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Microeconomics

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PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Answer FOUR questions

Question 1.

You own ω units of a safe asset, which yields one unit of the single consumption good in any state of the world. There is also a risky asset, which yields different amounts of the consumption good in the various states of the world. You can *ex ante* exchange safe and risky asset on a one-to-one basis in the financial markets. A *portfolio* is defined by an amount γ of the risky asset, which leaves you with the amount $\omega - \gamma$ of the safe asset.

Assume two states s_1 and s_2.

* s_1 is the bad state, which occurs with probability π, where the risky asset (gross) return rate is $v_1 < 1$ (i. e., the net return rate is $v_1 - 1 < 0$).

* s_2 is the good state, which occurs with probability $(1 - \pi)$, where the risky asset (gross) return rate is $v_2 > 1$ (i. e., the net return rate is $v_2 - 1 > 0$).

A portfolio γ induces the following contingent consumptions

* $x_1 = (\omega - \gamma) + \gamma\ v_1 = \omega + \gamma\,(v_1 - 1)$ in the bad state of the world;

* $x_2 = (\omega - \gamma) + \gamma\ v_2 = \omega + \gamma\,(v_2 - 1)$ in the good state of the world.

We assume that the risky asset has *positive net expected returns*, i. e., $\pi\,(v_1 - 1) + (1 - \pi)\,(v_2 - 1) > 0$.

Consider only the case where wealth ω is large enough so that preference-maximizing contingent consumptions are positive in both states of the world.

We are interested in the following three queries, for three types of preferences defined below.

Query A. As your initial wealth ω increases, how does the *amount* γ of the risky asset in your portfolio change?

Query B. As your initial wealth ω increases, how does the *share* $\frac{\gamma}{\omega}$ of the risky asset in your portfolio change?

Query C. How does the wealth expansion path in contingent consumption space look like?

1.1. Answer queries A-C for the case where your von Neumann-Morgenstern-Bernoulli (vNMB) utility function is of the CARA (constant absolute risk aversion) type. Justify your answer.

1.2. Answer queries A-C for the case where your vNMB utility function is of the CRRA (constant relative risk aversion) type. Justify your answer.

1.3. Consider now the case where your preferences are defined by a wealth-dependent "vNMB function," which has both consumption x and your initial wealth ω as arguments, namely $u = -\exp(-\frac{\beta}{\omega}x) \equiv -e^{-\frac{\beta}{\omega}x}$, where $\beta > 0$. Interpret in words.

1.4. Answer queries A-C for Preferences of 1.3 above. Justify your answer.

1.5. Compare your answers to 1.4 to those in 1.1-2, and comment.

Question 2.

The Second Theorem of Welfare Economics shows that, under appropriate assumptions, any Pareto optimal allocation can be obtained by competitive markets with appropriate income transfers – in particular an allocation which treats different economic agents "fairly". The problem is then to define what a "fair" allocation is. The usual definition is the following: given *I* agents with utility functions u^i, an allocation $(x^i)_{i=1}^I$ is fair if it is

- Pareto optimal, and
- envy free i.e. if, for all $i,k=1,\ldots,I$, $u^i(x^i)\ge u^i(x^k)$.

(a) In an exchange economy a fair allocation always exists but, as the following example shows, this is not true in a production economy. Consider an economy with two agents and two goods: labor/leisure and a consumption good. Both agents are endowed with 1 unit of time that they can use for leisure or labor. L^i denotes the labor time and ℓ^i the leisure time of agent *i* (*i* = 1,2). The utility functions of the two agents are

$$u^1(x^1,\ell^1)=\ell^1+\frac{11}{10}x^1 \quad \text{and} \qquad u^2(x^2,\ell^2)=\ell^2+2x^2$$

where x^i denotes the consumption good allocated to agent *i* (*i* = 1,2). The production function for the consumption good is

$$x=L^1+\frac{L^2}{10}$$

The main feature of this example is that agent 2 is less productive than agent 1. The feasible allocations of the economy are such that $x^1\ge 0$, $x^2\ge 0$, $0\le \ell^1\le 1$, $0\le \ell^2\le 1$.

(i) Show that if $\left((\bar{x}^1,\bar{\ell}^1),(\bar{x}^2,\bar{\ell}^2)\right)$ is Pareto optimal then $\bar{\ell}^1=0$.

(ii) Show that there is no Pareto optimal allocation with $\bar{\ell}^2<1$ and $\bar{x}^2>0$.

(iii) Use (i) and (ii) to show that there does not exist a fair allocation in this economy.

(b) This example suggests weakening the requirement for a fair allocation by taking into account the possible differences in agents' productivities. Consider a standard convex production economy with *N* goods

$$\mathcal{E}\left((u^i,\omega^i)_{i=1\ldots,I},\ (Y^j)_{j=1\ldots,J},\ (\theta^i_j)_{\substack{i=1,\ldots,I\\ j=1,\ldots,J}}\right)$$

with the usual notation, where the last good *N* is labor/leisure time. Each agent has an endowment $\omega^i_n\ge 0$ of each good, the endowment $\omega^i_N\ge 0$ of the last good being 1 unit of time which can be used for leisure or can be spent working. If agent *i* works L^i units of time, his effective labor is a_iL^i, while his leisure is $\ell^i=1-L^i$. Firms buy labor and not time, so that when agent *i* spends L^i hours working, he/she sells a_iL^i units of labor to the production sector. Let $u^i(x^i,\ell^i)$ denote the utility function of agent *i*, where $x^i\in\mathbb{R}^{N-1}_+$ is agent *i*'s consumption in goods other than leisure. An allocation $\left((x^i,\ell^i)_{i=1}^I,y\right)$ is called "consumption-fair" if

- it is Pareto optimal
- for all $i=1,\ldots,I$, $u^i(x^i,\ell^i)\ge u^i(x^k,1-\frac{a_k}{a_i}L^k)$ for all $k\ne i$ such that $1-\frac{a_k}{a_i}L^k\ge 0$,

that is, the no-envy free condition is made conditional on agent *i* contributing as much as agent *k* to the productive sector. If agent *i* cannot contribute as much effective labor as agent *k*, then he/she cannot envy agent *k*'s consumption. Show that there exists a consumption-fair allocation. [Hint: Consider a competitive equilibrium from an equal division of the initial resources in goods 1,…,*N*– 1 and of the ownership shares.]

Question 3.

Consider the problem of New City which is considering building a railway station and a railway line to Central City where a lot of its residents work and shop. If the city decides to build the railway it will at the same time provide services in the railway station: newspapers, food, drinks, etc. To simplify the problem let us model New City as an economy with three goods: good 1 is transportation by train, good 2 is services at the station, and the third good is a numeraire good which will be denoted by *m*, and which represents all other goods. The production of good 1 requires a fixed cost *C* of the numeraire good (for building the station and the tracks) and then one unit of *m* produces a_1 units of good 1. Good 2 is produced under constant returns, 1 unit of *m* produces a_2 units of good 2. There are *I* agents living in New City. Agent *i* has an endowment ω^i of the numeraire good and a quasi-linear utility function U^i of the form

$$U^i(x_1^i, x_2^i, m^i) = m^i + u^i(x_1^i) + v^i(x_2^i), \quad i = 1, \dots, I$$

where u^i and v^i are twice differentiable, strictly concave, increasing functions from $\mathbb{R}_+$ to $\mathbb{R}$. The endowments ω^i are assumed to be sufficiently large so that the non-negativity constraint on m^i will never bind for any agent in any of the scenarios considered in the exercise. We also assume that

$$a_1 \frac{du^i}{dx_1^i}(0) \geq 1, \quad a_2 \frac{dv^i}{dx_2^i}(0) \geq 1, \quad \lim_{x_1^i \to \infty} \frac{du^i}{dx_1^i}(x_1^i) = 0, \quad \lim_{x_2^i \to \infty} \frac{dv^i}{dx_2^i}(x_2^i) = 0 .$$

(a) For which values of the parameters C, a_1 and a_2 is it optimal to build the railway and provide the services? Use the notation $c_1 = \frac{1}{a_1}$, $c_2 = \frac{1}{a_2}$, $\phi^i = \left(\frac{du^i}{dx_1^i}\right)^{-1}$ and $\psi^i = \left(\frac{dv^i}{dx_2^i}\right)^{-1}$.

(b) To fix ideas, assume that all agents have the same preferences with $u^i(x_1^i) = \alpha \ln(1 + x_1^i)$, $v^i(x_2^i) = \beta \ln(1 + x_2^i)$, for $i = 1, \dots, I$, $\alpha > c_1$, $\beta > c_2$, and the parameters $(c_1, c_2, C, \alpha, \beta)$ satisfy the condition found in (a) with a strict inequality. The efficient way of building the railway would be to impose lump sum taxes on the agents to finance the fixed cost and then sell good 1 and good 2 at marginal cost. Unfortunately this is not possible. Adding new taxes requires a 2/3 majority in the City Council. The City Council typically contains 40% of Reps which by principle vote against any additional tax. Another solution is to create a Railway Authority which operates the line and the railway station, selling good 1 and good 2 above marginal cost and using the revenue to finance the fixed cost.

(i) Suppose that the price of the numeraire good is 1 and that the Railway Authority sells good 1 at price p_1 and good 2 at price p_2, with $c_1 < p_1 \leq \alpha$, $c_2 < p_2 \leq \beta$. Find

- **(i.1)** the quantity of each good which is sold,
- **(i.2)** the revenue raised (net of variable costs),
- **(i.3)** the utility of agent i, $i = 1, \dots, I$.

(ii) Let *P* be the set of pairs (p_1, p_2) such that the revenue raised by the Railway Authority covers the fixed cost *C* of producing good 1. To find the optimal combination (p_1^*, p_2^*) in *P* which maximizes the sum of the agents' utilities, the Railway Authority can restrict attention to prices such that $c_1 \leq p_1 \leq \alpha$, $c_2 \leq p_2 \leq \beta$. Assuming that *P* is non empty and the solution to the maximum problem of the Railway Authority is interior (with enough time we could derive for which values of the parameters these conditions hold) show that the optimal combination (p_1^*, p_2^*) satisfies

$$\frac{p_1^* - c_1}{p_1^*} \eta(p_1^*) = \frac{p_2^* - c_2}{p_2^*} \eta(p_2^*) = \frac{1 - \lambda}{\lambda}$$

where $\eta(p_\ell^*)$ is the price elasticity of the demand for good ℓ, $\ell = 1, 2$, and λ is the multiplier of the constraint $(p_1, p_2) \in P$. Interpret this relation, explaining why $\lambda > 1$.

Question 4.

An Incumbent Monopolist (IM) and a Potential Entrant (PE) play the following game. First the IM decides whether to be **passive** or **committed**. Commitment costs $**C** and the cost is non recoverable (sunk cost). Then the PE observes the action taken by the IM and decides whether to **enter** or **stay out**. If she stays out, her payoff is k (independent of whether the IM chose to be passive or committed), whereas the IM's payoff is M if passive and $(M - C)$ if committed. Assume that $M > C > 0$. If the PE decides to enter, then the two firms play a simultaneous Cournot game where the demand function is given by $D_{pass}(P) = a - bP$ if the IM chose to be passive and $D_{comm}(P) = c - dP$ if the IM chose to be committed. Production costs are zero for both firms.

The situation, however, is complicated by the fact that the PE's opportunity cost of entry k is known only to her and not to the IM. Thus we have a one-sided situation of incomplete information. Let K be the set of possible values of the opportunity cost of entry k.

(a) For the simple case where (i) $K = \{k_1, k_2\}$ and (ii) in the Cournot game each firm has only two possible choices of output (denote them by q_1 and q_2), apply the Harsanyi transformation and represent this incomplete-information situation as an extensive game. **You do not need to write the payoffs**: just show the structure of the game.

(b) Explain why in the game of part (a) above, the post-entry Cournot games are not subgames.

(c) Explain why a pure-strategy perfect-Bayesian (or weak sequential) equilibrium of the game of part (a) can be found by solving each Cournot game as if it were a subgame.

(d) Suppose that $K = \{1, 4, 6, 12\}$, the IM's beliefs are given by the uniform distribution over K, $a = 15$, $b = 5$,
$c = 6$, $d = 2$. Find a perfect-Bayesian, or weak sequential, equilibrium. [Hint: use the approach suggested in part (c); your answer must be conditional on the values of the parameters M and C.]

Now let us change the game. It is no longer a situation of incomplete information. First the IM decides whether to be passive or committed (as before, commitment costs $$C$ and is irreversible). Then Nature selects the opportunity cost of entry $k \in K$ (that is, the profit that the potential entrant could make in the best alternative investment) according to the cumulative distribution function F [thus, for every number x, $F(x)$ is the probability that the opportunity cost of entry k is less than or equal to x]. The value of k is revealed to both IM and PE (and becomes common knowledge between them). Then the PE decides whether or not to enter and if she enters then there is a simultaneous duopoly game (which we do not specify: it could be a Cournot game or a price-setting game). Let D_I and D_E be the incumbent's and entrant's profits, respectively, at the Nash equilibrium of the duopoly game following entry with a passive incumbent, and H_I and H_E be their respective profits at the Nash equilibrium of the duopoly game following entry with a committed incumbent (H_I **includes** the commitment cost C). **Assume that if she is indifferent between entering and not entering, the PE will choose to enter.**

(e) Draw the extensive form of this game for the case where $K = \{k_1, k_2\}$ (replace each duopoly game with the corresponding equilibrium payoffs; write all the payoffs).

(f) Assume that $K = [A, B]$ (the closed interval between A and B, $0 < A < B$) and $A < H_E < D_E < B$. Under what conditions is there commitment at every subgame-perfect equilibrium? Under what conditions are the subgame-perfect equilibria characterized by the fact that the incumbent is passive?

Question 5.

Adverse selection in the labor market.

There are two types of workers, $\theta = 1$ and $\theta = 2$, with an equal number of each type. For each type, utility depends on the wage *w* and the speed of work *e*. Specifically,

$$u(e, w, \theta) = w - \frac{e^2}{2\theta}$$

There are many firms competing for workers (free entry). Each firm runs exactly one assembly line at a chosen speed *e*. The marginal product of a worker of type θ at a firm with speed *e* is given by

$$y(e, \theta) = e + \theta .$$

Note that *e* (which can be any non-negative real number) is chosen by the firm that the worker joins; it is not subject to worker discretion (no moral hazard).

No firm can distinguish workers' types and there is team production, so that a firm cannot condition contracts on any given worker's output. Thus every firm will pay each of its workers the average product of its work force.

(a) Calculate the marginal rate of substitution (MRS) between *e* and *w* for each type of worker.

(b) In the *e*-*w* space (with *w* on the vertical axis), fix a point and draw the indifference curve of each type that goes through that point. Clearly state which curve corresponds to which type.

(c) In the *e*-*w* space (with *w* on the vertical axis), draw three zero-profit lines for a firm, one corresponding to the case where all the workers in the firm are of type $\theta = 1$, one corresponding to the case where all the workers in the firm are of type $\theta = 2$ and the third corresponding to the case where 50% of the workers are of one type and 50% of the other.

(d) As a benchmark, consider the case where firms **can** observe the workers' types. Calculate the possible wage-speed pairs across the industry when firms make zero profits and wages are such that, for each worker, the marginal cost (in terms of utility, that is, the marginal disutility) of additional speed equals his marginal benefit from additional speed $\frac{\partial y(e, \theta)}{\partial e}$. [Hint: in this case each firm would have a speed designed especially for a particular skill type θ.]

Now return to the case where firms **cannot** observe workers' types. For the next questions define an equilibrium as a situation where every firm makes zero profits and no firm can make positive profits by offering an appropriate wage *w* and setting up an appropriate speed *e.*

(e) Show that there cannot be a pooling equilibrium, where every firm offers the same wage and the same speed. [A graphical argument is sufficient.]

(f) Show graphically what a separating equilibrium must look like.

(g) How does a separating equilibrium compare to the benchmark case of part (d) for the $\theta = 1$ type? Show that at a separating equilibrium the $\theta = 2$ type work harder and have lower utility than in the benchmark case of part (d).