ON THE EFFECTS OF PRIVATE CAPITAL FALLING INTO THE PUBLIC DOMAIN

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this version: Oct, 2017

Abstract. The fact that some private capital eventually slides into the public domain —*e.g.* R+D investments as patents expire or taxed household savings and income that fund the institutional framework and public infrastructures— inefficiently distorts downwards the capital accumulation. This may have dire consequences on households' levels of consumption: in the neoclassical infinitely-live agents setup *with constant returns to scale* —as opposed to Romer (1986)— the planner would deliver a steady state consumption about 40% higher than the market one for standard parameters for the share of capital income, the consumption of physical capital, and discounting. This is established also for overlapping generations economies. I provide next a tax and transfers balanced policy able to decentralise the planner's steady state without resorting to the (impracticable) extension of property rights otherwise needed to address the problem. It consists of (i) subsidising the rental rate of capital by an amount equal to the depreciation/obsolescence rate of the capital sliding into the public domain, and (ii) taxing households' debt issued against future dividends.

The author thanks attendants to numerous seminar and conference presentations for useful feedback on this paper, and in particular David de la Croix, Pierre Pestieau and Victor Ginsburgh, for insightful remarks.

1. Introduction

Each generation passes on subsequent generations the results of its achievements, both physical (properties, infrastructures, facilities, ...) and intangible (technology, institutions, organisations, culture, ...),[1] the creation of all of them having required at any rate the use of available resources, and hence being capital in a broad sense. Some of this capital is passed on through the trade or bequest of individual property rights —this is notably the case for real state assets and production facilities, but also for intellectual property rights to the extent they have not expired yet— while some of it eventually just slides into the public domain —this is certainly the case for expired intellectual property rights, and also for institutional organisational organisational schemes (*e.g.* judiciary, urban planning,...) for which resources where used up,[2] but also any physical infrastructures publicly built out of taxed, privately held capital, *etc.*[3] This paper explores thus the consequences of at least some capital eventually escaping individual property rights.

I show in this paper that the progressive drift of proprietary investments into the

[1]Some of them actually fall into both categories like, for instance, cities, metropolitan areas or regions, with their combined nature of public infrastructures and organisations.

[2]Property rights over the technologies resulting from R+D investments are temporarily protected by law to allow the investor to get a return from the investment and, hence, supposedly to incentivise growth-enhancing R+D activities. There is a heated debate about whether the current patent system implementing those property rights is actually the most adequate for spurring innovation. For instance, Boldrin and Levine (2013) point that the evidence shows no correlation between the number of patents and productivity, and highlight that the rent-seeking nature of *patenting aims rather at preventing further innovation* from competitors, which typically builds on previous innovations. On the other hand, Gould and Gruben (1996) find that intellectual property protection is an important determinant of growth, although this seems to hinge on the openness of the country to international competition, without which it can be detrimental to growth. Also, in a Romer-style endogenous growth model Saint-Paul (2003) argues the crowding out effects of free blueprints on proprietary innovation and, hence, its negative impact on growth and welfare. At any rate, besides the issue of what drives innovation and what incentives are at play, there is the fact that technology is the result of investment, and is hence capital, but one whose property rights are protected only temporarily and thus eventually falls into the public domain.

[3]Strictly speaking, some commonly held physical capital is actually subject to property rights of state institutions —the *res publicae* and *res universitatis* of municipalities in Roman law, as opposed to things not subject to property rights at all by their nature (*res communes*) or by lack of appropriation (*res nullius*). Notwithstanding, for all purposes, I will consider it to be freely available in the public domain —at least for residents or citizens, depending on the case at hand— since the relevant feature characterising it is the fact of not being subject to *individual* property rights.

public domain distorts capital accumulation away from the optimal level that would be chosen by a utilitarian planner unconcerned by property rights. More specifically, the model shows *analytically* the essentials of why the gradual drift of capital into the public domain prevents the markets to deliver, under laissez-faire, the optimal level of capital accumulation.

The mechanism behind the result is simple enough. Some of the savings lent to firms as capital by households are used to create ways to increase productivity that will eventually become public domain[4] —*e.g.* R+D private investments, given the time limits to patents and copyrights or taxed savings or income funding the intangible organisational and legal framework in which the economy operates, as well as public infrastructures, but these are far from being the only examples.[5] As a result of this, firms effectively operate using not only the capital they borrow, but also the capital coming from prior private investments that has fallen into the public domain. Since only capital on which property rights can be enforced is remunerated,[6] savers do not take into account the impact that their loans to firms have on the future capital in the public domain, and hence on the future productivity of factors and on output through this channel.

The idea that firms may operate with more capital (intangible or not) than the one they have to remunerate might remind of the mechanism at the heart of the contribution made in Romer (1986). Notwithstanding, the mechanism above is distinct from the one underpinning Romer (1986). Indeed, increasing returns to scale are (among others) a *key* ingredient to the results in Romer (1986), while the misallocation of resources that depresses capital accumulation in this paper takes place even with a neoclassical, constant returns to scale technology. In a nutshell, Romer (1986) is driven by a technological assumption, while the results of this paper

[4]Some savings are even deliberately privately invested in capital intended, from the start, to be in the public domain, as it is the case for open source and shareware software. This paper is nonetheless not about such instances of capital deliberately accumulated to be freely available, but rather on the consequences of proprietary capital eventually sliding into the public domain.
[5]A related problem is that of firms' investment in the human capital of their employees. Such investments can be substantial, but they stop being "proprietary" for the firm if the employees quit for another job (since embodied in them). In a sense, such human capital investments by firms, while being proprietary to the employees, have the potentiality of becoming a *de facto* (excludable) public good provided by each firm to the industry.
[6]This is not to say that the productivity of public domain capital is not appropriated by anyone. It is indeed: it accrues firms' profits and eventually feeds into firms' owners wealth as distributed dividends. In effect, with proprietary capital eventually sliding into the public domain even constant returns to scale firms make profits. Nonetheless, even if these profits are distributed to the firms' owners, the latter will still fail to internalise the effect in their saving decision.

are driven by vanishing property rights —and the impossibility to restore them, as it will be seen. Moreover —although admittedly not as much a definitive conceptual difference as the previous one— the positive externality that investments have on everybody else's productivity is contemporaneous in Romer (1986), while in the case of capital falling into the public domain the externality exerted by the latter is intertemporal.

By sliding into the public domain, the productivity of the capital doing so is not remunerated to its original investors, but is rather fed into the profits of the firm operating with it for free. Even in an aggregate model, where the representative household is both the lender to the firm —so that it receives the returns to privately held capital— and the owner of the firm —so that it receives the distributed profits too— and therefore receives the entirety of the productivity of the capital used by the firm (whether privately or publicly held) the channel through which this productivity is received matters for the saving decision of the household. Namely, the productivity of capital in the public domain does not incentivise savings, while that of capital privately held does. This differentiated impact would be even more obvious with heterogeneous agents of which some are lenders and others owners, or all are both but to different extents. In another difference with Romer (1986), since technology is in the latter linearly homogeneous *"with respect to the factors that receive compensation"* firms do not make profits in Romer (1986) and therefore this differentiated impact of the remuneration, whether as return to loans or distributed dividends, on households' saving decision cannot be captured by the framework in Romer (1986).

As a consequence, private investments differ from those that a planner able to take into account the effect of public domain capital would choose. Specifically, in the case of infinitely-lived agents I explicitly show below that the market accumulates too little capital. Interestingly enough, in the overlapping generations case this is shown indirectly through the subsidy to capital returns required by the policy decentralising the planner's allocation.

In order to address the problem, I provide a policy allowing to steer decentralised choices towards the planner's allocation of choice. Since at the heart of the problem there lays an expiration of property rights,[7] it might seem that a simple all-encompassing extension of property rights would be enough. Nonetheless, since this is clearly impracticable —some of this capital cannot be appropriated (institutions, organisations,...) or is not advisable to be so (because, for instance, of

[7]And not of increasing returns as in Romer (1986).

the perverse incentives on innovation of extending indefinitely intellectual property rights pointed by the literature, see Boldrin and Levine (2013))[8]— it is important to devise an implementable policy that avoids running into generating additional inefficiencies. The policy put forward in this paper requires instead (i) to subsidise the rental rate of capital by an amount equal to the depreciation/obsolescence factor of the capital sliding into the public domain, (ii) to tax households' debt issued against the future dividends. While the first element of this policy —i.e. the subsidisation of capital returns— may be expected (although probably not the exact rate at which this needs to be done), its second element —i.e. taxing households' reliance on credit against future dividends distributed by firms— only makes full sense once it is understood the differentiated impact on the incentives to save of the return to privately held capital and the dividends received from the productivity of the unremunerated capital in the public domain.

In what follows, Section 2 presents the model with two variants for its demographics —infinitely-lived agents and overlapping generations respectively. Section 3, addressing the issue first in the infinitely-lived agents economy, establishes that the market necessarily under-accumulates capital due to part of it falling into the public domain. Section 4 addresses then the question for an overlapping generations setup, which provides additional insight on the way the externality operates and allows to provide a policy decentralising the planner's choice as a market outcome. This is done through a subsidy on capital returns, and a tax on debt issued against future profits. Section 5 concludes...

2. The model

Consider an economy in which part of savings or capital becomes, after some period of time, non-proprietary and falls into the public domain —*e.g.* the part of investments devoted to R+D that leads to proprietary technology for a limited amount of time only, or because of being taxed and used to fund the institutional and legal framework or some public infrastructure. Specifically, let N_t be the (possibly constant) population at t, and let k_t be the amount of savings lent to firms by the

[8]Many considerations about the public interest for patents to expire, to be entirely phased out, or even to invest in freely available technology —for instance, developing open source software as means to invest in network building— are not addressed in this paper (for the case against patents see Boldrin and Levine (2013); for an overview of the economics of open source see Lerner, Josh and Tirole (2005)). While extremely interesting, they address a different point from the one being made here.

representative household at t, so that the total investment at t used in production at $t+1$ is $N_t k_t$. Without loss of generality, assume that a fraction α of investments becomes public domain after one period, the complement fraction $1-\alpha$ remains proprietary and depreciates each period buy a factor $\delta \in (0,1)$. Public domain capital, on the other hand, depreciates or obsolesces each period by a factor $\phi \in (0,1)$, so that only a fraction ϕ of it remains productive after each period,[9] and is not reversible into consumption good.

Total capital available at any period t is therefore

$$K_t = N_{t-1}k_{t-1} + (1-\alpha)\sum_{i=2}^{+\infty}\delta^{i-1}N_{t-i}k_{t-i} + \alpha\sum_{i=2}^{+\infty}\phi^{i-1}N_{t-i}k_{t-i}$$

Notwithstanding, firms need to remunerate only

(1) the investment $N_{t-1}k_{t-1}$from savings made at $t-1$
(2) and proprietary older capital $(1-\alpha)\delta^{i-1}N_{t-i}k_{t-i}$, for $i = 2, 3, \ldots$ resulting from previous investments

that is to say

$$N_{t-1}k_{t-1} + (1-\alpha)\sum_{i=2}^{+\infty}\delta^{i-1}N_{t-i}k_{t-i}$$

but, crucially, not capital resulting from prior investments that has already slid into the public domain, i.e.

$$\alpha\sum_{i=2}^{+\infty}\phi^{i-1}N_{t-i}k_{t-i}.$$

For the sake of simplifying expressions and without loss of generality (see footnote 12 below), I will consider below —with no consequence for the main point of the paper— the extreme case in which $\alpha = 1$, so that capital available at t

$$K_t = N_{t-1}k_{t-1} + \sum_{i=2}^{+\infty}\phi^{i-1}N_{t-i}k_{t-i}$$

consists of newly saved capital and all the previously saved capital that has become public domain at previous periods (i.e. the sum in the second term in the right-hand side above).

[9]Note that the depreciation factor of proprietary capital δ and public domain capital ϕ need not be the same.

The production function $F(K_t, N_t)$ is neoclassical, i.e. returns to scale are constant in labor and available capital — both proprietary and in the public domain— as opposed to the overall increasing returns to scale that are the keystone of the setup considered in Romer (1986).

Finally, as for the demographics, I will consider next both the infinitely-lived agents and the overlapping generations setups. Specifically, a normalised unit of labor is supplied inelastically by each household

(1) when young, if the economy consists of 2-period-lived overlapping generations, and population changes each period by a constant factor n, so that, for all $i = 1, 2, \dots$
$$N_t = n^i N_{t-i}$$
(2) each period, if agents are infinitely-lived, with a constant population (*i.e.* $n = 1$) normalised to 1, so that $N_t = 1$, for all t.

3. Market underaccumulation: the infinitely-lived agents economy case

3.1 The planner's allocation for the infinitely-lived agents economy.

A planner is not constrained by property rights, but rather aims at maximising the increasing and concave utility u (discounted by a factor β) of the representative household from a sequence of consumptions c_t while satisfying, at each period t, the feasibility constraint, *i.e.* it chooses the sequence of nonnegative c_t and k_t solving
$$\max_{c_t, k_t} \sum_{t=1}^{+\infty} \beta^{t-1} u(c_t)$$
$$c_t + k_t \leq F(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t-i}, 1)$$
where $\beta \in (0, 1)$ and, in each constraint, *i.e.* for all $t = 1, 2, \dots$, trivially $k_{t-i} = 0$ for $i > t$, given some initial endowment $k_0 > 0$.

The planner's choice, therefore, must necessarily satisfy, for each $t = 1, 2, \dots$, and some positive $\lambda_t, \lambda_{t+1}, \dots$, the condition
$$\begin{pmatrix} \beta^{t-1} u'(c_t) \\ 0 \end{pmatrix} = \lambda_t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda_{t+1} \begin{pmatrix} 0 \\ -F_K^{t+1} \end{pmatrix} + \lambda_{t+2} \begin{pmatrix} 0 \\ -F_K^{t+2} \cdot \phi \end{pmatrix} + \dots$$

where F_K^{t+j} stands for the marginal productivity of capital at $t + j$, *i.e.*

$$F_K^{t+j} = F_K(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t+j-i}, 1)$$

and from which the next characterisation easily follows.

Proposition 1. *In the infinitely-lived agents economy in Section 2, a planner's allocation* $\{c_t, k_t\}_{t\in\mathbb{N}}$ *is characterised by*

$$1 = \sum_{j=1}^{+\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} F_K(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t+j-i}, 1)\phi^{j-1}$$

and the feasibility constraint, for all $t = 1, 2, \ldots,$ *given some initial endowment* $k_0 > 0$ *(and trivially* $k_{t-i} = 0$ *for* $i > t$*).*

We will now compare this necessary characterisation of the planner's choice with that of the market allocation next.

3.2 The market allocation for the infinitely-lived agents economy.

A household behaving competitively aims instead at maximising its utility under its budget constraint, choosing its sequences of consumptions c_t and savings k_t that are lent to firms as capital solving

$$\max_{c_t, k_t} \sum_{t=1}^{+\infty} \beta^{t-1} u(c_t)$$
$$c_t + k_t \leq w_t + r_t k_{t-1} + \pi_t$$

given the sequence of profits π_t it receives as the owner of the representative firm,[10] and the sequences of factor prices w_t and r_t, determined at equilibrium by their

[10]Each firm gets at t an equal share of the positive aggregate profits from the productivity of the capital already in the public domain, namely $\pi_t = F_K(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t-i}, 1) \sum_{i=2}^{+\infty} \phi^{i-1} k_{t-i}$. Free entry in the industry will drive *per firm* profits down to zero as an unbounded number of firms enter the market, but from the linear homogeneity of F *aggregate* profits remain constant at a positive level regardless the number of firms. It should be noticed that, acordingly, capital in the public domain is implicitly here nonproprietary but excludable (e.g. commons), which applies to any capital whose use may suffer from congestion (e.g. infrastructure, urban networks, judiciary, police forces,... but not technology).

marginal productivities, *i.e.*

$$w_t = F_L(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t-i}, 1)$$

$$r_t = F_K(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t-i}, 1)$$

where, for all $t = 1, 2, \dots$, trivially $k_{t-i} = 0$ for $i > t$, given some initial endowment $k_0 > 0$.

The household's choice therefore necessarily satisfies, for all t and some positive λ_t, λ_{t+1},

$$\begin{pmatrix} \beta^{t-1} u'(c_t) \\ 0 \end{pmatrix} = \lambda_t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda_{t+1} \begin{pmatrix} 0 \\ -F_K^{t+1} \end{pmatrix}$$

where F_K^{t+1} stands, as before, for the marginal productivity of capital at $t+1$, and from where the next characterisation easily follows.

Proposition 2. *In the infinitely-lived agents economy in Section 2, in which some private capital eventually falls into the public domain, a market allocation* $\{c_t, k_t\}_{t\in\mathbb{N}}$ *is characterised by*

$$1 = \beta \frac{u'(c_{t+1})}{u'(c_t)} F_K(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t+1-i}, 1)$$

and the budget constraint,[11] *for all* $t = 1, 2, \dots,$ *given some initial endowment* $k_0 > 0$ *(and trivially* $k_{t-i} = 0$ *for* $i > t$*).*

The characterisations provided in Propositions 1 and 2 allow to compare the planner's and market steady state allocations next.

3.3. Planner's vs market steady states in the infinitely-lived agents economy.

When it comes to comparing the steady state allocations of the economy that the market and the planner would deliver, in the framework of the infinitely-lived agents

[11] Which is equivalent to the feasibility constraint.

economy of Section 2, the previous characterisations point to a clear-cut result: at the steady state the market accumulates less capital than the planner would, as the next proposition establishes.

Proposition 3. *In the infinitely-lived agents economy in Section 2, in which some private capital eventually falls into the public domain, the market steady state level of capital $\bar{k}$ is unique and smaller than the planner's unique steady state level k^*.*

Proof. From the characterisations of Propositions 1 and 2 above

$$1 = \sum_{j=1}^{+\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} F_K(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t+j-i}, 1)\phi^{j-1}$$

$$1 = \beta \frac{u'(c_{t+1})}{u'(c_t)} F_K(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t+1-i}, 1)$$

it follows that each steady state —i.e. the planner's k^* and the market $\bar{k}$— is characterised respectively by

$$1 = \beta F_K(\frac{k^*}{1-\phi}, 1)\frac{1}{1-\beta\phi}$$
$$1 = \beta F_K(\frac{\bar{k}}{1-\phi}, 1)$$

From these conditions and the decreasing marginal productivity of capital that both $\bar{k}$ and k^* are unique. They also imply straightforwardly that $k^* > \bar{k}$ —since $\beta, \phi \in (0,1)$.[12] □

The impact, in the infinitely-lived agents economy, of private capital sliding into the public domain is therefore very clear: the market accumulates too little capital.

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[12] If not all capital slides into the public domain, i.e. $\alpha \in (0,1)$, then the respective steady states are characterised by

$$1 = \beta F_K([\frac{1-\alpha}{1-\delta} + \frac{\alpha}{1-\phi}]k^*, 1)[\frac{1-\alpha}{1-\beta\delta} + \frac{\alpha}{1-\beta\phi}]$$
$$1 = \beta F_K([\frac{1-\alpha}{1-\delta} + \frac{\alpha}{1-\phi}]\bar{k}, 1)$$

and the same conclusion follows.

In order to get a grasp of by how much might the market be missing the optimal level of capital because of the gradual slide of the latter (through a number of ways) into the public domain, consider a standard value for the share of income remunerating capital, *i.e.* $\alpha = 1/3$, for a Cobb-Douglas production function with normalised total factor productivity $F(K, L) = K^{\alpha}L^{1-\alpha}$, and a value of $\beta = .98$ corresponding approximately to a discounting by a rate of 2%, as well as a value of $\phi = .85$ corresponding to a 15% consumption of fixed capital.[13] In this case, the ratio of the planner's steady state level of capital to the market one is

$$\frac{k^*}{\bar{k}} = \left(\frac{1}{1-\phi\beta}\right)^{\frac{1}{1-\alpha}} = \left(\frac{1}{1-.85\cdot .98}\right)^{\frac{1}{1-\frac{1}{3}}} = 14.65$$

so that the market's steady state level of capital accumulation is way too low (the planner would choose to save/invest almost fifteen times more!). More informative of the potential impact of the households' well-being, in terms of consumption, the ratio of the planner's to the market is, at the steady state

$$\frac{c^*}{\bar{c}} = \frac{(14.65\bar{k})^{\frac{1}{3}} - 14.65\bar{k}}{\bar{k}^{\frac{1}{3}} - \bar{k}}$$

with

$$\bar{k} = (\alpha\beta)^{\frac{1}{1-\alpha}}(1-\phi) = (\frac{1}{3}\cdot .85)^{\frac{1}{1-\frac{1}{3}}}(1-.85) = .0226$$

so that

$$\frac{c^*}{\bar{c}} = \frac{(14.65\cdot .0226)^{\frac{1}{3}} - 14.65\cdot .0226}{.0226^{\frac{1}{3}} - .0226} = 1.3867$$

That is to say, for empirically reasonable values of basic parameters, the market fails to deliver the almost 40% more consumption that households would be allocated by a hypothetical planer. At any rate, the point of the exercise is clearly not precision, but rather to see that the inefficiency is substantial. It would be nonetheless interesting to have an estimate of the actual size of the inefficiency that followed from empirical data.

Addressing this same question, in the next section, in an overlapping generations setup instead, I will show how to offset in the market allocation the externality arising from capital sliding into the public domain.

[13]The consumption of fixed capital, or CFC, captures in national accounts the depreciation of aggregate capital stock. It is the difference between the gross investment (aggregate gross fixed capital formation) and net investment (net fixed capital formation) or between the Gross National Product and Net National Product. It has remained in the vicinity of 15% for Germany and the US, for instance, since the 80's (source: AMECO annual macro-economic database of the European Commission's Directorate General for Economic and Financial Affairs).

[[WHY NOT HERE?]]

4. Undoing the public domain distortion: the overlapping generations economy case

Because of the need, in the 2-period-lived (say, young and old) representative agent overlapping generations economy in Section 2, to distinguish variables relating to different generations as well as time periods, we will use superscript t to identify t's generation choice variables like, among others, the intertemporal profile of consumption c_0^t, c_1^t, or the amount k^t lent to firms by generation t's representative household for production at $t+1$.

As previously, I will characterise next the steady state allocation that the planner, unconstrained by property rights, would choose, and I'll compare it to the market steady state of the economy. From the systems of equations characterising the optimal steady state and the market steady state will follow the policy that is necessary to correct the depressing effect of capital sliding into the public domain on capital accumulation.

4.1 The planner's problem in the overlapping generations economy.

A utilitarian planner would choose an allocation of each period's output between consumption for the agents alive in the period and investment for future production that maximises a weighted sum of the utilities of all households under the each period's feasibility constraint —expressed below in *per* young terms, for a population growth factor n—

$$\max_{c_0^t, c_1^t, k^t \geq 0} \sum_{t=1}^{+\infty} \eta^{t-1} \big(u(c_0^t) + \beta u(c_1^t) \big)$$

$$c_0^t + \frac{c_1^{t-1}}{n} + k^t \leq F\Big(\frac{1}{\phi} \sum_{i=1}^{+\infty} \Big(\frac{\phi}{n}\Big)^i k^{t-i}, 1\Big), \forall t = 1, 2, \ldots$$

(with $k^{t-i} = 0$ for $i > t$ trivially) given some initial c_1^0, k^0, a discount factor η for future generations, the households' own discounting β of old age utility, and the depreciation/obsolescence factor ϕ for capital in the public domain.

From the problem above follows the next characterisation of the planner's choice linking, on the one hand, the contribution of savings k^t at any given period t to the marginal productivity of capital at all future periods $t+j$, for all $j = 1, 2, \dots,$ to, on the other hand, the marginal rates of intertemporal substitution of consumption for all agents between t and each $t+j$.

Proposition 4. *In the overlapping generations economy in Section 2, an allocation $\{c_0^t, c_1^t, k^t\}_{t\in\mathbb{N}}$ chosen by the planner satisfies*

$$1 = \frac{1}{\phi}\sum_{j=1}^{+\infty}\left[F_K\left(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+j-i}, 1\right)(\phi\beta)^j \prod_{h=0}^{j-1}\frac{u(c_1^{t+h})}{u(c_0^{t+h})}\right]$$

as well as each period feasibility constraint binding (with $k^{t-i} = 0$ for $i > t$ trivially).

Proof. The solution to the planner's problem is necessarily characterised, for some $\lambda^{t+i} > 0$ with $i = 0, 1, 2, \dots,$ by

$$\begin{pmatrix} \eta^{t-1}u'(c_0^t) \\ \eta^{t-1}\beta u'(c_1^t) \\ 0 \end{pmatrix} = \lambda^t \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda^{t+1}\begin{pmatrix} 0 \\ \frac{1}{n} \\ -\frac{1}{\phi}F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1)\frac{\phi}{n} \end{pmatrix}$$
$$+ \lambda^{t+2}\begin{pmatrix} 0 \\ 0 \\ -\frac{1}{\phi}F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+2-i}, 1)(\frac{\phi}{n})^2 \end{pmatrix}$$
$$+ \dots$$

for all $t = 1, 2, \dots,$ that is to say, by the following conditions on the marginal rates of substitution

(1) within generations

$$\frac{1}{\beta}\frac{u'(c_0^t)}{u'(c_1^t)} = \frac{\lambda^t}{\lambda^{t+1}} \cdot n \tag{1}$$

(2) and across generations

$$\frac{u'(c_0^t)}{u'(c_0^{t+i})} = \eta^i \frac{\lambda^t}{\lambda^{t+i}}$$

and

$$\frac{1}{\beta}\frac{u'(c_0^t)}{u'(c_1^{t+i})} = \eta^i \frac{\lambda^t}{\lambda^{t+i}} n$$

on top of

$$1 = \frac{1}{\phi}\sum_{j=1}^{+\infty}\left[F_K\left(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+j-i}, 1\right)(\frac{\phi}{n})^j \frac{\lambda^{t+j}}{\lambda^t}\right] \tag{2}$$

and

$$c_0^t + \frac{c_1^{t-1}}{n} + k^t = F(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1)$$

The necessary condition obtains then from repeated direct substitutions of the intra generational intertemporal marginal rate of substitution (1) into (2). □

From the previous characterisation it can be obtained that of the unique symmetric allocation that a planner treating equally all generations would choose, that is to say the characterisation of the planner's steady state in Proposition 5 next.

Proposition 5. *In the overlapping generations economy in Section 2, the egalitarian planner's steady state is characterised by the unique profile of life-cycle consumptions and savings* c_0, c_1, k *solving*

$$\begin{aligned} \frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} &= n = F_K(\frac{k}{n-\phi}, 1) + \phi \\ c_0 + \frac{c_1}{n} + k &= F(\frac{k}{n-\phi}, 1) \end{aligned} \tag{P}$$

given n, ϕ.

Proof. We will see first that the system above characterises any steady state, and then we will see that there is a solution to the system and only one.

From Proposition 4 and its proof, the symmetric limit allocation resulting from the planner treating all generations increasingly equally as $\eta \to 1$ —so that $\frac{u'(c_0^t)}{u'(c_0^{t+i})} \to 1$ for all t and all i, and hence so that $\eta^i \frac{\lambda^t}{\lambda^{t+i}} \to 1$ and thus $\frac{\lambda^t}{\lambda^{t+i}} \to 1$— is necessarily characterised by

$$\frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = n$$

—which implies $\phi\beta\frac{u'(c_1)}{u'(c_0)} < 1$ whenever $n > 1$,[14] so that the series next is convergent— and[15]

$$1 = \frac{1}{\phi} F_K\Big(\frac{k}{n-\phi}, 1\Big) \sum_{j=1}^{+\infty} \Big(\phi\beta\frac{u'(c_1)}{u'(c_0)}\Big)^j$$

i.e., replacing the series by its value and rearranging terms,

$$\frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = F_K\Big(\frac{k}{n-\phi}, 1\Big) + \phi$$

since $\phi\beta\frac{u'(c_1)}{u'(c_0)} < 1$.[16]

A planner steady state is clearly locally unique since it is a regular zero of the left-hand side of the planner's steady state equations

$$\begin{aligned} u'(c_0) - n\beta u'(c_1) &= 0 \\ F_k(\frac{k}{n-\phi}, 1) + \phi - n &= 0 \\ c_0 + \frac{c_1}{n} + k - F(\frac{k}{n-\phi}, 1) &= 0 \end{aligned}$$

In effect,

$$\begin{vmatrix} u''(c_0) & -n\beta u''(c_1) & 0 \\ 0 & 0 & F_{KK}(\frac{k}{n-\phi}, 1)\frac{1}{n-\phi} \\ 1 & \frac{1}{n} & 1 - F_K(\frac{k}{n-\phi}, 1)\frac{1}{n-\phi} \end{vmatrix} = \\ -F_{KK}(\frac{k}{n-\phi}, 1)\frac{1}{n-\phi}\Big[n\beta u''(c_1) + \frac{1}{n}u''(c_0)\Big] < 0.$$

But it is globally unique too, since if c_0, c_1, k and c_0', c_1', k' were two distinct steady states for the planner, then necessarily $k = k'$ —since the (injective) marginal productivity of capital must match $n - \phi$ for both of them— and $c_0 < c_0'$ would

[14]Indeed, equivalently $\beta\frac{u'(c_1)}{u'(c_0)} = \frac{1}{n} < 1$, since $n > 1$, from which $\phi\beta\frac{u'(c_1)}{u'(c_0)} < 1$ given that $\phi < 1$ too.

[15]Since $K_t = \sum_{i=1}^{+\infty} \phi^{i-1} k^{t-i} N_{t-i}$ and $N_t = n^i N_{t-i}$, then $\frac{K_t}{N_t} = \frac{1}{\phi}\sum_{i=1}^{+\infty}\big(\frac{\phi}{n}\big)^i k^{t-i}$ which at a steady state becomes $\frac{k}{n-\phi}$.

[16]Obtaining this characterisation from the one in Proposition 4 requires an argument in the limit as $\eta \to 1$, since for an equal weight for all generations in the planner's problem, i.e. $\eta = 1$, the planner's objective is not well defined.

imply $c_1 < c_1'$, which cannot be —since the *per* young aggregate consumption each period must match the common $F(\frac{k}{n-\phi}, 1) - k$. Therefore $c_0 = c_0'$ and from the feasibility constraint $c_1 = c_1'$ too. $\square$

In the next sections I will characterise now the market steady state.

4.2. Firms' problem.

The stock of capital available for production at t is generation t-1's aggregate savings in physical capital K^{t-1} plus the stock of depreciated capital available for production at $t-1$, that is to say ϕK_{t-1}, that is now in the public domain, *i.e.*

$$\begin{aligned} K_t &= K^{t-1} + \phi K_{t-1} \\ &= k^{t-1} N_{t-1} + \phi \sum_{i=1}^{+\infty} \phi^{i-1} k^{t-1-i} N_{t-1-i} \end{aligned}$$

or, in *per* young terms,

$$\frac{K_t}{N_t} = \frac{k^{t-1}}{n} + \frac{1}{n} \sum_{i=1}^{+\infty} (\frac{\phi}{n})^i k^{t-1-i}$$

Only the proprietary part of the stock of capital available for production at t in the first term of the right-hand side is remunerated by firms, while the fraction ϕ of capital built from all previous loans from households to firms and now in the public domain, represented by the remaining terms, is not.

Firms maximise profits at t choosing how much capital to borrow and how much labor to hire —which at equilibrium need be $K^{t-1} = k^{t-1} N_{t-1}$ (i.e. the aggregate of the amount k^{t-1} lent to firms by each of the N_{t-1} households at $t-1$) and N_t respectively— given the rental rate of capital r_t, the wage w_t, and the amount of private capital that has fallen into the public domain ϕK_{t-1}, that is to say

$$\max_{K^{t-1}, N_t} F(K^{t-1} + \phi K_{t-1}, N_t) - r_t K^{t-1} - w_t N_t$$

Note that, in its objective function above, the firm does not have to remunerate the non-proprietary capital ϕK_{t-1} in the public domain it uses. Factor prices are hence determined, at equilibrium, by

$$\begin{aligned} r_{t+1} &= F_K(K_{t+1}, N_{t+1}) \\ w_t &= F_L(K_t, N_t) \end{aligned} \tag{FP}$$

As a consequence, firms make at t the following aggregate profits

$$\pi_t = F_K(K^{t-1} + \phi K_{t-1}, N_t)\phi K_{t-1}$$

or, equivalently, the per old aggregate profits

$$\frac{\pi_t}{N_{t-1}} = F_K(\frac{k^{t-1}}{n} + \frac{1}{n}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-1-i}, 1) \cdot \sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-1-i} \qquad \text{(D)}$$

It is worth noting that —because of the linear homogeneity of the production function— since *aggregate* profits are positive at every period, free entry of firms in the market drives the level of *each firm*'s profits to zero, but not the level of *aggregate* profits π_t at any given period t, which remains positive as the product of the positive marginal productivity of capital and the positive amount of capital in the public domain. As a result, the ownership of the firms —which entitles to being distributed dividends— is traded across generations.

4.3 Households' problem in the overlapping generations economy.

As a consequence of the firms distributing as dividends the profits obtained from the non remunerated productivity of the capital in the public domain, firm ownership has a return and can therefore be used by households as a means of saving. The representative household born at t can therefore now transfer wealth *from its first period into the second* in three ways now: (i) lending to firms to get at $t+1$ the income r_{t+1} *per* unit of capital lent, (ii) holding real monetary balances[17] and, *moreover*, (iii) taking a stock in the ownership of firms in order to be distributed an equal share d_{t+1} of the aggregate profits π_{t+1} made by firms at $t+1$, as well as a proportional share of the resale value of the firms at $t+1$. Besides, we are going to assume that households can also transfer wealth *from its second period into the first* by (iv) borrowing from perfectly competitive financial intermediaries operating through the lives of all generations.

[17]Money is introduced in the model for the benchmark equilibrium to be optimal. In effect, in the Diamond (1965) setup at the foundation of the current one in this paper, in the absence of a bubbly asset in which to be able to save there is no hope for the market to implement the planner's choice, independently of whether the additional effect of public domain capital studied here is included or not. The reason is that it is the presence of such an asset which allows the market allocation to replicate the planner's link between the return to capital and the population growth factor. In more precise terms, generically, no non-monetary equilibrium can decentralise the planner's allocation, neither in Diamond (1965) nor in this paper's setup.

Thus, let k^t be the amount lent by the representative household born at t to firms, m^t be the household real balances, and s^t be the net saving in assets other than these two, that is to say the *net position* resulting from investing in firm ownership and borrowing from the financial intermediaries. If $s^t > 0$, household t is therefore investing in firms' ownership more than it may be borrowing from second period income. If $s^t < 0$ instead, household t is rather borrowing more from second period income than it is investing in firms' ownership.[18]

Household t's choice must therefore satisfy the budget constraints

$$\begin{aligned} c_0^t + k^t + s^t + m^t &\leq w_t \\ c_1^t \leq r_{t+1}k^t + d_{t+1} + s^{t+1}n &+ \frac{p_t}{p_{t+1}}m^t \end{aligned} \tag{BC}$$

where d_{t+1} is the *per* owner distributed profits —given the wage w_t, the rental rate of capital r_{t+1}, the level of prices during the household's lifetime p_t, p_{t+1}, the profits made by firms when old π_{t+1}, the *per* young net position s^{t+1} of generation $t+1$, and the population growth factor n, and moreover, the household's net position s^t —of savings in firms ownership minus borrowing against the future profits and resale value— must be bounded below by the present value of the revenue from firm ownership, *i.e.*

$$r_{t+1}s^t \geq d_{t+1} + s^{t+1}n.$$

Indeed, should the household choose to hold a negative net position between borrowing against firm ownership income —which is a possible equilibrium outcome if all generations did so— the amount borrowed cannot exceed the future income against which the borrowing takes place.

Therefore, agent t in principle solves

$$\begin{aligned} \max_{c_0^t, c_1^t, k^t, m^t \geq 0, s^t \in \mathbb{R}} & \quad u(c_0^t) + \beta u(c_1^t) \\ c_0^t + k^t + s^t + m^t &\leq w_t \\ c_1^t \leq r_{t+1}k^t + d_{t+1} + s^{t+1}n &+ \frac{p_t}{p_{t+1}}m^t \\ r_{t+1}s^t &\geq d_{t+1} + s^{t+1}n \end{aligned} \tag{H}$$

[18] For the ease of its interpretation, a positive s^t can be thought of as the amount payed by each household born at t to the households born at $t-1$ for the firm ownership, i.e. for the right to receive its dividends and the value of its resale to its n children paying each s^{t+1}. The claim on future dividends, and hence the possibility to borrow against them, is what allows to extend the interpretation of s^t to that of a net position that can be negative as well as positive.

Note that, necessarily, the first-order conditions with respect to c_0^t, c_1^t, k^t, m^t and s^t imply that the household's choice, at equilibrium, necessarily satisfies

$$\begin{pmatrix} u'(c_0^t) \\ \beta u'(c_1^t) \\ 0 \\ 0 \\ 0 \end{pmatrix} = \lambda_0^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \lambda_1^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{p_t}{p_{t+1}} \\ 0 \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -r_{t+1} \end{pmatrix}$$

for some $\lambda_0^t, \lambda_1^t, \mu^t > 0$, along with the binding constraints, or equivalently

$$\begin{aligned} \frac{1}{\beta}\frac{u'(c_0^t)}{u'(c_1^t)} = \frac{p_t}{p_{t+1}} = r_{t+1} \\ c_0^t + k^t + s^t + m^t = w_t \\ c_1^t = r_{t+1}k^t + d_{t+1} + s^{t+1}n + \frac{p_t}{p_{t+1}}m^t \\ r_{t+1}s^t = d_{t+1} + s^{t+1}n \end{aligned} \tag{HC}$$

It also follows from the first-order conditions above that the value of the firm for the household is $\mu^t = u'(c_0^t)/r_{t+1} > 0$.

The optimising behaviour of households in (HC) and firms in (D) and (FP), when compatible, determine a competitive equilibrium of this economy, as stated in the next section.

4.4. Competitive equilibrium of the overlapping generations economy.

A competitive equilibrium of the overlapping generations economy is therefore characterised by the following conditions.

Proposition 6. *In the overlapping generations economy in Section 2, in which some private capital eventually falls into the public domain, a competitive equilibrium is characterised by a consumption profile c_0^t, c_1^t, a loan to firms k^t, a net position in firms ownership and borrowing s^t, a real balance m^t, and distributed profits d_{t+1}, for each agent born in each period t, as well as prices p_t, for all t, such* *that*

$$\begin{aligned}
\frac{1}{\beta}\frac{u'(c_0^t)}{u'(c_1^t)} = \frac{p_t}{p_{t+1}} &= F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1) \\
c_0^t + k^t + s^t + m^t &= F_L(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1) \\
\frac{c_1^t}{n} = F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1)\frac{k^t}{n} &+ \frac{d_{t+1}}{n} + s^{t+1} + \frac{p_t}{p_{t+1}}\frac{m^t}{n} \\
F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1)s^t &= d_{t+1} + s^{t+1}n \\
d_{t+1} = F_K(\frac{k^t}{n} + \frac{1}{n}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1) &\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i} \\
\frac{p_t}{p_{t+1}}\frac{m^t}{m^{t+1}} &= n
\end{aligned} \tag{CE}$$

Proof. The first four lines follow from the household choice in (HC) with the factor prices replaced by the marginal productivities of factors according to the firms' behavior in (FP). The fifth line is the equilibrium *per* owner profits (D) distributed at each $t+1$.

The sixth line is equivalent to the feasibility of the allocation of resources for this economy, and can be obtained in the usual way adding up the budget constraints of the agents alive at any given period t —of which there are n young agents *per* old one— and taking into account the homogeneity of degree 1 of the production function, i.e. adding up

$$c_0^t + k^t + s^t + m^t = w_t$$

and

$$\frac{c_1^{t-1}}{n} = r_t\frac{k^{t-1}}{n} + \frac{d_t}{n} + s^t + \frac{p_{t-1}}{p_t}\frac{m^{t-1}}{n}$$

which with (FP) and the feasibility constraint

$$c_0^t + \frac{c_1^{t-1}}{n} + k^t = F(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1)$$

amounts to

$$\frac{p_t}{p_{t+1}} \frac{m^t}{m^{t+1}} = n$$

at any given t. $\square$

It should be noted, first, that the equilibrium conditions imply that different generations choose, necessarily, a different mix of monetary savings and a net position of stocks and borrowing for savings other than loans to firms, as the next proposition establishes.

Proposition 7. *In an overlapping generations economy in Section 2, in which some private capital eventually falls into the public domain, there is no competitive equilibrium in which the representative agent monetary savings* m^t*, and the net position in investing in firm ownership and borrowing* s^t*, are constant.*

Proof. In effect, should $m^t = m$ hold for some m and all t, then from the last equation in the system (CE) above

$$\frac{p_t}{p_{t+1}} = n$$

and should $s^t = s$ hold, then the no-arbitrage condition (NA) requires zero profits to be distributed, since

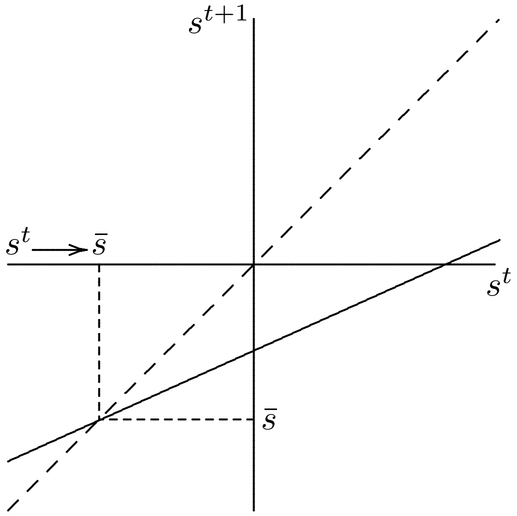
$$d_{t+1} = \Big[F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1) - n\Big] s = 0$$

according to the equilibrium condition $F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}\left(\frac{\phi}{n}\right)^i k^{t+1-i}, 1) = \frac{p_t}{p_{t+1}}$ and $\frac{p_t}{p_{t+1}} = n$ above. However, profits distributed at $t+1$ are positive, since

$$d_{t+1} = F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1)\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i} > 0$$

from which the conclusion follows. $\square$

Therefore a competitive equilibrium steady state will be an allocation where only the consumption profile, the loans to firms and the *total* (but not the composition) of savings in instruments other than loans to firms, $\bar{s}$, will stay constant, as shown in the next proposition. It is shown there too that, in the relevant case in which real balances and the value of the firm do not explode, (i) the share of real balances within $\bar{s}$ converges to zero, so that in the limit the net position of *ownership of the firm and borrowing replaces money* as the bubbly asset in the economy; and (ii) $\bar{s} < 0$, meaning that (in the limit) *each generation pays for the firm* not by saving but *by getting indebted* against the firm ownership income and repaying when old, the funds of the loan being provided to the lender by the repayment to the financial intermediaries from the previous generation.



Proposition 8. *In the overlapping generations economy in Section 2, in which some private capital eventually falls into the public domain, a competitive equilibrium steady state is characterised by a constant profile of consumptions c_0, c_1 and a constant loan to firms k, for all generations, as well as a constant growth factor of real balances m^{t+1}/m^t satisfying*

$$\begin{aligned} \frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = n\frac{m^{t+1}}{m^t} &= F_K(\frac{k}{n-\phi}, 1) \\ c_0 + k + \bar{s} &= F_L(\frac{k}{n-\phi}, 1) \\ c_0 + \frac{c_1}{n} + k &= F(\frac{k}{n-\phi}, 1) \end{aligned} \tag{M}$$

—where $\bar{s} = \frac{d}{r-n}$, with $d = r\frac{k}{n-\phi}\phi$ and $r = F_K(\frac{k}{n-\phi}, 1)$— which determines c_0, c_1, k, and $\frac{m^{t+1}}{m^t}$.

Moreover

$$s^t + m^t = \bar{s}$$

and, if $r < n$, the household net position in ownership of the firm and borrowing converge to $\bar{s} < 0$,[19] *while positive real balances converge to zero, i.e.*

$$\lim_{t \to +\infty} s^t = \bar{s} < 0$$
$$\lim_{t \to +\infty} m^t = 0$$

(*if $r \geq n$, savings invested in both real balances and the net position in ownership of the firm and borrowing diverge*).

Proof. From Propositions 6 and 7, a competitive equilibrium steady state is therefore characterised by the conditions next, where consumptions c_0^t, c_1^t, capital savings k^t, and distributed profits d_{t+1} —but not s^t or m^t (nor p_t, *a fortiori*)— stay constant at levels c_0, c_1, k, and d,

$$\frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = \frac{p_t}{p_{t+1}} = F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k, 1)$$
$$c_0 + k + s^t + m^t = F_L(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k, 1)$$
$$\frac{c_1}{n} = F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k, 1)\frac{k}{n} + \frac{d}{n} + s^{t+1} + \frac{p_t}{p_{t+1}}\frac{m^t}{n}$$
$$F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k, 1)s^t = d + s^{t+1}n$$
$$d = F_K(\frac{k}{n} + \frac{1}{n}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k, 1)\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k$$
$$\frac{p_t}{p_{t+1}}\frac{m^t}{m^{t+1}} = n$$

Note that, nonetheless, from the second line above, the aggregate $s^t + m^t$ has necessarily to be a constant, say $\bar{s}$, at a competitive equilibrium steady state, even though s^t and m^t are not.

[19]That is to say, ownership is debt-financed.

Furthermore, the steady state conditions imply —adding up the second, third (delayed 1 period), and fifth (divided by n) equations, after having substituted the sixth (delayed 1 period) into the third one— the feasibility of the allocation

$$c_0 + \frac{c_1}{n} + k = F(\frac{k}{n-\phi}, 1)$$

which can replace one of these equations, say the third one. Thus, after substituting the sixth into the equations in the first line and replacing the series by their value, a competitive equilibrium steady state is characterised by

$$\begin{aligned} \frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = n\frac{m^{t+1}}{m^t} &= F_K(\frac{k}{n-\phi}, 1) \\ c_0 + k + \bar{s} &= F_L(\frac{k}{n-\phi}, 1) \\ c_0 + \frac{c_1}{n} + k &= F(\frac{k}{n-\phi}, 1) \\ F_K(\frac{k}{n-\phi}, 1)s^t &= d + s^{t+1}n \end{aligned}$$

whose first three lines are those in (M) as requested. It remains to be checked that the no-abitrage condition in the last line above implies that $\bar{s}$ is the value claimed and that the convergence of s^t to $\bar{s}$ obtains. In effect, from the no-arbitrage condition at the steady state, which can be rewritten as

$$s^{t+1} = \frac{r}{n}s^t - \frac{d}{n}$$

—where $d = r\frac{\phi}{n-\phi}k$ and $r = F_K(\frac{k}{n-\phi}, 1)$ are, respectively, the profits distributed to each agent and the return to capital at a competitive equilibrium steady state— it follows that, whenever $r < n$, the value for s^t converges to

$$\bar{s} = \frac{d}{r-n} < 0.$$

as claimed. $\square$

A few remarks are now in order. Firstly, from conditions (P) in page 12 [?????] and (M) in page 20 [?????] above, it follows that the planner steady state cannot be decentralized through markets under laissez-faire. In particular, the market leads the agents to consume too early at the steady state, in the sense of choosing a intertemporal marginal rate of substitution smaller than the planner's, as made precise in the next proposition.

Proposition 9. *In the overlapping generations economy in Section 2, in which some private capital eventually falls into the public domain, the planner's steady state cannot be decentralized as a laissez-faire competitive markets outcome. In particular, the market makes the agents choose a profile of consumption whose intertemporal marginal rate of substation is smaller than the planner's.*

Proof. In effect, note first that since $s^t + m^t = \bar{s}$, for $m^t > 0$ it must be that $s^t < \bar{s} < 0$, so that s^t converges to $\bar{s}$ from the left, decreasing in absolute value. Therefore, $m^t = \bar{s} - s^t$ is decreasing, so that $m^t/m^{t+1} > 1$ which, from the equilibrium condition

$$\frac{p_t}{p_{t+1}} \frac{m^t}{m^{t+1}} = n$$

implies $p_t/p_{t+1} < n$.

Now, if $\bar{c}_0, \bar{c}_1$ is the competitive equilibrium steady state profile of consumption, while the profile chosen by the planner is c_0^*, c_1^*, it follows from the respective characterisations in (M) and (P) that

$$\frac{1}{\beta} \frac{u'(\bar{c}_0)}{u'(\bar{c}_1)} = \frac{p_t}{p_{t+1}} < n = \frac{1}{\beta} \frac{u'(c_0^*)}{u'(c_1^*)}$$

as claimed. $\square$

Interestingly enough, it is not immediate in the overlapping generations case —as opposed to what happened the infinitely-lived agents case— whether the market lends too few or too much capital to firms, compared to what the planner would choose. In effect, since at the competitive equilibrium steady state $p_t/p_{t+1} < n$, it follows from (M) and (P) respectively that

$$F_K(\frac{\bar{k}}{n-\phi}, 1) = \frac{p_t}{p_{t+1}} < n = F_K(\frac{k^*}{n-\phi}, 1) + \phi$$

that is to say

$$F_K(\frac{\bar{k}}{n-\phi}, 1) < n > F_K(\frac{k^*}{n-\phi}, 1)$$

so that the market level of capital $\bar{k}$ could, in principle, be smaller or bigger than the planner's k^*.

Nevertheless, it will follow from the policy decentralising the planner's steady state shown in the next section that savings need to be subsidised, so that —as in the infinitely-lived agents case— the market equilibrium leads to saving too little.

4.5. Market implementation of the planner's steady state through a subsidy on capital returns and a tax on debt.

If households see their returns from loans to firms subsidized by an amount equal to the depreciation/obsolescence factor ϕ and the debt issuance against future profits taxed by a factor $\sigma > 1$, the household born at t would face instead

$$\max_{c_0^t, c_1^t, k^t, m^t, s^t} u(c_0^t) + \beta u(c_1^t)$$
$$c_0^t + k^t + s^t + m^t \leq w_t$$
$$c_1^t \leq (r_{t+1} + \phi)k^t + d_{t+1} + \sigma s^{t+1} n + \frac{p_t}{p_{t+1}} m^t$$
$$(r_{t+1} + \phi)s^t \geq d_{t+1} + \sigma s^{t+1} n$$

given the wage w_t, the rental rate of capital r_{t+1}, the level of prices during his lifetime p_t, p_{t+1}, the profits received as dividends when owner d_{t+1}, the *per* young repayment of debt when old s_{t+1} and the population growth factor n.

As a consequence, the choice of a household born at t necessarily satisfies

$$\begin{pmatrix} u'(c_0^t) \\ \beta u'(c_1^t) \\ 0 \\ 0 \\ 0 \end{pmatrix} = \lambda_0^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \lambda_1^t \begin{pmatrix} 0 \\ 1 \\ -(r_{t+1} + \phi) \\ -\frac{p_t}{p_{t+1}} \\ 0 \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -(r_{t+1} + \phi) \end{pmatrix}$$

for some $\lambda_0^t, \lambda_1^t, \mu^t > 0$, along with the binding constraints, or equivalently

$$\frac{1}{\beta} \frac{u'(c_0^t)}{u'(c_1^t)} = \frac{p_t}{p_{t+1}} = r_{t+1} + \phi$$
$$c_0^t + k^t + s^t + m^t = w_t$$
$$c_1^t = (r_{t+1} + \phi)k^t + \frac{\pi_{t+1}}{N_t} + \sigma s^{t+1} n + \frac{p_t}{p_{t+1}} m^t$$
$$d_{t+1} + \sigma s^{t+1} n = (r_{t+1} + \phi)s^t$$

As before, firms distribute at $t+1$ to each household born at t dividends

$$d_{t+1} = F_K\left(\frac{k^t}{n} + \frac{1}{n} \sum_{i=1}^{+\infty} \left(\frac{\phi}{n}\right)^i k^{t-i}, 1\right) \sum_{i=1}^{+\infty} \left(\frac{\phi}{n}\right)^i k^{t-i}$$

and factor prices are

$$\begin{aligned} r_{t+1} &= F_K(K_{t+1}, N_{t+1}) \\ w_t &= F_L(K_t, N_t) \end{aligned}$$

The market clearing condition can again be obtained adding up the budget constraints of the agents alive at any given period t, of which there are n young agents *per* old one, *i.e.* adding up

$$c_0^t + k^t + s^t + m^t = w_t$$

and

$$\frac{c_1^{t-1}}{n} = (r_t + \phi)\frac{k^{t-1}}{n} + \frac{d_t}{n} + \sigma s^t + \frac{p_{t-1}}{p_t}\frac{m^{t-1}}{n}$$

which after taking into account the feasibility condition amounts to

$$m^t = \phi\frac{k^{t-1}}{n} + (\sigma - 1)s^t + \frac{p_{t-1}}{p_t}\frac{m^{t-1}}{n}$$

holding at any given t.

A competitive equilibrium is therefore characterised under such a policy by the following conditions.

Proposition 10. *In the overlapping generations economy in Section 2, in which some private capital eventually falls into the public domain, a competitive equilibrium under a policy that (i) subsidises the returns to capital by the depreciation/obsolescence factor ϕ and (ii) taxes debt issued against future profits by a factor $\sigma > 1$, is characterised by a consumption profile c_0^t, c_1^t, a loan to firms k^t, a net position in firms ownership and borrowing s^t, a real balance m^t, and distributed profits d_{t+1}, for each agent born in each period t, as well as prices p_t, for all t, such* *that*

$$\frac{1}{\beta}\frac{u'(c_0^t)}{u'(c_1^t)} = \frac{p_t}{p_{t+1}} = F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1) + \phi$$

$$c_0^t + k^t + s^t + m^t = F_L(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1)$$

$$\frac{c_1^t}{n} = \left[F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1) + \phi\right]\frac{k^t}{n} + \frac{d_{t+1}}{n} + \sigma s^{t+1} + \frac{p_t}{p_{t+1}}\frac{m^t}{n}$$

$$\left[F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1) + \phi\right]s^t = d_{t+1} + \sigma s^{t+1} n$$

$$d_{t+1} = F_K(\frac{k^t}{n} + \frac{1}{n}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1)\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}$$

$$m^t = \phi\frac{k^{t-1}}{n} + (\sigma - 1)s^t + \frac{p_{t-1}}{p_t}\frac{m^{t-1}}{n}$$

It is noteworthy that the argument underpinning Proposition 7 does not hold true under this policy, so that it does not rule out anymore the possibility of a steady state equilibrium in which s^t and m^t also are constant. As a matter of fact, the equilibrium that decentralises the planner's steady state is indeed an equilibrium in which s^t and m^t do stay constant, as the next proposition establishes. This policy finances a subsidy to the return to capital through a lump-sum tax and, interestingly enough, requires the use of a tax on household debt issued against future dividends too.

Proposition 11. *In the overlapping generations economy in Section 2, in which some private capital eventually falls into the public domain, the planner's steady state is decentralised as a competitive equilibrium steady state by a period-by-period balanced policy subsidising the returns to capital by the depreciation/obsolescence factor ϕ, by means of taxing debt issued on future profits at the positive rate*

$$\sigma - 1 = -\frac{d}{sn}$$

since $s < 0$ at a stable steady state.

Proof. Should there be values for c_0, c_1, k, s, m, d, and p_t/p_{t+1} such that

$$\begin{aligned}\frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} &= \frac{p_t}{p_{t+1}} = F_K(\frac{k}{n-\phi}, 1) + \phi \\ c_0 + k + s + m &= F_L(\frac{k}{n-\phi}, 1) \\ \frac{c_1}{n} &= \left[F_K(\frac{k}{n-\phi}, 1) + \phi\right]\frac{k}{n} + \frac{d}{n} + \sigma s + \frac{p_t}{p_{t+1}}\frac{m}{n} \\ d + \sigma s n &= \left[F_K(\frac{k}{n-\phi}, 1) + \phi\right] s \\ d &= F_K(\frac{k}{n-\phi}, 1)\frac{k}{n-\phi}\phi \\ m &= \phi\frac{k}{n} + (\sigma - 1)s + \frac{p_{t-1}}{p_t}\frac{m}{n}\end{aligned} \tag{1}$$

for a given σ, they would characterise a competitive equilibrium steady state under the policy of subsidising returns at a rate ϕ and taxing debt by a factor σ. In particular, the system including σ as endogenous variable and augmented by the additional equation balancing taxes and subsidies

$$\phi k + (\sigma - 1)ns = 0 \tag{2}$$

pins down a balanced policy implementing the planner's steady state.

In effect, a solution to (1) and (2) above satisfies $p_t/p_{t+1} = n$, so that

$$\frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = n = F_K(\frac{k}{n-\phi}, 1) + \phi$$

which is the first line of the planner's system in (P), and the equations in the second, third, fifth and sixth lines imply the feasibility of the allocation. The equation in the fourth line in (1) pins down the necessary σ to be

$$\sigma = 1 - \frac{d}{sn} > 1$$

given that $s < 0$ for a stable steady state, and hence it is a tax on debt since it increases the amount that households must repay.

The existence of a solution to (1,2) —and hence of a policy decentralizing the planner's steady state— follows from the existence of the latter. □

5. Discussion

[TBW]

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