

Adverse Selection Under Ambiguity*

Job Market Paper

Sarah Auster, EUI

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Abstract

This paper analyses a bilateral trade problem with asymmetric information and ambiguity aversion. There is a buyer and a seller who bargain over the exchange of an indivisible good. The seller is privately informed about the quality of the good, which determines both his valuation and the buyer's valuation. In the absence of ambiguity, the contract that maximizes the buyer's payoff is a posted price. In this paper, the buyer holds ambiguous beliefs about the quality of the good and therefore faces ambiguity over both the seller's willingness to accept an offer and her valuation of the good. The first type of ambiguity is exacerbated when the offered price is low, while the second type of ambiguity is exacerbated when the offered price is high. An ambiguity-averse buyer solves this conflict by proposing a screening menu rather than a posted price, thereby hedging against both types of ambiguity. Ambiguity aversion in this setting has opposing effects on the level of trade: the buyer's pessimism about the seller's valuation has a positive effect on trade, while the buyer's pessimism about her own valuation has a negative effect on trade. The paper separates the two effects and shows under which conditions an increase in ambiguity leads to an increase or decrease in trade.

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1 Introduction

In recent years, the lack of transparency in financial markets has received increasing attention by both regulators and academics. In the debate on whether opacity is harmful or beneficial for trade, various scholars have emphasized the role of ambiguity.¹ It has been argued that the lack of transparency inhibits traders' ability to understand the odds in such markets and thereby gives rise to

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¹For example, Easley and O'Hara (2010) and Cheng and Zhong (2012).

ambiguity, often perceived to inhibit the feasibility of efficient trade. While this presumption is correct for competitive markets, I show that it may fail when traders bargain bilaterally.² A prominent example for exchange environments in which terms of trade are negotiated bilaterally rather than in centralized platforms are over-the counter (OTC) markets.³ These markets are frequently referred to as dark markets because there is little or no information about past transaction prices and traded quantities (Duffie, 2012). This gives rise to the question how ambiguity affects negotiations in bilateral environments and under which conditions more transparency leads to better or worse trading outcomes.

I address this question in the context of a simple bilateral trade model where a buyer (she) and a seller (he) bargain over the exchange of an object with ambiguous quality. In contrast to previous work on ambiguity and bargaining (see Section 1.1), I consider a setting in which the object has a common value component, over which the seller is privately informed. Under this assumption, the buyer faces ambiguity over both her valuation and the seller's valuation, which has two major implications for the contracting problem. First, the presence of ambiguity in this setting changes the design of the contract that maximizes the buyer's payoff: while under a single prior the optimal contract is a posted price (Samuelson, 1984), the optimal contract under ambiguous beliefs is a screening menu. This menu hedges against ambiguity and can be interpreted as a non-linear pricing schedule with quantity discounts. Second, ambiguity aversion in this setting has opposing effects on the level of trade: the buyer's pessimism about her own valuation has a negative effect on trade, whereas the buyer's pessimism about the seller's willingness to accept an offer has a positive effect on trade. I separate the two effects and provide conditions under which one outweighs the other.

In the basic model, the seller possesses one unit of an indivisible good whose quality can be either high or low. The buyer's valuation exceeds the seller's valuation for both levels of quality, implying that there are certain gains from trade. In the benchmark model without ambiguity, the buyer either offers a pooling price and trades with probability one, or offers a price equal to the low-type seller's valuation and only trades the low-quality good (Samuelson, 1984). In this paper, the buyer follows a maxmin decision rule (Gilboa and Schmeidler, 1989), and thus evaluates her choices with the most pessimistic prior in a set of priors. The buyer faces ambiguity over the object's quality and therefore faces ambiguity over both the seller's valuation and her own valuation. The former determines the probability with which the seller accepts, while the latter determines the buyer's conditional expected valuation. The extent of both types of ambiguity is determined by the contract the buyer offers. If the buyer proposes the pooling price, she is sure to trade but

²More precisely, ambiguity inhibits trade in competitive markets if market participants are ambiguity-averse, sets of priors include a common prior and endowments are deterministic (see Rigotti et al., 2008 and De Castro and Chateauneuf, 2011).

³An OTC market is a stock exchange where transactions are made via telephone and computer rather than on the floor of an exchange.

faces ambiguity over her valuation, whereas if the buyer offers a price equal to the low-type seller's valuation, she is sure to buy low quality but faces ambiguity over the probability with which the seller accepts. An ambiguity-averse buyer solves this conflict by proposing a screening menu rather than a posted price. This is optimal because it balances the buyer's ex-ante expected utility across priors and thereby hedges against ambiguity.

I further show that the two types of ambiguity have opposing effects on the level trade. To separate these effects, I consider an alternative setting in which valuations are ambiguous but independent. The independent value setting allows for a separate variation of ambiguity over the buyer's valuation and ambiguity over the seller's valuation. I show that an increase in the former reduces trade, while an increase in the latter increases trade. The intuition is that ambiguity over the buyer's valuation leads the buyer to overweigh the event in which her valuation is low. This reduces her perceived gains from trade and thus has a negative effect on trade. On the other hand, ambiguity over the seller's valuation leads the buyer to overweigh the event in which the seller's valuation is high. This makes pooling more attractive and thus has a positive effect on trade. The most pessimistic prior in the independent value setting is generally the prior with maximal weight on the low-type buyer and the high-type seller. When values are perfectly correlated, these two cases are mutually exclusive. The buyer's most pessimistic prior thus depends on the contract she offers: if the buyer offers the pooling price, her most pessimistic prior has maximal weight on the low-quality good, while if the buyer offers a price equal to the low-type seller's valuation, her most pessimistic prior has maximal weight on the high-quality good. It is this feature that generates a contract different from the optimal contract under a single prior.

An interesting question is whether the behavioral implications of ambiguity aversion in this contracting problem differ from those of risk aversion. It is well established that screening menus can be optimal in Bayesian settings when traders have non-linear utility functions. This gives rise to the question how ambiguity-driven screening can be distinguished from screening in the standard framework. I show that the main difference between the two settings is how the dependence of traders' types affects the contracting problem and propose a simple comparative statics exercise to resolve this question. If the buyer is Bayesian and risk-averse, the optimal contract is determined by the well-known tradeoff between efficiency and rent extraction. This tradeoff does not depend on whether the buyer's valuation varies with the quality of the good or not. On the other hand, if the buyer is ambiguity-averse, the optimal contract is determined by the buyer's tradeoff between ambiguity over the seller's valuation and ambiguity over her own valuation, where the effect of the latter is determined by the degree to which the buyer's valuation depends on the quality of the good. A marginal change in the degree of dependence thus affects the optimal contract through a shift in the ambiguity tradeoff and thereby distinguishes ambiguity aversion from risk aversion in

this setting.

I extend the main results of the basic setup to a model with a continuum of types. I show that if the extent of ambiguity is sufficiently large, the optimal contract is a menu that perfectly separates all types, whereas if the extent of ambiguity is sufficiently small, the optimal contract is a posted price, bunching types at the bottom and/or at the top. In contrast to the binary type setting, there is an intermediate parameter region of ambiguity in which the optimal contract bunches a set of low-type sellers and perfectly separates the rest. The more ambiguity the buyer faces, the smaller the bunching region becomes. The intuition is that bunching maximizes the buyer's single prior payoff, while separation hedges against ambiguity. The more ambiguity the buyer faces, the more profitable hedging becomes. I derive conditions for these three cases and provide a complete characterization of the optimal contract.

For a given set of measures, the maxmin expected utility model may be seen as a limiting case of the smooth ambiguity model with infinite ambiguity aversion. The smooth ambiguity model, developed by Klibanoff et al. (2005), has very robust features and allows for a separation between ambiguity and ambiguity aversion. I extend the characterization of the optimal contract to this representation and show that also under smooth ambiguity aversion, the buyer's hedging motive against ambiguity gives rise to a screening equilibrium. I further show that the optimal contract under smooth ambiguity aversion is a convex combination of the optimal contract under maxmin preferences and ambiguity neutrality. As ambiguity aversion increases, the solution under smooth ambiguity aversion converges to the optimal contract under maxmin expected utility.

Coming back to the financial market example, there are various interesting implications to be drawn from this analysis. First, the paper shows that ambiguity in bilateral negotiations gives rise to price discrimination, suggesting that assets of higher quality are traded at higher prices and lower volume. The analysis further shows that ambiguity in bilateral contracting is beneficial for trade if and only if the positive effect of ambiguity over the seller's willingness to trade is large enough. Under further restrictions, this gives rise to cases in which bilateral trading under ambiguity yields a strictly more efficient allocation than competitive exchange. The paper provides an example in which competitive markets break down, while bilateral trade under ambiguity partially restores efficiency. This highlights a new aspect in the debate on the emergence of opaque off-exchange trading.

The rest of the paper is organized as follows. After a discussion of the related literature, Section 2 describes the contracting problem and characterizes the optimal contract. Section 2.1 shows the effect of ambiguity on trade and Section 2.2 discusses the differences between ambiguity aversion and risk aversion. Section 3 extends the main results to a framework with a continuum of types and

Section 4 characterized the optimal contract under smooth ambiguity aversion. Section 5 concludes.

1.1 Related Literature

The most related work to this paper is Bergemann and Schlag (2008 and 2011). They consider a monopoly pricing model in which the principal knows her type and follows a maxmin decision rule, or alternatively, a minimax regret criterion. The papers' main focus lies on the case of regret preferences because whenever the principal knows her valuation, ambiguity aversion does not affect the design of the optimal contract. This can be seen in the context of a special case in this paper: when the buyer's valuation does not depend on quality, the buyer only faces ambiguity over the seller's valuation and her most pessimistic prior is generally given by the prior with maximal weight on the high-quality good. This implies that the optimization problem under maxmin preferences corresponds to an optimization problem under a single pessimistic prior and the optimal contract is a posted price. The minimax regret criterion, on the other hand, generates a regret tradeoff that makes randomizing over prices optimal. My paper complements this result by demonstrating that deterministic prices may be dominated due to the principal's aversion to ambiguity rather than to foregone opportunities.

Furthermore, there is a recent literature on ambiguity and mechanism design in the Myerson-Satterthwaite (1983) environment with independent private values and two-sided private information, e.g. Bodoh-Creed (2012), Bose and Mutuswami (2012) De Castro and Yannelis (2012) and Wolitzky (2013). The main difference to my work is that this literature explores the interaction between ambiguity and rent extraction, whereas in my paper ambiguity enters the principal's objective, leaving incentives unaffected. Other papers that deal with games and mechanisms in ambiguous environments are Lo (1998), Mukerji (1998), Bose, Ozdenoren and Pape (2006), Lopomo et al. (2009), and Carroll (2013). Additionally, recent papers such as Bose and Renou (2013), Di Tillio et al. (2012) and Riedel and Sass (2011) introduce strategic ambiguity into games.

Finally, there is a considerable body of literature on ambiguity aversion and trade in partial or general equilibrium. Dow and Werlang (1992), Epstein and Wang (1994), Mukerji and Tallon (2001), and De Castro and Chateauneuf (2011) among others show conditions under which ambiguity aversion inhibits trade and leads to inefficient allocations and incomplete markets.

2 Optimal Contracting Under Ambiguity

Environment: A risk-neutral buyer makes an offer to a risk-neutral seller. The seller possesses one unit of an indivisible good which can be either of high quality or of low quality. The seller knows the quality of the object but the buyer does not. If the quality of the good is high (low), the

seller's valuation is c_h (c_l) and the buyer's valuation is v_h (v_l), where $c_h > c_l$ and $v_h > v_l$. For both types of good, the buyer's valuation exceeds the seller's valuation, implying that there are certain gains from trade: $v_i > c_i, i = l, h$. The ex-ante probability of quality being high is denoted by σ and the buyer's ex-post payoff is denoted by π . The buyer's ex-ante payoff for a single prior σ is given by $E_\sigma[\pi]$. In contrast to the standard bargaining model, the buyer does not have a single prior σ but holds ambiguous beliefs. The buyer is ambiguity-averse and follows a maxmin decision rule (Gilboa and Schmeidler, 1989).⁴ The buyer thus evaluates her choices with the worst prior in a set of priors. Her objective is given by

$$\Pi^{MEU} = \min_{\sigma \in \Sigma} E_\sigma[\pi],$$

where $\Sigma = [\underline{\sigma}, \bar{\sigma}]$ is the set of priors. If $\underline{\sigma} = \bar{\sigma}$ such that Σ is a singleton, the min-operator has no bite and the buyer is a standard subjective expected utility maximizer

Contract: The analysis corresponds to a monopolistic screening problem with an ambiguity-averse principal. As the seller faces no ambiguity, the revelation principle applies to this setting and the buyer can restrict her attention to a menu of contracts, $\{(\alpha_i, p_i)\}_{i=l,h}$, consisting of a trading probability α_i and a price p_i for each type of good.⁵ Price p_i is paid independent of whether the good is transferred or not and can be interpreted as an expected price. The buyer's and seller's ex-post expected payoffs are given by $\alpha v_i - p$ and $p - \alpha c_i$, respectively.

The buyer maximizes her objective subject to the participation and incentive constraints of both types of seller:

$$\begin{aligned} p_l - \alpha_l c_l &\geq 0, & (PC_l), \\ p_h - \alpha_h c_h &\geq 0, & (PC_h), \\ p_l - \alpha_l c_l &\geq p_h - \alpha_h c_l, & (IC_l), \\ p_h - \alpha_h c_h &\geq p_l - \alpha_l c_h, & (IC_h). \end{aligned}$$

Since the seller knows his type, the constraints of the optimization problem are not affected by ambiguity in this environment. This, and the fact that the buyer's objective is weakly decreasing in p_l and p_h , implies that the solution of the buyer's optimization problem satisfies the following well established properties (see for example Salanie, 1997, Chapter 2):

⁴The choice of representation is made for tractability and to facilitate the exposition. The main results in this paper are driven by the buyer's desire to hedge against ambiguity - a common feature of all ambiguity models - and thus, can easily be extended to other representations that allow for ambiguity aversion, e.g. smooth ambiguity model (Klibanoff et al., 2005, Section 4) or multiplier preferences (Strzalecki, 2011).

⁵The agent facing no ambiguity implies that the proof is equivalent to the proof of the revelation principle for Bayesian Nash equilibrium.

1. PC_h is binding: $p_h = \alpha_h c_h$.
2. IC_l is binding: $p_l = \alpha_h c_h + (\alpha_l - \alpha_h) c_l$.
3. PC_l and IC_h are slack.
4. $\alpha_l = 1$.

For notational convenience, let α denote the exchange probability of good h . With properties (1)-(4) the menu of contracts is completely characterized by α , independent of whether the buyer faces ambiguity or not:

$$\{(\alpha_l, p_l), (\alpha_h, p_h)\} = \{(1, \alpha c_h + (1 - \alpha) c_l), (\alpha, \alpha c_h)\}.$$

Note that there are two alternative interpretations of the basic model. In the interpretation followed throughout the paper, the good is indivisible and α is a lottery, determining the ex-ante probability of trade. In an alternative interpretation, the seller possesses one unit of a perfectly divisible good and α is a quantity. Under this interpretation, the menu is a non-linear pricing schedule with a quantity discount. Low quality is traded in large quantity at a low price, while high quality is traded in small quantity at a high price.

Optimization Problem: Given the properties stated above, the buyer's objective can be stated as a function of α . The buyer's expected utility under single prior σ is given by

$$E_\sigma[\pi] = \sigma \alpha (v_h - c_h) + (1 - \sigma)(v_l - \alpha c_h - (1 - \alpha) c_l).$$

The buyer's optimization problem is consequently

$$\max_{\alpha} \min_{\sigma \in \Sigma} \{\sigma \alpha (v_h - c_h) + (1 - \sigma)(v_l - \alpha c_h + (1 - \alpha) c_l)\}.$$

To derive the solution to this problem, consider first the case in which Σ is a singleton and the buyer is a subjective expected utility maximizer with single prior σ . Samuelson (1984) shows that the buyer's optimal contract is a first-and-final price p^* , which the seller either accepts or rejects. In the two-type setting this result can easily be seen by considering the buyer's objective. Since $E_\sigma[\pi]$ is linear in α , the optimization problem has a corner solution. $\alpha = 0$ corresponds to offering price $p^* = c_l$ and yields expected utility $(1 - \sigma)(v_l - c_l)$, while $\alpha = 1$ corresponds to offering the pooling price $p^* = c_h$ and yields expected utility $\sigma v_h + (1 - \sigma)v_l - c_h$. Pooling is optimal if and only if the probability that the seller's type is high is large enough, which is the case if

$$\sigma \geq \frac{c_h - c_l}{v_h - c_l}.$$

Under ambiguity and maxmin preferences, Samuelson’s result turns out to fail. Proposition 2.1 shows that if Σ is not a singleton, the buyer’s optimal contract is a posted price if and only if this price is optimal under all priors in Σ . Otherwise, the buyer optimally offers a menu.

Proposition 2.1. *Let $\tilde{\sigma} := \frac{c_h - c_l}{v_h - c_l}$ and $\tilde{\alpha} := \frac{v_l - c_l}{v_h - c_l}$. The optimal menu of contracts is $\{(1, \alpha^* c_h + (1 - \alpha^*) c_l), (\alpha^*, \alpha^* c_h)\}$ with*

$$\alpha^* = \begin{cases} 0 & \text{if } \bar{\sigma} \leq \tilde{\sigma}, \\ 1 & \text{if } \underline{\sigma} \geq \tilde{\sigma}, \\ \tilde{\alpha} & \text{otherwise.} \end{cases}$$

Proof See Appendix A.2.1.

$\tilde{\sigma}$ is the prior under which the buyer is indifferent between all $\alpha \in [0, 1]$. If $\underline{\sigma} \geq \tilde{\sigma}$ ($\bar{\sigma} \leq \tilde{\sigma}$), the contract that maximizes the buyer’s expected utility is a posted price equal to c_h (c_l) for all priors in Σ . Ambiguity aversion thus affects the buyer’s utility but not her choice of contract. Ruling these trivial cases out, the buyer optimally proposes a menu, characterized by $\tilde{\alpha}$. $\tilde{\alpha}$ is the contracting parameter for which the buyer’s ex-post expected payoff is constant across types, i.e.

$$\tilde{\alpha}(v_h - c_h) = v_l - \tilde{\alpha}c_h - (1 - \tilde{\alpha})c_l.$$

Offering $\tilde{\alpha}$ makes the buyer’s payoff independent of the type distribution and yields a ”safe payoff” equal to $\frac{(v_h - c_h)(v_l - c_l)}{v_h - c_l}$.

This is illustrated in Figure 1. Under the assumption $\underline{\sigma} < \tilde{\sigma} < \bar{\sigma}$, the expected utility of a buyer with single prior $\underline{\sigma}$ is downward sloping in α (solid line), while the expected utility of a buyer with single prior $\bar{\sigma}$ is upward sloping in α (dashed line). All expected utility functions $E_\sigma[\pi], \sigma \in (\underline{\sigma}, \bar{\sigma})$ lie in between these two benchmarks and intersect at $\tilde{\alpha}$. For any $\alpha < \tilde{\alpha}$, the buyer’s most pessimistic prior is $\bar{\sigma}$, while for any $\alpha > \tilde{\alpha}$, the buyer’s most pessimistic prior is $\underline{\sigma}$. Since $E_{\bar{\sigma}}[\pi(\alpha)]$ is upward sloping in α and $E_{\underline{\sigma}}[\pi(\alpha)]$ is downward sloping in α , the buyer’s utility is maximized at $\tilde{\alpha}$.

The characterization of the optimal contract demonstrates that if there is sufficient ambiguity, the buyer’s hedging motive against ambiguity gives rise to a screening equilibrium. Coming back to the introductory example of bilateral negotiations in OTC markets, this suggests that in the presence of ambiguity and asymmetric information, ambiguity-averse traders optimally screen their contracting partners by offering non-linear pricing schedules. Non-linear pricing is a feature commonly observed in OTC markets, for example in the form of dealers posting quotes which only hold for small quantities. The exchange of larger quantities is negotiated directly between dealer and trader and typically executed at a lower price (Jankowitsch et al., 2011).

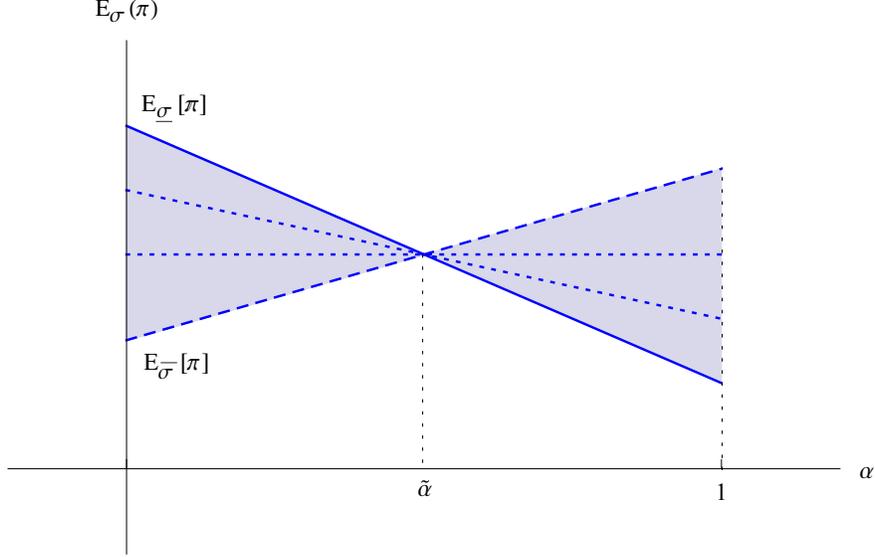


Figure 1: $E_\sigma[\pi]$ for $\sigma \in [\underline{\sigma}, \bar{\sigma}]$

2.1 Ambiguity and Trade

Ambiguity in this contracting problem has opposing effects on the ex-ante probability of trade. This section shows that ambiguity over the seller's valuation has a positive effect on trade, while ambiguity over the buyer's valuation has a negative effect on trade. To separate the two effects, I consider the benchmark case in which values are independent. The buyer's set of priors is given by $[\underline{\sigma}_s, \bar{\sigma}_s] \times [\underline{\sigma}_b, \bar{\sigma}_b]$, where σ_s (σ_b) denotes the probability with which the seller's (buyer's) valuation is high. Due to the independence of values, the buyer's most pessimistic prior does not depend on her contract choice but is generally given by $\{\bar{\sigma}_s, \underline{\sigma}_b\}$. To see this, consider the buyer's single prior payoff

$$E_{\sigma_s, \sigma_b}[\pi] = \sigma_s \alpha (\sigma_b v_h + (1 - \sigma_b) v_l - c_h) + (1 - \sigma_s) (\sigma_b v_h + (1 - \sigma_b) v_l - \alpha c_h - (1 - \alpha) c_l).$$

Note that

$$\begin{aligned} \frac{\partial E_{\sigma_s, \sigma_b}[\pi]}{\partial \sigma_s} &= -(1 - \alpha) (\sigma_b v_h + (1 - \sigma_b) v_l - c_l) \leq 0, \\ \frac{\partial E_{\sigma_s, \sigma_b}[\pi]}{\partial \sigma_b} &= (1 - \sigma_s + \sigma_s \alpha) (v_h - v_l) \geq 0. \end{aligned}$$

The buyer's single prior payoff is decreasing in σ_s and increasing in σ_b , for any $\alpha \in [0, 1]$. This implies that the buyer's most pessimistic prior is generally $\{\bar{\sigma}_s, \underline{\sigma}_b\}$, the prior with maximal weight on the high-type seller and low-type buyer. The buyer's optimization problem under ambiguity

thus corresponds to a standard optimization problem under single prior $\{\bar{\sigma}_s, \underline{\sigma}_b\}$ and is given by

$$\max_{\alpha} \quad \Pi^{MEU} = \bar{\sigma}_s \alpha (\underline{\sigma}_b v_h + (1 - \underline{\sigma}_b) v_l - c_h) + (1 - \bar{\sigma}_s) (\underline{\sigma}_b v_h + (1 - \underline{\sigma}_b) v_l - \alpha c_h - (1 - \alpha) c_l).$$

Samuelson's (1984) result applies and the problem has a corner solution. The optimal contract is characterized by

$$\alpha^* = \begin{cases} 0 & \text{if } \bar{\sigma}_s < \frac{c_h - c_l}{\underline{\sigma}_b v_h + (1 - \underline{\sigma}_b) v_l - c_l}, \\ 1 & \text{if } \bar{\sigma}_s \geq \frac{c_h - c_l}{\underline{\sigma}_b v_h + (1 - \underline{\sigma}_b) v_l - c_l}. \end{cases}$$

To see the opposing effects of ambiguity on trade, note that the extent of ambiguity over the buyer's valuation is measured by the size of the interval $[\underline{\sigma}_b, \bar{\sigma}_b]$, while the extent of ambiguity over the seller's valuation is measured by the size of the interval $[\underline{\sigma}_s, \bar{\sigma}_s]$.

Ambiguity over the buyer's valuation: Consider first an increase in ambiguity over the buyer's valuation by considering an expansion of $[\underline{\sigma}_b, \bar{\sigma}_b]$. Let $[\underline{\sigma}_b, \bar{\sigma}_b] \subset [\underline{\sigma}'_b, \bar{\sigma}'_b]$, or equivalently $\underline{\sigma}'_b \leq \underline{\sigma}_b$ and $\bar{\sigma}'_b \geq \bar{\sigma}_b$. Since

$$\frac{c_h - c_l}{\underline{\sigma}_b v_h + (1 - \underline{\sigma}_b) v_l - c_l} < \frac{c_h - c_l}{\underline{\sigma}'_b v_h + (1 - \underline{\sigma}'_b) v_l - c_l},$$

an increase in ambiguity over the buyer's valuation reduces the range of parameter values under which pooling is optimal. This implies that whenever the buyer does not offer the pooling price under $[\underline{\sigma}_b, \bar{\sigma}_b]$, she does not either under $[\underline{\sigma}'_b, \bar{\sigma}'_b]$:

$$\bar{\sigma}_s < \frac{c_h - c_l}{\underline{\sigma}_b v_h + (1 - \underline{\sigma}_b) v_l - c_l} \quad \Rightarrow \quad \bar{\sigma}_s < \frac{c_h - c_l}{\underline{\sigma}'_b v_h + (1 - \underline{\sigma}'_b) v_l - c_l},$$

An increase in ambiguity over the buyer's valuation thus leads to weakly less trade. The intuition is that the buyer's aversion to ambiguity over her own valuation implies that the buyer overweighs the probability with which her valuation is low. This diminishes the buyer's perceived gains from trade and thus has a negative effect on trade.

Ambiguity over the seller's valuation: Similarly, consider an increase in ambiguity over the seller's valuation by considering an expansion of the set $[\underline{\sigma}_s, \bar{\sigma}_s]$. Let $[\underline{\sigma}_s, \bar{\sigma}_s] \subset [\underline{\sigma}'_s, \bar{\sigma}'_s]$. Then

$$\bar{\sigma}_s > \frac{c_h - c_l}{\underline{\sigma}_b v_h + (1 - \underline{\sigma}_b) v_l - c_l} \quad \Rightarrow \quad \bar{\sigma}'_s > \frac{c_h - c_l}{\underline{\sigma}'_b v_h + (1 - \underline{\sigma}'_b) v_l - c_l},$$

by $\bar{\sigma}'_s \geq \bar{\sigma}_s$. This condition states that whenever the buyer offers the pooling price under $[\underline{\sigma}_s, \bar{\sigma}_s]$, she does so under $[\underline{\sigma}'_s, \bar{\sigma}'_s]$. An increase in ambiguity over the seller's valuation thus leads to weakly more trade. The intuition is that the buyer's aversion to ambiguity over the seller's valuation

implies that the buyer overweighs the probability with which the seller's valuation is high. Since under such priors pooling is optimal, this type of ambiguity has a positive effect on trade.

When values are perfectly correlated, the seller's and buyer's valuation cannot be simultaneously high and low. This implies that the buyer's most pessimistic prior depends on the contract she offers. The previous section showed that if the buyer offers a menu characterized by $\alpha < \tilde{\alpha}$, her most pessimistic prior is given by $\bar{\sigma}$, the prior with maximal weight on the high-quality good. On the other hand, if the buyer offers a menu characterized by $\alpha > \tilde{\alpha}$, her most pessimistic prior is given by $\underline{\sigma}$, the prior with maximal weight on the low-quality good. This implies that for low values of α , the buyer's aversion to ambiguity over the seller's valuation outweighs her aversion to ambiguity over her own valuation, whereas for large values of α , the reverse implication holds.

To see the net effect of ambiguity on trade, let α_{Σ}^* denote the buyer's optimal contracting parameter when her set of priors is Σ . Section 2 shows that α_{Σ}^* is equal to zero if $\underline{\sigma} < \bar{\sigma} < \tilde{\sigma}$, equal to $\tilde{\alpha}$ if $\underline{\sigma} < \tilde{\sigma} < \bar{\sigma}$, and equal to one if $\tilde{\sigma} < \underline{\sigma} < \bar{\sigma}$. An increase in ambiguity implies that $\underline{\sigma}$ decreases while $\bar{\sigma}$ increases. If the decrease in $\underline{\sigma}$ and the increase in $\bar{\sigma}$ is sufficiently large, the condition $\underline{\sigma} < \tilde{\sigma} < \bar{\sigma}$ eventually becomes satisfied and the buyer's optimal contract is characterized by $\tilde{\alpha}$. Whether an increase in ambiguity has a positive or negative effect on trade thus depends on the initial equilibrium contract: if $\alpha_{\Sigma}^* = 0$, an increase in ambiguity has a positive effect on trade, whereas if $\alpha_{\Sigma}^* = 1$, an increase in ambiguity has a negative effect on trade. If $\alpha_{\Sigma}^* = \tilde{\alpha}$, an increase in ambiguity has no effect. This implies that ambiguity is beneficial for trade if and only if the starting point is a situation in which only the low-quality good is traded. This is summarized in Proposition 2.2.

Proposition 2.2. *Let $\Sigma \subset \Sigma'$. Then $\alpha_{\Sigma}^* \leq \alpha_{\Sigma'}^*$ if $\tilde{\sigma} > \underline{\sigma}$ and $\alpha_{\Sigma}^* \geq \alpha_{\Sigma'}^*$ if $\tilde{\sigma} < \bar{\sigma}$.*

Proof See Appendix A.2.2.

Note that $\tilde{\sigma} = \frac{c_h - c_l}{v_h - c_l}$ is the relative difference in the seller's valuation across quality, while $\tilde{\alpha} = 1 - \frac{v_h - v_l}{v_h - c_l}$ is the inverse of the relative difference in the buyer's valuation across quality. The difference in the seller's valuation thus determines the sign of the effect of ambiguity on trade, while the difference in the buyer's valuation determines the extent of this effect. The larger $\frac{c_h - c_l}{v_h - c_l}$ is, the larger is the set of parameters for which ambiguity has a positive effect on trade, reflecting the positive effect of ambiguity over the seller's valuation. The larger $\frac{v_h - v_l}{v_h - c_l}$ is, the smaller is the contracting parameter $\tilde{\alpha}$ to which the solution under increasing ambiguity jumps, reflecting the negative effect of ambiguity over the buyer's valuation. The net effect is positive and sizable if $c_h - c_l$ is large and $v_h - v_l$ is small, while the reverse implication holds if $c_h - c_l$ is small and $v_h - v_l$ is large.

Remark: Note that if $v_l = v_h$, the buyer's type does not depend on the seller's type. Under this parameter restriction, the buyer only faces ambiguity over the seller's valuation and ambiguity always has a positive effect on trade. To see this, note that $\tilde{\alpha}$ is equal to one and the buyer's most pessimistic prior is generally given by the prior with maximal weight on the high-type seller, $\bar{\sigma}$. The buyer's optimization problem under maxmin preferences thus corresponds to the optimization problem of a subjective expected utility maximizer with single prior $\bar{\sigma}$. Samuelson's result applies and the optimal contract is a posted price. The buyer offers c_l if $\bar{\sigma} < \tilde{\sigma}$ and c_h if $\bar{\sigma} \geq \tilde{\sigma}$. As ambiguity increases, the latter condition eventually becomes satisfied, implying that an increase in ambiguity leads to weakly more trade. The intuition is that offering the pooling contract yields a safe payoff equal to the difference between the buyer's valuation and the pooling price. As ambiguity increases, the safe pooling option becomes increasingly attractive, implying that under $v_l = v_h$ ambiguity generally has a positive effect on trade.

Coming back to the introductory example of financial markets, these findings contribute a new aspect to the recent debate on opaque OTC trade (see for example Malamud and Rostek (2013) and the references therein). Proposition 2.2 shows that ambiguity in bilateral contracting is beneficial for trade if and only if the positive effect of ambiguity over the seller's willingness to trade is large enough. Under further restrictions, this gives rise to cases in which bilateral trading under ambiguity yields a strictly more efficient allocation than competitive exchange. To see this, suppose there is an equal number of buyers and sellers who can either trade competitively or be matched bilaterally. As in the basic model, sellers are privately informed about the quality of the good, while buyers are homogeneous and have ambiguous beliefs about the quality distribution. If traders meet bilaterally, the equilibrium contract is the one characterized in Proposition 2.1. If buyers and sellers trade competitively, the asset is exchanged at a market clearing price. Buyers face no ambiguity over their probability of trade but only over their valuation of the asset, and their most pessimistic prior has maximal weight on the low quality good, given by $\underline{\sigma}$.

Consider the case in which the buyers' set of priors satisfies

$$\underline{\sigma} < \frac{c_h - v_l}{v_h - v_l} < \underbrace{\frac{c_h - c_l}{v_h - c_l}}_{=\tilde{\sigma}} < \bar{\sigma}.$$

The first inequality is equivalent to $\underline{\sigma}v_h + (1 - \underline{\sigma})v_l - c_h < 0$, which precludes trade of the high-quality good under competitive exchange. The second inequality follows from $c_l < v_l$ and the third inequality together with the first implies that the optimal contract under bilateral trade is characterized by $\tilde{\alpha}$. The above condition thus implies that under bilateral contracting the high-quality asset is traded with positive probability, whereas under competitive exchange high quality is not traded at all. Since the condition requires $\underline{\sigma} < \bar{\sigma}$, the argument crucially relies on the presence of

ambiguity. In the context of financial markets, this suggests that if traders have ambiguous beliefs about the fundamentals, decentralized trading may yield strictly better outcomes than traditional exchanges, and thereby highlights a new aspect in the debate on why off-exchange trading may emerge.⁶

2.2 Ambiguity Aversion vs. Risk Aversion

An interesting question is whether the behavioral implications of ambiguity aversion in this contracting problem differ from those of risk aversion. It is well known that if utility functions are not restricted to be linear, screening can arise in equilibrium, even when there is no ambiguity. Given the long-standing debate on the observability of ambiguity aversion as opposed risk aversion (see for example Bayer et al., 2013, and references therein), the question arises how screening under ambiguity aversion differs from screening under risk aversion. This section demonstrates that a key difference between the two models is how the dependence of traders' valuations affects the equilibrium. It shows that the degree to which the buyer's valuation varies with the seller's valuation is crucial for the contracting problem under ambiguity aversion, while it does not affect the buyer's optimization problem under risk aversion. Along these lines, I propose a simple comparative statics exercise to distinguish ambiguity aversion from risk aversion in this contracting problem.

More concretely, I compare the properties of the equilibrium contract in two alternative settings. The first setting corresponds to the basic model in this paper where the buyer is risk-neutral and ambiguity-averse. The second specification is equivalent to the first, apart from the buyer's objective. The buyer in this specification is assumed to be Bayesian and risk-averse. Her utility function is restricted to be additively separable and given by $u(i, p) = v_i - g(p)$, where $i \in \{l, h\}$ and $g'(\cdot) > 0, g''(\cdot) > 0$. The buyer maximizes her expected utility subject to the participation and incentive constraints of the seller. The set of constraints is equivalent to the one in the basic model, implying that $p_h = \alpha c_h$ and $p_l = \alpha c_h + (1 - \alpha)c_l$. The buyer thus solves

$$\max_{\alpha} \sigma(\alpha v_h - g(\alpha c_h)) + (1 - \sigma)(v_l - g(\alpha c_h + (1 - \alpha)c_l)).$$

The first-order condition to this problem is

$$\sigma(v_h - g'(\alpha c_h)c_h) = (1 - \sigma)g'(\alpha c_h + (1 - \alpha)c_l)(c_h - c_l).$$

Note that $\sigma(v_h - g'(\alpha c_h)c_h)$ is decreasing in α , while $(1 - \sigma)g'(\alpha c_h + (1 - \alpha)c_l)(c_h - c_l)$ is increasing in α , implying that if $\sigma(v_h - g'(0)c_h) > (1 - \sigma)g'(c_l)(c_h - c_l)$ and $\sigma(v_h - g'(c_h)c_h) < (1 - \sigma)g'(c_h)(c_h - c_l)$,

⁶Note that a similar argument can be made when comparing ambiguous and unambiguous trade. Suppose that transparency allows traders to learn the distribution of assets, denoted by $\hat{\sigma}$. If $\hat{\sigma}v_h + (1 - \hat{\sigma})v_l - c_h < 0$, transparency precludes trade of the high-quality good (whether competitive or bilateral), whereas bilateral contracting under opacity may restore efficiency.

the buyer's problem has an interior solution and the optimal contract is a menu rather than a posted price.

In order to distinguish the screening equilibrium under risk aversion from that under ambiguity aversion, I consider a variation in the buyer's valuation of the low-quality good, v_l . In the model without ambiguity, a variation in v_l does not affect the equilibrium contract, which can easily be seen from the first-order condition above. The intuition is that under risk aversion, the optimal value of α balances the marginal efficiency gain of increasing trade with the high-type seller, $\sigma(v_h - g'(\alpha c_h)c_h)$, with the marginal incentive cost paid to the low-type seller $(1 - \sigma)g'(\alpha c_h + (1 - \alpha)c_l)(c_h - c_l)$. Since efficiency gains are determined by the gains from trade of the high-quality good, while incentive costs are determined by the variation in the seller's valuation, this tradeoff does not depend on the buyer's valuation of the low-quality good, v_l . Given that v_l determines how much the buyer's valuation varies with the seller's valuation, this implies that whether or not traders' values depend on each other is not relevant for the optimization problem in the absence of ambiguity.⁷

This stands in stark contrast to the case of ambiguity aversion. If the buyer is ambiguity-averse, the difference $v_h - v_l$ determines the magnitude of the effect of ambiguity over the buyer's valuation and therefore enters the ambiguity tradeoff. Since ambiguity over the buyer's valuation has a negative effect on trade (see Section 2.1), the larger $v_h - v_l$ is, the smaller is the optimal contracting parameter α^* . Under maxmin preferences, this can be directly seen from the equilibrium contract, characterized by $\tilde{\alpha} = 1 - \frac{v_h - v_l}{v_h - c_l}$.

Observation 2.3. *Assume $\alpha^* \in (0, 1)$.*

- *If $\underline{\sigma} = \bar{\sigma}$ and $u(i, p) = v_i - g(p)$, then $\frac{d\alpha^*}{dv_l} = 0$.*
- *If $\underline{\sigma} < \bar{\sigma}$ and $u(i, p) = v_i - p$, $\frac{d\alpha^*}{dv_l} > 0$.*

Proposition 2.3 shows that under ambiguity aversion, an increase in the buyer's valuation of the low-quality good leads to an increase in the optimal contracting parameter α^* , while under risk aversion this change has no effect. This comparative statics exercise thereby distinguishes ambiguity-driven screening from screening in the standard framework and highlights the critical role of the dependence of traders' valuations under ambiguity aversion as opposed to risk aversion.

Remark: *Note that the assumption of additive separability drives the result that in the absence of ambiguity, a variation in v_l has no effect on the optimal contract. Relaxing this assumption and*

⁷The same result holds for the case in which the good is perfectly divisible and α is interpreted as a quantity. To see this assume the buyer's utility function is given by $v_i(\alpha) - g(p)$, $i = l, h$, where $v'_i(\alpha) > 0$, $v''_i(\alpha) < 0$, $i = l, h$ and $v_l(\alpha) \leq v_h(\alpha)$, $\forall \alpha$. This setting is isomorphic to the canonical price discrimination model where a seller with a convex cost function screens a privately informed buyer (see for example Salanie, 1997, Chapter 2). Due to additive separability, also in this specification the optimal quantity of the high-quality good does not depend on the buyer's valuation of the low quality good, which can easily be seen from the first-order condition of the optimization problem.

allowing for a general utility function (quality and money as arguments) implies that this variation can affect the optimal contract through the cross derivative in the utility function. However, if the cross derivative is restricted to be weakly positive, e.g. Cobb-Douglas utility function, a marginal variation in v_l under risk aversion affects the optimal contracting parameter α^* in the opposite way compared to ambiguity aversion.

3 Continuous Types

This section shows how the results derived in the two-type setting extend to a model with a continuum of types. Suppose the seller's valuation is distributed on the interval $[0, 1]$ and let the buyer's valuation be a function of the seller's valuation. In order to facilitate the exposition, assume that gains from trade are constant across types: $v(c) = c + \Delta, \Delta > 0$. Appendix A.1 shows how the findings extend to other linear valuation functions.

Ambiguity: The buyer considers multiple distribution functions on $[0, 1]$ possible. The set of priors, denoted by \mathcal{F} , is defined by the set of all density functions, $f : [0, 1] \rightarrow \mathbb{R}_0^+$, that assign some minimal density g_c to each type $c \in [0, 1]$. Working with this general set of priors is complicated, because the extent of ambiguity is determined by an infinite set of parameters $g_c, c \in [0, 1]$. To make the analysis tractable, I assume that ambiguity is symmetric across types, i.e. $g_c = g, \forall c$:

$$\mathcal{F}^g = \left\{ f(c) : f(c) \geq g, \forall c, \text{ and } \int_0^1 f(c) = 1 \right\}, \quad g \in [0, 1].$$

Note that the symmetric structure on the set of priors does not imply that the distribution functions in the set are symmetric: within bound g , density functions in \mathcal{F}^g can be arbitrarily skewed to the right and to the left. The advantage of this structure is that the extent of ambiguity is captured by a single parameter g , where a larger g corresponds to a smaller set of priors: $\mathcal{F}^{g'} \subset \mathcal{F}^g, g' > g$. If $g = 0$, \mathcal{F}^g contains every possible distribution on the type space, while if $g = 1$, there is no ambiguity and the unique prior in \mathcal{F}^g is the uniform distribution on $[0, 1]$.

Contract: The optimal contract is characterized by a trading probability $\alpha(c)$ and a price $p(c)$ for each $c \in [0, 1]$. The participation and incentive constraints in this setting are

$$\begin{aligned} p(c) - \alpha(c)c &\geq 0, & \forall c, \\ p(c) - \alpha(c)c &\geq p(\tilde{c}) - \alpha(\tilde{c})c, & \forall c, \tilde{c}, \end{aligned}$$

which corresponds to the set of constraints in Samuelson (1984). Given that the seller truthfully

reveals, the buyer's and seller's ex-post payoffs are given by

$$\pi_b(c) = \alpha(c)v(c) - p(c) \quad \text{and} \quad \pi_s(c) = p(c) - \alpha(c)c,$$

respectively. Samuelson (1984, p. 997) shows that the set of constraints implies $\alpha'(c) \leq 0$ and $\pi'_s(c) = -\alpha(c)$. With these properties, the buyer's and seller's ex-post payoffs can be derived as a function of $\alpha(c)$ only. $\pi'_s(c) = -\alpha(c)$ together with $\pi_s(1) = 0$ implies

$$\pi_s(c) = \int_c^1 \alpha(u)du.$$

$\int_c^1 \alpha(u)du$ corresponds to the information rent paid to type c . The buyer's ex-post payoff is the difference between the expected gains from trade and the information rent paid to the seller, i.e.

$$\pi_b(c) = \alpha(c)\Delta - \int_c^1 \alpha(u)du.$$

Optimization Problem: The buyer solves

$$\max_{\alpha(c)} \Pi^{MEU} = \min_{f \in \mathcal{F}^g} \int_0^1 \left(\alpha(c)\Delta - \int_c^1 \alpha(u)du \right) f(c)dc, \quad \text{s.t.} \quad \alpha'(c) \leq 0.$$

The main challenge in finding the solution to this problem is to identify the minimizing prior for each function $\alpha(c)$ that satisfies the monotonicity constraint. The structure imposed on \mathcal{F} makes this task tractable: The set of minimizing distribution functions given $\alpha(c)$ is the set of distribution functions with maximal mass on the types that yield the lowest ex-post payoff for the buyer. This set contains every distribution function with a mass point $1 - g$ on some type that yields minimal ex-post payoff and density g on the rest of the type space. The buyer's payoff is consequently

$$\Pi^{MEU} = g \int_0^1 \pi_b(c)dc + (1 - g) \min_{c \in [0,1]} \pi_b(c).$$

This payoff is the weighted sum of the buyer's expected utility with a single uniform prior and the buyer's maxmin payoff without restrictions on the set of priors. Before presenting the full characterization of the contract that maximizes this objective, some intuition can be gained by considering the two extreme cases $g = 1$ (no ambiguity) and $g = 0$ (maximal ambiguity).

If $g = 1$, the buyer is a standard expected utility maximizer and the optimal contract is a posted

price (Samuelson, 1984). To see this, consider the buyer's payoff under a single uniform prior:⁸

$$\int_0^1 \pi_b(c)dc = \int_0^1 \left(\alpha(c)\Delta dc - \int_c^1 \alpha(u)du \right) dc = \int_0^1 \alpha(c)(\Delta - c)dc.$$

This payoff is maximized by the contract

$$\alpha^{NA}(c) = \begin{cases} 1 & \text{if } c \leq \Delta, \\ 0 & \text{if } c > \Delta, \end{cases}$$

which corresponds to a posted price $p^* = \Delta$.

If $g = 0$, the set of priors contains every possible distribution on $[0, 1]$ and the buyer's payoff is given by $\min_{c \in [0, 1]} \pi_b(c)$. The contract that maximizes this payoff is a contract under which the buyer's ex-post payoff is constant across types (see Proposition 3.1). Constant ex-post payoff requires

$$\pi_b'(c) = 0 \quad \implies \quad \alpha'(c)\Delta = \alpha(c).$$

The solution to this differential equation, together with $\alpha(0) = 1$, is

$$\alpha^{MA}(c) = e^{-\frac{c}{\Delta}},$$

a decreasing function of c that perfectly separates all types. The contract characterized by α^{MA} corresponds to a screening menu that makes the buyer indifferent between all types and thereby indifferent between all distributions over types. The intuition is that a decreasing schedule allows the buyer to balance her rent across types by exactly offsetting the reduced information rent paid to higher types by the increased trading probability with lower types. α^{MA} therefore hedges perfectly against the ambiguity in this contracting problem and may be seen as the continuous analogue of $\tilde{\alpha}$ in the binary type environment (Section 2).

For intermediate levels of ambiguity, the optimal contract compromises between maximizing the buyer's expected utility under a single uniform prior, $\int_0^1 \alpha(c)(\Delta - c)dc$, and limiting her worst-case payoff, $\min_c \pi_b(c)$. The ambiguity parameter g weighs the two objectives. Proposition 3.1 characterizes the buyer's optimal contract for all $g \in [0, 1]$. It shows that for intermediate levels of ambiguity the optimal contract exhibits partial bunching and partial separation, where the extent

⁸To see the second equality, note that changing the order of integration implies

$$\int_0^1 \int_c^1 \alpha(u)dudc = \int_0^1 \int_0^u \alpha(u)dcdu = \int_0^1 \alpha(u)udu.$$

of bunching decreases in the extent of ambiguity.

Proposition 3.1. *Assume $v(c) = c + \Delta$ and $\mathcal{F} = \mathcal{F}^g$ for some $g \in [0, 1]$.*

- **Perfect Separation:** if $g \leq \frac{1}{\Delta}e^{-\frac{1}{\Delta}}$,

$$\alpha^*(c) = e^{-\frac{c}{\Delta}}, \forall c.$$

- **Partial Separation:** if $g \in \left(\frac{1}{\Delta}e^{-\frac{1}{\Delta}}, \frac{1}{\Delta}e^{-\frac{1-\Delta}{\Delta}}\right)$,

$$\alpha^*(c) = \begin{cases} 1 & \text{if } c \leq \hat{c}, \\ \frac{\Delta - \hat{c}}{\Delta} e^{-\frac{c - \hat{c}}{\Delta}} & \text{if } c > \hat{c}, \end{cases}$$

where $\hat{c} = 1 + \Delta \ln(g\Delta)$.

- **Posted Price:** if $g \geq \frac{1}{\Delta}e^{-\frac{1-\Delta}{\Delta}}$,

$$\alpha^*(c) = \begin{cases} 1 & \text{if } c \leq \Delta, \\ 0 & \text{if } c > \Delta. \end{cases}$$

Proof See Appendix A.2.3.

There are three regions to be distinguished. First, if the extent of ambiguity is sufficiently small ($g \geq \frac{1}{\Delta}e^{-\frac{1-\Delta}{\Delta}}$), the buyer proposes α^{NA} , the optimal contract in the absence of ambiguity ($g = 1$). Under α^{NA} , the buyer's expected utility under a single uniform prior is maximized but the buyer's minimal ex-post payoff is equal to zero. Second, if the extent of ambiguity is sufficiently large ($g \leq \frac{1}{\Delta}e^{-\frac{1}{\Delta}}$), the buyer proposes α^{MA} , the optimal contract under maximal ambiguity ($g = 0$). By definition of α^{MA} , all types yield the same ex-post payoff, equal to

$$\bar{\Pi} = \pi_b(1) = \Delta \alpha^{MA}(1) = \Delta e^{-\frac{1}{\Delta}}.$$

Finally, for intermediate levels of ambiguity, the buyer faces a tradeoff between maximizing her expected utility under a single uniform prior and limiting her minimal ex-post payoff. She solves this tradeoff by partially separating the types. In particular, there is a separating region $(\hat{c}, 1]$ in which $\alpha(c)$ is strictly decreasing in c and the buyer's ex-post payoff is constant across types, while the rest of the types, $[0, \hat{c}]$, are bunched and trade with probability one. The larger the buyer's desired level of minimal ex-post payoff is, the smaller is the bunching region $[0, \hat{c}]$.

To see this, consider the optimal contract without ambiguity, α^{NA} . If the buyer wishes to raise her minimal ex-post payoff, she needs to raise $\alpha(c)$ for all $c \in (\Delta, 1]$. This implies that the

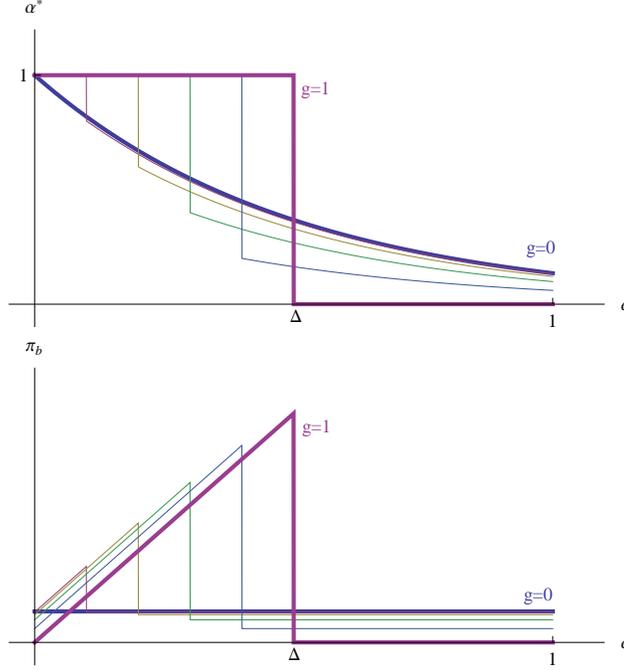


Figure 2: $\alpha^*(c)$ and $\pi_b(c)$ for different values of $g \in [0, 1]$

information rent paid to types $c \in [0, \Delta]$ increases, which consequently implies that the buyer's ex-post payoff when trading with the lowest type is negative:

$$\pi_b(0) = \Delta - \int_0^\Delta 1 du - \int_\Delta^1 \alpha(c) du < 0.$$

To increase $\pi_b(0)$, the buyer needs to decrease the information rent paid to the lowest type. This is optimally done by shrinking the bunching region $[0, \hat{c}]$ and by keeping ex-post payoff constant across the remaining types, $c \in (\hat{c}, 1]$. Constant ex-post payoff requires that $\alpha(c)$ is strictly decreasing on the interval $(\hat{c}, 1]$, as explained above. This implies that types in $(\hat{c}, 1]$ are perfectly separated and that the buyer obtains her minimal ex-post payoff either by trading with the lowest type or by trading with any type in $(\hat{c}, 1]$. The more the buyer wishes to raise her minimal ex-post payoff, the more she has to shrink the bunching region $[0, \hat{c}]$, illustrated in Figure 2. If $\hat{c} = \Delta$, the buyer's minimal ex-post payoff is equal to zero, while if $\hat{c} = 0$, the buyer's minimal ex-post payoff is equal to $\bar{\Pi}$.

The extent of ambiguity determines the degree of separation in equilibrium. The optimal marginal type is given by

$$\hat{c} = 1 + \Delta \ln(g\Delta).$$

\hat{c} is increasing in g , implying that the size of the bunching interval $[0, \hat{c}]$ is increasing in g . Since g is the inverse measure of the extent of ambiguity in this environment, this implies that the degree of separation in equilibrium increases in the extent of ambiguity the buyer faces. The intuition is that bunching maximizes the buyer's expected utility while separation allows the buyer to hedge against ambiguity. The more ambiguity the buyer faces, the more profitable hedging becomes.

4 Smooth Ambiguity Aversion

For a given set of measures, the maxmin expected utility model may be seen as a special case of the smooth ambiguity model with infinite ambiguity aversion. The smooth ambiguity model, developed by Klibanoff et al. (2005), not only has very robust features but also allows for a separation between ambiguity and ambiguity attitude. This section extends the characterization of the optimal contract to the case of smooth ambiguity aversion and shows how the properties of the solution are connected to the optimal contract under maxmin expected utility. The buyer's utility function is given by

$$\Pi^{SM} = E_{\mu} [\Phi (E_{\sigma}[\pi])],$$

where $\mu : [0, 1] \rightarrow [0, 1]$ is a subjective prior on a set of priors and $\Phi : \mathbb{R} \rightarrow \mathbb{R}$ is a function that weighs different realizations of the decision maker's expected utility $E_{\sigma}[\pi]$. Ambiguity is captured by the second-order belief μ , which roughly speaking measures the buyer's belief about a particular σ being the "correct" probability. Ambiguity attitude is captured by the function Φ . If Φ is linear, the buyer is ambiguity neutral and her preferences are observationally equivalent to those of a subjective expected utility maximizer. If, on the other hand, Φ is concave, the buyer is ambiguity-averse and prefers known risks over unknown risks. The degree of ambiguity aversion is measured by the coefficient of absolute ambiguity aversion $-\frac{\Phi''(x)}{\Phi'(x)}$.⁹

As in the basic model, the buyer proposes a menu of the form $\{(1, \alpha c_h + (1 - \alpha)c_l), (\alpha, \alpha c_h)\}$. The optimization problem is given by

$$\max_{\alpha} E_{\mu} [\Phi (\sigma \alpha (v_h - c_h) + (1 - \sigma)(v_l - \alpha c_h - (1 - \alpha)c_l))].$$

To make ambiguity matter assume that μ has positive mass on both $[0, \tilde{\sigma}]$ and $[\tilde{\sigma}, 1]$ and assume

⁹ $-\frac{\Phi''(x)}{\Phi'(x)}$ measures aversion to subjective uncertainty about ex-ante payoffs. Analogous to risk aversion, Klibanoff et al. (2005) define ambiguity aversion as an aversion to mean preserving spreads in the subjective probability distribution over the set of expected utility values.

that $\Phi'(\cdot) > 0$, $\Phi''(\cdot) < 0$. The first order condition to the buyer's optimization problem is given by

$$\begin{aligned} & \underbrace{\mathbb{E}_\mu [\Phi'(\mathbb{E}_\sigma[\pi]) [(c_h - c_l) - \sigma(v_h - c_l)] | \sigma < \tilde{\sigma}]}_{\equiv MC(\alpha)} \\ = & \underbrace{\mathbb{E}_\mu [\Phi'(\mathbb{E}_\sigma[\pi]) [\sigma(v_h - c_l) - (c_h - c_l)] | \sigma \geq \tilde{\sigma}]}_{\equiv MG(\alpha)}. \end{aligned}$$

The marginal cost of increasing α , $MC(\alpha)$, is the marginal decrease in expected utility in the event that pooling is not optimal ($\sigma < \tilde{\sigma}$), while the marginal gain of increasing α , $MG(\alpha)$, is the marginal increase in expected utility in the event that pooling is optimal ($\sigma \geq \tilde{\sigma}$). Concavity of Φ implies that the marginal cost is increasing in α , whereas the marginal gain is decreasing in α . This implies that there is a unique α that maximizes the buyer's expected payoff. The conditions for an interior solution are

$$MC(0) < MG(0) \quad \text{and} \quad MC(1) > MG(1).$$

Proposition 4.1 summarizes this result.

Proposition 4.1. *The optimal menu of contracts for a buyer with smooth ambiguity aversion is $\{(1, \alpha^* c_h + (1 - \alpha^*) c_l), (\alpha^*, \alpha^* c_h)\}$ with*

$$\alpha^* = \begin{cases} 0 & \text{if } MC(0) \geq MG(0), \\ 1 & \text{if } MC(1) \leq MG(1), \\ \text{s.th. } MC(\alpha^*) = MG(\alpha^*) & \text{otherwise.} \end{cases}$$

Proof See Appendix A.2.4.

Proposition 4.1 is the smooth counterpart of the equilibrium characterization under maxmin expected utility (Proposition 2.1). To see the connection between the two models, assume that the support of μ is Σ and assume that $\underline{\sigma} < \tilde{\sigma} < \bar{\sigma}$.¹⁰ Under maxmin preferences, the buyer's optimal contract, characterized by $\tilde{\alpha}$, makes the buyer's payoff unambiguous. Under smooth ambiguity aversion, the buyer compromises between maximizing the expected value of expected utility, $\mathbb{E}_\mu \mathbb{E}_\sigma[\pi]$, and limiting her exposure to ambiguity. $\mathbb{E}_\mu \mathbb{E}_\sigma[\pi]$ is maximized by offering a posted price (either equal to c_l or c_h), whereas ambiguity is eliminated by offering the menu characterized by $\tilde{\alpha}$. The optimal contract under smooth ambiguity aversion is a convex combination of the two. If offering the pooling price maximizes $\mathbb{E}_\mu \mathbb{E}_\sigma[\pi]$, then α^* is in the interval $[\tilde{\alpha}, 1]$, otherwise α^* is in the interval $[0, \tilde{\alpha}]$. The more ambiguity-averse the buyer is, the closer is α^* to $\tilde{\alpha}$. This is summarized in Proposition 4.2.

Proposition 4.2. *Assume $-\frac{\Phi''(x)}{\Phi'(x)} = \gamma$.*

¹⁰Otherwise the solution is the same posted price under both representations.

- If $E_\mu[\sigma] < \tilde{\sigma}$, then $\alpha^* \in [0, \tilde{\alpha}]$ and $\frac{d\alpha^*}{d\gamma} \geq 0$.
- If $E_\mu[\sigma] > \tilde{\sigma}$, then $\alpha^* \in [\tilde{\alpha}, 1]$ and $\frac{d\alpha^*}{d\gamma} \leq 0$.

Proof See Appendix A.2.5.

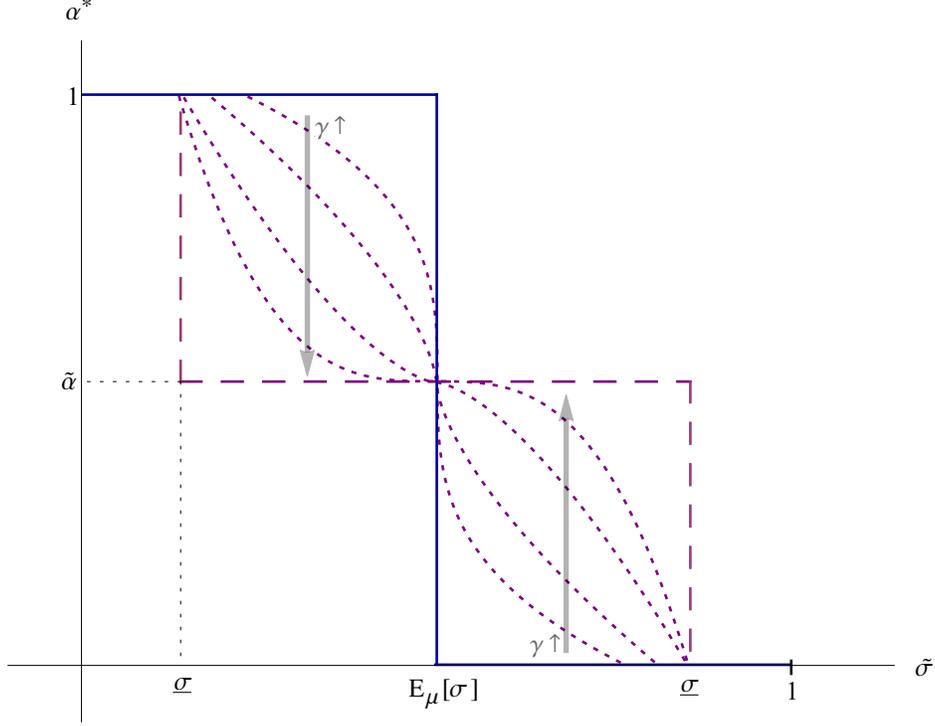


Figure 3: α^* for different levels of γ .

Proposition 4.2 is illustrated in Figure 3. The figure shows the optimal contracting parameter α^* as a function of $\tilde{\sigma}$ for different degrees of ambiguity aversion. The solid curve represents the optimal contract under ambiguity neutrality ($\gamma = 0$). If $\tilde{\sigma} < E_\mu[\sigma]$, the optimal contract is characterized by $\alpha^* = 1$, whereas if $\tilde{\sigma} > E_\mu[\sigma]$, the optimal contract is characterized by $\alpha^* = 0$. α^* as a function of $\tilde{\sigma}$ therefore has a single step at $E_\mu[\sigma]$. The dashed curve represents the optimal contract choice under maxmin expected utility ($\gamma \rightarrow \infty$). If $\tilde{\sigma} < \underline{\sigma}$ ($\tilde{\sigma} > \bar{\sigma}$), the maxmin buyer's optimal contracting parameter is given by $\alpha^* = 1$ ($\alpha^* = 0$). Otherwise, the optimal contract is a menu characterized by $\tilde{\alpha}$. α^* as a function of $\tilde{\sigma}$ therefore has two steps, one at $\underline{\sigma}$ and one at $\bar{\sigma}$. The dotted curves in between these two benchmarks show α^* as a function of $\tilde{\sigma}$ for intermediate levels of ambiguity aversion ($\gamma \in (0, \infty)$). If $\tilde{\sigma} < \underline{\sigma}$ ($\tilde{\sigma} > \bar{\sigma}$), the optimal contract under ambiguity neutrality coincides with the optimal contract under maxmin preferences, and the degree of ambiguity aversion is irrelevant. The interesting parameter region is therefore $\underline{\sigma} < \tilde{\sigma} < \bar{\sigma}$. If $\underline{\sigma} < \tilde{\sigma} < E_\mu[\sigma]$, the optimal contract under ambiguity neutrality is characterized by $\alpha^* = 1$, whereas under maxmin preferences

it is characterized by $\alpha^* = \tilde{\alpha}$. For intermediate levels of ambiguity aversion, the buyer compromises between the two and offers a contract characterized by $\alpha^* \in [\tilde{\alpha}, 1]$. Similarly, if $E_\mu[\sigma] < \tilde{\sigma} < \bar{\sigma}$, the optimal contracting parameter under ambiguity neutrality is $\alpha^* = 0$, whereas under maxmin preferences it is $\alpha^* = \tilde{\alpha}$. For intermediate levels of ambiguity aversion, the optimal contracting parameter consequently lies in the interval $[0, \tilde{\alpha}]$. The more ambiguity-averse the buyer is, the closer is α^* to $\tilde{\alpha}$.

5 Conclusion

This paper demonstrates the implications of ambiguity aversion on optimal contracting with asymmetric information and correlated values. It shows how non-linear pricing hedges against ambiguity over the valuations of the contracting parties and provides conditions for which the optimal contract under ambiguity differs from the optimal contract under any single prior. The paper further demonstrates that when types are continuous, low-quality sellers are bunched while the remaining sellers are price discriminated, and shows that the degree of bunching decreases in the extent of ambiguity.

Moreover, the analysis demonstrates that ambiguity does not necessarily inhibit trade, a rather surprising result given the previous literature on ambiguity and trade, e.g. Dow and Werlang (1992), De Castro and Chateauneuf (2011), etc. The paper provides conditions under which ambiguity is beneficial for trade and shows that under further restrictions, the trading outcome under bilateral negotiations and ambiguity may be strictly more efficient than in competitive markets. In the context of financial markets, this suggests that if opacity gives rise to ambiguity, opaque OTC markets may implement strictly better trading outcomes than traditional exchange markets, and thereby highlights a new aspect in the debate on why opaque off-exchange trading may emerge.

An interesting question for future research is how the results derived in this paper extend to trading environments with more than two contracting parties. One promising approach is to introduce ambiguity into an auction setting with interdependent values. In an auction with interdependent values, bidders compete for a good of uncertain value and receive private signals, determining each other's valuations. If bidders face ambiguity over the signal distribution, they face ambiguity over their valuation of the good and over the probability to win, and are thus confronted with a decision problem akin to the one presented in this paper. The results of this work suggest that if the bidders' aversion to ambiguity over the probability to win outweighs their aversion to ambiguity over their valuation, the presence of ambiguity may lead to strictly higher bids, thereby benefiting the auctioneer.

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A Appendix

A.1 Continuous Types: Generalization

The buyer's valuation function is given by $v(c) = \underline{v} + mc$, where $m \in [0, 1)$ and $\underline{v} \geq 1 - m$.¹¹

Proposition A.1. *Assume $v(c) = mc + \underline{v}$ and $\mathcal{F} = \mathcal{F}^g$ for some $g \in [0, 1]$. There are two thresholds $0 < \underline{g} < \bar{g} < 1$ such that*

- **Perfect Separation:** if $g \leq \underline{g}$,

$$\alpha^*(c) = \left(1 - \frac{(1-m)c}{\underline{v}}\right)^{\frac{m}{1-m}}.$$

- **Partial Separation:** if $g \in (\underline{g}, \bar{g})$,

$$\alpha^*(c) = \begin{cases} 1 & \text{if } c \leq \hat{c}, \\ \frac{\underline{v} - \hat{c}}{\underline{v} - (1-m)\hat{c}} \left(\frac{\underline{v} - (1-m)c}{\underline{v} - (1-m)\hat{c}}\right)^{\frac{m}{1-m}} & \text{if } c > \hat{c}, \end{cases}$$

$$\text{where } \hat{c} = \frac{1}{1-m} \left(\underline{v} - \left(\frac{\underline{v} - (1-m)}{g^{1-m}} \right)^{\frac{1}{2-m}} \right).$$

- **Posted Price:** if $g \geq \bar{g}$,

$$\alpha^*(c) = \begin{cases} 1 & \text{if } c \leq \frac{\underline{v}}{2-m}, \\ 0 & \text{if } c > \frac{\underline{v}}{2-m}. \end{cases}$$

The thresholds are $\underline{g} = \underline{v}^{-\frac{2-m}{1-m}} (\underline{v} - (1-m))^{\frac{1}{1-m}}$ and $\bar{g} = \left(\frac{\underline{v}}{2-m}\right)^{-\frac{2-m}{1-m}} (\underline{v} - (1-m))^{\frac{1}{1-m}}$.

¹¹ $\underline{v} \geq 1 - m$ guarantees $v(c) > c, \forall c$, i.e. strict gains from trade.

The proof for Proposition A.1 is essentially the same as the proof for Proposition 3.1. To derive the optimal contract, consider first the contract that yields constant ex-post payoff:

$$\pi'_b(c) = 0 \quad \Leftrightarrow \quad -\alpha'(c)[\underline{v} - (1 - m)c] = m\alpha(c).$$

The solution to this differential equation is $\alpha(c) = D[\underline{v} - (1 - m)c]^{\frac{m}{1-m}}$. With $\alpha(1)(\underline{v} - (1 - m)) = \bar{\pi}$, this gives

$$\alpha(c) = \frac{\bar{\pi}}{\underline{v} - (1 - m)} \left(\frac{\underline{v} - (1 - m)c}{\underline{v} - (1 - m)} \right)^{\frac{m}{1-m}}.$$

To derive $\bar{\pi}$ as a function of \hat{c} note that in contrast to the setting in Section 3, type 0 is not necessarily in the set $\mathcal{C}^{min}(\alpha^*)$. To see this note that if the seller offers the contract that maximizes her unambiguous payoff, $\pi_b(0)$ is strictly positive. The buyer's payoff with unambiguous beliefs is $\int_0^1 \alpha(c)((m - 1)c + \underline{v} - c)$, which is maximized by offering a posted price $p^* = \frac{\underline{v}}{2-m}$. This implies $\pi_b(0) = \underline{v} - \frac{\underline{v}}{2-m} > 0$. Thus, there are two cases to be distinguished.

First, suppose $0 \notin \mathcal{C}^{min}(\alpha^*)$. In this case, $\hat{c} = \frac{\underline{v}}{2-m}$ because if $\hat{c} < \frac{\underline{v}}{2-m}$ ($\hat{c} > \frac{\underline{v}}{2-m}$), the buyer could marginally increase the unambiguous payoff without affecting her minimal ex-post payoff by marginally increasing (decreasing) \hat{c} . The buyer's objective as a function of $\bar{\pi}$ then is:¹²

$$\Pi^{MEU} = g \frac{\underline{v}}{2-m} \left(\frac{\underline{v}}{2} - \bar{\pi} \left(\frac{\underline{v}}{2-m} \frac{1}{\underline{v} - (1-m)} \right)^{\frac{1}{1-m}} \right) + \bar{\pi}.$$

Note that Π^{MEU} is linear in $\bar{\pi}$, implying that the optimal $\bar{\pi}$ is either equal to zero or it is bound above by $\pi_b(0)$. The former is the case if $\frac{\partial \Pi^{MEU}}{\partial \bar{\pi}} > 0$, which is true if

$$g > \left(\frac{\underline{v}}{2-m} \right)^{-\frac{2-m}{1-m}} (\underline{v} - (1-m))^{\frac{1}{1-m}}. \quad (\text{G})$$

Second, suppose inequality (G) is not satisfied such that $0 \notin \mathcal{C}^{min}(\alpha^*)$. The constraint $\pi_b(0) = \bar{\pi}$ implies

$$\bar{\pi}(\hat{c}) = (\underline{v} - \hat{c}) \left(\frac{\underline{v} - (1-m)}{\underline{v} - (1-m)\hat{c}} \right)^{\frac{1}{1-m}}.$$

¹²To derive the buyer's objective, note that

$$p \left(\frac{\underline{v}}{2-m} \right) = \frac{\underline{v}}{2-m} - \bar{\pi} \left(\frac{\underline{v}}{2-m} \frac{1}{\underline{v} - (1-m)} \right)^{\frac{1}{1-m}} - \bar{\pi}.$$

Using $p(\hat{c}) = \underline{v} - \bar{\pi}(\hat{c})$, the buyer solves the problem

$$\max_{\hat{c}} \quad \Pi^{MEU} = gm\frac{\hat{c}}{2} + \bar{\pi}(\hat{c}).$$

The first order condition to this problem is

$$gm\hat{c} = -\bar{\pi}'(\hat{c}),$$

where $\bar{\pi}'(\hat{c}) = -m\hat{c}(\underline{v} - (1-m))^{1-m}(\underline{v} - (1-m)\hat{c})^{-\frac{2-m}{1-m}}$. To derive the bounds for an interior solution, note that $-\bar{\pi}'(\hat{c})$ is convexly increasing and intersecting $gm\hat{c}$ at the origin. This implies $gm\hat{c} < -\bar{\pi}'(\hat{c})$ for all $c \in \left[0, \frac{\underline{v}}{2-m}\right]$ if

$$gm < -\bar{\pi}''(0) \quad \Rightarrow \quad g < \underline{v}^{-\frac{2-m}{1-m}}(\underline{v} - (1-m))^{1-m}.$$

On the other hand, $gm\hat{c} > -\bar{\pi}'(\hat{c})$ for all $c \in \left[0, \frac{\underline{v}}{2-m}\right]$ if

$$gm\frac{\underline{v}}{2-m} > -\bar{\pi}'\left(\frac{\underline{v}}{2-m}\right) \quad \Rightarrow \quad g > \left(\frac{\underline{v}}{2-m}\right)^{-\frac{2-m}{1-m}}(\underline{v} - (1-m))^{1-m},$$

which coincides with condition (G). Finally, if g is within these two bounds, the problem has an interior solution. Solving the first order condition for \hat{c} yields

$$\hat{c} = \frac{1}{1-m} \left(\underline{v} - \left(\frac{\underline{v} - (1-m)}{g^{1-m}} \right)^{\frac{1}{2-m}} \right),$$

completing the characterization.

A.2 Proofs

A.2.1 Proof of Proposition 2.1

The buyer maximizes $\min_{\sigma \in \Sigma} \mathbb{E}_{\sigma}[\pi]$. To identify the minimizing prior, consider

$$\frac{\partial \mathbb{E}_{\sigma}[\pi]}{\partial \sigma} = \alpha(v_h - c_l) - (v_l - c_l),$$

which is negative if $\alpha < \tilde{\alpha}$ and positive if $\alpha \geq \tilde{\alpha}$. This implies that $\bar{\sigma} \in \arg \min_{\sigma \in \Sigma} \{\mathbb{E}_{\sigma}[\pi(\alpha)]\}$ if $\alpha \leq \tilde{\alpha}$ and $\underline{\sigma} \in \arg \min_{\sigma \in \Sigma} \{\mathbb{E}_{\sigma}[\pi(\alpha)]\}$ if $\alpha \geq \tilde{\alpha}$. The buyer consequently maximizes the step function

$$\Pi^{MEU} = \begin{cases} \bar{\sigma}\alpha(v_h - c_h) - (1 - \bar{\sigma})(v_l - \alpha c_h + (1 - \alpha)c_l) & \text{if } \alpha \leq \tilde{\alpha}, \\ \underline{\sigma}\alpha(v_h - c_h) - (1 - \underline{\sigma})(v_l - \alpha c_h + (1 - \alpha)c_l) & \text{if } \alpha > \tilde{\alpha}. \end{cases}$$

Note further that $E_\sigma[\pi(\alpha)]$ is weakly increasing in α if and only if $\sigma \geq \tilde{\sigma}$:

$$\frac{\partial E_\sigma[\pi(\alpha)]}{\partial \alpha} = \sigma(v_h - c_l) - (c_h - c_l).$$

If $\bar{\sigma} \leq \tilde{\sigma}$, both parts of the step function are weakly decreasing in α and Π^{MEU} is maximized at $\alpha = 0$. Similarly, if $\underline{\sigma} \geq \tilde{\sigma}$, both parts of the step function are weakly increasing in α and Π^{MEU} is maximized at $\alpha = 1$. If $\underline{\sigma} < \tilde{\sigma} < \bar{\sigma}$, Π^{MEU} is increasing in α on the interval $[0, \tilde{\alpha}]$ and decreasing in α on the interval $[\tilde{\alpha}, 1]$, and therefore maximized at $\tilde{\alpha}$. \square

A.2.2 Proof of Proposition 2.2

Suppose $\tilde{\sigma} > \underline{\sigma}$. If $\tilde{\sigma} < \bar{\sigma}$, then $\alpha_{\Sigma}^* = \tilde{\alpha}$. Since $\underline{\sigma} < \tilde{\sigma} < \bar{\sigma}$ implies $\underline{\sigma}' < \tilde{\sigma} < \bar{\sigma}'$, $\alpha_{\Sigma'}^* = \tilde{\alpha}$. An increase in ambiguity has no effect.

If $\tilde{\sigma} > \bar{\sigma}$, then $\alpha_{\Sigma}^* = 0$. This implies either $\underline{\sigma}' < \bar{\sigma}' < \tilde{\sigma}$ or $\underline{\sigma}' < \tilde{\sigma} < \bar{\sigma}'$, in which case $\alpha_{\Sigma'}^* = 0$ or $\alpha_{\Sigma'}^* = \tilde{\alpha}$, respectively. Hence, $\alpha_{\Sigma}^* \leq \alpha_{\Sigma'}^*$. The proof for the case $\tilde{\sigma} < \bar{\sigma}$ is analogous.

A.2.3 Proof of Proposition 3.1

The optimal contract α^* will be derived in a series of lemmas. Let $\Pi^{NA} := \int_0^1 \alpha(c)[\Delta - c]dc$ and $\Pi^{AA} := \min_c \left\{ \alpha(c)\Delta - \int_c^1 \alpha(u)du \right\}$ denote the buyer's maxmin payoff for no ambiguity and maximal ambiguity, respectively. Further, let the set of types that yield the buyer's lowest expected ex-post payoff given contract α be denoted by

$$\mathcal{C}^{min}(\alpha) = \left\{ c : c \in \min_{\tilde{c}} \left\{ \alpha(\tilde{c})\Delta - \int_{\tilde{c}}^1 \alpha(u)du \right\} \right\}.$$

The buyer solves

$$\max_{\alpha(c)} g \int_0^1 \alpha(c)[\Delta - c]dc + (1 - g) \min_{c \in [0,1]} \left\{ \alpha(c)\Delta - \int_c^1 \alpha(u)du \right\} \quad \text{s.t.} \quad \alpha'(c) \leq 0.$$

The first lemma shows that in the optimal contract, the minimal ex-post payoff is non-negative.

Lemma A.2. *Assume $v(c) = c + \Delta$ and $\mathcal{F} = \mathcal{F}^g$ for some $g \in [0, 1]$. $\pi_b(c) \geq 0, c \in \mathcal{C}^{min}(\alpha^*)$.*

Proof Π^{NA} is maximized by the contract

$$\alpha(c) = \begin{cases} 1 & \text{if } c \leq \Delta, \\ 0 & \text{if } c > \Delta. \end{cases}$$

The set of minimizing types given this contract is $\mathcal{C}^{min}(\alpha^{NA}) = \{0 \cup (\Delta, 1]\}$, where $\pi_b(c) = 0, \forall c \in \mathcal{C}^{min}(\alpha^{NA})$. This implies that any contract $\tilde{\alpha}$ with $\pi_b(c) < 0, c \in \mathcal{C}^{min}(\tilde{\alpha})$ is strictly dominated by

α^{NA} . Hence, a minimizing type in the optimal contract α^* yields a payoff that is weakly greater than zero. \square

Lemma A.3. *Assume $v(c) = c + \Delta$ and $\mathcal{F} = \mathcal{F}^g$ for some $g \in [0, 1]$. For any $\bar{\pi} \in \left[0, \Delta e^{-\frac{1}{\Delta}}\right]$, there exists a strictly decreasing function $\alpha^{\bar{\pi}} : [0, 1] \rightarrow [0, 1]$ such that $\alpha^{\bar{\pi}}(c)\Delta - \int_c^1 \alpha^{\bar{\pi}}(u)du = \bar{\pi}, \forall c$.*

Proof $\alpha(c)\Delta - \int_c^1 \alpha(u)du$ is constant in c if

$$\pi'_b(c) = \alpha'(c)\Delta + \alpha(c) = 0.$$

The solution to the differential equation $-\alpha'(c)\Delta = \alpha(c)$ is $\alpha(c) = De^{-\frac{c}{\Delta}}$. The level of constant ex-post payoff is equal to $\bar{\pi}$ if $\pi_b(1) = \bar{\pi}$, i.e. if

$$\alpha(1)\Delta = \bar{\pi} \quad \Rightarrow \quad D = \frac{\bar{\pi}}{\Delta} e^{\frac{1-c}{\Delta}}.$$

Consequently

$$\alpha^{\bar{\pi}}(c) = \frac{\bar{\pi}}{\Delta} e^{\frac{1-c}{\Delta}}.$$

Note that $\alpha^{\bar{\pi}}(c)$ is decreasing in c and thus, satisfies the monotonicity constraint. Finally, $\alpha(c)$ may not exceed one for any c , which is satisfied if

$$\alpha^{\bar{\pi}}(0) \leq 1 \quad \Rightarrow \quad \bar{\pi} \leq \Delta e^{-\frac{1}{\Delta}}.$$

Lemma A.4. *Assume $v(c) = c + \Delta$ and $\mathcal{F} = \mathcal{F}^g$ for some $g \in [0, 1]$. Let $\pi^* = \pi_b(c), c \in \mathcal{C}^{min}(\alpha^*)$. Then $\alpha^*(c) \geq \alpha^{\pi^*}(c)$ for all c .*

Proof Suppose not. Let \tilde{c} be the largest type such that $\alpha^*(c) < \alpha^{\pi^*}(c)$. Then $\alpha^*(\tilde{c}) - \int_{\tilde{c}}^1 \alpha(u)du \geq \pi^*$ implies

$$\int_{\tilde{c}}^1 \alpha^*(u)du < \int_{\tilde{c}}^1 \alpha^{\pi^*}(u)du,$$

a contradiction. \square

Lemma A.5. *Assume $v(c) = c + \Delta$ and $\mathcal{F} = \mathcal{F}^g$ for some $g \in [0, 1]$. There exists some $\hat{c} \in [0, \Delta]$ and some $\bar{\pi} \in \left[0, \Delta e^{-\frac{1}{\Delta}}\right]$ such that the optimal contract α^* satisfies $\alpha^*(c) = 1, \forall c \leq \hat{c}$ and $\alpha^*(c) = \alpha^{\bar{\pi}}(c), \forall c < \hat{c}$.*

Proof For given minimal ex-post payoff $\bar{\pi}$, the buyer's optimization problem is

$$\max_{\alpha(c)} \Pi^{NA} \quad \text{s.t.} \quad \alpha(c)\Delta - \int_c^1 \alpha(u)du \geq \bar{\pi}, \forall c \quad \text{and} \quad \alpha'(c) \leq 0.$$

If $\bar{\pi} = 0$, the optimal contract is α^{NA} (see the proof of Lemma A.2), i.e. $\hat{c} = \Delta$. If $\bar{\pi} > 0$, the optimal contract given $\bar{\pi}$ can be derived by considering a less restricted optimization problem and then showing that the solution to this problem satisfies all constraints of the original problem:

$$\max_{\alpha(c)} \Pi^{NA} \quad \text{s.t.} \quad \alpha(0)\Delta - \int_0^1 \alpha(u)du \geq \bar{\pi} \quad \text{and} \quad \alpha(c) \geq \alpha^{\bar{\pi}}(c), \forall c.$$

Let λ denote the Lagrange-multiplier of the constraint $\pi_b(0) \geq \bar{\pi}$ and let $\mu_c, c \in [0, 1]$ denote the Lagrange-multipliers of the constraints $\alpha(c) \geq \alpha^{\bar{\pi}}(c)$. First, we can show that the constraint $\pi_b(0) \geq \bar{\pi}$ is binding. Suppose not.

$$\frac{\partial \mathcal{L}}{\partial \alpha(c)} = \Delta - c + \mu_c > 0 \quad \text{for all} \quad c < \Delta,$$

implying that $\alpha^*(c) = 1, \forall c < \Delta$. Then

$$\pi_b(0) = \Delta - \int_0^\Delta 1dc - \int_\Delta^1 \alpha(c)dc = 0 - \int_\Delta^1 \alpha(c)dc \leq 0,$$

a contradiction. The binding constraint implies that $\lambda > 0$. Let \hat{c} denote the type such that

$$\Delta - \hat{c} - \lambda = 0,$$

Since $\frac{\partial \mathcal{L}}{\partial \alpha(c)} = \Delta - c - \lambda + \mu_c > 0$ for all $c < \hat{c}$, we have $\alpha^*(c) = 1, \forall c \leq \hat{c}$. For the remaining types, the constraint $\alpha(c) \geq \alpha^{\bar{\pi}}(c)$ is binding as

$$\frac{\partial \mathcal{L}}{\partial \alpha(c)} = \Delta - c - \lambda + \mu_c = 0, \quad c > \hat{c}$$

requires $\mu_c > 0$ for all $c > \hat{c}$. Finally, we need to check whether this solution satisfies the original constraints of the principal's optimization problem, i.e. monotonicity and $\pi_b(c) \geq \bar{\pi}, \forall c$. Given that $\alpha^{\bar{\pi}}(c)$ is strictly decreasing, monotonicity is not violated. To see that $\pi_b(c) \geq \bar{\pi}, \forall c$, note that for $c \leq \hat{c}$

$$\pi_b(c) = \Delta - \int_c^{\hat{c}} 1dc - \int_{\hat{c}}^1 \alpha^{\bar{\pi}}(c)dc \geq \Delta - \int_0^{\hat{c}} 1dc - \int_{\hat{c}}^1 \alpha(c)dc = \pi_b(0) = \bar{\pi},$$

while for $c > \hat{c}$, $\pi_b(c) \geq \bar{\pi}$ is satisfied by definition of $\alpha^{\bar{\pi}}(c)$. □

Lemmas A.2-A.5 allow us to derive the optimal contract α^* . First, the minimal ex-post payoff $\bar{\pi}$ can be expressed as a function of \hat{c} :

$$\begin{aligned}\bar{\pi} &= \pi_b(0), \\ \Leftrightarrow \bar{\pi} &= \Delta - \hat{c} - \int_{\hat{c}}^1 \frac{\bar{\pi}}{\Delta} e^{\frac{1-u}{\Delta}} du, \\ \Leftrightarrow \bar{\pi} &= \Delta - \hat{c} + \bar{\pi} - \bar{\pi} e^{\frac{1-\hat{c}}{\Delta}}, \\ \Leftrightarrow \bar{\pi} &= (\Delta - \hat{c})e^{-\frac{1-\hat{c}}{\Delta}}.\end{aligned}$$

This implies

$$\alpha(c) = \begin{cases} 1 & \text{if } c \leq \hat{c}, \\ \frac{\Delta - \hat{c}}{\Delta} e^{-\frac{c-\hat{c}}{\Delta}} & \text{if } c > \hat{c}. \end{cases}$$

To derive the buyer's objective as a function of \hat{c} note that

$$p(\hat{c}) = \hat{c} + \int_{\hat{c}}^1 \frac{\Delta - \hat{c}}{\Delta} e^{-\frac{c-\hat{c}}{\Delta}} = \Delta - \bar{\pi}(\hat{c}),$$

where $\bar{\pi}(\hat{c}) = (\Delta - \hat{c})e^{-\frac{1-\hat{c}}{\Delta}}$. Then the buyer's optimization problem is

$$\max_{\hat{c}} g \left[\hat{c} \left(\frac{1}{2} \hat{c} + \Delta - p(\hat{c}) \right) + (1 - \hat{c})\bar{\pi}(\hat{c}) \right] + (1 - g)\bar{\pi}(\hat{c}),$$

or simply

$$\max_{\hat{c}} \frac{1}{2} g \hat{c}^2 + \bar{\pi}(\hat{c}).$$

The first order condition to the problem is

$$g\hat{c} = -\bar{\pi}'(\hat{c}),$$

where $\bar{\pi}'(\hat{c}) = -\frac{\hat{c}}{\Delta} e^{-\frac{1-\hat{c}}{\Delta}}$. To derive the bounds for an interior solution, note that $g\hat{c}$ is linearly increasing in \hat{c} and $-\bar{\pi}'(\hat{c})$ is convexly increasing in \hat{c} , both of them intersecting at the origin:

$$-\bar{\pi}''(\hat{c}) = e^{-\frac{1-\hat{c}}{\Delta}} \left(\frac{1}{\Delta} + \frac{\hat{c}}{\Delta^2} \right) > 0, \quad -\bar{\pi}'''(\hat{c}) = e^{-\frac{1-\hat{c}}{\Delta}} \left(\frac{2}{\Delta^2} + \frac{\hat{c}}{\Delta^3} \right) > 0.$$

This implies that $g\hat{c} \geq -\bar{\pi}'(\hat{c})$ on the interval $[0, \Delta]$ if

$$-\bar{\pi}''(0) \geq g \quad \Rightarrow \quad g \leq \frac{1}{\Delta} e^{-\frac{1}{\Delta}}.$$

On the other hand, $g\hat{c} \leq -\bar{\pi}'(\hat{c})$ on the interval $[0, \Delta]$ if

$$g\Delta \geq -\bar{\pi}'(\Delta) \quad \Rightarrow \quad g \geq \frac{1}{\Delta} e^{-\frac{1-\Delta}{\Delta}}.$$

Finally, if $g \in \left(\frac{1}{\Delta} e^{-\frac{1}{\Delta}}, \frac{1}{\Delta} e^{-\frac{1-\Delta}{\Delta}}\right)$, the problem has an interior solution. Solving the first order condition for \hat{c} yields

$$\hat{c} = 1 + \ln(g\Delta).$$

To see that \hat{c} is a maximum, note that

$$\frac{\partial^2 \Pi}{\partial \hat{c}^2} (1 + \ln(g\Delta)) = - \left(\frac{1}{\Delta} + \ln(g\Delta) \right) < 0 \quad \text{for all } g \in \left(\frac{1}{\Delta} e^{-\frac{1}{\Delta}}, \frac{1}{\Delta} e^{-\frac{1-\Delta}{\Delta}} \right),$$

making the characterization complete. □

A.2.4 Proof of Proposition 4.1

Define $m(\sigma) := \sigma(v_h - c_h) - (1 - \sigma)(c_h - c_l)$ as the marginal utility for a single prior σ . Note that $m(\sigma)$ is linearly increasing in σ and that $m(\tilde{\sigma}) = 0$. Taking the first and second derivative of the buyer's objective yields

$$\frac{\partial \Pi^{SM}}{\partial \alpha} = E_\mu [\Phi'[EU_\sigma(\alpha)]m(\sigma)], \quad \frac{\partial^2 \Pi^{SM}}{\partial \alpha^2} = E_\mu [\Phi''[EU_\sigma(\alpha)]m(\sigma)^2] < 0,$$

where the last inequality follows directly from concavity of Φ . This shows that there is unique α solving the buyer's optimization problem. If $\frac{\partial \Pi^{SM}}{\partial \alpha} < 0$ for all $\alpha \in [0, 1]$, the optimal α is equal to zero. This is the case if

$$E_\mu [\Phi'[EU_\sigma(0)]m(\sigma)] < 0,$$

or equivalently $MC(0) > MG(0)$. If $\frac{\partial \Pi^{SM}}{\partial \alpha} > 0$ for all $\alpha \in [0, 1]$, the optimal α is equal to one. This is the case if

$$E_\mu [\Phi'[EU_\sigma(1)]m(\sigma)] > 0,$$

or equivalently $MC(1) < MG(1)$. Otherwise the optimal α is equal to α^* such that

$$E_\mu [\Phi'[EU_\sigma(\alpha^*)]m(\sigma)] = 0.$$

□

A.2.5 Proof of Proposition 4.2

Constant absolute ambiguity aversion implies that the buyer's preferences can be represented by $\Phi(x) = -\frac{1}{\gamma}e^{-\gamma x}$ (see Klibanoff et al., 2005).

Suppose first $E_\mu[\sigma] > \tilde{\sigma}$. Note that $E_\mu E_\sigma[\pi(\alpha)]$ increases in α and that $\text{Var}_\mu[E_\sigma[\pi(\alpha)]] > 0$ for all $\alpha < \tilde{\alpha}$. This implies that the (degenerate) expected utility distribution induced by any $\alpha < \tilde{\alpha}$ is second-order stochastically dominated by the (degenerate) distribution induced by $\tilde{\alpha}$. Since Φ is strictly concave, this implies $\alpha^* \in [\tilde{\alpha}, 1]$.

The optimal value of α^* is characterized by

$$E_\mu \left[e^{-\gamma E_\sigma[\pi(\alpha^*)]} m(\sigma) \right] = 0,$$

if the solution is interior (otherwise a marginal change in γ has no effect). The implicit function theorem implies

$$\frac{d\alpha^*}{d\gamma} = -\frac{E_\mu \left[e^{-\gamma E_\sigma[\pi(\alpha^*)]} m(\sigma) E_\sigma[\pi(\alpha^*)] \right]}{\gamma E_\mu \left[e^{-\gamma E_\sigma[\pi(\alpha^*)]} m(\sigma)^2 \right]}.$$

$\frac{d\alpha^*}{d\gamma} \leq 0$, if the numerator is positive on the interval $[\tilde{\alpha}, 1]$. Suppose not. Then

$$E_\mu \left[-e^{-\gamma E_\sigma[\pi(\alpha^*)]} m(\sigma) E_\sigma[\pi(\alpha^*)] \mid \sigma \leq \tilde{\sigma} \right] > E_\mu \left[e^{-\gamma E_\sigma[\pi(\alpha^*)]} m(\sigma) E_\sigma[\pi(\alpha^*)] \mid \sigma \geq \tilde{\sigma} \right].$$

But

$$\begin{aligned} E_\mu \left[-e^{-\gamma E_\sigma[\pi(\alpha^*)]} m(\sigma) E_\sigma[\pi(\alpha^*)] \mid \sigma \leq \tilde{\sigma} \right] &\leq E_{\tilde{\sigma}}[\pi(\alpha^*)] E_\mu \left[-e^{-\gamma E_\sigma[\pi(\alpha^*)]} m(\sigma) \mid \sigma \leq \tilde{\sigma} \right], \\ &= E_{\tilde{\sigma}}[\pi(\alpha^*)] E_\mu \left[e^{-\gamma E_\sigma[\pi(\alpha^*)]} m(\sigma) \mid \sigma \geq \tilde{\sigma} \right], \\ &\leq E_\mu \left[e^{-\gamma E_\sigma[\pi(\alpha^*)]} m(\sigma) E_\sigma[\pi(\alpha^*)] \mid \sigma \geq \tilde{\sigma} \right], \end{aligned}$$

which follows from $\frac{\partial E_\sigma[\pi(\alpha)]}{\partial \sigma} > 0, \forall \alpha \in [\tilde{\alpha}, 1]$, where with slight abuse of notation $\underline{\sigma} = \arg \min\{\sigma : \mu(\sigma) > 0\}$ and $\bar{\sigma} = \arg \max\{\sigma : \mu(\sigma) > 0\}$. A contradiction. Hence, $\frac{d\alpha^*}{d\gamma} \leq 0$.

The proof for the case $E_\mu[\sigma] > \tilde{\sigma}$ is analogous, where the above inequalities are reversed. Just note that second-order stochastic dominance implies $\alpha^* \in [0, \tilde{\alpha}]$ and that $\frac{\partial E_\sigma[\pi(\alpha)]}{\partial \sigma} < 0, \forall \alpha \in [0, \tilde{\alpha}]$. \square