University of California, Davis
Department of Economics
Microeconomics

Date: June 25, 2012
Time: 5 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Please answer four questions (out of five)

Question 1.

Let there be L goods (which may be interpreted as varieties of a particular product, with the understanding that the consumer may wish to consume many varieties), with price vector $(p_1,..., p_L) \in \Re^L_{++}$. Define the *arithmetic mean of prices* as

$$P^A(p_1,..., p_L) = \frac{\sum_{k=1}^{L} p_k}{L},$$

and the *quadratic mean of prices* as

$$P^Q(p_1,..., p_L) = \sqrt{\frac{\sum_{k=1}^{L} [p_k]^2}{L}},$$

a convex function of $(p_1,\ldots p_L)$.

Consider the function:

$$\theta : \Re^L_{++} \to \Re : \theta(p_1,..., p_L) = P^A(p_1,..., p_L) + c\left[P^A(p_1,..., p_L) - P^Q(p_1,..., p_L)\right],$$

where the parameter c is a nonnegative real number. Define

$$f : \Re^{L+1}_{++} \to \Re : f(p_1,..., p_L, u) = \theta(p_1,..., p_L)u.$$

1.1. Show that $P^A(p,..., p) = P^Q(p,..., p) = p$ and that, in general, $P^Q \geq P^A$. (Remark. Here and in what follows, p denotes a positive real number, not a vector.)

1.2. Show that f displays all the properties of an expenditure function (i. e., of the value function of the consumer's expenditure minimization problem), except possibly being increasing in u or in some prices.

1.3. Let $c = 0$. Argue that in this case f is a legitimate expenditure function. What type of preferences generate this expenditure function?

1.4. We return to the general case where $c \geq 0$, but we assume that c and $(p_1,\ldots, p_L)$ are such that f satisfies all the properties of a legitimate expenditure function. What can you say about the underlying preferences?

1.5. We maintain the assumptions that $c \geq 0$, and that c and $(p_1,\ldots,p_L)$ are such that f satisfies all the properties of a legitimate expenditure function. Obtain the Walrasian demand function for good j.

1.6. Assume now, as it is often assumed in the monopolistic competition literature, that when expressing her demand for good j, the consumer perceives the price indices P^A and P^Q as given, i. e., independent of the single price p_j: such a perception is approximately justified if L is large and p_j is not too large relative to the other prices. For given wealth w, graph the resulting Walrasian demand curve for good j in the (x_j, p_j) space.

1.7. Compute the price elasticity of the demand curve obtained in 1.6 above. What is the value of this elasticity at a point where the prices of all goods are the same (as is often the case at the equilibria of symmetric models of monopolistic competition)?

1.8. Comment, in particular comparing your answers to questions 1.4 and 1.6 above.

Question 2.

Postulate the expected utility hypothesis, and assume the decision maker's von Neumann-Morgenstern-Bernoulli (vNMB) utility function is

$$u : X \to \Re : u(x) = b\left[a + \frac{x}{c}\right]^{1-c} \quad , \qquad (2.1)$$

where *a*, *b* and *c* are real numbers, $c \neq 0$, satisfying

$$\frac{b[1-c]}{c} > 0 , \qquad (2.2)$$

and the domain *X* is defined as

$$X := \{x \in \Re_{++} : a + \frac{x}{c} > 0\} . \qquad (2.3)$$

2.1. Argue that the decision maker is strictly risk-averse.

2.2. Show that the preferences described by the following vNMB utility functions are special cases of the ones defined by (2.1)

$$(A). \quad u^R : X \to \Re : u^R(x) = \frac{x^{1-\eta}}{1-\eta}, X = \Re_{++}, \eta > 0, \eta \neq 1. \qquad (2.4)$$

$$(B). \quad u^N : X \to \Re : u^N(x) = \frac{[x+k]^{1-\eta}}{1-\eta}, X = \{x \in \Re_{++} : x > -k\}, k \in \Re, \eta > 0, \eta \neq 1. \qquad (2.5)$$

$$(C). \quad u^Q : X \to \Re : u^Q(x) = -[g - x]^2, X = \{x \in \Re_{++} : x < g\}. \qquad (2.6)$$

2.3. Show that the preferences described by the following vNMB utility function are a limit case of the ones defined by (2.4)

$$(D). \quad u^D : X \to \Re : u^D(x) = \ln x, X = \Re_{++}. \qquad (2.7)$$

2.4. For which values of the parameters *a*, *b*, *c* does the vNMB of (2.1) display:

- Increasing Absolute Risk Aversion?

- Decreasing Absolute Risk Aversion?

- Increasing Relative Risk Aversion?
- Decreasing Relative Risk Aversion?

Illustrate when possible by referring to the utility functions (2.4)-(2.7)

2.5. Postulate that nature randomly chooses one of two "states or the world", s_1 or s_2. State s_1 is the bad state, which occurs with probability π, whereas s_2 is the good state, which occurs with probability $(1 - \pi)$. For $j = 1, 2$, denote by x_j consumption contingent to state j and by P_j the price of consumption contingent to state j. Assume that $P_1/ P_2 > \pi/[1 - \pi]$. Denote by W the wealth of the consumer. Consider only the case where the solution to the consumer optimization problem is interior to the consumption set, which is defined by the property that both x_1 and x_2 must belong to the domain of the relevant vNMB utility function

> **2.5.1.** For the vNMB utility function (2.1), show that the wealth expansion path is a straight line, with positive slope, in (x_1, x_2) space.
>
> **2.5.2.** For the vNMB utility functions given by (2.4) to (2.7), does an increase in the wealth of the consumer lead her to bear absolutely more risk? Relatively more risk? Explain.

2.6. Assume a population of individuals that have different wealth levels but are otherwise identical, with identical preferences defined by a vNMB utility function of the form (2.1), facing the same probability π of the bad state, and the same prices P_1 of P_2 for the contingent consumption commodities. Does their aggregate demand for the contingent consumption commodities admit a positive representative consumer? Argue your answer.

Question 3

When some agents are either envious or altruistic, their utility depends not only on their own consumption but also on the consumption of other agents. Intuition might suggest that such external effects will lead to inefficiency of the market system. Let us show that this intuition may not always be correct.

(a) Let us first study *envy*. Consider a two-agent, two-good economy in which the agents have endowments $\omega^1 = (\omega^1_x, \omega^1_y), \omega^2 = (\omega^2_x, \omega^2_y)$. Let $w = (w_x, w_y) = (\omega^1_x + \omega^2_x, \omega^1_y + \omega^2_y)$ denote the aggregate endowment.

(i) Suppose that the utility functions of the two agents are

$$u_1(x_1, y_1, x_2, y_2) = \ln(x_1 y_1) - a \ln(x_2 y_2),\ a > 0, \quad u_2(x_2, y_2) = \ln(x_2 y_2)$$

Thus agent 1 is envious of agent 2: the higher the utility of agent 2, the lower the utility of agent 1. Agent 2 only cares about her own consumption. Write the constrained maximum problem which gives the Pareto optima of this economy, maximizing the utility of agent 1 subject to a guaranteed utility for agent 2 and the feasibility constraints. Show that the guaranteed utility constraint for agent 2 is always binding. Find the FOCs which must be satisfied at a Pareto optimum.

(ii) Show that the competitive equilibria of this economy, in which each agent takes prices and the action of the other agent as given are the same as the competitive equilibria of the economy in which both agents have Cobb Douglas utility i.e. the utility of agent 2 is the same as in (i), and agent 1's utility is $\tilde{u}_1(x_1, y_1) = \ln(x_1 y_1)$. [No need to calculate to show this.] Thus competitive equilibria exist for all possible endowments.

(iii) Show that a competitive equilibrium of the economy in (i) satisfies the FOCs for Pareto optimality. Explain why you cannot deduce from this that the competitive equilibrium is Pareto optimal.

(iv) Thus to establish Pareto optimality we need to do a direct proof. Let us show this in a slightly more general setting. Assume that the first agent has a utility function of the form

$$u_1(x_1, y_1, x_2, y_2) = h(\tilde{u}_1(x_1, y_1), u_2(x_2, y_2))$$

where u_2 is the utility function of agent 2, and $\tilde{u}_1$ denotes the utility that agent 1 derives from his own consumption. $\tilde{u}_1$ and u_2 are increasing. The function $h : \mathbb{R}^2 \to \mathbb{R}$ is increasing in the first variable (the own consumption of the agent) and decreasing in the second variable (the consumption of the other agent). Prove that a competitive equilibrium is Pareto optimal. [Hint: Appropriately modify the usual proof by contradiction.] Thus envy by agent 1 does not lead to inefficiency of a competitive equilibrium.

(b) Let us now study *altruism.*

(i) Consider the same two-agent economy as in a(i) but now assume that $a < 0$: since it is easier to work with positive parameters you may use $b = -a$ as the parameter in the utility of agent 1 who is now altruistic. Consider the maximum problem characterizing the Pareto optima of the economy. Show that there exists a utility level v_2^* such that if $v_2 < v_2^*$ then the constraint $u_2(x_2, y_2) \geq v_2$ is not binding. Conclude that there is no Pareto optimal allocation for which $u_2(x_2, y_2) < v_2^*$. Explain the intuition for this result.

(ii) Use (i) to show that the First Theorem of Welfare Economics does not hold in economies in which agents are altruistic. To make the argument, you can use the Edgeworth box for the economy $\mathcal{E}(\tilde{u}_1, u_2, \omega^1, \omega^2)$.

(iii) What type of government intervention would increase efficiency when the competitive equilibrium is not Pareto optimal?

Question 4

Consider an economy with a private and a public good in which agents have different tastes for the public good: we study an example where the tastes give rise to an extreme form of the free-rider problem. To fix ideas suppose there are $I \geq 2$ agents indexed by $i = 1, \dots I$. Each agent initially owns one unit of the private good. The private good can be transformed into a public good with a constant returns technology, one unit of private good producing one unit of public good. The preferences of the agents are given by

$$u_i(x_i, y_i) = x_i - \frac{a_i}{y}, \qquad i = \dots, I$$

where x_i is agent i's consumption of the private good and y is the quantity of the public good in the economy, and the preference parameters $(a_i)_{i=1}^{I}$ satisfy

$$0 < a_I < \dots < a_2 < a_1 < 1$$

(a) To determine the equilibrium with voluntary contributions we first study each agent's optimal contribution (from her point of view). Let z_i denote agent i's contribution and let $Z^{-i} = \sum_{j \neq i} z_j$ denote the contribution of all other agents. Derive the optimal contribution of agent i given Z^{-i}.

(b) Show that there is a unique voluntary contributions equilibrium in which agent 1 is the only agent contributing to the provision of the public good: all other agents free ride on agent 1's contribution. [Hint: show this first for the case $I = 2$.]

(c) Find the Pareto optimal level y^* of the public good and show that $\hat{y} < y^*$: explain why the difference $y^* - \hat{y}$ increases with the number of agents.

(d) Suppose instead that the level of the public good is decided by majority voting i.e. $\tilde{y}$ is the level chosen if there is no other level y' that a majority of agents prefer to $\tilde{y}$, each agent contributing $\tilde{y}/I$ to the cost.

(i) Find the most preferred level of the public good for agent i.

(ii) Find the level $\tilde{y}$ chosen by majority voting.

(iii) Assume that I is odd. Show that if the median and the mean of the numbers $a_1, \dots, a_I$ coincide then $\tilde{y} = y^*$ while if the median is smaller (respectively greater) than the mean then $\tilde{y} < (>) y^*$. Compare this result with the outcome with voluntary contributions and comment.

Micro Prelim June 2012
Question 5

Two travelers returning home from a remote island discover that the identical antiques they bought have been smashed in transit. The airline wants to reimburse the travelers in an amount equal to the price they paid. However, this is private information to the travelers. In principle, they could claim that they paid more than they actually did and the airline would have no way of finding out. The airline manager, having taken a course in game theory, sets up the following game to elicit the value of the article. The two travelers are put in separate rooms and asked to independently fill in a compensation claim between $2 and $100 (in increments of $1; thus only integer amounts). The airline will then reimburse each traveler at the smallest of the two claims; in addition, if the claims differ, a reward of $2 will be paid to the person making the smaller claim and a penalty of $2 is deducted from the reimbursement of the larger claimant (for example, if Player 1 claims $78 and Player 2 claims $55 then Player 1 gets 55 − 2 = $53 while Player 2 gets 55 + 2 = $57).

For parts (a)-(e) assume that each player is selfish and greedy in the sense that each player only cares about how much money he gets and prefers more money to less.

(a) Represent this game using a matrix. Since I don't expect you to have the time or inclination to fill in almost 10,000 entries, show the pattern by showing the payoffs when the travelers claim $2, $3, $4, $98, $99 and $100 (and every combination of these). [Hint: exploit the symmetry of the problem.]

(b) Do the players have a dominant strategy?

(c) What do you get by applying the procedure of iterative elimination of weakly dominated strategies?

(d) Find all the pure-strategy Nash equilibria of this game and prove that they are Nash equilibria and that there are no other Nash equilibria.

(e) Assuming that the true value of the damaged article is indeed between $2 and $100, does this mechanism (or game) induce the travelers to reveal the true price they paid?

(f) Given a strategy profile (*m*,*n*) define the regret of Player 1, denoted by $R_1(m,n)$, as the difference between the maximum amount of money he could have got (given that player 2 chose *n*) and the amount of money he actually gets at (*m*,*n*).

(f.1) Find $R_1(m,n)$ for all $(m,n) \in \{2,...,100\} \times \{2,...,100\}$.

(f.2) What are the maximum possible regret and the minimum possible regret (over the entire set $\{2,...,100\} \times \{2,...,100\}$)?

(f.3) Defining player 2's regret $R_2(m,n)$ similarly (as the difference between the maximum amount of money she could have got - given that Player 1 chose *m* - and the amount of money she actually gets at (*m*,*n*)), write the matrix of regrets. As before you only need to show the pattern by showing regrets when the travelers claim $2, $3, $4, $96, $97, $98, $99 and $100 (and every combination of these; please note that, relative to part a, I have added $96 and $97). [Hint: exploit the symmetry of the problem.]

(f.4) Suppose that each player chooses in a MinMax way in the following sense: for each claim the player finds the maximum possible regret and then chooses a claim that minimizes the maximum regret. What strategy profiles are consistent with both players choosing in a MinMax way?

(f.5) Suppose that it is common knowledge between the players that they choose in a MinMax way (as explained in f.4). Apply the following iterative deletion procedure: in each round eliminate all those claims that are weakly dominated in the current matrix. What do you get by applying this iterative deletion procedure?