Answer Keys for Micro Prelim August 24, 2017

Question 1 - Answer Key

Assume that your grandmother pays for your studies. You probably agree that this assumption is more realistic than most assumptions we make in economics given the cost of living and our "generous" financial support at UC Davis. Imagine the following story: You pay a visit to your grandmother to thank her for her truly generous support. She asks what path-breaking insights you have learned in 200A. After mentioning that the central topic of 200A is consumer theory you proudly tell her that you learned that when prices go up then demand may go up or down. She looks at you with some consternation and starts to wonder whether supporting your studies makes any sense. After all, she knew this all along without having studied 200A. Since you rely on her support, her doubts naturally alarm you. You desperately try to search your memory for something less trivial to tell her. You vaguely recall the "compensated law of demand":

For any $(p, w), (p', w')$ with $w' = p' \cdot x(p, w)$ we have

$$(p' - p) \cdot (x(p', w') - x(p, w)) \leq 0$$

with strict inequality if $x(p, w) \neq x(p', w')$.

(With this notation, $p \in \mathbb{R}^L_{++}$ is a price vector, where L is the number of commodities in the economy; $w \in \mathbb{R}_+$ denotes the consumer's wealth; $x(p, w)$ denotes Walrasian demand at prices p and wealth w. Throughout, we assume that $x(p, w)$ is single-valued in $\mathbb{R}^L$ for any p and w.)

a.) In order to discuss the "compensated law of demand" with your grandmother, you need a verbal interpretation of it. Give a verbal interpretation of the "compensated law of demand".

    Prices and demand move in opposite directions if wealth is adjusted after the price change so as to make the initial consumption bundle just affordable at new prices.

b.) Your grandmother asks in what sense the "compensated law of demand" is a law. You recall the following proposition that we proved in class:

Proposition: Suppose that the Walrasian demand function $x(p, w)$ is homogeneous of degree zero and satisfies Walras' law. Then $x(p, w)$ satisfies the weak axiom of revealed preference if and only if it satisfies the compensated law of demand.

We recall the weak axiom of revealed preference: The Walrasian demand function $x(p, w)$ satisfies the weak axiom of revealed preference if for any (p, w) and (p', w'),

$$\text{if } p \cdot x(p', w') \leq w \text{ and } x(p', w') \neq x(p, w), \text{ then } p' \cdot x(p, w) > w'.$$

Let's see whether you are able to prove the first direction of the proposition: If $x(p, w)$ satisfies the weak axiom of revealed preference then it satisfies the compensated law of demand. I will guide you step-by-step through a proof:

1. Consider the case in which $x(p', w') = x(p, w)$. This should be easy.
2. Consider now the non-trivial case $x(p', w') \neq x(p, w)$. Rewrite

   $$(p'-p)\cdot(x(p', w') - x(p, w)) = p'\cdot(x(p', w') - x(p, w)) - p\cdot(x(p', w') - x(p, w)).$$

   Consider separately each term of the right-hand side. Derive the signs of each of the two terms using (some) assumptions of the proposition and the fact that we assumed $w' = p' \cdot x(p, w)$ for the compensated law of demand. To derive the sign of the second term of the right-hand side you will need to make use of the weak axiom of revealed preference.
3. Put everything together to conclude the proof of the first direction.

`This is just as in MWG, proof of Proposition 2.F.1 (i), p. 31.`

c.) Given the characterization of the compensated law of demand by the weak axiom of revealed preference, you need to explain to your grandmother the meaning of the weak axiom of revealed preference. Provide a verbal interpretation of the weak axiom of revealed preference.

`If the new consumption bundle is affordable at old prices and the old wealth level and the new consumption bundle differs from the old one, then it must be that the old consumption bundle is not affordable anymore at new prices and the new wealth level.`

d.) The weak axiom of revealed preference may be viewed as a condition on the "consistency of consumer choice". We may be inclined to label a consumer violating this condition as being "irrational" (unless a more complex setting is considered). So the above proposition characterizes the compensated law of demand in terms of consistency of consumer choice. But is it really a characterization in terms of rational consumer choice in the sense of having complete and transitive preferences over consumption bundles? That is, is the following conjecture true?

Conjecture: Suppose that the Walrasian demand function $x(p, w)$ is homogeneous of degree zero and satisfies Walras' law. Then $x(p, w)$ satisfies the compensated law of demand if and only if the consumer has complete and transitive preferences over consumption bundles.

Consider both directions of the conjecture separately. Moreover, consider the special case of $L = 2$ separately.

(You don't have to present a detailed proof. You can make use of results learned in class. The line of arguments should be clear.)

`The famous Hicks example discussed in class and in MWG Example 2.F.1 shows that consumer choice satisfying the weak axiom of revealed preference may violate transitivity. Thus, we conclude` from above proposition that it is not true that if $x(p,w)$ satisfies the compensated law of demand then the consumer must have transitive preferences. Conversely, a consumer with complete and transitive preferences over consumption bundles satisfies the strong axiom of revealed preference. The strong axiom of revealed preference implies the weak axiom of revealed preference. Hence, complete and transitive preferences imply the compensated law of demand (under the additional assumptions of the conjecture).

In case of $L=2$ the strong and weak axioms of revealed preference coincide. Moreover, we also know from class that the strong axiom of revealed preference is essentially equivalent to a complete and transitive preference over consumption bundles. Thus, the conjecture is true for $L=2$.

Question 2: Answer Key

1. Suppose not: there exists a competitive equilibrium (p, x, y) such that (x, y) is not in the core of the economy. Fix $\mathcal{H} \subseteq \mathcal{I}$ and $(\hat{x}^i, \hat{y}^i)_{i \in \mathcal{H}}$ that satisfy the four conditions in the definition of the core. By the fourth condition, local non-satiation of preferences and individual rationality of the competitive allocation, it must be true that

$$p \cdot \hat{x}^i \geqslant p \cdot w^i + \sum_j s^{i,j} p \cdot y^j = p \cdot w^i + p \cdot y^i,$$

for all $i \in \mathcal{H}$. By the third condition, this inequality is strict for some $i \in \mathcal{H}$, so, adding up,

$$\sum_{i \in \mathcal{H}} p \cdot \hat{x}^i > \sum_{i \in \mathcal{H}} p \cdot w^i + \sum_{i \in \mathcal{H}} p \cdot y^i.$$

By the first condition and profit maximality at the equilibrium allocation, $p \cdot y^i \geqslant p \cdot \hat{y}^i$ for all $i \in \mathcal{H}$. Substituting and factoring p out, we get that

$$p \cdot \sum_{i \in \mathcal{H}} \hat{x}^i > p \cdot \sum_{i \in \mathcal{H}} (w^i + \hat{y}^i),$$

which contradicts the second condition.

2. (a) Let $\bar{w}$ be Pareto efficient, and let $(p, \bar{x})$ be a competitive equilibrium of economy $\{\mathcal{I}, (u^i, \bar{w}^i)_{i \in \mathcal{I}}\}$. By individual rationality, each $\bar{x}^i$ solves

$$\max_x \left\{ u^i(x) : p \cdot x \leqslant p \cdot \bar{w}^i \right\} q. \qquad (*)$$

This implies that for each i, $u^i(\bar{x}^i) \geqslant u^i(\bar{w}^i)$. Since $\bar{x}$ is a feasible allocation and $\bar{w}$ is efficient, none of the previous inequalities can be strict, so in effect they all hold with equality. This implies that each $\bar{w}^i$ solves program $(*)$ and, since market clearing becomes tautological, that $(p, \bar{w})$ is a competitive equilibrium of the same economy.

   (b) Given $\hat{x}$, which is Pareto efficient, it suffices to let $\hat{w}^i = \hat{x}^i$ for all i. Feasibility of this policy is immediate, and the result follows from part (a).

Question 3: Answer Keys

1. For each i, just let $\hat{w}^i = \bar{x}^i$. Condition (a) is immediate by market clearing in the local equilibria. Condition (b) follows from the fact that any competitive equilibrium allocation is weakly Pareto superior to the endowments of the economy.

2. The only assumption needed is that, in the original economy $\{\mathcal{I}, (u^i, w^i)_{i\in\mathcal{I}}\}$,

$$\sum_i w^i \gg I \times w_*.$$

   The formal claim is that, under such assumption, *there exists* $(\hat{w}^i)_{i\in\mathcal{I}}$ *such that:*

   (a) $\sum_i \hat{w}^i = \sum_i w^i$*; and*

   (b) in economy $\{\mathcal{I}, (u^i, \hat{w}^i)_{i\in\mathcal{I}}\}$ *the distribution of wealth is biased in favour only of agent* 1 *and permits subsistence*

   To prove the claim, one simply lets $\hat{w}^i = w_*$ for all $i \geqslant 2$, and

$$\hat{w}^1 = \sum_i w^i - (I - 1) \times w_*.$$

   Feasibility of this reallocation is immediate, as is the fact that it permits subsistence. That $\hat{w}^1 \gg \hat{w}^i$ for all $i \geqslant 2$ implies the bias.

ANSWER TO QUESTION 4

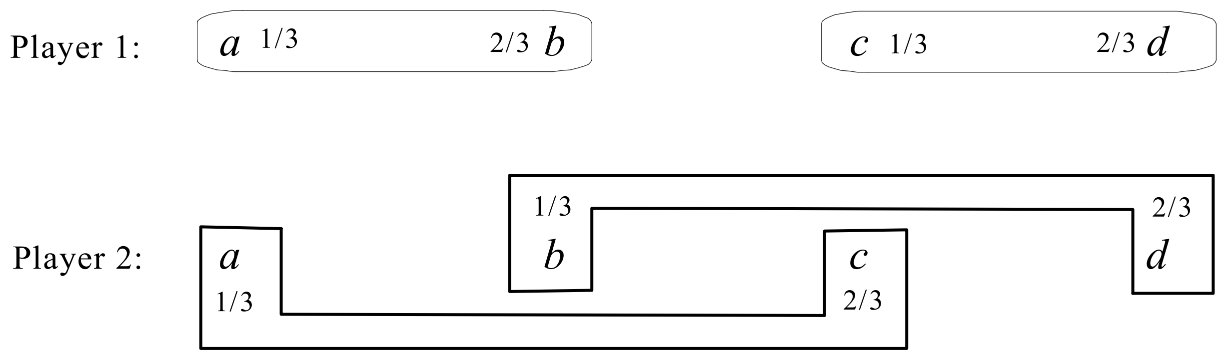
(a) Let M_i mean that player i is a morning person and N_i mean that she is a night person. Then there are four possible states: $a = (M_1, M_2)$, $b = (M_1, N_2)$, $c = (N_1, M_2)$ and $d = (N_1, N_2)$.

		Player 2: E	Player 2: L
Pl 1	E	5,5	5,4
	L	4,5	0,0

		Player 2: E	Player 2: L
Pl 1	E	5,1	5,4
	L	4,1	0,0

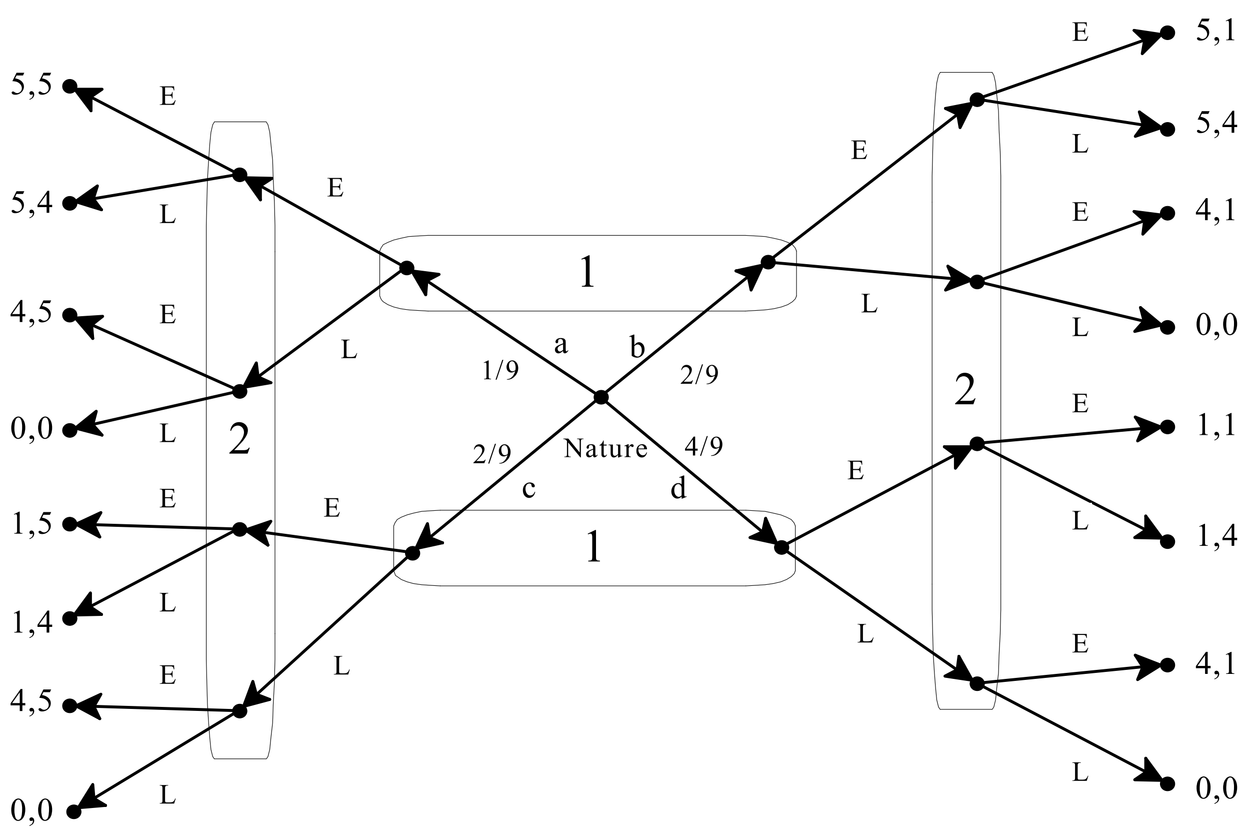
		Player 2: E	Player 2: L
Pl 1	E	1,5	1,4
	L	4,5	0,0

		Player 2: E	Player 2: L
Pl 1	E	1,1	1,4
	L	4,1	0,0



(b) The common prior is $\begin{pmatrix} a & b & c & d \\ \frac{1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{4}{9} \end{pmatrix}$

(c) The game is as follows:

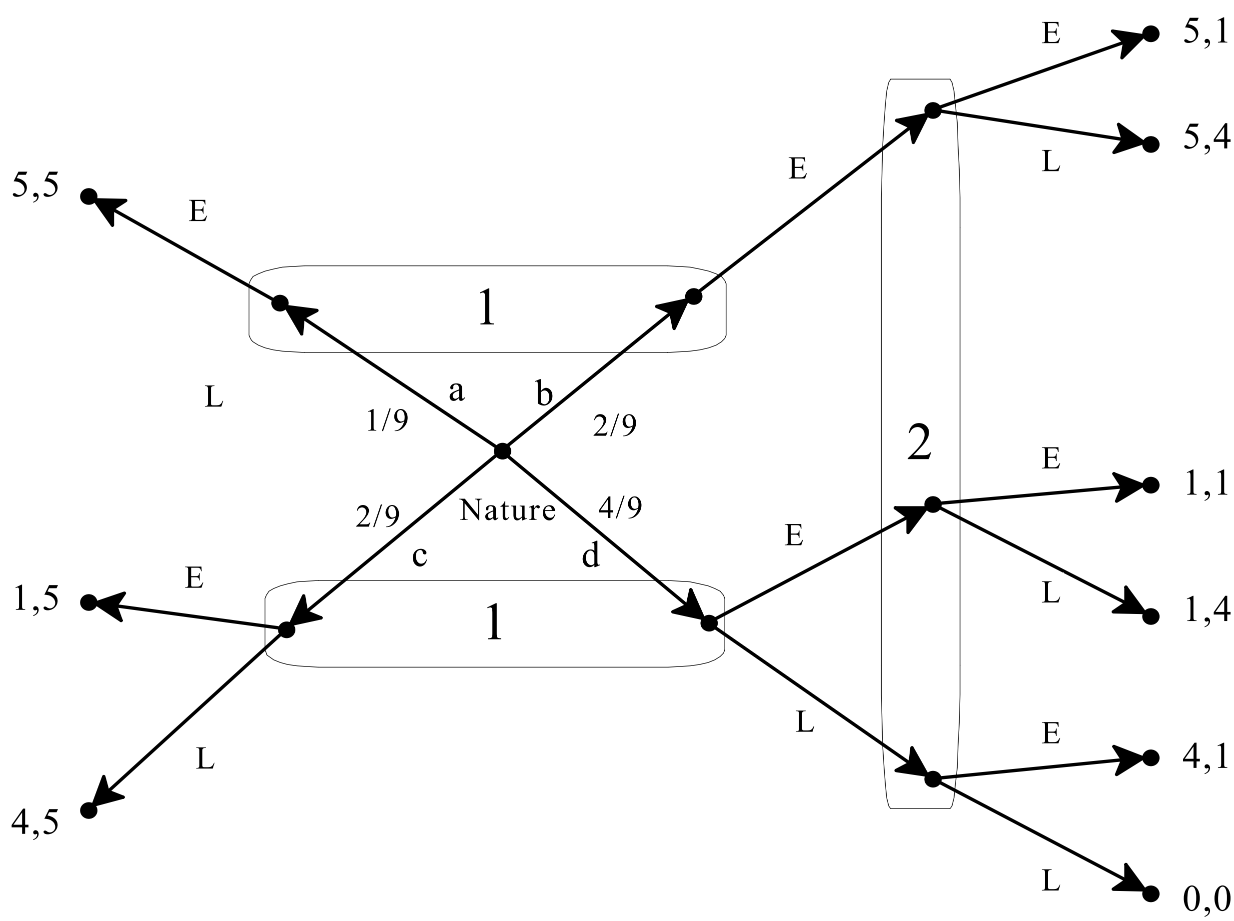


(d) **(d.1)** Player 2 has four pure strategies: (E,E), (E,L), (L,E) and (L,L).

(d.2) A possible mixed strategy for Player 1 is $\begin{pmatrix} (E,E) & (E,L) & (L,E) & (L,L) \\ p & q & r & 1-p-q-r \end{pmatrix}$ with your favorite choice of *p, q* and *r*.

(d.3) A possible behavior strategy for Player 2 is $\left(\begin{array}{cc|cc} E & L & E & L \\ p & 1-p & q & 1-q \end{array}\right)$ where the first choice refers to the top information set and the second to the bottom information set.

(e) At Player 1's top information set *E* is a strictly dominant choice, thus it must be selected with probability 1 at every WSE. Similarly, for Player 2 at her information set on the left *E* is a strictly dominant choice, thus it must be selected with probability 1 at every WSE. Thus we can simplify the game as follows:



Now, there is no WSE where Player 1 chooses E with probability 1 at the bottom information set, because, by Bayes' rule, Player 2 at the right information set would have to assign probability $\frac{1}{3}$ to the top node and probability $\frac{2}{3}$ to the third node from the top and thus with *E* she would get $\frac{1}{3}1+\frac{2}{3}1=1$ and with *L* she would get $\frac{1}{3}4+\frac{2}{3}4=4$ so that sequential rationality requires that she play *L* there [hence Player 2's best response to Player 1's (*E,E*) is (*E,L*)]; but then *E* is not sequentially rational for Player 1 at the bottom information set, because his beliefs there would have to be probability $\frac{1}{3}$ on the left node and probability $\frac{2}{3}$ on the right node, so that *E* would yield $\frac{1}{3}1+\frac{2}{3}1=1$ while *L* would yield a higher payoff, namely $\frac{1}{3}4+\frac{2}{3}0=\frac{4}{3}$.

Now let us consider the case where Player 1 chooses L with probability 1 at the bottom information set. Then, by Bayes' rule, Player 2 at the right information set has to assign probability $\frac{1}{3}$ to the top node and probability $\frac{2}{3}$ to the bottom node and thus with *E* she would get $\frac{1}{3}1+\frac{2}{3}1=1$ and with *L* she would get $\frac{1}{3}4+\frac{2}{3}0=\frac{4}{3}$ so that sequential rationality requires that she play *L* there [hence Player 2's best response to Player 1's (*E,L*) is (*E,L*)]. It remains to check if *L* is sequentially rational for Player 1 at the bottom information set: *E* gives $\frac{1}{3}1+\frac{2}{3}1=1$ and *L* gives a higher payoff, namely $\frac{1}{3}4+\frac{2}{3}0=\frac{4}{3}$. Hence *L* is sequentially rational. Thus we have found a pure-strategy WSE: $\big((E,L),(E,L)\big)$ with the following beliefs: at both information sets of Player 1 probability $\frac{1}{3}$ on the left node and probability $\frac{2}{3}$ on the right node, at both information sets of Player 2 probability $\frac{1}{3}$ on the top node and probability $\frac{2}{3}$ on the bottom node. We have also shown that there is no other pure-strategy WSE.

Now let us check if there are any mixed-strategy WSE. It cannot be sequentially rational for Player 1 to play both E and L with positive probability at his bottom information set, because it would require him to get the same payoff from E and from L. However, E gives him $\frac{1}{3}1+\frac{2}{3}1=1$, no matter what Player 2 plays at her right information set and L gives him $\frac{1}{3}4+\frac{2}{3}(\text{some convex combination of 0 and 4})\geq\frac{4}{3}$, so that L is always better than E. Thus Player 1 must play L with probability 1 at his bottom information set. Given this, we saw above that for Player 2 L is strictly better than E at her right information set and thus it cannot be sequentially rational for her to mix either. It follows that the pure-strategy WSE found above is the unique WSE.

(f) Since, whatever strategy profile is played, every information set is reached with positive probability, every WSE is also a sequential equilibrium. To prove that the WSE found in part (f) is a sequential equilibrium, consider the sequence of completely mixed behavior strategy profiles whose n^{th} element is $\sigma_n=\left(\begin{array}{cc|cc||cc|cc} \multicolumn{2}{c}{\text{Pl. 1 top}} & \multicolumn{2}{c}{\text{Pl. 1 top}} & \multicolumn{2}{c}{\text{Pl. 2 left}} & \multicolumn{2}{c}{\text{Pl. 2 right}} \\ E & L & E & L & E & L & E & L \\ 1-\frac{1}{n} & \frac{1}{n} & \frac{1}{n} & 1-\frac{1}{n} & 1-\frac{1}{n} & \frac{1}{n} & \frac{1}{n} & 1-\frac{1}{n} \end{array}\right)$ whose corresponding system of beliefs for Player 2 (using Bayes' rule) is a follows (where $\ell_1,\ell_2,\ell_3,\ell_4$ are the nodes at Player 2's left information set, numbered from top to bottom, and r_1,r_2,r_3,r_4 are the nodes at Player 2's right information set, numbered from top to bottom):

$$\mu_n=\left(\begin{array}{cccc|cccc} \ell_1 & \ell_2 & \ell_3 & \ell_4 & r_1 & r_2 & r_3 & r_4 \\ \frac{1}{3}(1-\frac{1}{n}) & \frac{1}{3}\frac{1}{n} & \frac{2}{3}\frac{1}{n} & \frac{2}{3}(1-\frac{1}{n}) & \frac{1}{3}(1-\frac{1}{n}) & \frac{1}{3}\frac{1}{n} & \frac{2}{3}\frac{1}{n} & \frac{2}{3}(1-\frac{1}{n}) \end{array}\right)$$

Clearly, $\lim_{n\to\infty}\sigma_n=\left(\begin{array}{cc|cc||cc|cc} \multicolumn{2}{c}{\text{Pl. 1 top}} & \multicolumn{2}{c}{\text{Pl. 1 top}} & \multicolumn{2}{c}{\text{Pl. 2 left}} & \multicolumn{2}{c}{\text{Pl. 2 right}} \\ E & L & E & L & E & L & E & L \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$ and

$$\lim_{n\to\infty}\mu_n=\left(\begin{array}{cccc|cccc} \ell_1 & \ell_2 & \ell_3 & \ell_4 & r_1 & r_2 & r_3 & r_4 \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & \frac{2}{3} \end{array}\right).$$