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Abstract

As competitive reforms are introduced into the electric power industry, much attention has focused on the potential market organization of the industry's transmission sector. This paper analyzes the incentives for grid investment which result from various proposed transmission network regimes. In particular, we focus on "transmission congestion contracts" within a contract network regime such as proposed by William Hogan. We formalize a rule for awarding these new property rights to investors, and show that under certain conditions, this contract network approach can effectively deter detrimental investments, some of which are encouraged under other regimes. However, when these conditions are not met, market participants may still find it profitable to undertake network alterations detrimental to the network as a whole.

1 Introduction

Led by initiatives on both the Federal ([5]) and State ([4], [2]) level, the electric power industry is going through a process of fundamental restructuring and realignment. This process has lead to an acceptance of the need to redefine the concepts of transmission access, pricing and ownership in order to promote robust and efficient competition. This task is complicated by the fact that the transmission sector of the industry is still generally regarded as a natural monopoly, and also by a perceived need of investors in generation for some certainty about future costs of transmission services. Traditional approaches to transmission access and pricing have focused on "contract path" and cost-recovery-based transmission tariffs which ignore the economic and physical realities of the transmission of electricity.

Nodal spot prices can provide a more economic approach to access and pricing. Under this paradigm the customer, instead of paying for "transportation" separately, finds the transmission charge bundled into the price of power and access guaranteed. Although nodal spot prices create

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temporal price risks, these can be handled with "contracts for differences." Locational price uncertainties can be "hedged" through futures contracts, or, as we will see shortly, through "transmission congestion contracts" within a contract network regime.

The potential benefits of such systems for transmission customers and for system efficiency have been widely explored, less so their implications for investment in the grid itself. In fact, to even consider investment incentives one must first specify what property rights will be allocated to someone who increases or in any way modifies the transmission capacity of the grid. The fact remains that if customers, under any system, are buying transmission services, someone must own the rights to collect the payments for these services. This leads to an array of unresolved questions: how is capacity defined?, who owns it?, what rights does ownership confer?, and what rights are conferred on those who modify the grid? The answers to these questions bear directly on the long term incentives for investment in the grid itself.

Several critiques of the nodal spot market approach to transmission services have been offered ([16], [11]). A key concern has been the incentives created for parties making investments in the grid. One of the fundamental problems is that efficient nodal prices contain network congestion and loss components. If transmission owners collect revenues based upon these prices, they can have an incentive to increase, rather than reduce, congestion and losses. Such problems complicated by the presence of network externalites, economies of scale in transmission investments, and the inherent barriers to entry along a given transmission path which tend to prevent a single transmission path from having a number of competing owners (see [1]). Under some nodal-spot-market-based transmission rates, an investor could therefore build a line to be intentionally congested and profit from it even though the net social benefit to the system as a whole decreases due to the congestion.

In this article we address these investment incentive issues in the context of a contract network regime. This concept, developed by William Hogan ([7][9]), allows market participants to purchase and trade *transmission congestion contracts* (TCCs), which pay the owner the locational price difference between the two nodes specified in the contract.[1] To date, no procedure for allocating new TCCs to agents who invest in the grid has been published. The nature of the investment incentives under such a regime were therefore uncertain.

In this paper we present an allocation rule which is based upon the concept of feasible dispatch. This requires making a formal correspondence between the set of all TCCs and a network dispatch. This is possible because each TCC is defined by a power flow between two nodes. By adding together (consolidating) the power flows associated with all allocated TCCs we find a set of injections and demands at the various nodes in the network corresponding to some particular dispatch. Under the concept of feasible allocation, an investor in the grid is allowed to select any set of TCCs which, when combined with the existing set, corresponds to a dispatch which is feasible under the constraints of the newly modified grid. An expander who creates an intentionally congested line which effectively reduces the feasible set of dispatches would therefore be required to accept whatever TCCs that exactly cancel the flows that are no longer feasible in the resulting, lower capacity network. The concept of feasibility therefore provides some check on the incentive to create congestion.

We show that the utilization of such a rule can reduce or, under ideal circumstances, eliminate

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[1] Ownership of TCCs entails potential obligations as well as rights since the value of a TCC may sometimes be negative, implying a payment by the contract holder to the grid.

the incentives for a detrimental grid expansion. We also show that TCCs, when combined with contracts for differences, can immunize grid participants from detrimental grid modifications even when they are made by an irrational investor. The nature of these ideal circumstances, depends upon allocated TCCs matching the actual dispatch of the system, and the unlikelihood that this will occur gives rise to a series of policy and implementation questions. These may imply the desirability of redefining TCCs or the rules for their allocation.

In section 2 we develop a generalized model of a nodal spot market which allows for the presence of transmission congestion contracts and contracts for differences. In section 3 we describe a method for awarding new transmission congestion contracts based upon the principle of feasible dispatches. Our results about the investment incentive properties of this allocation scheme are presented in Section 4. Section 5 extends the methodology of previous sections to the include the presence of transmission line losses. In section 6 we discuss the likelihood and applicability of the condition of contracts matching dispatch, which is necessary for the results of section 4. Our conclusions are presented in section 7.

2 Defining Network Property

2.1 Network Model

We will utilize the notation and conventions of Schweppe, et. al.[15], and Hogan [7]to describe the concepts involved in the allocation of network-based revenues. For most of the paper, we will use a simplified network model that deals with only real power flow and assumes no losses. This simplified representation is convenient for describing the ideas behind defining and allocating transmission capacity. Section 5 discusses these concepts in the context of a network model generalized to include line losses.

The network consists of N nodes indexed by $i, j = 1, \ldots, N$ and K market participants, or agents, indexed by $k = 1, \ldots K$. Associated with the nodes are power injections q_i and spot prices p_i. Supply to the grid is indicated by a positive injection, while consumption from the grid is indicated by $q_i < 0$. Often we will be interested in the a complete set of q_i or p_i for $1 \leq i \leq N$ which we will represent with the vectors q and p, respectively. Since agents may buy and sell power at more than one node, injection vectors are useful for describing agents, and we denote the injections of agent k by q^k. The sum of all agent's injection vectors is called the system dispatch, thus $q = \sum\limits_k q^k$ is the system dispatch.

An *optimal dispatch*, q^*, maximizes social surplus, which is calculated from the costs of production and the benefits of consumption. We represent both costs and benefits by a cost function $C(q)$ that takes negative values in the case of benefits from use. Each agent has a cost function $C^k(q^k)$. We define $W(q) = \max_{q^k} - \sum_k C^k(q^k) \; s.t. \sum_k q^k = q$, as the social surplus which results from a dispatch q, where it is assumed that q_i, the power at each node, is supplied at least cost. Associated with the optimal dispatch is an *optimal price vector* p^* which induces the optimal dispatch, q^*.

When computing the optimal dispatch it is necessary to limit consideration to feasible dispatches, by which we mean dispatches that satisfy all thermal limits, and all voltage, stability, and contingency constraints. Nothing in this paper depends on the particulars of the way these limits and constraints are determined, except that we will need the set of feasible dispatches to be convex in order to to place an upper bound on transmission contract revenue[2]. Because of these constraints on feasibility, prices will differ between nodes to reflect the effects of out of merit dispatch.

Because optimal nodal prices will vary due to congestion[3], there will generally be a *merchandising surplus* equal to $p \cdot q = \sum_{i=1}^{N} p_i q_i$ (see Wu, et. al.[17]). This surplus will be collected by some Independent Grid Operator (IGO), but need not be kept by the IGO as profit. An unresolved issue surrounding the organization of a nodal spot market is the distribution of these network revenues, or merchandising surplus. Almost all proposals link the distribution of merchandising surplus with the problem of cost recovery for the owners of network assets, and it is foreseen that some or all of that surplus would be redistributed to the owners of network assets. The nature of this redistribution has been a subject of some controversy. Ideally, nodal spot prices, and therefore the merchandising surplus, should reflect the effects of congestion and losses on system operations. However, if the revenue distributed to grid owners is based upon this surplus, there is a potential incentive for such owners to create, rather than mitigate, congestion [3]. This is a problem with most conventional schemes based on nodal spot markets for rewarding grid ownership. In the following two sections we discuss the two currently most prominent concepts for rewarding grid ownership.

2.2 Link Based Rights

One approach for distributing the merchandising surplus is to pay each link owner the price difference between the nodes served by that link times the power flow on that link. Without losses, this quantity is $q_{ij}(p_j - p_i)$, where q_{ij} is the power flow from i to j. Oren, et. al. [11]have called this form of revenue distribution *Link Based Rights* (LBRs). When a new line is constructed, its builder would receive the rights to revenue from flows on that line. This approach has been used in Chile and Argentina. From a transmission owner's perspective, LBRs can be thought of as the adaptation of the impacted mega-watt mile approach (see [12]) to nodal spot markets. Under impacted mega-watt mile transmission pricing, the marginal and, when congested, capacity costs of each transmission line would be allocated amongst agents proportional to the "impact" the transactions of those agents imposes on that line. Owners of transmission lines therefore earn revenues proportional to the flows on their lines. With Link Based Rights, this proportion is based on the nodal prices of the nodes connected by their line.

A serious problem with LBRs is the incentives they produce for investment in the grid (see Oren, et. al.[11]). A grid expansion which maximizes transmission revenue under link based rights is by no means an optimal one. Incentives exist to increase congestion by either degrading the capacity of links, or constructing new lines. Because of the externalities created by grid modifications, the negative effects of such detrimental actions can be shifted to remote players. The classic example of such an investment is the construction of a low capacity line connecting two supply nodes which had previously served a single demand node (see figure 1). The transfer capacity into the demand

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[2] This has been shown to be true for commonly used approximations to the optimal power flow problem and, while still an open research question, has been conjectured to be true in general over the relevant range of dispatches (see [8].

[3] When we include losses, this will provide an addition reason for prices to differ.

node from the cheaper supply node can be reduced by the addition of the weak link, while that link still earns substantial revenue under a link-based rights regime. In figure 1 the cheaper supply node (2) supplies all 900 megawatts of power before the "expansion", but because of loop flow and the limited capacity of the new line, can supply only 500MWs after the "expansion." In spite of this, if the link owner has a right to the congestion charges on that line he will earn (5 cents - 3 cents)*100,000kWs/hr.[4]

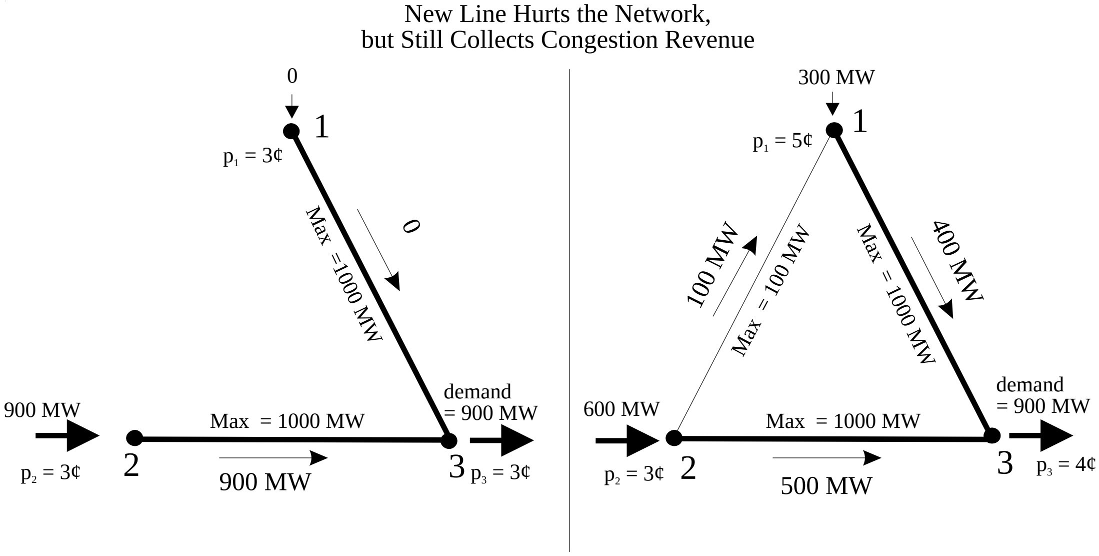

Figure 1: "Bad" modifications can still be profitable

2.3 Transmission Congestion Contracts

A second approach to distributing network revenues is the Contract Network system developed by William Hogan. This approach utilizes *Transmission Congestion Contracts* (TCCs) which pay the holder the spot price difference between nodes times a quantity specified in the contract. In a lossless network, this price difference is due solely to congestion, hence the name. A TCC with magnitude t from i to j would pay its owner $t \cdot (p_j - p_i)$. Since contract quantities do not depend upon actual flows, TCCs, unlike LBRs, need not be limited to nodes physically connected by a single link. TCCs can be written on any pair of nodes in a network.

Unlike LBRs, no immediately intuitive method for awarding TCCs to new investors offers itself. The incentives for investment in the grid when network revenues are distributed through TCCs have heretofore also been ambiguous. There is a perceived need for transmission cost certainty, or

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[4] The nodal spot prices show in figure 1 are optimal assuming that the admittances of all lines are equal, that the marginal cost of generation is 5 cents at node 1 and 3 cents at node 2, and that the demander has an inelastic demand for 900 MWs.

alternatively, a hedge against locational price differences for investors in power plants who would in turn sign long-term supply contracts with customers at other nodes. In the presence of a market-making pool, such a supply contract would take the form of a contract for differences (CFD). A supplier at node i who enters into a CFD with a consumer at node j agrees to pay the consumer the difference between a negotiated strike price and the true spot price, in exchange for a fixed payment. The customer thereby locks in a constant price for power. If the spot prices at nodes i and j are always the same, the supplier is also guaranteed a minimum revenue stream since he will be paid the spot price by the pool for his generation. The CFD therefore eliminates temporal price risks for the two parties of the contract. However, if the spot prices of nodes i and j sometimes differ, the supplier is exposed to locational price risk. If the price for power is higher at the consumer's node than at the supplier's and is above the strike price at the consumer's node, the supplier will suffer a marginal loss equal to the difference between the nodal prices since he cannot earn enough from his spot sales to compensate for his obligations under his CFD. Transmission congestion contracts were developed to eliminate this price risk. Under a TCC, the grid would pay the supplier an amount equal to the price difference between nodes i and j. This is exactly the marginal loss the supplier could suffer under the CFD. Through the combined application of a CFD and a matching TCC, both the supplier and the consumer can eliminate price uncertainty.

Mechanism	**Payment**	
	Supplier i $q_i = q$	Consumer j $q_j = -q$
Spot Market	$p_i q$	$-p_j q$
CFD f at strike price p_c	$(p_c - p_j)f$	$-(p_c - p_j)f$
Total	$p_c f + p_i q - p_j f$	$-p_c f - p_j q + p_j f$
TCC t from i to j	$(p_j - p_i)t$	-
Total with $f = t$	$p_c t + p_i q - p_i t$	$-p_c t - p_j q + p_j t$
When Contracts Match Dispatch, $t = q$.	$p_c q$	$-p_c q$

The price-insulation of trading partners by the contractual arrangement displayed in the table, depends on each agent's contracted quantities exactly offsetting their power injection. Such a matching of contract and spot obligations which provides a lower bound on revenues. As we show in the following sections, when the group has such protection of their benefit and profit levels, it becomes possible to force someone who makes a detrimental alteration to the grid to suffer the consequences.

TCCs and CFDs are essentially financial instruments, and it has been argued (see Outhred, [13]) that more conventional financial instruments such as forward contracts can provide the same level of price certainty with less complication. When price hedging is accomplished through a contract network, however, rules for allocating new TCCs can be designed which essentially *forces* those who make detrimental grid modifications to take "positions" opposite to those of existing contract holders. These compulsory TCC acquisitions have negative values and thereby deter those who would make detrimental modifications.

2.4 Transmission Contract Model

To facilitate the analysis in this paper, we introduce some additional notation to better represent individual payoffs under a contract network regime and to reflect the dispatch which is implied by a set of TCCs. In the notation of Hogan and of Wu et. al. a TCC of size t_{ij} from node i to node j is denoted by t_{ij}. To simplify notation when summing sets of TCCs, we choose instead to define a TCC of quantity τ from i to j as an N-vector where the *ith* and *jth* elements are the only non-zero elements. Thus $t = (0, -\tau, 0, \ldots, 0, \tau, 0)$, where $t_i = -\tau$, and $t_j = \tau$ is such a TCC. The combination of all TCCs owned by agent k can be expressed as the vector sum $t^k = \sum\limits_{t \in \mathcal{T}^k} t$, where $\mathcal{T}^k$ is the set of all TCCs owned by agent k. Since $p \cdot t$ gives the revenue from a single TCC, $p \cdot t^k$ gives the total revenue from an agent's set of TCCs. (The centered dot will indicate the standard vector dot product throughout this paper.)

As with TCCs, we will also represent contracts for differences as vectors. A CFD covering q at node i will be represented by a vector, f, whose only non-zero component is $f_i = q$, where $f_i < 0$ indicates an obligation to pay f_i times the difference between the spot price at i and the strike price and $f_i > 0$ indicates a contract to receive that difference. An agent's CFDs can be consolidated into a single vector f^k by taking the vector sum of all his CFDs.

We define the *net contract position,* n^k, of an agent to be the sum of all TCCs and CFDs owned by that agent. Thus $n^k = t^k + f^k$. The nodal contract revenue of agent k is therefore $p \cdot n^k$. Note that we have omitted any CFD components that are referenced to a negotiated strike price, because these payments are independent of spot market price fluctuations. As we described above, by setting dispatch, q^k, equal to contracts, n^k, agent k can remove all spot price fluctuation from his income stream[5].

Definition: For any group, g, of agents we say that "*contracts match dispatch*" if $n^g = -q^g$, where $n^g = \sum_{k \in g} n^k$ is the group's consolidated contract vector and $q^g = \sum_{k \in g} q^k$ is the group's dispatch vector.

This means that the group's net contract position exactly *cancels* the group's spot position. Note that the group may be as small as a single agent or as large as the set of all agents. We will frequently be examining the effects of changes in nodal prices or dispatch on the net benefit of one or more participants. Consequently it is useful to define notation for the resulting change in net benefit.

Definition: The *net benefit* of an agent is $NB^k(p, n^k) = p \cdot q^k + p \cdot n^k - C^k(q^k)$, market revenue plus contract revenue minus cost, where q is defined to be optimal for p.

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[5] Of course under some circumstances agent k will find it profitable to set $q_i^k \neq t_i^k$, we are simply pointing out that matching output to the contract position provides a lower bound on revenue

The net benefit of a group is just the sum of its agent's net benefits. Note that this definition of net benefit ignores the fixed payments that are part of CFDs and also ignores any fixed payments associated with purchasing TCCs. Because such fixed payments are by definition *un*associated with nodal price fluctuations, and because we are only concerned with *changes* in net benefit, these can have no effect on any of our results.

Definition: The change in net benefit of the group holding net contract position n, both before and after the price changes from p to p', is $\Delta NB(n) = NB(p', n) - NB(p, n)$. Only contracts in position n are considered when computing the NBs even if new ones acquired during the interval.

Using the above definitions, we now state two intuitive results about the elimination of price risk for agents with contract positions matching their dispatch. The first is the well understood concept, illustrated by table 1, involving matching CFDs, TCCs, and injections.

Lemma 1 *For any agent whose contracts,* n^k*, match its dispatch,* $\Delta NB(n^k) \geq 0$ *for any price change. If the agent's cost function,* $C^k(q^k)$*, is strictly convex, then net benefit must increase.*

Proof (part 1): The first part of the lemma asserts only a weak inequality and is proven by simply considering the agent's possible strategy of keeping q fixed. This strategy fixes costs. Because contracts match dispatch, $n^k = -q^k$, the first two terms in net benefit cancel for any price change. Thus under this strategy NB (temporarily redefined to consider suboptimal q) is unaffected by price and the lemma's weak inequality holds as an equality. The agent cannot do worse (achieve lower NB) with the optimal q.

Proof (part 2): This part of the proof relies on convexity, which is generally a characteristic of cost and benefit functions. All variables refer to agent k.

$$\Delta NB(n) = [p' \cdot q(p') - p \cdot q(p)] + [p' \cdot n - p \cdot n] - [C(q(p')) - C(q(p))] \tag{1}$$

when $n = -q(p)$, (1) reduces to

$$\Delta NB(n) = p'[q(p') - q(p)] - [C(q(p')) - C(q(p))] \tag{2}$$

At a nodal equilibrium, $q(p')$ will be set such that $p' = C'(q(p'))$. Substituting for p' in (2) along with the convexity of $C(q)$ gives

$$\Delta NB(n) = C'(q(p'))[q(p') - q(p)] - [C(q(p')) - C(q(p))] \tag{3}$$

if $C(q)$ is strictly convex, then the RHS of (3) is strictly positive.□

We now extend lemma 1 to cover a group of traders. For example, this lemma would cover the pair of trading partners in Table 1 even if they had not written a CFD. In this case the supplier would own the entire symmetric TCC and the consumer would own no TCC at all. Thus lemma 1 would not apply to either party individually, but lemma 2 would cover the pair of them together. This lemma will be extremely useful in the proofs of our main results.

Lemma 2 *For any group of agents, g, whose contracts, n^g, match its dispatch, $\Delta NB(n^g) \geq 0$ for any price change.*

Proof: First note that changing the distribution of rights does not effect the dispatch because the revenue from contracts is not linked to dispatch, but only to nodal prices. Since contracts match dispatch for the group as a whole it is possible to redistribute them so that they match individually. Since this does not change dispatch by group members, the group's NB is unaffected, because redistribution does not change the sum of the contract revenues. Now consider the price change. By lemma 1, each member of the group will have $\Delta NB^k \geq 0$, so this is also true for $\Delta NB^g = \sum_{k \in g} \Delta NB^k$. Now redistribute the rights in their original pattern. Again this leaves ΔNB^g unaffected.[6]$\square$

Lemmas 1 and 2 show how transmission contracts, either in conjunction with other contracts or within a group, can insulate an agent or group of agents from price fluctuations. One possible reason for a shift in nodal prices would be an alteration to the grid. If such an alteration results in a decrease in total net benefit, and existing players are insulated from this price change, the difference must be made up by either the agent who altered the grid, or the grid operator itself. In the following section, we describe a rule for allocating new transmission contracts created by an alteration. This rule provides the grid operator with the same level of protection against detrimental grid alterations that transmission contracts provide to agents using the grid.

3 Feasibility and the Allocation of TCCs

The incentive for grid modification comes partly from nodal price effects of the modification and partly from the value of TCCs that are allocated to the modifier in response to the modification. The magnitude of these incentives depends on the extent to which the modifier intends to make use of the modification, and on the rule for allocating TCCs.

In this section, we describe a method for allocating new transmission congestion contracts in response to a grid modification. This rule, first suggested by Hogan [10], is based upon the concept that the nodal injections implied by an aggregated set of TCCs should constitute a feasible dispatch (though not necessarily the actual dispatch).

According to a result proved by Hogan [7] and confirmed by Wu et. al. [17], any feasible set of contracts has a convenient financial property. The result (theorem 1) states that as long as the set of allocated TCCs represents a feasible dispatch, the revenue collected by contracts holders will not exceed the network's merchandising surplus produced by an optimal dispatch. It would therefore appear that feasibility provides a fairly appealing constraint on the total set of allocated contracts.[7]

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[6] It should be noted that this argument assumes that agents are pure price takers in this nodal spot market. This is a common assumption the spot pricing literature. Issues of market power, while important, are beyond the scope of this paper.

[7] The positive mechandising surplus should not be confused with an ability to cover the capital cost of the grid out of congestion revenues. These concepts are unrelated and Perez-Arriaga et. al. [14] have shown that the marketing surplus alone will likely not recover full network cost.

It is also well known that since the grid has tremendous market power in a Poolco system, the grid can certainly increase its profits above what is achieved at the optimal dispatch.[8] This necessitates divorcing the grid's profits from the marketing surplus created by the dispatch. When this is done and profit is instead secured by some other mechanism, the need to maintain a positive cash flow from marketing surplus is eliminated.

The observation that the marketing surplus does not impose a necessary constraint on the set of TCCs led Oren *et al.* [11] to conclude that "the feasibility condition in Hogan's (1992) formulation is unnecessary and meaningless." This conclusion is based upon the assumption that "the feasibility condition is only intended to serve as a solvency condition for the market maker." However, feasibility can play a second important role, that of defining the allowable allocations to grid investors.

3.1 Feasible and Optimal Dispatches

So far we have discussed the consolidation of CFDs and TCCs for groups of agents, which we accomplish simply by summing the contract vectors of individual agents. A special case of this, which is needed for the definition of a feasible allocation, is the group of all participating agents. We denote the complete set of allocated TCCs by, $\mathcal{T}$, and consolidate them into a single vector by summing all agent's TCC vectors: $T = \sum_k t^k$. Since $p \cdot t$ gives the revenue from a single TCC, $p \cdot T$ gives the revenue from the set of all allocated TCCs.

When CFDs are consolidated over the entire market they completely cancel each other out, since CFDs are bilateral agreements with one party holding the negative of the CFD held by the other. Therefore, $\sum_k f^k = 0$. Thus the net contract position of the entire market, $n = \sum_k n^k$, is simply T the TCC vector. Recall that $NB^k(p, n^k)$ represents the net benefit of an agent in this market. Since $T = \sum_k n^k$ we can define $NB(p, T)$ to be the aggregate net benefit of all the agents in the market.

$$NB(p,T) \;=\; \sum_k NB^k(p,n^k) \;=\; p \cdot T + p \cdot q - \sum_k C^k(q^k)$$

A supplier of a vector of power, q would desire transmission contracts $-t$, to offset these power injections.The *negative* of the consolidated vector of transmission contracts,$-T$, therefore implies a dispatch of the grid. Considering $-T$ as a dispatch, we can then ask whether on not it is a feasible dispatch, and answer that question just as we would for any real dispatch. This leads us to the rule for the allocation of new contracts as a result of grid modifications.

Definition: The *feasibility allocation rule* grants a modifier of the grid the right to take any set of contracts $\widetilde{\mathcal{T}}$ such that the corresponding dispatch, $-(\mathcal{T} + \widetilde{\mathcal{T}})$ is feasible under the new grid configuration, where $\mathcal{T}$ is the previously allocated set of contracts.[9]

Example: Consider a one-line, two-node network whose capacity is raised from 1 to 2 MWs. Assume that before the expansion, the total TCC allocation consisted of a 1 MW TCC from node 1 to node 2. After completing the expansion, the expander will be free to take any TCC such that

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[8] Wu et. al. demonstrated the potential for one such abuse in [17].

[9] A more detailed examination of the feasibility rule can be found in [3]

the total allocation of TCCs is no more than 2 MWs in either direction. Thus he can take 1 MW TCC from node 1 to 2, or a 3 MW TCC from node 2 to 1, or anything in between. □

This rule leaves an infinite number of possible TCC combinations from which investors may choose. However it establishes an upper bound on total contract revenue. By theorem 1, this occurs when the allocated rights match the actual dispatch. For a given ex-post dispatch, we can therefore place an upper bound on the potential revenues of the additional contracts selected by an investor.

3.2 Maximum Contract Revenues

All of the following results concerning incentives for grid modification, assume that contracts match dispatch for some set of participants. We have already seen that when a participant has a matching set of rights, he is immune to nodal price fluctuations, but this is not reason enough for participants to consistently acquire matching contracts. One might suppose that the builder of a new high-capacity transmission line would want to acquire rights in excess of the expected power flow on that line in order to earn the maximum revenue from his rights and thus from his investment. This might lead to a feasible total allocation of rights that exceeds the actual system dispatch. The following theorem shows that such a motivation does not exist, at least not if the investor can take a TCC that increases in capacity over time in a way that matches the actual system dispatch. As long as it is possible to secure a set of TCCs that continuously match system dispatch, that set will provide the maximum total revenue to the contract holder. This result provides some motivation for the assumption that contracts match dispatch at the system level, but it does not address the assumption of matching at any lower level of aggregation.

Theorem 1 *The TCC revenue from a consolidated set of feasible contracts, T, under the optimal dispatch, (p^*, q^*), is no greater than the merchandising surplus. In other words, $p^* \cdot T \leq -p^* \cdot q^*$.*

This theorem was first proven for the DC load flow approximation to lossless networks by Hogan [7] and subsequently by Wu, et. all [17]. In the appendix, we prove it for an approximation to lossy networks. Under an optimal dispatch, no feasible set of contracts generates more revenue than the set that matches that dispatch.

Corollary 1 $NB(p^*, T) \leq W(q^*)$ *for all* T. *If contracts match dispatch,* $T = -q^*$, *then* $NB(p^*, T) = W(q^*)$.

Proof: By definition, $NB(p^*, T) = \; p^* \cdot T + p^* \cdot q^* - \sum_k C^k(q^k)$. Since $p^* \cdot T - p^* \cdot q^* \leq 0$, we have the first result. When $T = -q^*$, we have the second result.

Besides providing some motivation for the assumption of matching rights, theorem 1 plays a key role in proving all of the following results. It does this by helping to place a limit on the benefit that can be gained from the acquisition of new TCCs when the grid is modified.

4 Investment under a Contract Network Regime

The allocation of TCCs based upon the concept of feasible dispatch under the modified grid provides a structure for defining and creating property rights on an electric network. In this section, we turn our attention to determining the value of those property rights and the resulting incentives for making investments in the grid. We will show that when the current set of allocated contracts, corresponding to some feasible dispatch, matches the current dispatch of the system, the property rights generated by a detrimental modification to the grid will be of negative value.

This result is driven by the ability of TCCs to insulate market participants from the negative affects of price fluctuations. In section 2, we outlined how TCCs in combination with contracts for differences, can remove the risk of spot market price fluctuations from any bilateral agreement between grid users. Price risk is removed when the transaction is fully contracted: when the CFD and TCC and spot transaction quantities are all equal. Even in the absence of contracts for differences, TCCs can remove spot price risk on an aggregate level when the injections of the system are matched by the consolidation of all TCCs.

All of the following results concern incentives for grid modification and consequently depend on the rights allocation rule. These results all concern a contract network, by which we mean all trades are made within the context of a nodal spot market and that network modifiers are subject to the above defined feasibility allocation rule for TCCs. Three of the results concern *detrimental modifications* of the grid, which we define as one that decreases the social surplus under optimal dispatch; we write this a $\Delta W = W(q') - W(q) < 0$. When contracts match dispatch for the set of all participants, the net benefit of the participants equals the social surplus. We now have the machinery to state and prove the following results about investment incentives under a feasibility allocation rule.

Theorem 2 *If the consolidated set of all transmission contracts, T, matches optimal dispatch q, then the new set of rights, $\tilde{T}$, allocated under the feasibility rule to the maker of a detrimental modification ($\Delta W < 0$) will have a negative value which is greater in magnitude than the loss in social surplus: $p' \cdot \tilde{T} < \Delta W$, where p' is the optimal price vector after modification and q' is optimal for p'.*

Proof: We will proceed by proving a result that is slightly stronger than the theorem. First apply corollary 2 to the dispatch before and after the modification.

$$\begin{aligned} NB(p, T) &= W(q) \\ NB(p', T + \tilde{T}) = NB(p', T) + p' \cdot \tilde{T} &\leq W(q') \end{aligned}$$

Subtracting "before" from "after" we have

$$\begin{aligned} \Delta NB(T) + p' \cdot \tilde{T} &\leq \Delta W \\ p' \cdot \tilde{T} &\leq \Delta W - \Delta NB(T) \end{aligned}$$

By by Lemma 2, $\Delta NB(T) \geq 0$, which proves the theorem. □

Theorem 2 shows that the revenue from any set of contracts selected by the detrimental investor under the feasibility allocation rule will have negative value. From this result, two conclusions about the incentives for grid investment immediately follow.

Corollary 2 *If the consolidated set of all transmission contracts matches the current dispatch, then an outside party, with no prior interest in the grid, will not invest in a detrimental alteration of the grid.*

Proof: Since the investor was an outsider, his only revenue comes from the rights he receives which by theorem 2 is negative.

Theorem 2 and corollary 2 shows the feasible allocation rule provides some protection against bad modifications under the assumption that the rights which have been allocated match the current dispatch of the system. These results do not exclude the possibility that an agent with current interests in the market may find it profitable to modify the grid in a detrimental way. Such an incentive would exist when the agent's gains from current transmission contracts and spot market injections outweigh the revenues from the new transmission contracts, which must be negative. An extreme example of such a scenario is monopoly ownership of all TCCs. If one company owned all TCCs, then a bad modification could increase congestion costs on a part of the network used only by others. This would raise both the merchandising surplus and the monopolist's contract revenue (thus theorem 1 is not violated) in spite of decreasing the social surplus. Stronger conclusions about investment incentives can be drawn under stronger assumptions about the contracting of current users of the grid.

Corollary 3 *If the consolidated set of all transmission contracts matches the current dispatch, then no group of agents whose contracts match their dispatch will find it profitable to make a detrimental alteration of the grid.*

Proof: Let g indicate the relevant group of agents and let $NB^g(p, n^g)$ be the net benefit of that group and $NB^{-g}(p, n^{-g})$ be the net benefit of everyone else. Since contracts match dispatch for g, by complimentarily they must also match for those not in g. By lemma 2, $\Delta NB^{-g}(n^{-g}) \geq 0$. By theorem 2, $\Delta NB^g(n^g) + \Delta NB^{-g}(p, n^{-g}) + p' \cdot \tilde{T} \leq \Delta W < 0$. Therefore, $\Delta NB^g(n^g) + p' \cdot \tilde{T} < 0$. □

It is only under a special case of corollary 3 that the potential for profit from detrimental grid modifications is completely eliminated. We will say that a network market is *completely contracted* if every agent's net contract position, n^k equals their spot position, q^k. As was shown in section 2, TCCs and CFDs used in combination can eliminate all spot price fluctuations from an agent's net benefit. If all agents are so insulated, then none can be harmed by a detrimental modification. Consequently, when a market is completely contracted, no one will find it profitable to modify the grid in a detrimental way. This is just corollary 3 applied to each agent considered as a group.

Up to this point we have focused our attention on the potential for incentives to do damage to the grid. Theorem 2 also illustrates a difficulty with beneficial expansions: namely that the full benefit of them will most likely not be captured by the investor making the expansion. The value of the additional rights, $p' \cdot \tilde{T}$ will always be less than or equal to the change in social surplus.

In the cases where it is less than the change in social surplus, "free-riders" will gain benefit from the expansion. Proponents of the "PoolCo" proposal in California have conceded the possible need for "regulatory procedures" to guide investment when "no coalition of grid users is able to agree to pay for a grid expansion that appears to be beneficial to the system as a whole [6]." A better understanding of the investment incentives provided by a given market structure is the first step in identifying the necessary scope of such regulation.

5 Feasibility of Transmission Contracts with Losses

The conventional form of a TCC as developed by Hogan becomes problematic in networks with losses. Hogan developed a TCC to pay its owner only the *congestion* component of the spot price difference between nodes. The cost of providing the added power needed for a pair-wise transaction would be folded into the spot price. However, this does not solve the problem of having allocated contracts match the actual dispatch. By definition, the set of all contracts which were allocated assuming no losses cannot equal a dispatch in which there are losses

One way to accommodate this difficulty is to make TCCs asymmetric. Assume that a pair-wise injection from i to j causes losses of $\alpha\%$. Then a TCC could be designed to pay $t_{ij}((1-\alpha)p_j - p_i)$, where the nodal prices would include the loss component. But the losses caused by any pair-wise injection depend upon the dispatch of the entire network, so no fixed α can be assigned to an i-j pair.

We would instead suggest the use of a nodal transmission contract, t_i which would simply pay its holder $t_i p_i$. A conventional t_{ij} TCC could be put together by combining a t_j with a negative t_i. Feasibility would then be calculated and enforced in the same way as it was in the lossless network, and the consolidation of all nodal contracts would be energy balanced after accounting for losses. If T is the vector of allocated contracts, and Q is a new feasible dispatch, then the grid modifier could obtain the incremental contract vector $Q - T$. The components of this vector are of course the nodal transmission contracts t_i. The feasibility of Q would be calculated using the appropriate lossy flow equation, so the injections will not sum to zero.

With this new definition it is clearly possible for contracts to match dispatch individually or collectively just as before. The only problem in carrying out our previous set of proofs lies in the possibility that theorem 1 (revenue adequacy) is no longer valid. Fortunately, as the appendix shows, it is valid for lossy networks where losses are determined by the lossless approximation to the lossy flows. This approximation is very close in real-world networks, and it is quite likely that a more advanced proof would show the theorem to be true even for actual losses.

It is interesting to note that the lossy case has once again illustrated the advantages of a "half" TCC, which appeared naturally in our first table describing contracts matching dispatch. With losses, injection vectors are also the most convenient way to represent sets of rights, since it would be impossible in the presence of losses to decompose an incremental contract allocation into a set of symmetrical pairs. We would now claim that this nodal-vector view of TCC's is a more natural one. Interestingly, from the market participants perspective, these nodal units are equivalent to forward contracts at individual nodes. Thus TCCs can be interpreted as a special form of forward contracts, as was argued by Oren, et. al. [11]. We now see that the difference between the standard forward approach, and the TCC approach does not lie in the nature of the financial instrument, but in the allocation rule. We now turn to that allocation rule and the possibility that contracts match dispatch.

6 The problems with assuming rights match dispatch

We have shown that transmission congestion contracts awarded under a feasibility rule for new expansions can provide the incentives to deter some bad modifications to the grid when the existing transmission contracts match the dispatch of the system. When a network market is "completely contracted", with all spot transactions having an offsetting forward contract, then no party will have an incentive to make a detrimental modification to the grid under this system. In this section we discuss the implications of these two possibilities: aggregate matching and complete contracting.

The possibility that the allocated set of transmission contracts matches the dispatch of the system is the less heroic of the two. Of course this can only be true on average since dispatch is constantly changing while the set of allocated contracts presumably will not be. The fact that the highest potential revenue from a set of contracts is reached when those contracts match the dispatch provides a powerful incentive to match contracts to an expected dispatch. This assumes that the dispatch itself is not being influenced by a desire to *mismatch* dispatch and transmission contracts, a possibility warned against by Wu, et al[17].

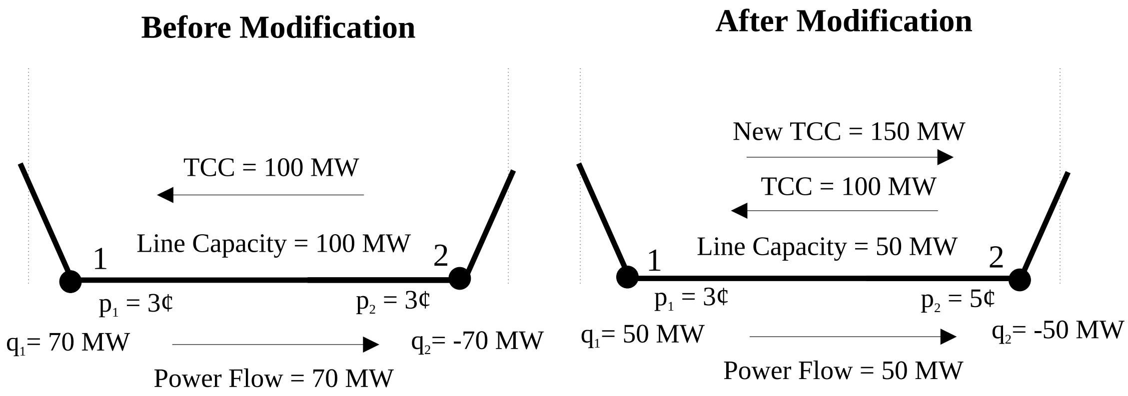

Figure 2: "Worthless" transmission contracts can give bad incentives

A potential weakness of this "money on the table" argument is the fact that, while matching the optimal dispatch maximizes contract revenues, other sets of rights may produce an equal payout. In particular, it is easily imaginable that agents may hold excess contracts on uncongested paths. These contracts would yield no current revenue but may do so in the future as load approaches capacity on that path. Our analysis does not address such a possibility, but several examples lead us to believe that excess existing rights can give an outsider an incentive to do something detrimental to the grid. These "bad" incentives appear to arise when rights of no value are being held in a direction counter to actual power flow, but may well appear in other circumstances.

The example of figure **??** illustrates this point. Before the modification, some agent holds a transmission contract $t = (-100, 100, \ldots, 0)$ for 100 MW from node 2 to node 1. This contract is feasible, but does not match the actual injections at these two nodes. The contract earns no revenue since the nodal prices at 1 and 2 are equal. The transmission contract revenue of this contract allocation could therefore be as great (but not greater) as the allocation which matches the dispatch. A modification which effectively reduces the capacity of the line between 1 and 2 to 50 MW congests that line and drives the nodal spot price at 2 up to 5 cents. Because of the existing 100 MW of contracts from 2 to 1, the agent who made this detrimental modification can receive a 150 MW transmission contract from 1 to 2, cancelling out the existing contracts and reaching the line's new capacity of 50 MW. This new contract earns \$3000. The detrimental modification was therefore profitable, even for an outsider. Note that the holder of the existing 100 MW right loses \$2000. This indicates that if maximizing existing contract revenue doesn't provide enough incentive to match contracts to dispatch, potential exposure to future losses may.

The possibility that a nodal market would be completely contracted seems considerably less likely. In the case of traditional TCCs, this would imply contracts for differences matching each TCC as well as individual supply and consumption. If the half-TCCs of section 5 were utilized, the presence of a complete set of matching CFDs would not be necessary since an individual could "match" his dispatch at one node using only a half-TCC. Even if contracting were not complete, some significant percentage of contracts matched to actual dispatch could potentially provide a serious enough penalty to prevent bad investments.

The desirability of having contracts match dispatch as closely as possible reveals a benefit of classifying contracts by time-of-use. The trade-off here lies in the accuracy of a contract versus the liquidity of a market for any given contract, or contracts in general. The analysis of this paper demonstrates one potential benefit of sub-dividing contracts temporally. For example a transmission contract might represent injections for daytime weekdays in the summer.

Another key aspect in the dynamics of grid expansions which is not treated in the analysis above is the *timing* of contract allocations. One question to be resolved should such a system be adopted is whether the feasibility analysis on new transmission contracts should be performed as soon as an investment is proposed, or only when it is completed. If agents are allowed to exchange rights for other rights which are also feasible under an existing grid, there could be a flurry of such swaps made just before the contracts for the new expansion are allocated. Just how much, if any, of this "interim" contract trading should be allowed will need to be resolved.

7 Conclusions

A properly functioning competitive electricity market should produce locational prices which differ according to the effects of transmission congestion and losses on the optimal dispatch of the system. When the revenues earned by the owners of transmission assets are linked to these locational price differences, there is a potential incentive on the part of transmission owners to increase or even create congestion in the system rather than mitigate it. The manner in which ownership of transmission "property" in a competitive electricity market is defined will play a key role in shaping those incentives. Many of the currently proposed concepts of transmission property, such as "link-based rights" do provide an incentive to create congestion in a network.

One prominent concept of transmission ownership is the contract network regime utilizing transmission congestion contracts. In this paper, we have presented the most intuitive method for allocating new property under a contract network and analyzed the resulting investment incentives of such a system. We do not advocate or oppose the contract network concept. We only seek to shed light on the myriad of claims and speculations about the incentives produced by this approach. We have shown that, if TCCs are awarded according to the concept of feasible dispatch, the contract network system can eliminate some of the investment incentive problems that arise under other transmission property concepts. The complete elimination of these perverse incentives, however, relies upon conditions which are extremely unlikely. These conditions involve the matching of agents' contract positions in this market with their actual spot consumption or supply.

If the implied dispatch corresponding to the allocated set of transmission contracts matches, on aggregate, the actual dispatch of the system, then agents with no current commercial interest in the network cannot profit from grid investments which create congestion. While this may seem a modest achievement, it is one that is not accomplished by other proposed property rights schemes. If transmission contracts and contracts for differences matches the actual dispatch on an individual, rather than aggregate level, then no party could find it profitable to create congestion in the grid. While the exact matching of transmission contracts to the dispatch of the system is virtually impossible, particularly if contracts encompass large blocks of time, institutional adjustments could be made to the allocation of contracts or contract revenues to help improve the degree of matching. The prospect of contracts matching dispatch on an individual basis is even more unlikely.

The concept of awarding property on the basis of feasible nodal injections goes further towards capturing the external costs of reducing network capacity than any other approach examined to date. It is important to note that the advantageous investment incentive aspects from the contract network regime are due more to the process of allocation of rights than to the nature of the property rights themselves. It is possible that these beneficial properties could be transported to other forms of network property.

8 Appendix

The revenue adequacy theorem, was first proven by Hogan [7] for lossless networks. At that time he conjectured that it was true for lossey networks as well. His proof of the lossless case depends on the set of feasible dispatches, FD, being convex. He conjectured that FD was also convex for lossey networks and that this would allow the same proof. However, defining the set of feasible lossy dispatches, FLD, so that it is convex requires a small trick, as can be seen from this simple example. Consider the one line network from i to j in which a feasible dispatch is 100MW injected at i and 90MW recovered at j. Since the zero dispatch is always feasible, for FLD to be convex it must include the power-averaged dispatch (50,45). But this implies a loss of exactly half that of the (100,90) dispatch and we know that loss is proportional to the square of the power flow, so the loss should be only 1/4 as much. Therefore the dispatch (50,45) is not feasible (though (50, 47.5) is) unless we allow for the "disposal" of unused power. This is equivalent to relaxing the energy balance constraint to be an inequality rather than equality constraint. The expansion of the FLD to include such wastefull dispatches presents no problem, because, with power costly to produce, such a dispatch will never appear as a solution to the optimal dispatch problem. We now show that the FLD with the relaxed energy balance constraint is convex when using the DC load flow with quadratic losses approximation.

Following Hogan, and Scwheppe et. al, we designate bus 0 as the swing bus, with output q_0. Let $\underline{q}$ be the $n-1$ vector of injections $(q_1, \ldots, q_{n-1})$. Considering power flows, $K(\underline{q})$, losses, $L(\underline{q})$, and thermal line limts $\overline{z}$[10], the optimal dispatch problem is

$$\begin{aligned} &\max_{q_0,\underline{q}} W(q) \\ \text{s.t. } &q_0 + e^t\underline{q} - L(\underline{q}) = 0 \qquad \text{(energy balance)} \\ &K(\underline{q}) \le \overline{z} \qquad \text{(thermal flow limits)} \end{aligned} \tag{4}$$

Using the DC load flow approximation with a quadratic estimate of losses (see [15], appendix D), we can define $K(\underline{q}) = K\underline{q}$ and $L(\underline{q}) = \underline{q}^t L\underline{q}$. By relaxing the energy balance constraint, we can rewrite 4 as

$$\begin{aligned} &\max_{q_0,\underline{q}} W(q) \\ \text{s.t. } &q_0 + e^t\underline{q} - \underline{q}^t L\underline{q} \ge 0 \\ &K\underline{q} \le \overline{z} \end{aligned} \tag{5}$$

Since losses are always non-negative, we know that $\underline{q}^t L\underline{q} \ge 0$. The matrix L is therefore positive semi-definite. The constraint set formed by the relaxed energy balance constraint in (5) is therefore convex. Since the line flow constraints are linear, the overall feasible set, S, defined by (5) is also convex.

To prove thereom (1), we first assume that it is false. In this case, there exists a contract vector T, corresponding to a feasible dispatch, such that $p^* \cdot T > -p^* \cdot q^*$. Because of the definition of contract set T, the corresponding dispatch is $D = -T$, so we have $p^*(D - q^*) < 0$. At equilibrium the optimal nodal prices are given by, $p_i^* = -\frac{\partial W(q^*)}{\partial q_i^*}$, or in vector notation, $p^* = -\nabla W(q^*)$. Substituting for p^* in our inequality we have:

$$\nabla W(q^*) \cdot (D - q^*) \quad > 0 \tag{6}$$

However, if q^* is optimal for (5), (6) cannot be true for any feasible direction $(D - q^*)$. Since the S is convex, D being feasible implies $V = D - q^*$ is a feasible direction. This contradicts 6, which contradicts our assumption, and proves theorem (1) for the DC load flow approximation with quadratic losses. □

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[10] Hogan also includes a voltage constraint, the feasible set of which is believed to be convex over the relevant region of q.

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