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Microeconomics

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Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Please answer four questions (out of five)

Question 1. The number of firms in monopolistic competition

ANSWER KEY

Consider an industry with J firms, each of them producing a distinct variety of a differentiated product, with the understanding that consumers may wish to consume all varieties. Denote by $(p_1,\ldots, p_J)$ the vector of prices charged by the J firms.

The demand for variety j, addressed to firm j by consumers, is given by the expression

$$f_j(p_1,\ldots,p_J) = a[P(p_1,\ldots,p_J)]^{\sigma-1} p_j^{-\sigma}, \qquad (1)$$

where the price index $P(p_1,\ldots,p_J)$ is defined as

$$P(p_1,\ldots,p_J) = \left[\sum_{j=1}^{J} [p_j]^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \qquad (2)$$

and the magnitude a, related to the wealth of the consumers in the economy, is taken as given by all firms.

1.1. What do (1) and (2) suggest on the preferences of the consumers?

ANSWER. Demand functions as those given by (1) and (2) are related to CES preferences. If a were the wealth of a consumer, then (1) would be her Walrasian demand for the utility function

$$u(x_1,\ldots,x_J) = \left(\sum_{j=1}^{J} x_j^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \sigma \in (-\infty,0)\cup(0,1)\cup(1,\infty). \qquad \text{(A1)}$$

Because a positive representative consumer exists under identical, homothetic preferences, (1) would be the aggregate demand from a consumption section with aggregate wealth a and individual utility functions given by (A1).

But a is not assumed to coincide with wealth: it could for example be a fraction of aggregate wealth, in which case the preferences of the consumers could also involve goods other than varieties (1,…, J), with a (A1) providing a subutility function for these varieties.

For j = 1,.., J, firm j's cost function is given by

$$C(y_j) = \begin{cases} 0 \text{ , if } y_j = 0, \\ p_0 y_j + F \text{ , if } y_j > 0, \end{cases}$$

where $p_0 > 0$, $F > 0$, and y_j denotes the amount of output (variety j) produced by firm j.

1.2. Comment on the cost function. Is this technology consistent with perfectly competitive (price-taking) behavior? Explain.

ANSWER. This technology displays constant marginal cost, at level p_0, and a fixed, avoidable (i. e., nonsunk) cost, at level F. This implies that the average cost is decreasing. Perfectly competitive behavior is incompatible with positive production with this technology. Trivially, one could have price-taking profit maximization with market price less than or equal to p_0 and zero output.

In what follows, assume that $\sigma > 1$, and let firm j (j = 1,…, J) maximize its profits under the perception that the price index P is fixed, but that firm j can freely choose its own price p_j . (In other words, firm j views the prices of other firms as given and, in addition, considers the price index P as independent from its own price, a perception that is approximately justified if J is large and p_j is not too large relative to other prices.)

1.3. Write and solve firm j's profit maximization problem. Interpret the parameter σ in terms of the competitiveness of the industry.

ANSWER. The profit maximization problem of the firm can be written in two steps

Step 1. Choose p_j order to maximize $[p_j - p_0][p_j]^{-\sigma} aP^{\sigma-1} - F$.

Step 2. If profits are nonnegative at the solution p_j^* to Step 1, then set the price equal to p_j^* and the quantity equal to $[p_j^*]^{-\sigma} aP^{\sigma-1}$. Otherwise, set the quantity equal to zero.

(Remark. If profits are zero at the solution p_j^* to Step 1, then both $y_j = [p_j^*]^{-\sigma} aP^{\sigma-1}$ and $y_j = 0$ maximize profits; we will focus on the positive solution in this case.)

The solution to Step 1 satisfies

$$[p_j]^{-\sigma} aP^{\sigma-1} - \sigma[p_j - p_0][p_j]^{-\sigma-1} aP^{\sigma-1} = 0,$$

or dividing through by $[p_j]^{-\sigma-1} aP^{\sigma-1}$, $p_j - \sigma[p_j - p_0] = 0$, which yields $p_j = \dfrac{\sigma}{\sigma-1} p_0$, same for all firms, i. e. ,

$$p_j = p := \frac{\sigma}{\sigma-1} p_0 . \qquad \text{(A2)}$$

The markup = [PRICE-MARGINAL COST]/PRICE is then

$$\frac{p - p_0}{p} = \frac{1}{\sigma}, \qquad \text{(A3)}$$

equal to the degree of monopoly, which is the reciprocal of the absolute value of the price elasticity of the demand curve faced by the firm. Hence, the higher σ, the higher the degree of competitiveness in the industry.

1.4. It is often postulated in monopolistic competition analysis that the entry of firms drives profits to zero. Accordingly, let the number *J* of firms (and varieties) adjust so that, for each firm, output is positive but profits are zero. Determine the number *J* of firms as a function of *a*, *F* and σ under this condition. (For simplicity, here and in what follows we let *J* be a positive real number: we do not restrict *J* to be an integer.)

ANSWER.

Using (2) and (A2), we can write

$$P(p,..., p) = \left[Jp^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = J^{\frac{1}{1-\sigma}} p,$$

so that, from (1), the equilibrium output of a firm becomes

$$a\left[J^{\frac{1}{1-\sigma}} p \right]^{\sigma-1} p^{-\sigma} = aJ^{-1} p^{\sigma-1} p^{-\sigma} = \frac{a}{Jp},$$

and the zero-profit condition becomes $[p - p_0]\dfrac{a}{Jp} = F,$

or, using (A3),

$$J = \frac{a}{\sigma F}. \qquad \text{(A4)}$$

1.5. The aim is now to relate the number of firms to the parameter σ. As indicated above, *a* is related to the wealth of buyers in the industry. We now assume that, through general-equilibrium effects that are left unspecified, *a* is actually a function $\tilde{a}(\sigma)$, with elasticity satisfying

$$0 \le \tilde{a}'(\sigma)\frac{\sigma}{\tilde{a}(\sigma)} < 1, \qquad (3)$$

which admits the possibility that $\tilde{a}'(\sigma) = 0$.

In addition, we consider two cases concerning *F*.

CASE 1. *F* is a fixed parameter. This is the conventional case.

CASE 2. *F* is decreasing in σ, more precisely

$$\tilde{F}(\sigma) = \frac{b}{\sigma},$$

where $b > 0$. The motivation for Case 2 is that *F* is an advertising cost that varies with σ.

Does the number of firms increase or decrease with σ in Case 1? In Case 2? Argue your answer, and comment.

ANSWER. Case 1. From (A4), $\tilde{J}(\sigma) = \frac{\tilde{a}(\sigma)}{\sigma F}$, and

$$\tilde{J}'(\sigma) = \frac{\tilde{a}'(\sigma)\sigma F - F\tilde{a}(\sigma)}{\sigma^2 F^2} = \frac{\tilde{a}(\sigma)}{\sigma^2 F}\left[\tilde{a}'(\sigma)\frac{\sigma}{\tilde{a}(\sigma)} - 1\right] < 0,$$

by (3), which is somewhat counterintuitive: higher competitiveness (higher σ) yields fewer firms. An explanation is that, as σ increases, the markup goes down, making it harder to cover the fixed costs unless the number of firms decreases and the output per firm increases.

Case 2. Now $\tilde{J}(\sigma) = \frac{\tilde{a}(\sigma)}{\sigma F} = \frac{\tilde{a}(\sigma)}{b}$, nondecreasing in σ, and actually increasing if $\tilde{a}'(\sigma) > 0$. This is more intuitive, but it relies on the assumption that, as competitiveness increases, the fixed cost goes down.

(a) Let $(p_1, p_2, r, w) \gg 0$ denote the price of good 1, good 2, capital and labor respectively. To maximize profit, firm 1 must first minimize cost which implies $k_1 = \ell_1$. The profit of firm 1 is then $\Pi_1 = p_1(\ell_1)^2 - (w+r)\ell_1$. $\frac{d\Pi_1}{d\ell_1} = 2p_1\ell_1 - (w+r)$, which is positive when $\ell_1 > \frac{w+r}{2p_1}$. When $\ell_1 \to +\infty$, $\Pi_1 \to +\infty$ and therefore there is no maximum for the profit.

The cost function is $c(y_1) = (r+w)\sqrt{y_1}$. If the firm sells y_1 at price $p_1 = \frac{r+w}{2\sqrt{y_1}} = c'(y_1)$, the profit is

$$\frac{r+w}{2\sqrt{y_1}}\, y_1 - (r+w)\sqrt{y_1} = -\frac{1}{2}(r+w)\sqrt{y_1}$$

The loss is thus $\frac{1}{2}(r+w)\sqrt{y_1}$, which increases with the scale of production.

(b) The Pareto optimal allocation must maximize

$$\sqrt{x_1 x_2}$$

subject to

$$x_1 \le k_1^2$$
$$k_1 \le \ell_1$$
$$x_2 \le \sqrt{k_2 \ell_2}$$
$$k_1 + k_2 \le 1$$
$$\ell_1 + \ell_2 \le 1$$
$$\ell_1 \ge 0 \quad \ell_2 \ge 0 \quad k_1 \ge k_2 \ge 0$$

Since the utility of the representative agent is 0 if one good is not produced while it can be positive, and since the utility is strongly monotone, the non-negativity constraints are not binding and the other constraints are binding, so that the P.O. allocation

maximizes $\sqrt{k_1^2 \sqrt{(1-k_1)(1-k_1)}}$ (since $k_1 = \ell_1$ & $y_1 = k_1^2$, $\ell_2 = k_2 = 1 - k_1$)

or equivalently maximizes $k_1^2(1-k_1) \equiv \varphi(k_1)$

$\varphi'(k_1) = 2k_1(1-k_1) - k_1^2 = 2k_1 - 3k_1^2$, which leads to the variations of φ of the form

	0		$2/3$		1
$\varphi'(k_1)$		$+$		$-$	
$\varphi(k_1)$	0	$\nearrow$	$\varphi(2/3)$	$\searrow$	

Thus the maximum is obtained for $k_1^* = \frac{2}{3} = \ell_1^*$ $y_1^* = \frac{4}{9}$

$k_2^* = \ell_2^* = \frac{1}{3}$ $y_2^* = \frac{1}{3}$.

(c) $\left((\bar{x}_1, \bar{x}_2), (\bar{y}_1, \bar{k}_1, \bar{\ell}_1)(\bar{y}_2, \bar{k}_2, \bar{\ell}_2)(\bar{p}_1, \bar{p}_2, \bar{r}, \bar{w})\right)$

is a marginal cost pricing equilibrium if

(i) $(\bar{k}_1, \bar{\ell}_1)$ minimizes $c_{\bar{r},\bar{w}}(\bar{y}_1)$

(ii) $\bar{p}_1 = c'_{\bar{r},\bar{w}}(\bar{y}_1)$ (marginal cost pricing)

(iii) $(\bar{y}_2, \bar{k}_2, \bar{\ell}_2)$ maximizes $\bar{p}_2 y_2 - \bar{r} k_2 - \ell_2$ s.t. $y_2 = \sqrt{k_2 \ell_2}$

(iv) $(\bar{x}_1, \bar{x}_2)$ maximizes $\sqrt{x_1 x_2}$ subject to

$\bar{p}_1 x_1 + \bar{p}_2 x_2 = \bar{r} + \bar{w} + \bar{\pi}_2 - \bar{T}$

where $\bar{\pi}_2$ is the profit of firm 2 (here 0) and $\bar{T}$ is the subsidy to firm 1

(v) $\bar{T} + \bar{p}_1 \bar{y}_1 - \bar{r}\bar{k}_1 - \bar{w}\bar{\ell}_1 = 0$ (tax = subsidy = loss of firm 1)

(vi) $\bar{y}_1 = \bar{x}_1$, $\bar{y}_2 = \bar{x}_2$, $\bar{\ell}_1 + \bar{\ell}_2 = 1$, $\bar{k}_1 + \bar{k}_2 = 1$

(d) firm 1 : from (a) $\bar{k}_1 = \bar{\ell}_1 = \sqrt{\bar{y}_1}$ $\bar{p}_1 = \dfrac{\bar{r}+\bar{w}}{2\sqrt{\bar{y}_1}}$

firm 2 : $\max\ \bar{p}_2 \sqrt{k_2 \ell_2} - \bar{w}\,\ell_2 + \bar{r}\,k_2$ (we look for an interior equilibrium)

FOCs : $\bar{p}_2 \dfrac{\sqrt{\ell_2}}{2\sqrt{k_2}} = \bar{r}$ $\bar{p}_2 = \dfrac{\sqrt{k_2}}{2\sqrt{\ell_2}} = \bar{w}$

$\Rightarrow \dfrac{\ell_2}{k_2} = \dfrac{\bar{r}}{\bar{w}}$ and $p_2^2 = 4\bar{r}\bar{w}$ (condition on prices imposed by constant returns to scale)

the scale of production adapts to the demand and

$\ell_2 = \sqrt{\dfrac{\bar{r}}{\bar{w}}}\, y_2$, $k_2 = \sqrt{\dfrac{\bar{w}}{\bar{r}}}\, y_2$.

Consumer income of the consumer $\bar{r}+\bar{w} - \dfrac{\bar{r}+\bar{w}}{2}\sqrt{\bar{y}_1}$

(the loss of the firm is subsidized by a lumpsum tax on the consumer and the profit of firm 2 is zero).

Cobb. Douglas demand :

$x_1 = \dfrac{\bar{r}+\bar{w} - \dfrac{\bar{r}+\bar{w}}{2}\sqrt{\bar{y}_1}}{2p_1}$ $x_2 = \dfrac{\bar{r}+\bar{w} - \dfrac{\bar{r}+\bar{w}}{2}\sqrt{\bar{y}_1}}{2p_2}$

market clearing : $\bar{x}_1 = \bar{y}_1 \iff$

$\bar{y}_1 = \dfrac{\bar{r}+\bar{w} - \dfrac{\bar{r}+\bar{w}}{2}\sqrt{\bar{y}_1}}{2\,\dfrac{\bar{r}+\bar{w}}{2\sqrt{\bar{y}_1}}} \iff \bar{y}_1 = \sqrt{\bar{y}_1}\left(1 - \dfrac{\sqrt{\bar{y}_1}}{2}\right)$

$\iff \sqrt{\bar{y}_1} = 1 - \dfrac{\sqrt{\bar{y}_1}}{2}$

$\iff \dfrac{3}{2}\sqrt{\bar{y}_1} = 1$

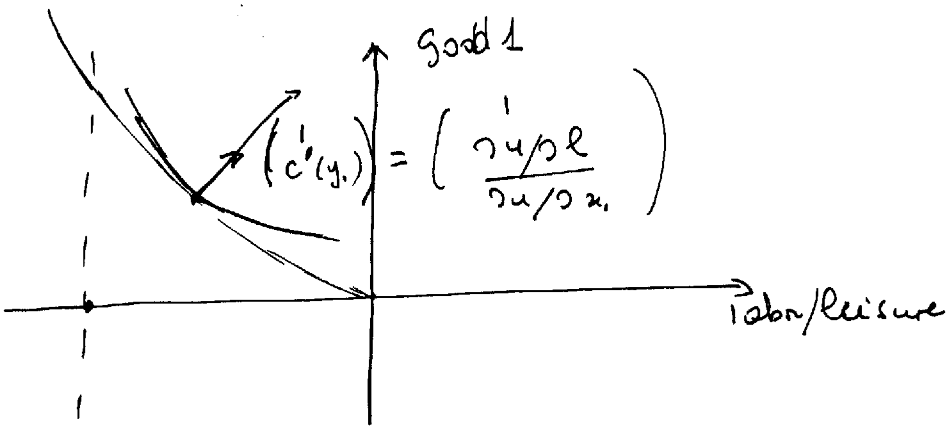
$$\boxed{\bar{y}_1 = \frac{4}{9}}$$

Thus $\bar{l}_1 = \bar{k}_1 = \frac{2}{3}$ $\quad \bar{k}_2 = \bar{l}_2 = \frac{1}{3} = \bar{y}_2$ $\quad \bar{r} = \bar{w}_4 = 1$ (normalization)

$\bar{p}_1 = \frac{2}{2 \times 2/3}$ $\quad \bar{p}_1 = \frac{3}{2}$ $\quad \bar{p}_2 = 2$

check: $\bar{x}_2 = \frac{2 - \frac{2}{2} \times \frac{2}{3}}{2 \times 2} = \frac{2 - \frac{2}{3}}{4} = \frac{1}{3}$

(e) Suppose that there are only 2 goods: labor and good 1 and one consumer providing the labor and consuming good 1. If the price of good 1 is the marginal cost of production, the rate of transformation in production and the marginal rate of substitution for the consumer are equalized, which is a necessary condition for Pareto optimality. However, since the production set is non convex, the condition is not sufficient so that marginal cost pricing equilibrium may or may not be Pareto optimal. In the example above the condition turns out to be sufficient



(a) competitive equilibrium

- firm 1 profit maximization with a positive level of production

imposes $p = w$

- firm 2 $\max_{L_2 \geq 0} 2p\sqrt{L_2} - wL_2 \Rightarrow L_2 = \frac{p^2}{w^2}$ $y_2 = 2\frac{p}{w} = z$

profit $\pi_2 = \frac{p^2}{w}$

since $p = w$ $L_2 = 1$ $y_2 = 2 = z$

- consumer 1 $\max \sqrt{x\,l}$ (takes z as given)

$px = w(3 - l) + 0$ with $p = w$

$x^1 = \frac{3}{2}$ $l^1 = \frac{3}{2}$ $L^1 = 3 - 3/2 = 3/2$

- consumer 2 $\max \sqrt{xl}$

$px = w(3 - l) + \frac{p^2}{w}$

$x^2 = \frac{1}{2}$ $\frac{3w + \frac{p^2}{w}}{p} = 2$ if $p = w$

$l^2 = 2$ $L^2 = 1$

(indices of consumers in superscripts, indices of firms in subscripts)

At equilibrium firm 2 uses 1 unit of labor and firm 1 units 3/2 units of labor. The total production is $3/2 + 2$, which equals the total demand. The level of pollution is $z = 2$.

At equilibrium $\bar{u}^1 = \sqrt{3/2 \times 3/2} - 2/8 = 3/2 - \frac{1}{4} = \frac{5}{4}$

$\bar{u}^2 = 2 - 2/8 = 2 - 1/4 = \frac{7}{4}$

(b) equilibrium with pollution limited at z.

- firm 1 still imposes $p = w$ that we normalize to 1
- firm 2 : if $z \leq 2$: max $2p\sqrt{L_2} - wL_2$ $\quad 2\sqrt{L_2} \leq z$

it is easy to see that it produces at the maximum allowed

$$L_2 = \frac{z^2}{4} \qquad y_2 = z \qquad \Pi_2 = pz - w\frac{z^2}{4} = z - \frac{z^2}{4} \quad (p = w = 1)$$

- consumer 1 : $x_1^1 = l^1 = 3/2 \quad L^1 = 3/2 \quad u^1 = \frac{3}{2} - \frac{z}{8}$
- consumer 2 $\quad x^2 = l^2 = \frac{1}{2}\left(3 + z - \frac{z^2}{4}\right) \quad L^2 = \frac{3}{2} - \frac{z}{2} + \frac{z^2}{8}$

At equilibrium firm 2 uses $\frac{z^2}{4}$ units of labor to produce z units of goods. Firm 1 uses $3/2 + 3/2 - \frac{z}{2} + \frac{z^2}{8} - \frac{z^2}{4}$ $= 3 - \frac{z}{2} - \frac{z^2}{8}$ to produce the same quantity of good and markets clear. The utility of agent 2 is $u^2 = \frac{1}{2}\left(3 + z - \frac{z^2}{4}\right) - \frac{z}{8}$

Thus $\boxed{u^1(z) = \frac{3}{2} - \frac{z}{8} \qquad u^2(z) = \frac{3}{2} + \frac{3z}{8} - \frac{z^2}{8}}$

(c) (i) If $z < 2$ agent 1 is better off than in the laissez faire equilibrium of (a), and the smaller z the better off agent 1 is.

For agent 2 $\quad u^2(z) - u^2(2) = \frac{3}{2} + \frac{3z}{8} - \frac{z^2}{8} - \frac{7}{4}$

$$= -\frac{1}{4} + \frac{3}{8}z - \frac{z^2}{8}$$

Let $\varphi(z) = -\frac{1}{4} + \frac{3}{8}z - \frac{z^2}{8} \qquad \varphi'(z) = \frac{3}{8} - \frac{z}{4}$

variations of φ:

z	0		z^*		$3/2$		2
$\varphi'(z)$		+				−	
$\varphi(z)$	$-\frac{1}{4}$	↗	0	↗	$\frac{1}{32}$	↘	0

Since φ is increasing from $-\frac{1}{4}$ to $+\frac{1}{32}$ when z varies from 0 to $3/2$, there exists $z^* \in [0, \frac{3}{2})$ such that, if $z^* \leq z < 2$, $\varphi(z) \geq 0$. Thus if $z^* \leq z < 2$ both agents are better off than in the laissez faire equilibrium

(ii) If $z \in (\frac{3}{2}, 2)$, there exists $\tilde{z} \in (z^*, 3/2)$ such that $\varphi(\tilde{z}) = \varphi(z)$. If pollution is limited at $\tilde{z} + \varepsilon$, agent 2 is strictly better off than if it is limited at z and agent 1 is also better since there is less pollution

(iii) However in the interval $[z^*, 3/2]$ the utility of agent 2 is increasing in z and the utility of agent 1 is decreasing so that the agents cannot agree on the level of pollution to impose. Agent 1 always want to reduce z to decrease the externality but agent 2 has two conflicting interests: as a consumer he/she wants to decrease z but as a owner of the firm he/she wants to increase z to produce more and increase the profit. In the interval $(z^* \; 3/2)$ the income effect is stronger than the externality effect.

(d) If the planner can make a transfer t from agent 2 to agent 1, then

$$x^1 = \ell^1 = \frac{3+t}{2} \qquad u^1 = \frac{3+t}{2} - \frac{z}{8}$$

$$x^2 = \ell^2 = \frac{3 + z - z^2/4 - t}{2} \qquad u^2 = \frac{3+z-z^2/4 - t}{2} - \frac{z}{8}$$

Eliminating t

$$u_1 + u_2 = \frac{6 + z - z^2/4}{2} - \frac{2z}{8} = \frac{3}{2} + \frac{z}{4} - \frac{z^2}{8}$$

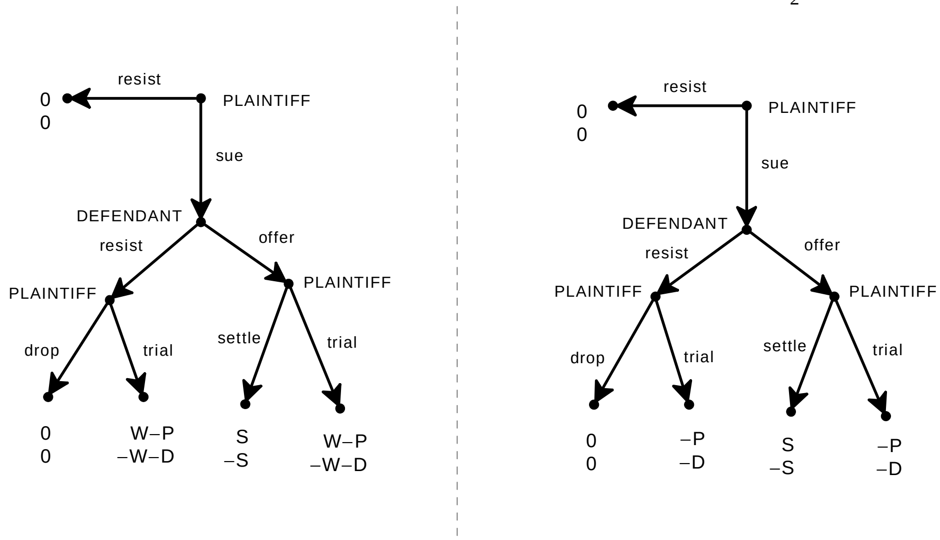
$$\psi(z) = \frac{3}{2} + \frac{z}{4} - \frac{z^2}{8} \qquad \psi'(z) = \frac{1}{4} - \frac{z}{4}$$

The maximum of $u_1 + u_2$ is for $z = 1$, in which case $\psi(z) = u^2(1) - u^2(0) = 0$. That is $u_1 + u_2$ is maximized for $z = z^* = 1$. Thus $t < 0$ and the transfer goes from agent 1 to agent 2 to compensate the loss of profit due to a severe reduction in the production of good 2.

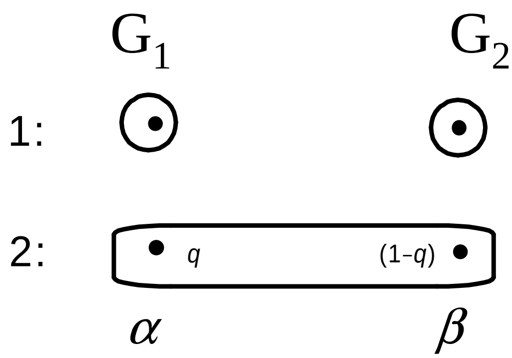
ANSWER KEY FOR QUESTION 4

(a) Let G_1 and G_2 be the following games (in G_1 the defendant is negligent and in G_2 he is not):

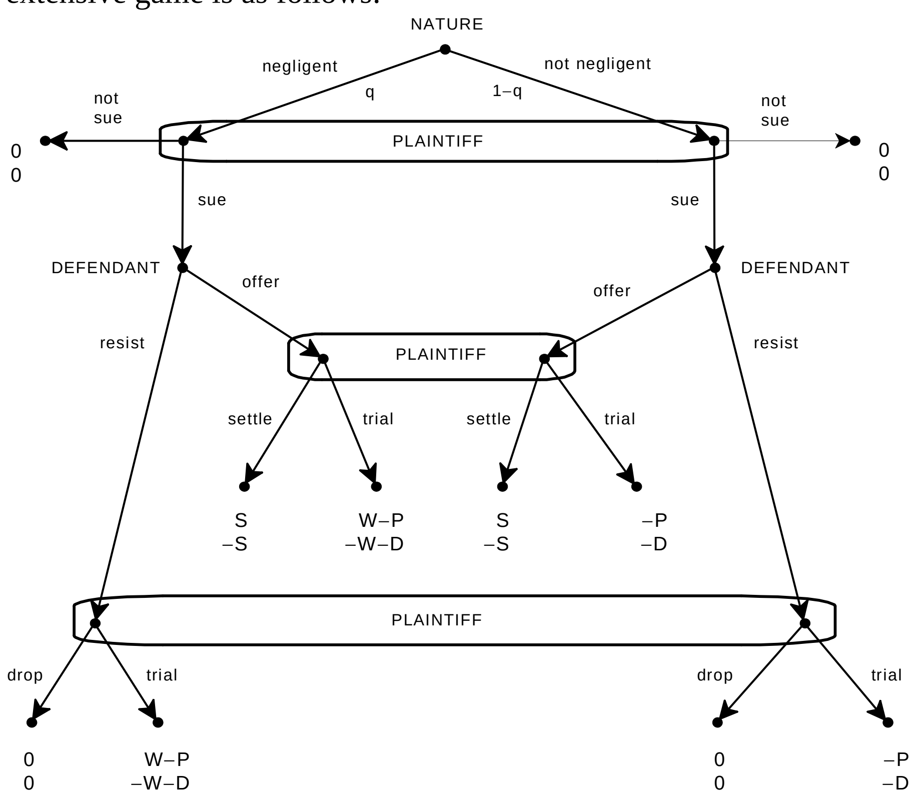
GAME G_1 GAME G_2



Then the situation can be represented as follows (state α is the state where the defendant is negligent and state β is the state where the defendant is not negligent):



(b) The extensive game is as follows:



(c) Plaintiff's strategies: (1) (sue; if offer settle; if resist drop), (2) (sue; if offer settle; if resist go to trial), (3) (sue; if offer go to trial; if resist drop), (4) (sue; if offer go to trial; if resist go to trial), (5) (not sue; if offer settle; if resist drop), (6) (not sue; if offer settle; if resist go to trial), (7) (not sue; if offer go to trial; if resist drop), (8) (not sue; if offer go to trial; if resist go to trial). [The last four can be considered as one "plan of action" which we can call "Not sue".]

(d) There are two possibilities for a separating equilibrium: (1) the defendant's strategy is "resist if negligent and offer if not negligent" and (2) the defendant's strategy is "offer if negligent and resist if not negligent". In both cases we assume that the plaintiff's strategy involves suing with probability 1.

Consider case (1) first. By Bayes' rule, at the bottom information set the plaintiff must attach probability 1 to the negligent type and thus, by sequential rationality, must choose "trial" (because $W - P > 0$). Similarly, by Bayes' rule, at the middle information set the plaintiff must attach probability 1 to the non-negligent type and thus by sequential rationality must choose "settle". But then the negligent type of the defendant gets $-(W+D)$ by resisting and would get $-S$ by offering. Since, by assumption, $S < W$ ($< W + D$), the choice of resisting is not sequentially rational.

Consider now case (2). By Bayes' rule, at the bottom information set the plaintiff must attach probability 1 to the non-negligent type and thus by sequential rationality must choose "drop". But then the negligent type of the defendant gets a negative payoff by offering, while he would get 0 by resisting. Hence the choice of offering is not sequentially rational.

(e) There are two candidates for a pure-strategy pooling equilibrium: (1) both types of the defendant choose "offer" and (2) both types of the defendant choose "resist".

Consider case (1) first (both types of the defendant choose "offer"). In order for "offer" to be sequentially rational for the non-negligent type, it cannot be that the plaintiff's strategy involves "settle" at the middle information set (the non-negligent type would get either 0 or $-D$ by resisting and both payoffs are grater than $-S$) and/or "drop" at the bottom information set. That is, **it must be that the plaintiff chooses "trial" at both information sets.** By Bayes' rule, at the middle information set the plaintiff must attach probability q to the negligent type and probability $(1-q)$ to the non-negligent type. Hence at the middle information set "trial" is sequentially rational if and only if $qW - P \geq S$, that is, $q \geq \frac{S+P}{W}$. In order for "trial" to be sequentially rational at the bottom information set, the plaintiff must attach sufficiently high probability (namely $p \geq \frac{P}{W}$) to the negligent type. This is allowed by weak sequential equilibrium because the bottom information set is not reached. Finally, in order for "sue" to be sequentially rational it must be that $qW - P \geq 0$, that is, $q \geq \frac{P}{W}$, which is implied by $q \geq \frac{S+P}{W}$. Thus

there is a pooling equilibrium of type (1) with ((sue,trial,trial),(offer,offer)) if and only if $q \geq \frac{S+P}{W}$.

Now consider case (2) (both types of the defendant choose "resist"). [Note: since in part (f) the restriction $S < W - P$ does not hold, we will carry out the analysis below at first without imposing the restriction.] If the plaintiff's strategy involves "drop" at the bottom information set, then it is indeed sequentially rational for both types of the defendant to choose "resist". Now, "drop" is sequentially rational in this case if and only if $qW - P \leq 0$, that is, $q \leq \frac{P}{W}$. Then "sue" is also sequentially rational, since the Plaintiff's payoff is 0 no matter whether he sues or does not sue. Thus there is a pooling equilibrium of type (2) with ((sue,x,drop),(resist,resist)) if and only if $q \leq \frac{P}{W}$ and appropriate beliefs as follows (p is the probability on the left node of the unreached middle information set):

either x = settle and any p if $W \leq S+P$ or $p \leq \frac{S+P}{W}$ if $W > S+P$

or x = trial and $p \geq \frac{S+P}{W}$, which requires $W \geq S+P$.

Since it is assumed that $W > S+P$, we can conclude that

((sue,settle,drop),(resist,resist)) is an equilibrium if and only if $q \leq \frac{P}{W}$ with $p \leq \frac{S+P}{W}$

((sue,trial,drop),(resist,resist)) is an equilibrium if and only if $q \leq \frac{P}{W}$ with $p \geq \frac{S+P}{W}$

If, on he other hand, $q \geq \frac{P}{W}$ then "trial" is sequentially rational at the bottom information set. Then, in order for the non-negligent type of the defendant to choose "resist" it must be that the plaintiff's strategy involves "trial" also at the middle information set, for which we need him to assign probability $p \geq \frac{S+P}{W}$ to the negligent type (which is possible, since the middle information set is not reached); of course, this requires $W \geq S+P$. Thus,

((sue,trial,trial),(resist,resist)) is an equilibrium if and only if $q \geq \frac{P}{W}$ with $p \geq \frac{S+P}{W}$

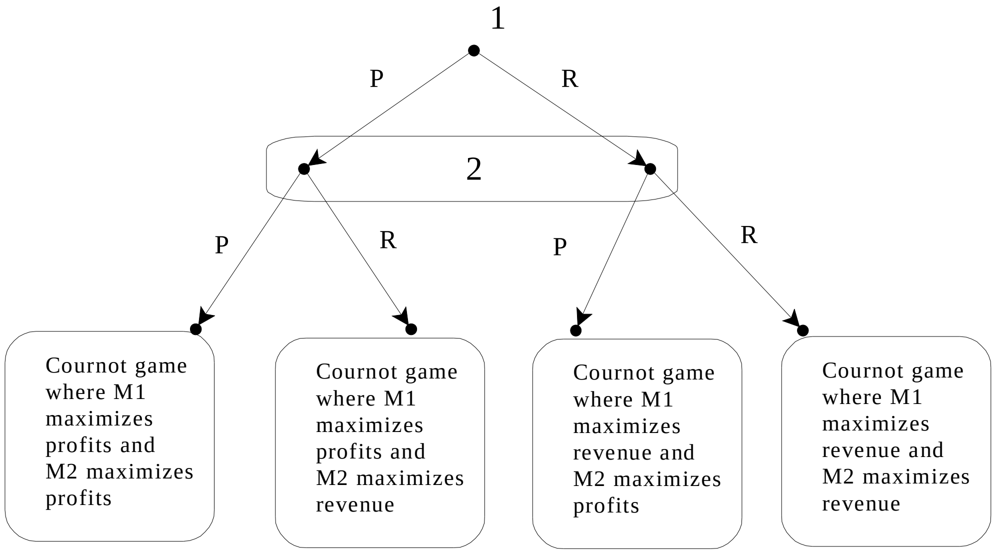
(f) Note that here the restriction $W - P > S$ does **not** hold. In this case, $q < \frac{S+P}{W} = \frac{80+70}{100} = \frac{3}{2}$ and thus, by the previous analysis, there is no pooling equilibrium of type (1), namely ((sue,trial,trial),(offer,offer)).

Also, since $S+P>W$ there is no pooling equilibrium of type (2) with ((sue,trial,trial),(resist,resist)) or with ((sue,trial,drop),(resist,resist)).

However, there is a pooling equilibrium of type (2) with ((sue,settle,drop),(resist,resist)) with any beliefs at the middle information set, since "settle" strictly dominates "trial" there (and, of course, belief q = 1/12 on the left node of the bottom information set).

ANSWER KEY FOR QUESTION 5

(a) In the figure below, *P* stands for Profit-sharing contract and *R* for Revenue-sharing contract. *Mi* stands for Manager of firm *i*.



(b) Number the subgames games 1 to 4 from left to right.

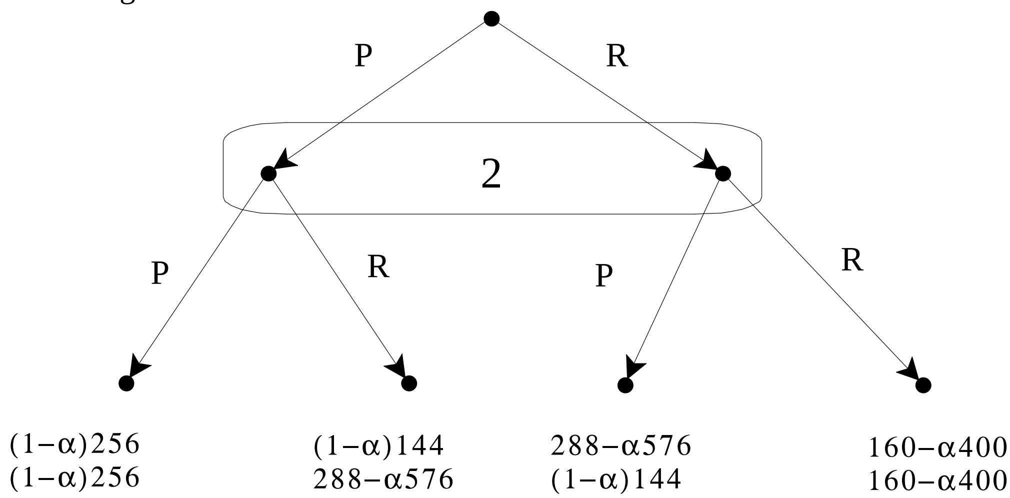
Subgame 1: q_1 is chosen to maximize $\alpha\Pi_1(q_1,q_2)=\alpha\left(q_1(60-q_1-q_2)-12q_1\right)$ and q_2 is chosen to maximize $\alpha\Pi_2(q_1,q_2)=\alpha\left(q_2(60-q_1-q_2)-12q_2\right)$. Solving $\frac{\partial\Pi_1}{\partial q_1}=0$ and $\frac{\partial\Pi_2}{\partial q_2}=0$ gives $q_1=q_2=16$. Player 1's payoff is $(1-\alpha)\Pi_2(16,16)=(1-\alpha)256$ and the same is true for player 2.

Subgame 2: q_1 is chosen to maximize $\alpha\Pi_1(q_1,q_2)=\alpha\left(q_1(60-q_1-q_2)-12q_1\right)$ and q_2 is chosen to maximize $\alpha R_2(q_1,q_2)=\alpha q_2(60-q_1-q_2)$. Solving $\frac{\partial\Pi_1}{\partial q_1}=0$ and $\frac{\partial R_2}{\partial q_2}=0$ gives $q_1=12$ and $q_2=24$. Player 1's payoff is $(1-\alpha)\Pi_1(12,24)=(1-\alpha)144$ and player 2's payoff is $\Pi_2(12,24)-\alpha R_2(12,24)=288-\alpha 576$.

Subgame 3: this is the same as subgame 2, with the roles reversed. Thus Player 1's payoff is $288-\alpha 576$ and Player 2's payoff is $(1-\alpha)144$.
(In this game q_1 is chosen to maximize $\alpha R_1(q_1,q_2)=\alpha q_1(60-q_1-q_2)$ and q_2 is chosen to maximize $\alpha\Pi_2(q_1,q_2)=\alpha\left(q_2(60-q_1-q_2)-12q_2\right)$. Solving $\frac{\partial R_1}{\partial q_1}=0$ and $\frac{\partial\Pi_2}{\partial q_2}=0$ gives $q_1=24$ and $q_2=12$. Player 1's is $\Pi_1(24,12)-\alpha R_1(24,12)=288-\alpha 576$ and player 2's payoff is $(1-\alpha)\Pi_2(24,12)=(1-\alpha)144$.)

Subgame 4: q_1 is chosen to maximize $\alpha R_1(q_1,q_2)=\alpha q_1(60-q_1-q_2)$ and q_2 is chosen to maximize $\alpha R_2(q_1,q_2)=\alpha q_2(60-q_1-q_2)$. Solving $\frac{\partial R_1}{\partial q_1}=0$ and $\frac{\partial R_2}{\partial q_2}=0$ gives $q_1=20$ and $q_2=20$. Player 1's is $\Pi_1(20,20)-\alpha R_1(20,20)=160-\alpha 400$ and similarly for Player 2.

Thus the game reduces to:



The normal form is

	Profit contract	Revenue contract
Profit contract	$(1-\alpha)256$, $(1-\alpha)256$	$(1-\alpha)144$, $288-\alpha 576$
Revenue contract	$288-\alpha 576$, $(1-\alpha)144$	$160-\alpha 400$, $160-\alpha 400$

The Nash equilibria of this game are as follows:

1. If $\alpha < \frac{1}{16}$ then *R* (a Revenue contract) is a strictly dominant strategy for each player and thus (*R*,*R*) is the only Nash equilibrium
2. If $\alpha = \frac{1}{16}$ then *R* is a weakly dominant strategy for each player; there are 3 Nash equilibria: (*R*,*R*), (*R*,*P*) and (*P*,*R*).
3. If $\frac{1}{16} < \alpha < \frac{1}{10}$ there are two Nash equilibria: (*R*,*P*) and (*P*,*R*).
4. If $\alpha = \frac{1}{10}$ then *P* is a weakly dominant strategy for each player; there are 3 Nash equilibria: (*P*,*P*), (*R*,*P*) and (*P*,*R*).
5. If $\alpha > \frac{1}{10}$ then *P* is a strictly dominant strategy for each player and thus (*P*,*P*) is the only Nash equilibrium.

(c) From the calculations for Subgame 1, we get that in the past each firm had a profit of 256. When α is small, the only equilibrium involves a revenue contract for each manager, yielding an income of at most 160 for each owner. Thus delegation has reduced the owners' incomes. This is a Prisoners' Dilemma situation: when one of the firms delegates with a revenue contract then the other must too, giving rise to a Pareto inefficient situation (from the point of view of the owners only).