**University of California, Davis**
**Department of Economics**
**Macroeconomics**

# PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE, August 31, 2017

**Directions:** Answer all questions. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. Point totals for each question are given in parentheses. You have 5 hours to complete the exam and an additional 20 minutes of reading time.

**Question 1** (*10 points*)

Consider an pure exchange economy that consists of two islands, $i = \{1, 2\}$. Each island has a large population of infinitely-lived, identical agents, normalized to the unit. There is a unique consumption good, say, coconuts, which is not storable across periods. Agents' preferences are given by

$$u_i(c^i) = \sum_{t=0}^{\infty} \beta^t ln(c_t^i), \forall i,$$

where $c_t^i$ is consumption in period $t$ for the typical agent in island $i$ and $\beta \in (0, 1)$. The total endowment of coconuts in the economy in period $t$ is given by the sequence $\{e_t\}_{t=0}^{\infty}$, such that $e_t > 0$ for all $t$. But due to weather conditions in this economy, we have $e_t^1 = e_t$, if $t$ is even, and $e_t^1 = 0$, if $t$ is odd. (Naturally, $e_t^2 = e_t - e_t^1$.) Agents cannot do anything to boost the production of coconuts, but they can trade coconuts, so that the consumption of the typical agent in island $i$, in period $t$, is not necessarily equal to the production of coconuts on that island in that period. Assume that shipping coconuts across islands is costless.

a) Define an Arrow Debreu equilibrium (ADE) and a sequential markets equilibrium (SME) for this economy.

b) Assuming that $e_t = 2 - e^{-t}$, and using any method you like, characterize the ADE prices, $\{\hat{p}_t\}_{t=0}^{\infty}$. Are prices increasing or decreasing in $t$, and why?

**Question 2** (*20 points*)

Consider the following extension of the Mortensen-Pissarides model in continuous time. Labor force is normalized to 1, but there are two types of workers, Type 1, with measure $\pi \in (0,1)$, and Type 2, with measure $1-\pi$. The two types of workers have different productivities: when a type 1 worker is matched with a firm, she can produce $p > 0$ units of the numeraire good per unit of time, but when a type 2 worker is matched with a firm, she cannot produce anything (the type 2 is a total lemon). Although firms would clearly prefer to match with Type 1 workers, they can only observe the worker's type after they have matched. This implies that the matching process is "unbiased", i.e., when a firm matches with a worker, the probability that this worker is of Type 1, depends only on the relative measure of Type 1 workers in the pool of unemployed.[1] On the flip side, this assumption means that the arrival rate of jobs to a worker does not depend on her type (since firms cannot discriminate, even though they would like to).

Once a match has been formed, the worker's type is immediately revealed. If the worker is of type 1, the two parties negotiate over the wage as in the baseline model (with $\beta \in (0,1)$ denoting the worker bargaining power) and production starts right away. If the worker is type 2, clearly, there is no need for any negotiation, since there is no production and no surplus to split. In this case, by law, the firm must pay the worker a fixed wage $w_m$ per unit of time (think of it as the minimum wage), until it can prove that the worker is a lemon. The firm will eventually be able to prove this in a court of law, but the court decision takes a random amount of time. Specifically, the decision of the court arrives at a Poisson rate $a > 0$. When the decision is made, the firm can (finally) fire the unproductive worker and stop paying her the amount $w_m$.

Let the measure of unemployed workers of Type i be $u_i$, and let the total measure of unemployed workers be $u = u_1 + u_2$. There is a very large measure of (identical) firms that can enter the market and search for workers. A firm can enter the labor market with exactly one vacancy, and the total measure of vacancies $v$ will be determined endogenously by free entry. A CRS matching function brings together unemployed workers and vacant firms, and, due to the "unbiased" matching technology assumed here, the total number of matches depends only on $v, u$, i.e., $m = m(u, v)$, and it is increasing in both arguments. As is standard, let $\theta \equiv v/u$ denote the market tightness.

To close the model, we will make a few more standard assumptions. While a firm is searching for a worker it has to pay a search (or recruiting) cost, $pc > 0$, per unit of time. Productive jobs are exogenously destroyed at Poisson rate $\lambda > 0$, and, as already explained, unproductive jobs are terminated (through the legal process) at the rate $a > 0$. To avoid weird equilibria, assume that $a > \lambda$. All agents discount future at the rate $r > 0$, and all unemployed workers enjoy a benefit $z > 0$ per unit of time. We will impose $p > w_m > z$.

a) Describe the Beveridge curve (the relationship between unemployment, $u_i$, and market tightness, $\theta$) for each type of worker.[2]

[1] Suppose there are $u$ unemployed workers out there looking for jobs, and 75% of them are of Type 1. Then, **conditional on meeting a worker**, the probability that this worker is a Type 1 is 75%.

[2] **Hint:** This economy will have two Beveridge curves, one for each type. To find them equate the inflows and outflows out of the pool of unemployment for each type.

b) For $i = 1, 2$, use your findings in part (a) to define the fraction of Type $i$ workers who are unemployed (i.e., the unemployment rate *within* the Type i population). Denoting this term by $\gamma_i$, show that $\gamma_1 < \gamma_2$.

c) Let $V, J_1, J_2$ denote the value functions of a firm that is vacant, matched with a Type 1 worker, and matched with a Type 2 worker, respectively. Also, let $U_i, W_i$, $i = 1, 2$, denote the value functions of a Type $i$ worker who is unemployed or employed, respectively. Describe these value functions.

d) Exploit the free entry condition (i.e., $V = 0$) in order to provide the job creation (JC) curve for this economy.[3]

e) Describe the wage curve ($WC$) for this economy.

f) Discuss shortly the existence and uniqueness of equilibrium (no need to go into great detail and formal proofs).

[3] **Hint:** The JC curve will contain the expectation of the firm about which worker type it will meet, and it should relate $\theta$ with the (endogenous) wage paid to type-1 workers, $w_1$. Everything else that appears in that equation should be a parameter.

**Question 3** (*20 points*)

Consider the standard growth model in discrete time. There is a large number of identical households normalized to 1. Each household maximizea life-time discounted utility

$$U(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t u(c_t), \ \beta \in (0,1).$$

Each household has an initial capital $k_0 > 0$ at time 0, and one unit of productive time in each period that can be devoted to work. Final output is produced using capital and labor, according to a production function, $F$, which has the standard properties discussed in class, most notably, it is increasing in both arguments and exhibits CRS. This technology is owned by firms (whose measure does not really matter because of the CRS assumption). Output can be consumed ($c_t$) or invested ($i_t$). Households own the capital (so they make the investment decision), and they rent it out to firms. Let $\delta \in (0,1)$ denote the depreciation rate of capital. Households own the firms, i.e., they are claimants to the firms' profits, but these profits will be zero in equilibrium. The function $u$ also has the usual nice properties, which I will not spell out here since you will not need them explicitly.

In this economy there is a government that collects taxes and (for simplicity) throws the tax revenues into the ocean. The government can implement one of the follwoing two alternative taxation systems, let us call them System A and System B. System A is a proportional tax, $\tau \in [0,1]$, on agents' capital income. In other words, if the government implements System A, it collects a fraction $\tau$ of all the income that agents earn by renting out their capital to firms. System B is a proportional tax, $\tau \in [0,1]$, on agents' **investment**. In other words, if the government implements System B, it collects a fraction $\tau$ of all the resources that agents choose to allocate into investment.

a) Write down the problem of the household recursively, under both taxation systems.[4] Pay special attention to the budget constraints. These constraints will not be the same under the two specifications. Also, notice that I am not asking you to define a RCE in detail; just state the representative agent's problem within a RCE environment.

b) Describe the steady state equilibrium capital stock under taxation System A, for any given $\tau \in [0,1]$. Denote this object by $K_A^*(\tau)$.

c) Describe the steady state equilibrium capital stock under taxation System B, for any given $\tau \in [0,1]$. Denote this object by $K_B^*(\tau)$.

d) Assume that $F(K,N) = K^a N^{1-a}$, $a \in (0,1)$. Provide closed form solutions for the terms $K_A^*(\tau), K_B^*(\tau)$, described in parts (b),(c).[5]

e) Plot the terms $K_A^*, K_B^*$, calculated in part (d), against $\tau \in [0,1]$ and in the same graph. Discuss shortly.

[4] Here, the firms face a static problem. I am not asking you to explicitly spell it out, but this problem is critical for the determination of the various prices.

[5] **Hint:** Here, it is more convenient to work directly with $F$, i.e., do not work with the auxiliary function $f$ that we introduced in the lectures.

f) Describe the governemnt's total tax revenue in steady state under System B, $T_B$. Plot $T_B$ as a function of the tax rate $\tau$ (this is the so-called Laffer curve). Discuss the shape (i.e., the monotonicity) of the Laffer curve for the various values of $a$ and $\tau$.

**Question 4** (*20 points*)

Consider the following decentralized real business cycle model with no trend growth. There is a continuum of households and the representative household's preferences are given by:

$$\ln\left(c_t - hC_{t-1}\right) - \frac{1}{2}n_t^2 \tag{1}$$

Where $c$ is household consumption and $C$ is aggregate consumption (which the household takes as given). The household is infinitely lived and maximizes utility subject to their budget constraint. The household budget constraint can be written as:

$$c_t + k_{t+1} - (1-\delta)k_t = w_t n_t + r_t^k k_t + \pi_t \tag{2}$$

where $w$ is the real wage, $n$ is hours worked, $k$ is capital, $r^k$ is the rental price of capital and $\pi$ are profits from firms.

The representative firm produces output using capital $k$ and labor $n_t$:

$$y_t = A_t k_t^{\alpha} n_t^{1-\alpha} \tag{3}$$

Total factor productivity $A$ follows a Markov chain over the set $A = \{a_1, .....a_N\}$ with transition probabilities given by $p_{ij}$. The aggregate resource constraint is

$$Y_t = C_t + I_t$$

where upper case letters denote aggregate variables.

a) Write down the household's problem in recursive form and write down the firm's maximization problem. Derive the household's first order conditions and the firm's optimal hiring rules.

b) Carefully define a recursive competitive equilibrium. Take care to distinguish between the aggregate and individual state variables and explain any market clearing conditions.

c) Linearize the consumption Euler equation you found in part (a) around the deterministic steady state.

d) With reference to your answer in part (c) (if you can), discuss how a TFP shock affects consumption in this model. Would the dynamics of consumption be different if consumption preferences were given by $\ln\left(c_t - hc_{t-1}\right)$? (Hint: note the second term is now household consumption at $t-1$, not aggregate consumption. You also do not need to derive anything for this question).

e) What is the labor supply elasticity in this model? Given this value, how well will the model match the data? Explain.

**Question 5** (*20 points*)

The question asks you to consider the effects of shocks to household preferences in the New Keynesian model.

*Households*

The representative household's per period utility function is:

$$\left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi}\right) Z_t \tag{4}$$

where $Z$ is a household preference shock, $N$ is hours worked and $C$ is real consumption. The linearized equilibrium conditions are:

$$E_t\hat{c}_{t+1} - \hat{c}_t = \frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1}) - \frac{1}{\sigma}(1-\rho)\hat{z}_t \tag{5}$$

$$\hat{w}_t = \sigma\hat{c}_t + \psi\hat{n}_t \tag{6}$$

*And for firms*

$$\hat{y}_t = \hat{n}_t \tag{7}$$

$$\hat{w}_t = \hat{\phi}_t \tag{8}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \kappa\tilde{y}_t \tag{9}$$

$\kappa$ is inversely related to the degree of degree of price stickiness and $\tilde{y}_t$ is the output gap (relative to the model with flexible prices)

$$\tilde{y}_t = \hat{y}_t - \hat{y}_t^n \tag{10}$$

*Resource constraint*

$$\hat{y}_t = \hat{c}_t \tag{11}$$

*Policy:*

$$\hat{i}_t = \phi_\pi\hat{\pi}_t \tag{12}$$

where $\phi_\pi > 1$. Preference shocks follow an AR(1) process (in percentage deviations from steady state)

$$\hat{z}_t = \rho\hat{z}_{t-1} + e_t \tag{13}$$

In percentage deviations from steady state: $\hat{z}_t$ is the preference shock, $\hat{\phi}_t$ is real marginal cost, $\hat{c}_t$ is consumption, $\hat{w}_t$ is the real wage, $\hat{n}_t$ is hours worked, $\hat{y}_t$ is output. In deviations from steady state: $\hat{i}_t$ is the nominal interest rate, $\hat{\pi}_t$ is inflation.

a) Using the equilibrium conditions above, show that this model can be represented by the standard 3 equations

$$E_t\tilde{y}_{t+1} - \tilde{y}_t = \frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1} - \hat{r}_t^n) \tag{14}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \kappa \tilde{y}_t \tag{15}$$

$$\hat{i}_t = \phi_\pi \hat{\pi}_t \tag{16}$$

Where the natural real rate of interest is:

$$\hat{r}_t^n = (1-\rho)\hat{z}_t \tag{17}$$

and where $\hat{z}_t$ follows the process in equation 13. (Hint: you may find it useful to start by showing that the natural rate of output is constant in this model).

b) Using the method of undetermined coefficients, find the response of the output gap and inflation to an exogenous decrease in $\hat{z}_t$ when prices are sticky and monetary policy follows the Taylor Rule above. To do this, guess that the solution for each variable is a linear function of the shock $\hat{z}_t$:

$$\tilde{y}_t = \Lambda_y \hat{z}_t$$

$$\hat{\pi}_t = \Lambda_\pi \hat{z}_t$$

c) Interpret your results. In particular, explain how, and why, preference shocks affect the output gap and inflation. Briefly comment on how a decrease in $\hat{z}_t$ relates to typical recessions we see in the data.

d) Instead of following the Taylor Rule above, policy is now set optimally. Derive the optimal monetary policy rule under discretionary policy. (Hint: As in class, assume that the loss function has quadratic terms for the output gap and inflation, with a relative weight $\vartheta$ on the output gap. For simplicity, assume the steady state is efficient). What is the optimal path for the output gap and inflation in response to preference shocks under this policy (for this last part you do not need to derive anything)?

e) Now suppose the central bank has access to a credible commitment technology. How would your answer to (d) change if the central bank followed the optimal policy rule under commitment? You do not need to derive anything, but briefly explain how commitment policy differs from discretionary policy and whether this would change the optimal path for the output gap and inflation.

**Question 6** (*10 points*)

This question is about fiscal policy in the baseline real business cycle model. You do not need to derive anything and keep your answers clear and concise.

a) TFP shocks can explain the positive correlation of GDP, consumption and investment in the data. Government consumption shocks cannot explain these facts. Is this statement true or false? Explain. (Note: in this model, government consumption shocks are defined as the purchase of consumption goods by the government. This spending is not productive and does not enter the household's utility function.)

b) The government wants to stimulate private consumption and GDP. They propose a temporary, debt-financed, tax cut. In the RBC model, will a tax cut have the desired effect on the economy? Start by considering lump sum taxes, and then discuss how this result might change for other types of taxes.