of lotteries.

Write down the GP functional form, and explain the notion of "cost of self-control" in their model.

b) Describe the basic new behavioral phenomenon GP try to model.

c) Show that GP-style menu preferences satisfy their Set Betweenness axiom.

d) Consider the following preference pattern:

$$\{hard\} \sim \{hard, easy\} \succ \{easy\} \succ \{easy, cake\} \succ \{hard, cake\}.$$

These are motivated by the following story: only the cake is tempting; in absence of temptation, act according to committment preferences; in presence of the cake, overcoming temptation requires will-power effort that aggravates hard task more than easy task.

i) Can these preferences be explained in the GP model?

ii) Show that these preferences can be explained by a non-linear generalization of the GP model.

2. (Subjective Probability)

a) State Savage's representation theorem, restricting your attention to acts with a finite number of consequences. State each axiom in a formally precise manner, and interpret its content intuitively.

b) Savage's theorem contains a representation of "qualitative probability" relations with particular structure; state this representation, and point out what additional structure is added.

c) Let $\mathbf{N}$ denote the set of natural numbers and Σ denote the family of all sets $A \subseteq \mathbf{N}$ such that A or its complement A^c is finite. Define a relation $\trianglerighteq_{fin}$ on the family of *finite* subsets of Σ that orders sets lexicographically according to their greatest elements: $A \equiv B$ iff $A = B$, and $A \triangleright B$ iff, for some $k \geq 0$, the k largest elements of A and B agree, and the $k+1$st element of A is larger than the $k+1$st element of B, or if B has only k elements. For example, $\{5\} \triangleright \{4,3,2\}$, $\{5,4,3\} \triangleright \{5,4,2\}$, and $\{5,4,3\} \triangleright \{5,4\}$.

i) Show that the ordering $\trianglerighteq_{fin}$ can be extended to a qualitative probability $\trianglerighteq$ on all of Σ.

ii) Can the qualitative probability $\trianglerighteq$ be represented by a finitely additive probability measure? (Hint: to answer this question, it suffices to focus on the subrelation $\trianglerighteq_{fin}$)

Part 2

3. Consider a two-period finance ($t = 0, 1$) economy with I agents, such that the real side of the economy is deterministic: agents' endowments are riskless, of the form $\omega^i = (\omega^i_0, \omega^i_1) \in \mathbb{R}^2_{++}$. However transactions use money, the government monetary policy may introduce variability in the purchasing power of money, and this affects the payoff of the nominal bonds that agents use to borrow and lend. A monetary policy is described by the amounts of money injected in the economy, M_0 at date 0 and $M_{\mathbf{1}} = (M_1, \ldots, M_S)$ at date 1 (we assume that the random variable $M_{\mathbf{1}}$ can take at most S values and there is a probability distribution $\rho = (\rho_s)_{s \in S}$ on these values; we also assume that the monetary policy is perfectly anticipated by the agents in the private sector). In this economy, simple "quantity theory equations" hold: in each date/state $s = (0, \ldots, S)$, $p_s \sum_{i=1}^{I} x^i_s = M_s$, where p_s is the price of the (composite good) in terms of money and x^i_s is the consumption of agent i in date/state s. Agents are risk averse with separable utility functions of the form

$$u^i(x^i) = v^i(x^i_0) + \delta E v^i(x^i_{\mathbf{1}})$$

where δ is the discount factor and v^i is concave increasing.

(a) Prove that the Pareto optimal allocations of the economy just described are deterministic, i.e. each agent' consumption is the same in all monetary states at date 1.

(b) Suppose that, despite the presence of monetary shocks, agents only use for trading the nominal bond which pays one unit of money at date 1 whatever happens. Prove that generically an equilibrium of the monetary economy is Pareto optimal if and only if the monetary policy $(M_0, M_{\mathbf{1}})$ is non random.

(c) Suppose that the situation described by this model is one where monetary shocks regularly occur and additional nominal securities have been introduced in such a way that markets are complete. Show that the monetary policy is neutral in the sense that it does not affect the real side of the economy.

4. Consider an exchange economy with two periods ($t = 0, 1$) and uncertainty described by S states of nature at date 1. There is one good at each date and agent i has the initial endowment $\omega^i \in \mathbb{R}_+^{S+1}$. The preferences of each agent are characterized by a utility function $u^i : \mathbb{R}_+^{S+1} \longrightarrow \mathbb{R}$ which is smooth, differentiably strictly quasi-concave and has indifference surfaces which do not intersect the axes. There are J securities with a payoff matrix V at date 1. The financial markets are incomplete ($J < S$).

(a) Write out the definition of a financial market equilibrium and the first-order conditions which are satisfied at an equilibrium.

(b) Explain from the first-order conditions why an equilibrium is not likely to be Pareto optimal.

(c) Suppose now that you want to prove formally that an equilibrium with incomplete markets is typically suboptimal. Explain what you are able to prove and give an outline of the proof.

(d) What are the non-generic cases that you know where a financial market equilibrium is Pareto optimal despite the fact that the financial markets are incomplete?

Part 3

5. Consider a public good game with $N = \{1, ..., n\}$ players. Player i's wealth is denoted by $w_i \in \mathbb{R}_+$. She can contribute an amount $c_i \in [0, w_i]$ to the public good. Player i's payoff function is given by

$$u_i(c_i, c_{-i}) = \sum_{j \in N} c_j + w_i - c_i + (w_i - c_i)\left(\sum_{j \in N} c_j\right).$$

The first term, $\sum_{j \in N} c_j$, is interpreted as the total provision of the public good. The second term, $w_i - c_i$, is the amount of wealth spent on "private" goods. Finally, the last term, $(w_i - c_i)\left(\sum_{j \in N} c_j\right)$, reflects the interaction between the amount of the public good and player i's private consumption. The greater the amount of public good, the more she values her private consumption.

Assume that $w_i = w > 0$ for all $i \in N$.

(a) Find a Nash equilibrium of the game.

(b) Is the Nash equilibrium of (a) unique? Explain your answer.

(c) If each player contributes half of her wealth to the public good, is she better or worse off as compared to the Nash equilibrium in (a)?

(d) As the number of players increases, how does the total amount contributed in the Nash equilibrium of (a) change?

Allow now w_i to differ among players.

Assume further that players choose contribution levels from a grid. I.e., each players' contribution is restricted to be a non-negative multiple of some small amount $\delta > 0$ that is at most w_i. Assume that δ is such that all the specific contribution levels you need in below arguments are multiples of δ.

Sometimes the context does not justify the modeler to focus on an equilibrium convention. Rather, some set valued solution concepts like iterative deletion of dominated actions or correlated rationalizability may be used.

(e) Show that any contribution of more than $\frac{w_i}{2}$ is strictly dominated for player i.

(f) Show that if $n = 3$, then every contribution of at most $\frac{w_i}{2}$ is rationalizable.

(g) Show that if $n = 3$ and $w_1 = w_2 < \frac{1}{3}w_3$, then the unique correlated rationalizable contribution of players 1 and 2 is 0 and the unique rationalizable contribution of player 3 is w_3.

6. During World War II the Germans used the Enigma coding system. They did not expect somebody to break the code. The British employed a team of experts including the mathematician Alan Turing who eventually managed to break the code. Top secret military messages become transparent to the Allies. One of the deciphered messages was Germany's intention to raid Coventry during the night of 14-15 of November, 1940. Churchill decided not to evacuate Coventry, fearing this would make the Germans aware that the code was broken, and lead them to abandon the Enigma and thus prevent the Allies from continuing to decipher their secret communication.

As simplified version of this historical incident is presented in the game tree (see next page). We assume that the British ability to decode the Enigma or not is decided by a prior move of nature. Then the Germans - not knowing whether the British are able to decode the Enigma or not - take a decision to use the Enigma or not for the upcoming raid on Coventry. The raid is successful if and only if the Enigma is used (for coordination) and the British do not evacuate Coventry. The British can not evacuate Coventry if the Enigma is not used or not decoded. A successful raid on Coventry yields a payoff of 1 to the Germans and a payoff of -1 to the British. After the raid on Coventry the Germans can decide whether to use the Enigma further for another raid or not. The British could counter attack successfully wining the war. Such counter attack is successful if and only if the Germans use the Enigma and the British attack. A successful counter attack yields 10 to the British and -10 to the Germans. For a complete description of the model see the extensive form game on the the next page.[1]

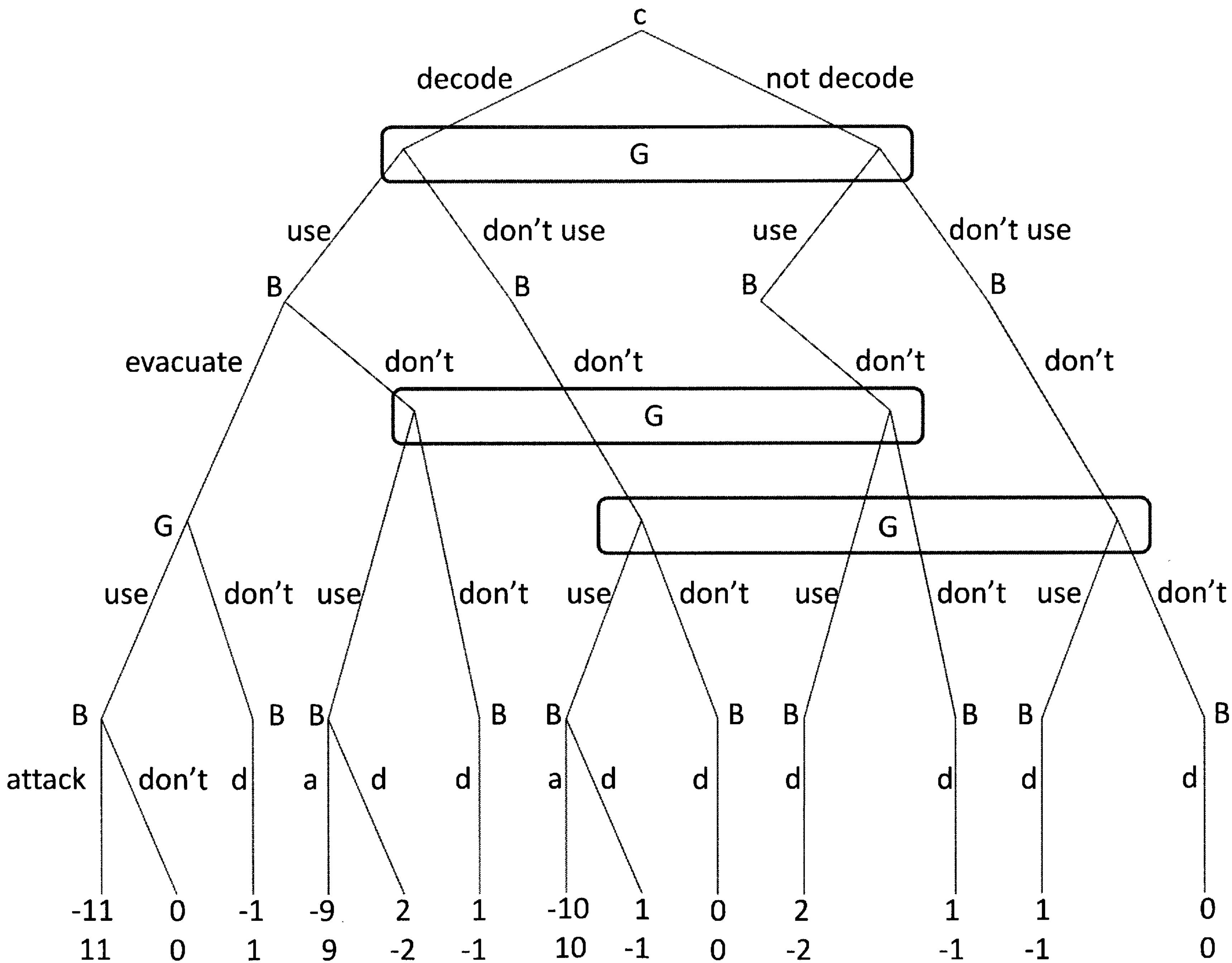
Denote by μ the prior probability that the British are able to decode the Enigma.

(a) Find a (weak) sequential equilibrium of the game. (Hint: Simplify the game first by deleting dominated strategies. Think also about different values of μ.)

(b) Is the equilibrium of (a) unique?

(c) Suppose the British are able to decode the Enigma but the Germans don't know. What "reputation" do the British cryptographers like to have? That is, what do the British like the Germans believe about the British ability to decode the Enigma? (I.e., conditional of being able to decode the Enigma, what value of μ do the British prefer?)

[1] "G" stands for Germans, "B" stands for British, "c" for chance (nature). The upper payoffs belongs to the Germans, the lower ones to the British.


c
decode
not decode
G
use
don't use
use
don't use
B
B
B
B
evacuate
don't
don't
don't
don't
G
G
G
use
don't
use
don't
use
don't
use
don't
use
don't
B
B
B
B
B
B
B
B
B
B
attack
don't
d
a
d
d
a
d
d
d
d
d
d
-11
11
0
0
-1
1
-9
9
2
-2
1
-1
-10
10
1
-1
0
0
2
-2
1
-1
1
-1
0
0