University of California, Davis
Department of Economics
Microeconomics

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Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Please answer any four questions (out of five)

Question 1. The number of firms in monopolistic competition

Consider an industry with J firms, each of them producing a distinct variety of a differentiated product, with the understanding that consumers may wish to consume all varieties. Denote by $(p_1,\ldots, p_J)$ the vector of the prices charged by the J firms.

The demand for variety j, addressed to firm j by consumers, is given by the expression

$$f_j(p_1,..., p_J) = a[P(p_1,..., p_J)]^{\sigma-1} p_j^{-\sigma}, \qquad (1)$$

where the price index $P(p_1,..., p_J)$ is defined as

$$P(p_1,..., p_J) = \left[\sum_{j=1}^{J} [p_j]^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \qquad (2)$$

and the magnitude a, related to the wealth of the consumers in the economy, is taken as given by all firms.

1.1. What do (1) and (2) suggest on the preferences of the consumers?

For $j = 1,.., J$, firm j's cost function is given by

$$C(y_j) = \begin{cases} 0 \text{ , if } y_j = 0, \\ p_0 y_j + F \text{ , if } y_j > 0, \end{cases}$$

where $p_0 > 0$, $F > 0$, and y_j denotes the amount of output (variety j) produced by firm j.

1.2. Comment on the cost function. Is this technology consistent with perfectly competitive (price-taking) behavior? Explain.

In what follows, assume that $\sigma > 1$, and let firm j ($j = 1,\ldots, J$) maximize its profits under the perception that the price index P is fixed, but that firm j can freely choose its own price p_j . (In other words, firm j views the prices of other firms as given and, in addition, considers the price index P as independent from its own price, a perception that is approximately justified if J is large and p_j is not too large relative to other prices.)

1.3. Write and solve firm j's profit maximization problem. Interpret the parameter σ in terms of the competitiveness of the industry.

1.4. It is often postulated in monopolistic competition analysis that the entry of firms drives profits to zero. Accordingly, let the number J of firms (and varieties) adjust so that, for each firm, output is positive but profits are zero. Determine the number J of firms as a function of a, F and σ under this condition. (For simplicity, here and in what follows we let J be a positive real number: we do not restrict J to be an integer.)

1.5. The aim is now to relate the number of firms to the parameter σ. As indicated above, a is related to the wealth of buyers in the industry. We now assume that, through general-equilibrium effects that are left unspecified, a is actually a function $\tilde{a}(\sigma)$, with elasticity satisfying

$$0 \le \tilde{a}'(\sigma)\frac{\sigma}{\tilde{a}(\sigma)} < 1, \qquad (3)$$

which admits the possibility that $\tilde{a}'(\sigma) = 0$.

In addition, we consider two cases concerning F.

CASE 1. F is a fixed parameter. This is the conventional case.

CASE 2. F is decreasing in σ, more precisely

$$\tilde{F}(\sigma) = \frac{b}{\sigma},$$

where $b > 0$. The motivation for Case 2 is that F is an advertising cost that varies with σ.

Does the number of firms increase or decrease with σ in Case 1? In Case 2? Argue your answer, and comment.

Question 2

Consider an economy with 2 consumption goods, good 1 and 2, and two factors of production, capital and labor. There is a representative agent who owns the factors of production, normalized to one unit each. Thus $\omega = (1, 1)$. The representative agent's utility function is $u(x_1, x_2) = \sqrt{x_1 x_2}$. Good 1 is produced by a firm with production function $y_1 = (\min\{k_1, l_1\})^2$. Good 2 is produced by the representative firm 2 with production function $y_2 = \sqrt{k_2 l_2}$. The representative agent and the representative firm 2 actually stand in for a large number of agents and firms producing good 2, so we can assume that consumers take prices as given, and firm 2 maximizes profit taking prices as given. However since there are increasing returns in the production of good 1, it is more efficient to produce it in a single firm, which is thus a "natural" monopoly. Actually even if we assumed that firm 1 maximized profit taking prices as given we could not find a competitive equilibrium. Thus we assume that, to prevent it from using its market power, a planner takes control of firm 1 instructing it to minimize cost (taking factor prices as given) and sell its output at marginal cost. The planner subsidizes the firm for its losses, using lump-sum taxation of the consumer to finance the subsidy. An equilibrium where the consumer and firm 2 behave as in a competitive equilibrium, firm 1 minimizes cost, the price of good 1 is the marginal cost of production, and markets clear, is called a marginal cost pricing equilibrium.

(a) First let us justify some assertions of the text above. Show that there is no solution to maximizing the profit of firm 1 if it takes input and output prices as given. Then show that if the firm sells its output at marginal cost (once the cost is minimized) it makes a loss. Calculate the loss as a function of the quantity produced and the input prices.

(b) Find the Pareto optimal allocation for this economy. (When you assert that a function is maximized, be sure that you justify your assertion.)

(c) Express formally the conditions on the actions of the consumer and firms and the market prices that must be satisfied in order that $\big((\bar{x}_1, \bar{x}_2), (\bar{y}_1, \bar{k}_1, \bar{l}_1), (\bar{y}_2, \bar{k}_2, \bar{l}_2), (\bar{p}_1, \bar{p}_2, \bar{r}, \bar{w})\big)$ is a marginal cost pricing equilibrium.

(d) Find the marginal cost pricing equilibrium. Is it Pareto optimal?

(e) Using a simple Robinson Crusoe economy with two goods and increasing returns in production explain as best as you can the normative property of the marginal cost pricing equilibrium.

Question 3

Consider an economy with two goods, labor and a consumption good. There are two firms producing the consumption good from labor. Firm 1's production function is $y_1 = L_1$ and firm 2's production function is $y_2 = 2\sqrt{L_2}$, where y_j is the production of firm j and L_j is the labor used by firm j, $j = 1, 2$. Firm 2's technology produces chemical emissions in quantity proportional to its level of production. The units are such that the amount z of pollution associated with a level of production y_2 is $z = y_2$. There are two (types of) agents ($i = 1, 2$) with the same utility function

$$u(x, \ell, z) = \sqrt{x\ell} - \frac{z}{8},$$

where x is the amount of consumption good, ℓ is the amount of time used as leisure, and z is the level of pollution. Each agent is endowed with 3 units of time which can be used as labor or leisure. Agent 1 owns firm 1 while agent 2 owns firm 2.

(a) Calculate the competitive equilibrium with externality for this economy. Show that the level of pollution is $\bar{z} = 2$.

(b) Suppose the government can limit pollution by imposing a limit z on the emissions of firm 2. (If firm 2 produces more emissions than it is allowed, the firm is closed by the government and the owner does not receive any profit). Firms and consumers behave like in (a) (except that firm 2 needs to take the limit z into account). Find the resulting equilibrium as a function of the ceiling z on emissions. Let $u_1(z)$ and $u_2(z)$ the utility of both agents at this equilibrium.

(c) Show the following properties of the equilibrium found in (b)

(i) There is z^* such that if $z^* \leq z < 2$ both agents are better off than in the competitive equilibrium of (a) (the "laisser-faire" equilibrium)

(ii) There is a level $\hat{z} > z^*$ such that for any z in $[\hat{z},2]$ there is an equilibrium with $\tilde{z} \in [z^*, \hat{z}]$ which is better from the point of view of both agents.

(iii) In the interval $[z^*, \hat{z}]$ there is no consensus among the two agents on the optimal level of pollution. Explain where the disagreement comes from.

[Hint: study carefully the function $\phi(z) = u_2(z) - u_2(2)$]

(d) Suppose that in addition to controlling the level of emissions of firm 2 the government can redistribute income among the agents with lump sum taxes and subsidies. Show that the set of achievable utilities can be written as

$$u_1 + u_2 = \phi(z)$$

Find the level of emission which maximizes $u_1 + u_2$ and relate it to the discussion in question (c).

Question 4.

There are two parties to a potential lawsuit: the owner of a chemical plant and a supplier of safety equipment. The chemical plant owner (from now on called the *plaintiff*) alleges that the supplier (from now on called the *defendant*) was negligent in providing the safety equipment. The defendant knows whether or not he was negligent, while the plaintiff does not know. The plaintiff believes that there was negligence with probability q. These beliefs are common knowledge between the parties. The plaintiff has to decide whether or not to sue. If she does not sue then nothing happens and both parties get a payoff of 0. If the plaintiff sues then the defendant can either offer an out-of-court settlement of $\$S$ or resist. If the defendant offers a settlement, the plaintiff can either accept (in which case her payoff is $\$S$ and the defendant's payoff is $-\$S$) or go to trial. If the defendant resists then the plaintiff can either drop the case (in which case both parties get a payoff of 0) or go to trial. If the case goes to trial then legal costs are created in the amount of $\$P$ for the plaintiff and $\$D$ for the defendant. Furthermore (if the case goes to trial), the judge is able to determine if there was negligence and, if there was, requires the defendant to pay $\$W$ to the plaintiff (and each party has to pay its own legal costs), while, if there was no negligence, the judge will drop the case without imposing any payments to either party (but each party has to pay its own legal costs). Each party is "selfish and greedy" (that is, only cares about its own wealth and prefers more to less) and is risk neutral.

Assume the following about the parameters: $0 < q < 1$, $0 < D < S$, $0 < P < S < W - P$.

(a) Represent this situation of incomplete information using states and information partitions (the only two agents are the plaintiff and the defendant). Be clear about what each state represents or describes.

(b) Apply the Harsanyi transformation to represent the situation in part (a) as an extensive-form game. [Don't forget to subtract the legal expenses from each party's payoff if the case goes to trial.]

(c) Write down all the strategies of the plaintiff.

(d) Prove that there is no **pure-strategy** weak sequential equilibrium which (1) is a **separating** equilibrium and (2) involves suing.

You might want to answer question (f) before question (e) to gain some insight, but you will be duplicating some effort.

(e) For what values of the parameters (q, S, P, W, D) are there **pure-strategy** weak sequential equilibria which (1) are **pooling** equilibria and (2) involve suing? Consider all types of pooling equilibria and prove your claim.

(f) For the case where $q = \frac{1}{12},\ P = 70,\ S = 80,\ W = 100$ find all the pure-strategy weak sequential equilibria which (1) are **pooling** equilibria and (2) involve suing.

Question 5.

Is it better for the shareholders to remunerate CEOs with profit-sharing contracts or other types of contracts? One of the issues is, of course, moral hazard on the part of the CEO. In this question we consider instead a different issue, namely a strategic one.

Consider the following situation. There are two individuals, 1 and 2. Each individual is the sole owner of a firm in a Cournot duopoly. The inverse demand function is $P = 60 - Q$ and both firms have the same cost function characterized by a constant marginal cost equal to 12 and zero fixed cost. Individuals 1 and 2 **simultaneously and independently** decide whether to appoint a manager with a profit-sharing contract (the manager of firm *i* gets the fraction α of the profit of firm *i*) or a revenue-sharing contract (the manager of firm *i* gets the fraction α of the revenue of firm *i*), where $0 < \alpha < 1$. The value of α is given exogenously (is not a choice variable). The contracts are then made public (that is, they become common knowledge among everybody in the industry) and afterwards the managers simultaneously compete in output levels. The objective of each manager is to maximize her own income. The objective of each owner is to maximize his own net income (= profit of the firm minus the payment to the manager).

- **(a)** Sketch the extensive-form game.
- **(b)** Find the pure-strategy subgame-perfect equilibria of this game for every value of α.
- **(c)** In the past, each owner used to run the firm himself and the industry was a Cournot duopoly. Now they have delegated the running of the firms to managers. Suppose that α is small (say $\alpha = \frac{1}{100}$). Has delegation led to an increase or a decrease in the income of the owners of the firms?