MICRO PRELIM AUGUST 2009 – ANSWER KEYS

QUESTION 1. We start with the following facts, valid for all three preference types below.

Fact 1. *γ is constant (resp. increasing) iff so is $x_2 - x_1$.*

Proof. Clearly, $x_2 - x_1 = \gamma[(v_2 - 1) - (v_1 - 1)] = \gamma(v_2 - v_1)$, where $v_2 - v_1 > 0$ because $v_2 > 1 > v_1$. Hence, $\frac{d(x_2 - x_1)}{d\gamma} > 0$.

Fact 2. *$\frac{\gamma}{\omega}$ is constant (resp. decreasing) iff so is $\frac{x_2}{x_1}$.*

Proof. $\frac{x_2}{x_1} = \frac{\omega + \gamma(\nu_2 - 1)}{\omega + \gamma(\nu_1 - 1)} = \frac{1 + \frac{\gamma}{\omega}(\nu_2 - 1)}{1 + \frac{\gamma}{\omega}(\nu_1 - 1)}$. Because $v_2 - 1 > 0$ and $v_1 - 1 < 0$, we have that $\frac{d\frac{x_2}{x_1}}{d\frac{\gamma}{\omega}} > 0$.

Fact 3. *Preference maximization is characterized by the* FOC $\frac{u'(x_1)}{u'(x_2)} = \frac{1-\pi}{\pi}\frac{\nu_2 - 1}{1 - \nu_1}$.

Proof. The decision maker chooses γ in order to maximize

$$\pi u(\omega + \gamma(v_1 - 1)) + (1-\pi)\, u(\omega + \gamma(v_2 - 1)).$$

The stated FOC follows from differentiability and interiority, guaranteed by the proviso that ω is large enough. (We may want to use a partial-derivative notation for the preference type of 1.3 below.)

Fact 4. $\frac{1-\pi}{\pi}\frac{\nu_2 - 1}{1 - \nu_1} \equiv \psi > 1$.

Proof. The inequality follows from the assumption of positive net expected returns. We introduce the symbol ψ for convenience.

Putting facts 3 and 4 together, we have

$$\frac{u'(x_1)}{u'(x_2)} = \psi > 1, \qquad (1)$$

which under the strict concavity of u implies that $x_2 > x_1$, i. e., the (risk averse) consumer always bears some risk, given the positive net expected returns.

1.1. Answer queries A-C for the case where your von Neumann-Morgenstern-Bernoulli (vNMB) utility function is of the CARA (constant absolute risk aversion) type. Justify your answer.

ANSWER. Now $u = -e^{-rx}$ for some $r > 0$, where r is the coefficient of ARA. Expression (1) becomes

$$\frac{re^{-rx_1}}{re^{-rx_2}} = \psi > 1,$$

i. e., $e^{-r(x_1 - x_2)} = \psi > 1$, or: $-r(x_1 - x_2) = \ln\psi > 0$.

It follows that $x_2 - x_1 = \frac{\ln\psi}{r} > 0$. Hence, the difference $x_2 - x_1$ does not vary with ω. It follows from Fact 1 that γ does not change with ω either. Hence, $\frac{\gamma}{\omega}$ decreases with ω, which implies that so does $\frac{x_2}{x_1}$.

A wealth expansion path is a line of slope 1, parallel to (and above) the certainty line.

1.2. Answer queries A-C for the case where your vNMB utility function is of the CRRA (constant relative risk aversion) type. Justify your answer.

ANSWER. Now either $u = \frac{x^{1-r}}{1-r}$ for $r > 0$, $r \neq 1$, or $u = \ln x$. In the first case, expression (1) becomes $\frac{x_1^{-r}}{x_2^{-r}} = \psi > 1$, or $\left(\frac{x_2}{x_1}\right)^r = \psi > 1$, i. e., $\frac{x_2}{x_1} = \psi^{\frac{1}{r}} > 1$, an expression valid for the $\ln x$ case by setting $r = 1$. Hence, the ratio $\frac{x_2}{x_1}$ does not vary with ω. It follows from Fact 2 than neither does the share $\frac{\gamma}{\omega}$, and that both $x_2 - x_1$ and γ increase with ω.

A wealth expansion path is a ray through the origin of slope greater than 1.

1.3. Consider now the case where your preferences are defined by a wealth-dependent vNMB function, which has both consumption x and your initial wealth ω as arguments, namely $u = -\exp(-\frac{\beta}{\omega}x) \equiv -e^{-\frac{\beta}{\omega}x}$, where $\beta > 0$. Interpret in words.

ANSWER. It looks like a CARA function but with the modification that the coefficient of ARA is not constant with respect to wealth, but it is instead given by $\frac{\beta}{\omega}$, i. e., it is inversely proportional to wealth. In other words, the wealthier you are, the lower your coefficient of absolute risk aversion.

1.4. Answer queries A-C for Preferences of 1.3 above, Justify your answer.

Expression (1) now becomes $\frac{\frac{\beta}{\omega}e^{-\frac{\beta}{\omega}x_1}}{\frac{\beta}{\omega}e^{-\frac{\beta}{\omega}x_2}} = \psi > 1$, i. e., $e^{-\frac{\beta}{\omega}[x_1 - x_2]} = \psi > 1$, or

$$\frac{\beta}{\omega}[x_2 - x_1] = \ln\psi > 0 . \qquad (2)$$

We can write $x_2 - x_1 = \gamma(v_2 - v_1)$. Hence, (2) can be written

$$\frac{\beta}{\omega}\gamma(v_2 - v_1) = \ln\psi > 0 ,$$

where, as noted above, $v_2 - v_1 > 0$. Therefore, $\frac{\gamma}{\omega} = \frac{\ln\psi}{\beta(v_2 - v_1)}$, constant with respect to ω. Hence, by Fact 2, so is $\frac{x_2}{x_1}$.

Again, a wealth expansion path is a ray through the origin of slope greater than 1.

1.5. Compare your answers to 1.4 to those in 1.1-2, and comment.

The functional form in 1.3 looks like CARA, and it behaves indeed as a CARA function for changes in π, v_1 and v_2 as long as wealth ω is kept constant. But for changes in ω *ceteris paribus*, it behaves as a CRRA function, with wealth expansion paths that are rays through the origin. We can say that a "constant absolute risk aversion coefficient" inversely proportional to wealth mimics a "constant relative risk aversion coefficient."

Answer Key - September 2009

2. (a) (i) Consider an allocation where $\bar{l}^1 > 0$.

Then the change $\Delta l^1 = -\varepsilon$ $\Delta x^1 = \varepsilon$ $\Delta l^2 = 0$ $\Delta x^2 = 0$ is feasible and $\Delta u^1 = \frac{1}{10}\varepsilon > 0$, $\Delta u^2 = 0$. Thus such an allocation cannot be P.O.

(ii) Consider an allocation with $\bar{l}^2 < 1$ and $\bar{x}^2 > 0$. Then the change $\Delta l^2 = +\varepsilon$ $\Delta x^2 = -\frac{\varepsilon}{10}$, $\Delta l^1 = 0$, $\Delta \bar{x}^1 = 0$ is feasible and such that $\Delta u^1 = 0$, $\Delta u^2 = \varepsilon - \frac{2\varepsilon}{10} > 0$.
Thus such an allocation cannot be P.O.

(iii) A Pareto optimal allocation is thus such that $\bar{l}_1 = 0$ $\bar{l}_2 = 1$ or $\bar{l}_1 = 0$ and $\bar{x}_2 = 0$. In the first case, the no-envy requirements are

$$\left.\begin{array}{l} \frac{11}{10}\bar{x}^1 \geq 1 + \frac{11}{10}\bar{x}^2 \iff \frac{11}{10}(\bar{x}^1 - \bar{x}^2) \geq 1 \\ 1 + 2\bar{x}^2 \geq 2\bar{x}^1 \iff 2(\bar{x}^1 - \bar{x}^2) \leq 1 \end{array}\right\} \Rightarrow \frac{10}{11} \leq \bar{x}^1 - \bar{x}^2 \leq 0.5$$

which is clearly impossible. In the second case

$$\left.\begin{array}{l} \frac{11}{10}\bar{x}^1 \geq \bar{l}^2 \\ \bar{l}^2 \geq 2\bar{x}^1 \end{array}\right\} \Rightarrow 2\bar{x}^1 \leq l^2 \leq \frac{11}{10}\bar{x}^1$$

which is also impossible since $\bar{x}^1$ must be positive ($\bar{l}^1 > 0 \Rightarrow \bar{x} > 0$)

(b) Consider a competitive equilibrium $\left((\tilde{x}^i, \tilde{\ell}^i)_{i=1}^{I}, \tilde{y}, \tilde{p} \right)$ of the economy in which the agents initial endowment in the first N-1 goods are (2)

$$\tilde{\omega}_n^i = \frac{1}{I} \sum \omega_n^i , \quad n = 1, \dots, N-1$$

the ownership shares of all firms are equally distributed

$$\theta_j^i = \frac{1}{I} , \quad j = 1, \dots J .$$

and the time endowment stays $\omega_N^i = 1$, $\forall i = 1 \dots I$. The wage is normalized to 1 and $\tilde{p} \in R_+^{N-1}$ is the price vector for the consumption goods. By the First Thm of Welfare Economics the allocation is Pareto optimal. To show that it is consumption fair note that $(\tilde{x}^k, \tilde{\ell}^k)$ satisfies the budget constraint

$$\tilde{p} \cdot \tilde{x}^k = \tilde{p} \cdot \tilde{\omega}^k + a^k (1 - \tilde{\ell}^k) + \frac{1}{I} \sum_{j=1}^{J} \tilde{\pi}^j$$

where $\tilde{\pi}^j$ denotes the profit of firm j. $\tilde{x}^k$ is feasible for agent i if agent i works $\hat{L}^i$ units of time where

$$a^i \hat{L}^i = a^k \tilde{L}^k = a^k (1 - \tilde{\ell}^k)$$

If $\frac{a^k}{a^i} \tilde{L}^k > 1$, agent i cannot generate $\tilde{x}^k$ and thus cannot "envy" agent k. If $\hat{L}^i \leq 1$, then the allocation $(\tilde{x}^k, 1 - \hat{L}^i)$ is budget feasible for agent i and, by revealed preference, $u^i(\tilde{x}^i, \tilde{\ell}^i) \geq u^i(\tilde{x}^k, 1 - \hat{L}^i) = u^i\left(\tilde{x}^k, 1 - \frac{a^k}{a^i} \tilde{L}^k\right)$

Thus the allocation is conditionally "envy free". Since the economy is convex and $\left((\tilde{x}^i, \ell^i)_{i=1}^{I}, \tilde{y}, \tilde{p} \right)$ exists, a consumption fair allocation exists.

3. (a) Given that the utility functions are quasi linear, if the sum of the agents utilities is higher when the railway is built and $(x_1^i)_{i=1}^I, (x_2^i)_{i=1}^I$ are chosen optimally than without building it, then it is worthwhile building it. The surplus can be distributed in such a way that all agents are better off. The condition is thus

$$\max_{x \in \mathbb{R}_+^{2I}} \left\{ \sum_{i=1}^I w^i - \frac{\sum x_1^i}{a_1} - \frac{\sum x_2^i}{a_2} - C + \sum u^i(x_1^i) + \sum v^i(x_2^i) \right\} \geq \sum_{i=1}^I w^i$$

or with the notation $c_1 = \frac{1}{a_1}$, $c_2 = \frac{1}{a_2}$

$$\max_{\substack{x_1^i \geq 0 \\ x_2^i \geq 0}} \sum_{i=1}^I u^i(x_1^i) + \sum_{i=1}^I v^i(x_2^i) - c_1 \sum_{i=1}^I x_1^i - c_2 \sum_{i=1}^I x_2^i - C \geq 0$$

The FOCs for the optimal choice of $(x_1^i, x_2^i)_{i=1}^I$ are:

$$u^{i\prime}(x_1^i) = c_1 \iff x_1^i = \varphi^i(c_1) \qquad i = 1, \dots, I \qquad \text{with } \varphi^i = (u^{i\prime})^{-1}$$

$$v^{i\prime}(x_2^i) = c_2 \iff x_2^i = \psi^i(c_2) \qquad i = 1, \dots, I \qquad \psi^i = (v^{i\prime})^{-1}$$

Note that $c_1 \in (0\ u^{i\prime}(0))$ which is the domain where φ^i is defined and $c_2 \in (0, v^{i\prime}(0))$ which is the domain where ψ^i is defined.

The condition required for making it worthwhile to build the railway is thus:

$$\sum_{i=1}^I u^i(\varphi^i(c_1)) + \sum_{i=1}^I v^i(\psi^i(c_2)) - c_1 \sum_{i=1}^I \varphi^i(c_1) - c_2 \sum_{i=1}^I \psi^i(c_2) - C \geq 0$$

(b) (i) (α) demand function of agent i :

(i.1)

$$\max_{x_1^i \geq 0 \;\; x_2^i \geq 0} \; w^i - p_1 x_1^i - p_2 x_2^i + \alpha \ln(1 + x_1^i) + \beta \ln(1 + x_2^i)$$

FOCs:

$$\frac{\alpha}{1 + x_1^i} \leq p_1 \qquad = \text{ if } x_1^i > 0$$

$$\frac{\beta}{1 + x_2^i} \leq p_2 \qquad = \text{ if } x_2^i > 0$$

$$x_1^i = \begin{cases} 0 & \text{if } p_1 \geq \alpha \\ \frac{\alpha}{p_1} - 1 & \text{if } p_1 \leq \alpha \end{cases} \iff x_1^i = \max\left\{ \frac{\alpha}{p_1} - 1, 0 \right\}$$

$$x_2^i = \begin{cases} 0 & \text{if } p_2 \geq \beta \\ \frac{\beta}{p_2} - 1 & \text{if } p_2 \leq \beta \end{cases} \iff x_2^i = \max\left\{ \frac{\beta}{p_1} - 1, 0 \right\}$$

(i.2) (β) revenue

$$R = \begin{cases} 0 & \text{if } p_1 \geq \alpha \quad p_2 \geq \beta \\ I(p_1 - c_1)\left(\frac{\alpha}{p_1} - 1\right) & \text{if } c_1 \leq p_1 \leq \alpha, \quad p_2 \geq \beta \\ I(p_2 - c_2)\left(\frac{\beta}{p_2} - 1\right) & \text{if } p_1 \geq \alpha \quad c_2 \leq p_2 \leq \beta \\ I(p_1 - c_1)\left(\frac{\alpha}{p_1} - 1\right) + I(p_2 - c_2)\left(\frac{\beta}{p_2} - 1\right) & \text{if } c_1 \leq p_1 \leq \alpha, \; c_2 \leq p_2 \leq \beta \end{cases}$$

(i.3) (γ) utility of agent i:

$$u^i = \begin{cases} w^i & \text{if } p_1 \geq \alpha \quad p_2 \geq \beta \\ w^i - p_1\left(\frac{\alpha}{p_1} - 1\right) + \alpha \ln \frac{\alpha}{p_1} & \text{if } c \leq p_1 \leq \alpha,\ p_2 \geq \beta \\ w^i - p_2\left(\frac{\beta}{p_2} - 1\right) + \beta \ln \frac{\beta}{p_2} & \text{if } p_1 \geq \alpha \quad c_2 \leq p_2 \leq \beta \\ w^i - p_1\left(\frac{\alpha}{p_1} - 1\right) - p_2\left(\frac{\beta}{p_2} - 1\right) + \alpha \ln \frac{\alpha}{p_1} + \beta \ln \frac{\beta}{p_2} & \end{cases}$$

(ii) Assuming that the optimal combination (p_1^*, p_2^*) is such that $c_1 < p_1 < \alpha$, $c_2 < p_2 < \beta$, it must satisfy the FOCs for the maximum problem

$$\max \quad \sum w^i - I p_1\left(\frac{\alpha}{p_1} - 1\right) - I p_2\left(\frac{\beta}{p_2} - 1\right) + I \alpha \ln \frac{\alpha}{p_1} + I \beta \ln \frac{\beta}{p_2}$$

$$\text{subject to} \quad I (p_1 - c_1)\left(\frac{\alpha}{p_1} - 1\right) + I\left(\frac{\beta}{p_2} - 1\right)(p_2 - c_2) \geq C \qquad \lambda$$

The FOCs are

$$I - \frac{I\alpha}{p_1} + \lambda I\left(\frac{\alpha}{p_1} - 1\right) - \lambda I \left(\frac{p_1 - c_1}{p_1^2}\right)\alpha = 0$$

$$I - \frac{I\beta}{p_2} + \lambda I\left(\frac{\beta}{p_2} - 1\right) - \lambda I \left(\frac{p_2 - c_2}{p_2^2}\right)\beta = 0.$$

or, dividing each relation by the demand on the related good

$$-1 + \lambda + \lambda \frac{p_1 - c_1}{p_1} \eta(p_1) = 0$$

$$-1 + \lambda + \lambda \frac{p_2 - c_2}{p_2} \eta(p_2) = 0.$$

Note that because of the separability of the utility function and the quasi linear form, the demand in good i, $i=1,2$, only depends of the price of good i. Thus the optimal combination (P_1^*, P_2^*) must satisfy

$$1 + \frac{P_1 - c_1}{P_1}\eta(P_1) = \frac{1}{\lambda} \iff \frac{P_1 - c_1}{P_1}\eta(P_1) = \frac{1}{\lambda} - 1 = \frac{1-\lambda}{\lambda}$$

and
$$\frac{P_2 - c_2}{P_2}\eta(P_2) = \frac{1-\lambda}{\lambda}$$

Since the elasticity of demand is negative, λ must be more than 1. To see why suppose the fixed cost C increases by ε: the social utility decrease by λ. If there was no distorsion, λ would be equal to 1 since one dollar spent on the railway is one dollar less to spend on other goods and the m.u. of income is 1 by quasi-linearity. Since the fixed cost is financed by excise taxes which are distortive, ($P_1 > c_1$ is akin to an excise tax). the social cost is higher and $\lambda > 1$.

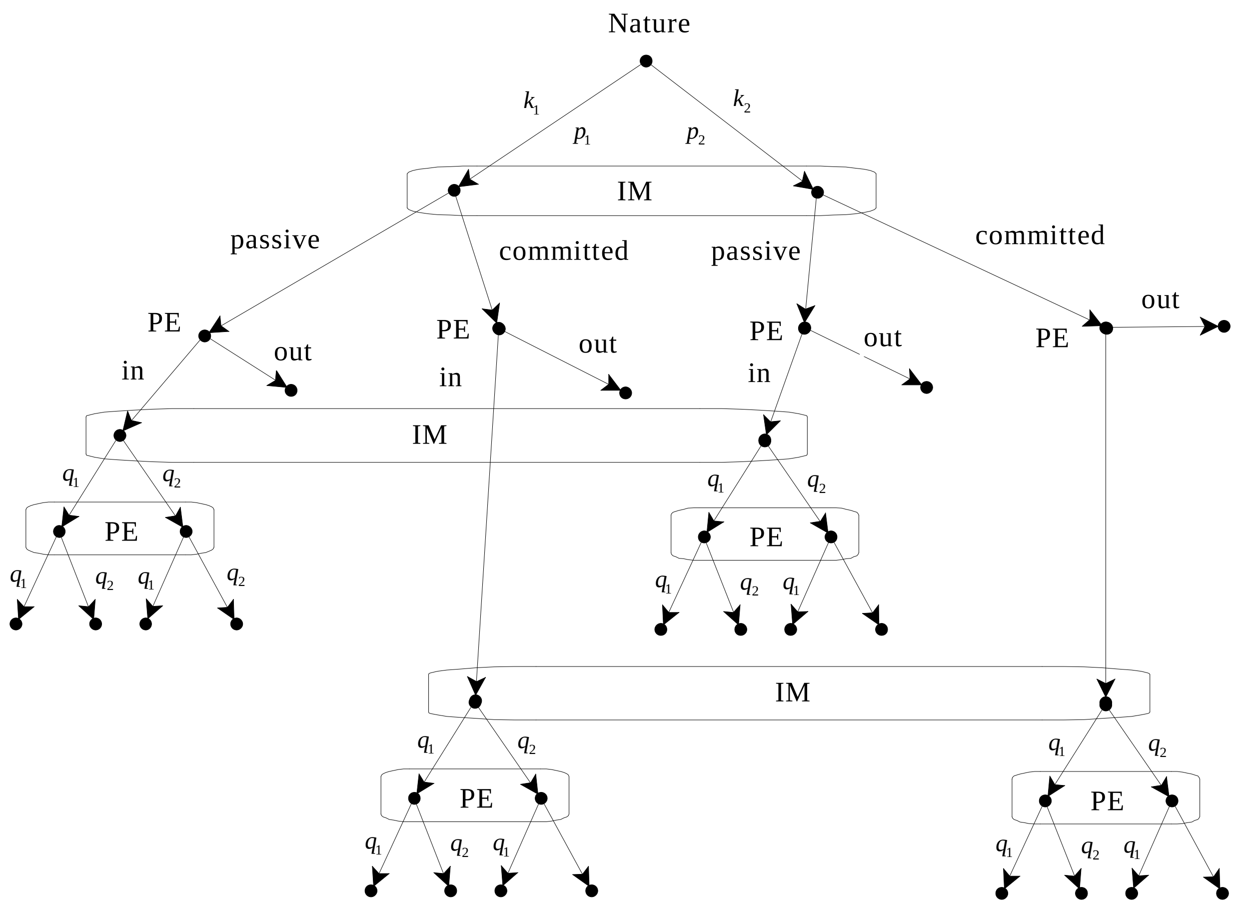
The formula $\frac{P_1 - c_1}{P_1}\eta(P_1) = \frac{P_2 - c_2}{P_2}\eta(P_2) = \frac{1-\lambda}{\lambda}$

implies that the goods with a lower demand elasticity is taxed more than the other. Although the fixed cost is associated with the production of good 1, it may be that it is mainly financed by the profit made on selling good 2, if the demand for good 2 is inelastic (e.g. increasing the price →

of the latte does not change the demand much by increasing the price of the railway ride drives quickly the inhabitants of New City back to their cars).

Answer keys for Questions 4 and 5 Micro Prelim August 2009

4. **(a)** In a one-sided incomplete information situation the uninformed player's beliefs are common knowledge. Let p_i be the probability that the IM assigns to k_i . Then the extensive form is as follows:



(b) If the PE chooses to enter, the IM will still be uncertain as to the value of k (although this value does not enter the payoffs in the Cournot game). Starting at the node where the IM chooses its level of output and taking everything that follows that node, one would "cut" the IM's information set and therefore not get a subgame.

(c) Fix one of the after-entry Cournot games and let $\left(q_{IM}^*, q_{PE}^*\right)$ be a Nash equilibrium of it. Then

$$\pi_{IM}\left(q_{IM}^*, q_{PE}^*\right) \geq \pi_{IM}\left(q, q_{PE}^*\right) \text{ for every } q \quad (1.a)$$

and

$$\pi_{PE}\left(q_{IM}^*, q_{PE}^*\right) \geq \pi_{PE}\left(q_{IM}^*, q\right) \text{ for every } q. \quad (1.b)$$

A perfect Bayesian equilibrium consists of a system of beliefs and a strategy profile. Let s be the probability that the IM assigns to being at the left node of his information set after the PE has entered and $(1-s)$ be the probability that he assigns to being at the right node. *Note that payoffs are the same after the left node and after the right node (they are independent of k).* Let $\hat{q}_{IM}$ be the IM's choice of output at that information set and $\hat{q}_{PE}$ be the PE's choice of output at her two information sets, that is, *we impose the constraint that the PE makes the same choice at those two information sets*; in other words, we make the assumption that the

PE's choice of output in the post-entry interaction does not depend on the opportunity cost of entry. Sequential rationality requires

$$s\,\pi_{IM}\left(\hat{q}_{IM},\hat{q}_{PE}\right)+(1-s)\,\pi_{IM}\left(\hat{q}_{IM},\hat{q}_{PE}\right)\geq s\,\pi_{IM}\left(q,\hat{q}_{PE}\right)+(1-s)\pi_{IM}\left(q,\hat{q}_{PE}\right) \text{ for every } q$$

which is the same as

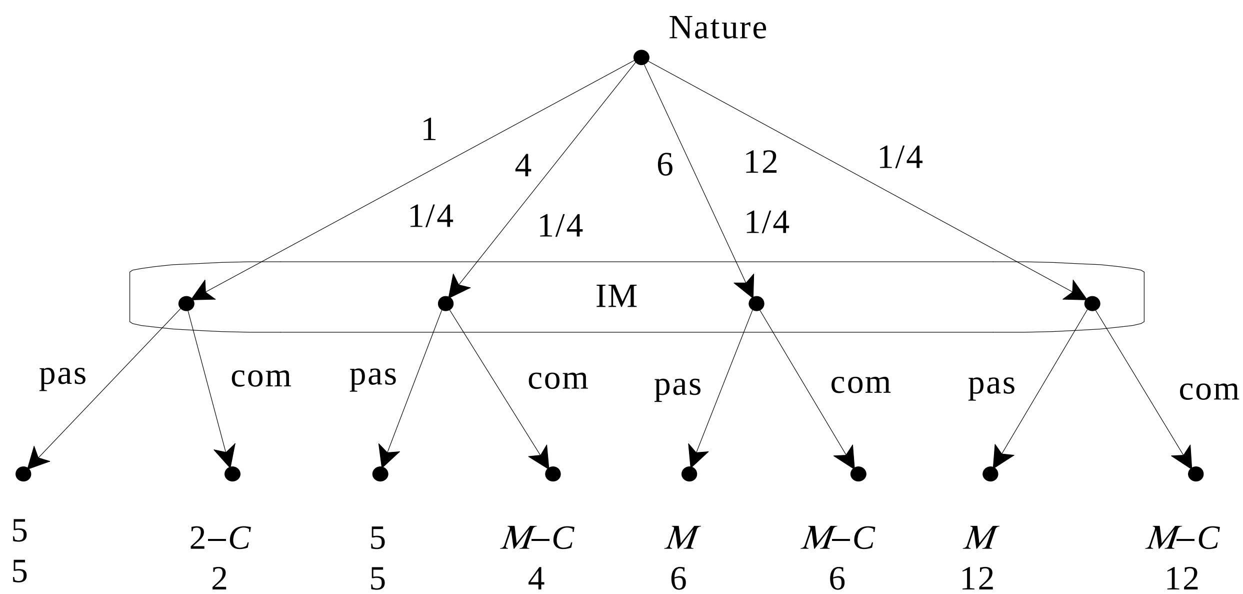
$$\pi_{IM}\left(\hat{q}_{IM},\hat{q}_{PE}\right)\geq \pi_{IM}\left(q,\hat{q}_{PE}\right) \text{ for every } q \qquad (2.a)$$

Similarly, sequential rationality for the PE requires (note: given that we are looking at pure strategies, Bayes' rule requires the PE to assign probability 1 to the node that follows the IM's choice at each of the PE's information sets)

$$\pi_{PE}\left(\hat{q}_{IM},\hat{q}_{PE}\right)\geq \pi_{PE}\left(\hat{q}_{IM},q\right) \text{ for every } q. \qquad (2.b)$$

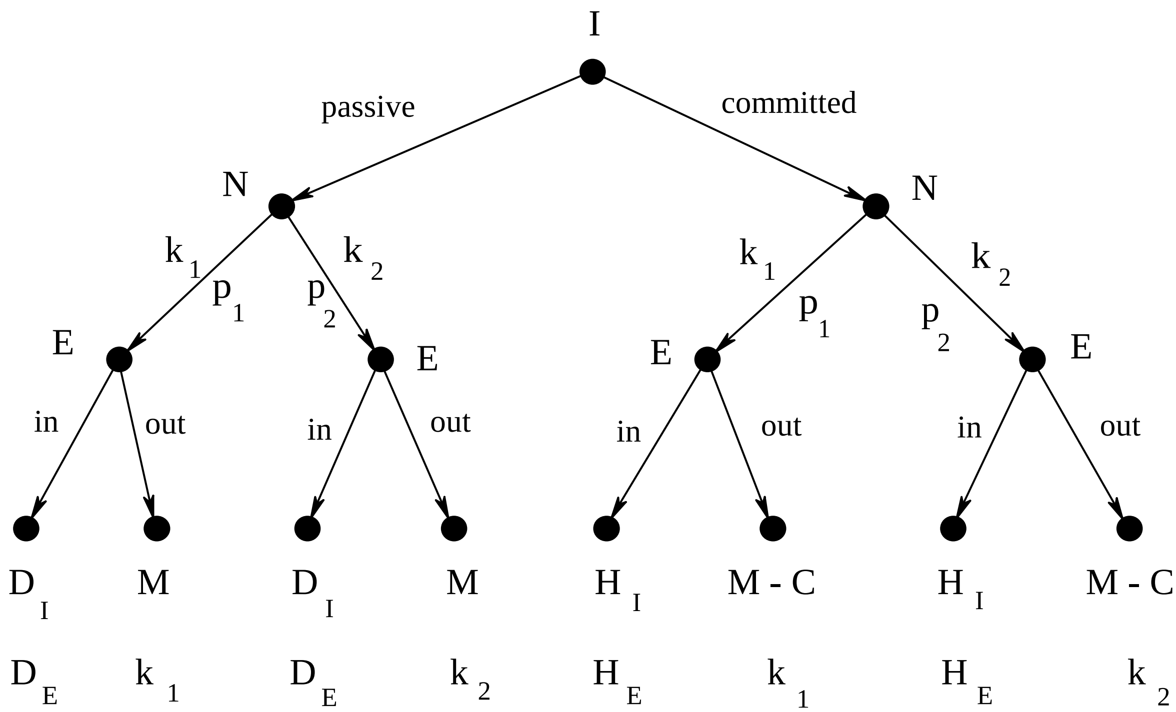
Clearly, a pair of output levels that satisfies (1a) and (1b) satisfies also (2a) and (2b).

(d) Using the fact established in part (c) we can find a perfect Bayesian equilibrium by pretending that the Cournot games are subgames. The Cournot game when the IM is passive has a unique Nash equilibrium given by $q_{IM}=q_{PE}=\frac{a}{3}=5$ with corresponding profits $\pi_{IM}=\pi_{PE}=\frac{a^2}{9b}=5$. The Cournot game when the IM is passive has a unique Nash equilibrium given by $q_{IM}=q_{PE}=\frac{c}{3}=2$ with corresponding profits $\pi_{IM}=\pi_{PE}=\frac{c^2}{9d}=2$. Thus the PE will enter only in the following cases: (i) the incumbent chose commitment and $k = 1$, (ii) the incumbent chose to be passive and either $k = 1$ or $k = 4$. Hence the game reduces to



Thus the IM's expected profits if he chooses to be passive are $\frac{2}{4}5+\frac{2}{4}M$ and his expected profits if he chooses commitment are $\frac{1}{4}(2-C)+\frac{3}{4}(M-C)=\frac{1}{2}+\frac{3M}{4}-C$. Thus he will choose commitment if $\frac{1}{2}+\frac{3M}{4}-C>\frac{1}{2}5+\frac{1}{2}M$, that is, if $M>8+4C$; he will choose to be passive if $M<8+4C$ and will be indifferent if $M=8+4C$.

(e) The game is as follows (I denotes the incumbent, N nature and E the potential entrant):



(f) Suppose the IM chose to be passive. Then the entrant will enter if and only if $D_E \geq k$. Thus *ex ante* the probability of entry, if the IM is passive, is Prob$\{k \leq D_E\}$ = $F(D_E)$. Thus the incumbent's expected profits if he chooses to be passive are:

$$F(D_E)\, D_I + [1 - F(D_E)]\, M \qquad (3).$$

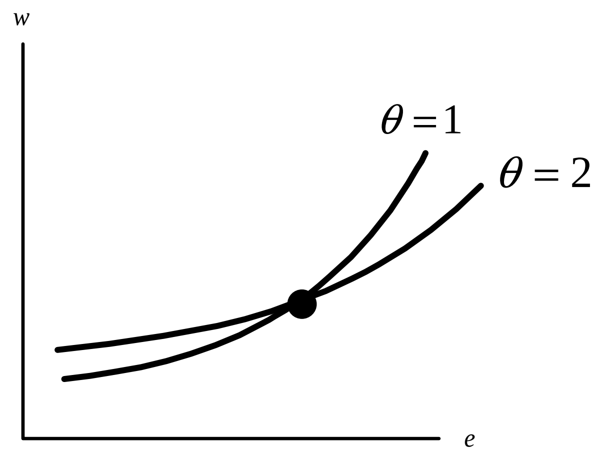
Similarly, if the IM is committed, entry occurs with probability $F(H_E)$ and the IM's expected profits are:

$$F(H_E)\, H_I + [1 - F(H_E)]\, (M - C) \qquad (4).$$

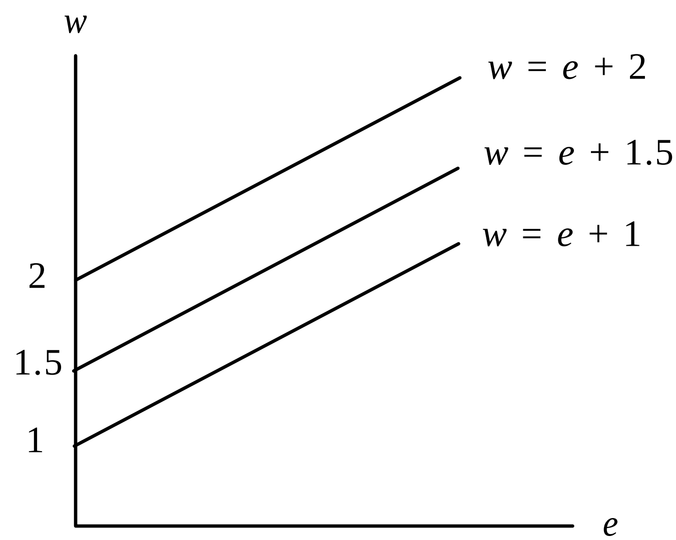
Thus the incumbent will choose to be passive if (3) > (4), and will choose to commit if (4) > (3) and be indifferent otherwise.

5. **(a)** $\frac{\partial u}{\partial w} = 1$ and $\frac{\partial u}{\partial \theta} = -\frac{e}{\theta}$. Hence $MRS(\theta) \overset{def}{=} \frac{dw}{de} = \frac{e}{\theta}$.

(b) The indifference curve of the θ = 2 type is flatter [$MRS(2) < MRS(1)$]. See the following figure:

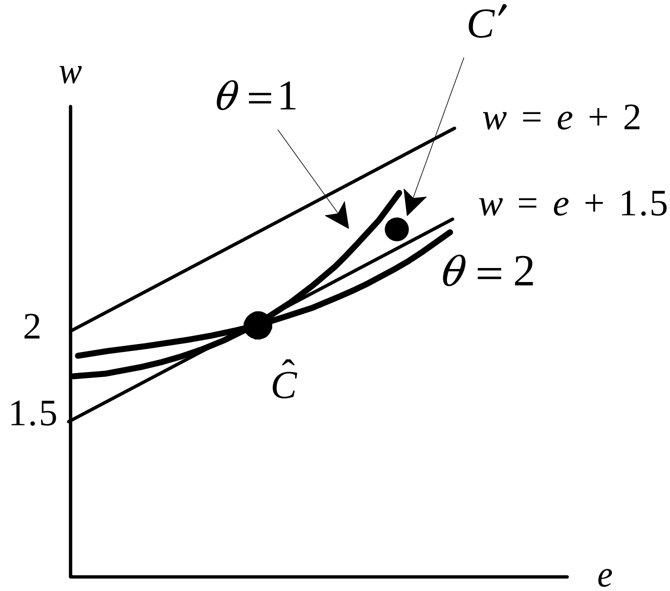


(c) See the following figure:

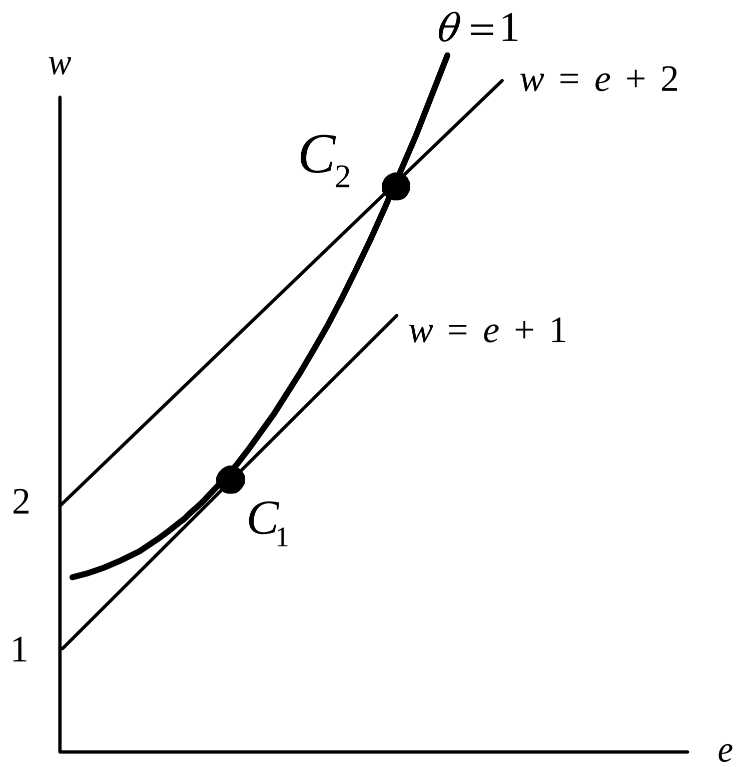


(d) The wage of a worker of skill θ at a firm operating at speed e would equal his marginal product at the firm, i.e. $e + \theta$. This would also ensure that a worker of skill θ could find a firm with a speed that is just right for him, i.e. where his marginal cost of additional speed, $\frac{e}{\theta}$, equals his marginal benefit from additional speed, $\frac{\partial y(e,\theta)}{\partial e} = 1$. So workers with θ = 1 would go to firms offering employment contract $\boxed{e = 1, w = 2}$ and workers with θ = 2 would go to firms offering the contract $\boxed{e = 2, w = 4}$

(e) Suppose that contract $\hat{C} = (\hat{e}, \hat{w})$ is a pooling equilibrium. Then it must lie on the average zero-profit line of equation $w = e + 1.5$. Draw the indifference curves of the two types through point $\hat{C}$. The indifference curve for the θ = 2 type has slope $\frac{\hat{e}}{2}$, while the indifference curve for the θ = 1 type has slope $\hat{e}$. Thus there is a contract $C' = (e', w')$ between the two indifference curves such that the $\theta = 2$ type would be prefer C' to $\hat{C}$ while the $\theta = 1$ type would prefer $\hat{C}$ to C' (see figure below). Hence C' would attract only the $\theta = 2$ workers. If C' is close to $\hat{C}$ then e' is close to $\hat{e}$ and w' is close to $\hat{w} = \hat{e} + 1.5$. The profit from contract C' would be (since only type $\theta = 2$ apply) $e' + 2 - w' \approx \hat{e} + 2 - \hat{w} = 0.5 > 0$ and thus a firm offering contract C' would be making positive profits.



(f) A separating equilibrium would consist of two contracts: $C_1 = (e_1, w_1)$ offered by firms chosen by type θ =1 and $C_2 = (e_2, w_2)$ offered by firms chosen by type θ =2. By the zero-profit condition, contract C_1 must be on the line of equation $w = e + 1$ and contract C_2 must be on the line of equation $w = e + 2$. Furthermore, the indifference curve for type $\theta = 1$ through contract C_1 must be tangent to the line of equation $w = e + 1$. The incentive compatibility constraints for type $\theta = 1$ requires that contract C_2 be on the indifference curve for type $\theta = 1$ through contract C_1 (and thus at the intersection of this curve and the line of equation $w = e + 2$ (se figure below).



(g) The tangency condition for the $\theta = 1$ type requires $e_1 = 1$ and $w_1 = 2$. Thus the $\theta = 1$ type gets the same contract at a separating equilibrium as in the benchmark case of part (e). Since the contract $(e = 2, w = 4)$ for the $\theta = 2$ type in the benchmark case of part (e) lies on a higher indifference curve for the $\theta = 1$ type than contract C_1, contract C_2 cannot be $(e = 2, w = 4)$ (because of the incentive compatibility constraint for the $\theta = 1$ type). Hence it must be that at contract C_2 the slope of the indifference curve of the $\theta = 2$ type (given by $\frac{e_2}{2}$) is higher than 1 (the slope of the line of equation $w = e + 2$), so that $e_2 > 2$, that is, type $\theta = 2$ work harder at a separating equilibrium than at the benchmark case of part *e*.

A little bit of calculation shows that $e_2 = 1 + \sqrt{2}$ and $w_2 = 3 + \sqrt{2}$. In fact, the indifference curve of the $\theta = 1$ type through contract $C_1 = (1, 2)$ is given by the equation $w - \frac{e^2}{2} = 2 - \frac{1}{2}$; thus it is $w = \frac{3 + e^2}{2}$. Equating this to $w = e + 2$ gives $e_2 = 1 + \sqrt{2}$ and $w_2 = 3 + \sqrt{2}$. Thus the utility of type $\theta = 2$ at contract C_2 is $w_2 - \frac{e_2^2}{4} = 3 + \sqrt{2} - \frac{(1 + \sqrt{2})^2}{4} = 2.957$ which is less than the utility at the contract of part (e), namely $4 - \frac{2^2}{4} = 3$.