Perils of quantitative easing [1]

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Abstract

Monetary policy conducted in a manner akin to quantitative easing compromises the control of the central bank over the stochastic path of inflation.

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In the wake of the financial crisis that occurred in 2008-2009, central banks responded with large cuts in nominal interest rates. As interest rates fell to their effective lower bound (at or near to zero) and conventional interest rate policies were exhausted, attention shifted to unconventional policies.

Unconventional policies take many forms, and, while a general distinction can be made between quantitative policies, that focus on the expansion of the balance sheet of the central bank bank and the related creation of reserves, and credit policies, that concern the allocation of credit across asset in the balance sheet, much of the commentary on unconventional monetary policy has emphasized the effect both policies are designed to have on easing credit and liquidity conditions [1].

Here, we show that the distinction between credit and quantitative policies has significant implications for the ability of monetary policy to target the stochastic path of inflation. Specifically, we show that, like conventional policies, credit policies pin down the rate of inflation, while, under quantitative policies, indeterminacy prevails.

[1] In the US, the Federal Reserve reduced the target federal funds rate from 5.25 percent to effectively zero and also implemented a number of other programs and policies that led to significant changes to the Federal Reserve's balance sheet, according to Bernanke (2009), Goodfriend (2009) or Reis (2009). Some of these tools are closely tied to the central bank's traditional role as the lender of last resort and were designed to support the liquidity of financial institutions and foster improved conditions in financial markets. In light of improved functioning of financial markets, many of the new programs have expired or been closed. These include the Money Market Investor Funding Facility (MMIFF), the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF), the Commercial Paper Funding Facility (CPFF), the Primary Dealer Credit Facility (PDCF), the Term Securities Lending Facility (TSLF), and the Term Auction Facility (TAF). The temporary liquidity swap arrangements between the Federal Reserve and other foreign central banks were used on and off.

Other Fed unconventional policies were designed to try to stimulate the economy. The Fed first used Large Scale Asset Purchases (LSAP I and LSAP II) which sought to purchase assets directly from the distressed markets in order to reduce yields on those instruments and, therefore, reduce borrowing costs for private borrowers. Starting in November 2008 with $600 billion in agency mortgage-backed securities and agency debt, the FOMC then extended LSAP I between March 2009 to March 2010 to purchase an additional $850 billion to purchases of agency securities, and a further $300 billion to acquiring longer-term Treasury securities. LSAP II, beginning in November 2010, consisted of further purchases of $600 billion in longer-term Treasury securities. The most recent unconventional easing policy is the Maturity Extension Program (MEP) that, unlike the LSAP, does not aim to expand the size of the Fed balance sheet, but rather extends the average maturity of the Fed's Treasury holdings. MEP is essentially a new version of Operation Twist, implemented in the early 1960s, which sought to "twist" the yield curve by lowering the longer-term yields lower while keeping the short rates at existing levels. Under MEP, the Fed bought about $667 billion of longer-term Treasury securities and reduced its holdings of short-term Treasury bills by an equivalent amount.

The Fed describe their unconventional easing policy "Credit Easing" (CE) and, according to Federal Reserve Chairman Ben Bernanke, CE is conceptually distinct from "Quantitative Easing" (QE): the focus of CE was on the composition of the Fed's balance sheet, with the size being largely incidental, as opposed to the emphasis on the size under QE. As Bernanke (2009) explained, "The Federal Reserve's approach to supporting credit markets is conceptually distinct from quantitative easing, the policy approach used by the Bank of Japan from 2001 to 2006. Our approach – which could be described as 'credit easing'–resembles quantitative easing in one respect: It involves an expansion of the central bank's balance sheet. However, in a pure QE regime, the focus of policy is the quantity of bank reserves that are liabilities of the central bank; the composition of loans and securities on the asset side of the central bank's balance sheet is incidental." In fact, motivated by the desire to communicate with the markets about the new policies, the Fed "committed to providing the public as much information as possible about the uses of its balance sheet, plans regarding future uses of its balance sheet, and the criteria on which the relevant decisions are based." As part of this, the New York Federal Reserve published how (in terms of portfolio weights) the total LSAP purchases would be distributed across maturity sectors. Moreover, each two-week-long round of asset purchases would began every-other Wednesday when the SOMA Desk would announce, along with the specific days on which it would be conducting the auctions, the maturity sectors in which it would be buying over the subsequent two weeks.

Thw Bank of Japan (BoJ), according to Ugai (2007) or Maeda, Fujiwara, Mineshima, and Taniguchi (2005), set new operational targets for monetary policy in terms of financial intermediaries' holdings of central bank reserves, known as "Current Account Balances" (CABs) and, to the achieve these targets, the BoJ made outright purchases of a long-term Japanese government bonds, stocks held by commercial banks, from October 2002 to September 2003, and asset-backed securities (July 2003 to March 2006).

Other central banks recently started to employ quantitative easing policies. The Bank of England's "Asset Purchase Facility" (APF) was first established in January 2009 and, as in Bank of England (2010), the first instructions "to buy high-quality assets financed by the issuance of Treasury Bills" came in March 2009. The ECB has not, technically, made use of quantitative policies. This is because they sterilize, or intend to sterilize, the monetary effects of any purchases of assets made under its Securities Markets Program or "Outright Monetary Transactions" (OMT) programs.

To examine this issue, we consider a stochastic cash-in-advance economy where prices are flexible. In our baseline analysis, we make the assumption of non-Ricardian seigniorage policy for the central bank so that we consider an environment that typically yields a determinate price level. Our conclusion, that, surprisingly, had gone unnoticed, is that monetary policy determines the path of expected or average inflation, but not the distribution of possible paths of inflation. The stochastic path of inflation is determined by the manner in which monetary authorities adjust their portfolios over time. We show that indeterminacy is pervasive under pure QE without adequate targets.

Under normal operations, monetary policy sets a target for the short-term (here one period) interest rates, and conducts open market operations or repo transactions, using as collateral Treasury securities, with various maturities, but to conform to an ex-ante determined overall portfolio composition that has an exclusive focus on Treasuries of short maturity. Indeed, the fiscal theory of the price level in Woodford (1994) takes for granted that monetary authorities trade exclusively in short term nominally risk free bonds, and it fails to highlight the importance of this assumption for the claim of determinacy of the path of prices; this is also the case in Dubey and Geanakoplos (2003).

The possible multiplicity of stochastic inflation paths at equilibrium was clear in Bloise, Drèze, and Polemarchakis (2005) and Nakajima and Polemarchakis (2005); there, in a Ricardian specification, the indeterminacy in a monetary economy is characterized by the price level and a nominal martingale measure, while Magill and Quinzii (2012) emphasized the role of inflationary expectations [2].

First, Drèze and Polemarchakis (2000) pointed out the need for "comprehensive monetary policy" that restricts the stochastic path of the term structure of interest rates in order to determine the path of inflation, a theme that was later developed in Adao, Correia, and Teles (2011) and Magill and Quinzii (2012), Magill and Quinzii (2013); they did not make the connection with quantitative easing.

Under unconventional quantitative policies, the asset side of the portfolio is more varied and is not ex ante specified, and it is dependent on market forces, and ultimately market expectations that determine the value of the assets held by the central bank. Variation in the value of the balance sheet of the central bank then determines the stochastic path in which money is injected or withdrawn determining the path of inflation. Under credit policies, it is the explicit target for the composition of the balance sheet that allows the monetary authority to target the stochastic path of inflation: the target for the composition of the portfolio guarantees the necessary restrictions to

[2] The multiplicity or indeterminacy of equilibria in stochastic, monetary economies with operative cash-in-advane-constraints is not conceptually different from the indeterminacy of Walrasian allocations in Cass (1985) and the work that followed.

obtain determinacy.

The indeterminacy that we discuss in the baseline model is nominal; while the central bank loses the control of inflation, the indeterminacy does not affect the attainable equilibrium allocations (a real indeterminacy would also affect allocations). If the central bank were to switch to a money supply, rather than interest rate, policy the indeterminacy becomes real and affects consumption allocations. Moreover, while our baseline model assumes that asset markets are complete and prices are flexible, if prices are sticky or the asset market is incomplete, unconventional monetary policies similarly lead to a real indeterminacy. [3].

1 The analytical argument

Does monetary policy determine the stochastic path of prices and rates of inflation?

Monetary policy is it conventional if open market operations are restricted to treasury bills: short term, nominally risk-free assets. It involves *quantitative easing* if open market operations are extended to government bonds of different maturities or bonds issued by the private sector.

Quantitative easing generates indeterminacy indexed by a *nominal pricing measure*over states of the world. This measure determines the distribu-

[3] As already touched upon, there is a vast and important literature on indeterminacy of monetary equilibria. Sargent and Wallace (1975) discussed the indeterminacy of the initial price level under interest rate policy; Lucas and Stokey (1987) derived the condition for the uniqueness of a recursive equilibrium with money supply policy; Woodford (1994) analyzed the dynamic paths of equilibria associated with the indeterminacy of the initial price level under money supply policy. In this paper, we give the exact characterization of the indeterminacy in stochastic economies in terms of the nominal equivalent martingale measure and show that there is a continuum of recursive equilibria under QE with interest rate policy.

The type of indeterminacy in our paper does not derive from the stability of a steady state or the infinity of the horizon;Benhabib and Farmer (1999) is a useful survey of the literature on indeterminacy arising from the stability of a steady state. To restrict attention to equilibria that stay in a neighborhood of a steady state, its stability would be important. This is related to recent discussions of the Taylor rule: although, as long as fiscal policy is Ricardian, the coefficient in the Taylor rule does not change the degree of indeterminacy, it affects the number of locally bounded equilibria as in Woodford (1999) and Benhabib, Schmitt-Grohe, and Uribe (2001); Benhabib, Schmitt-Grohe, and Uribe (2002) examine how non-Ricardian fiscal policy interacts with the Taylor rule to obtain a unique equilibrium.

Carlstrom and Fuerst (1998) discussed the indeterminacy of sticky-price equilibria when the nominal interest rates are zero. Here, we discuss the indeterminacy in more general case.

tion of rates of inflation, up to a moment that is determined by the risk-free rate and non-arbitrage.

Fiscal policy is *Ricardian* if it is restricted to satisfy an intertemporal budget constraint or transversality condition; equivalently, if public debt vanishes for all possible, equilibrium or non-equilibrium, values of price and rates of interest. It is *non-Ricardian*, if it is not restricted to satisfy an intertemporal budget constraint; in particular, outside money or initial liabilities of the public towards the private sector are not taxed back.

Ricardian policy leaves the initial price level indeterminate as well.

Determinacy and, by extension, monetary and financial stability, require monetary policy that is conventional and, possibly, fiscal policy that is non-Ricardian.

The results hold whether monetary policy sets nominal interest rates or money supplies; or whether prices are flexible or sticky and competition imperfect.

The indeterminacy is nominal only as long as prices are flexible, monetary policy sets nominal rates of interest, and the asset market is (effectively) complete; otherwise, there are, generically real effects.

1.1 A finite economy

With uncertainty, activity extends over dates $t = 0, 1$, while a final third date, $t = 2$, serves for accounting purposes.

Commodities are perishable and production and consumption extend the specification of the elementary economy.

Uncertainty is described by states of the world, $s \in \{\ldots, s, \ldots\}$, and realizes at $t = 1$. Events are

$$0, \underbrace{\ldots, s, \ldots}_{t=1}, \underbrace{\ldots, s, \ldots}_{t=2}.$$

The sequence of budget constraints that a household faces over time are

$$p_0 z_o^i + m_0^i + \textstyle\sum_s q_s f_s^i \leq w^i,$$

$$p_s z_s^i + m_s^i + \tfrac{1}{i+r_s} b_s^i \leq f_s^i + m_0^i,$$

$$0 \leq m_s^i + b_s^i,$$

and cash-in advance constraints are

$$p_0 z^i_{0-} \leq m^i_0,$$

$$p_s z^i_{s-} \leq m^i_s.$$

By non-arbitrage,

$$\frac{1}{1+r} = \sum_s q_s.$$

Equivalently, the optimization problem of a household is to

$$\max u^i(c^i_0, \ldots, c^i_s, \ldots),$$

subject to the budget constraint

$$p_0 z^i_0 + \sum_s \tilde{p}_s z^i_s + \frac{r_0}{1+r_0} m^i_0 + \sum_s \frac{r_s}{1+r_s} \tilde{m}^i_s \leq w^i,$$

and the cash-in-advance constraints

$$p_0 z^i_{0-} \leq m^i_0,$$

$$\tilde{p}_s z^i_{s-} \leq \tilde{m}^i_s,$$

where

$$\tilde{p}_s = q_s p_s, \quad \text{and} \quad \tilde{m}_s = q_s m_s.$$

Equilibrium requires that, across households,

$$z^a_0 = 0, \quad \ldots, z^s_0 = 0, \ldots,$$

and

$$m^a_0 = M_0, \quad \ldots m^a_s = M^a_s$$

or, equivalently,

$$m^a_0 = M_0, \quad \ldots \tilde{m}^a_s = \tilde{M}^a_s.$$

Non-ricardian policy determines the overall price level.

Quantitative easing leaves the measure

$$\mu = (\ldots, \mu_s = \frac{q_s}{\sum_s q_s}, \ldots)$$

indeterminate.

It is determined under conventional monetary policy by the conditions

$$f^a_s = f^a_{s'}, \quad \text{all} \ \ s, s'.$$

An example A representative individual derives utility from leisure and consumption according to the cardinal utility index

$$u(l,c) = ln(\bar{l} - l) + \ln c,$$

where $\bar{l}$ is the endowment in leisure, and the intertemporal utility function of the individual is

$$u(l_0, c_0, \dots, l_s, c_s, \dots) =$$

$$ln(\bar{l}_0 - l_0) + \ln c_0 + \delta \sum_s ln(\bar{l}_s - l_s) + \ln c_s.$$

Labor input produces output, the consumption good,with constant marginal cost of 1.

The real wage is 1 and the price of consumption is p.

The sequence of budget constraints is

$$p_0 c_0 + m_0 + \sum_s q_s f_s \leq p_0 l_0 + w,$$

$$p_s c_s + m_s + \tfrac{1}{1+r_s} b_s \leq p_s l_s + m_0 + f_s,$$

$$0 \leq m_s + b_s,$$

and the cash-in-advance constraints are

$$p_0 l_0 \leq m_0,$$

$$p_s l_s \leq m_s.$$

Equivalently, the individual faces the cumulative budget constraint

$$p_0 c_0 + \sum_s \tilde{p}_s c_s \leq \frac{1}{1+r_0} p_0 l_0 + \sum_s \frac{1}{1+r_s} \tilde{p}_s l_s + t,$$

where

$$\tilde{p}_s = q_s p_s.$$

From the maximization of individual utility,

$$c_0 = \tfrac{1}{2(1+\delta)} \tfrac{1}{p_0} \tau,$$

$$l_0 = \bar{l}_0 - \tfrac{1}{2(1+\delta)} \tfrac{1+r_0}{p_0} \tau,$$

$$c_s = \tfrac{\pi_s \delta}{2(1+\delta)} \tfrac{1}{\tilde{p}_s} \tau,$$

$$l_s = \bar{l}_s - \tfrac{\pi_s \delta}{2(1+\delta)} \tfrac{1+r_s}{\tilde{p}_s} \tau,$$

where

$$\tau(p_0, r_0, \ldots, p_s, r_s, \ldots) = \frac{1}{1+r_0} p_0 \bar{l}_0 + \sum_s \frac{1}{1+r_s} \tilde{p}_s \bar{l}_s + w.$$

Equilibrium in the consumption and labor markets requires that

$$c_0 = l_0,$$

$$c_s = l_s.$$

It follows that

$$p_0 = \tfrac{1}{l_0}(2 + r_0) \tfrac{1}{2(1+\delta)} \tfrac{1}{1-a} w,$$

$$\tilde{p}_s = \tfrac{1}{l_s}(2 + r_s) \tfrac{\pi_s \delta}{2(1+\delta)} \tfrac{1}{1-a} w,$$

where

$$a(r_0, \ldots, r_s, \ldots) = \frac{(2+r_0)}{(1+r_0)} \frac{1}{2(1+\delta)} + \sum_s \frac{(2+r_s)}{(1+r_0)} \frac{\pi_s \delta}{2(1+\delta)}$$

and

$$c_0 = l_0 = \tfrac{\bar{l}_0}{2+r_0},$$

$$c_s = l_s = \tfrac{\bar{l}_s}{2+r_s}.$$

Given w, that is, with a non-ricardian specification, equilibrium in the goods markets, determines the price level or, equivalently, p_0; but, it does not determine the decomposition of $\tilde{p}_s = q_s p_s$.

Since, at equilibrium,

$$\frac{r_s}{1+r_s} p_s l_s = m_0 + f_s,$$

the requirement that

$$f_s = f_{s'}, \quad \text{all } s, s',$$

together with the non-arbitrage condition

$$\frac{1}{1+r_0} = \sum_s q_s$$

completes the determination of equilibrium prices; in particular,

$$q_s = \frac{\pi_s}{1+r_0} \quad \text{or} \quad p_s = (1+r_0)(2+r_s) \frac{\pi_s \delta}{2(1+\delta)} \frac{1}{1-a} w.$$

1.2 A stochastic dynamic economy

Time is discrete and extends into the infinite future: $t = 0, 1 \ldots.$

Elementary states of the world are finitely many: $s = 0, \ldots, S$.

An event at date t is $s^t = (s_0, \ldots, s_t)$.

An (immediate) successor of a date-event $s^t = (\bar{s}_0, \ldots, \bar{s}_t)$ is

$$s^{t+1} = (\bar{s}_0, \ldots, \bar{s}_t, s_{t+1}) = (s^t, s_{t+1}),$$

and , inductively,

$$s^{t+k} = (\bar{s}_0, \ldots, \bar{s}_t, s_{t+1}, \ldots, s_{t+k}) = (s^t, s_{t+1}, \ldots, s_{t+k}).$$

Conditional on a date-event, probabilities of successors are

$$f(s^{t+1}|s^t)$$

and, inductively,

$$f(s^{t+k}|s^t) = f(s^{t+k}|s^{t+k-1})f(s^{t+k-1}|s^t).$$

At a date-event, a perishable input, labor, $l(s^t)$, is employed to produce a perishable output, consumption, $y(s^t)$, according to a linear technology:

$$y(s^t) = a(s^t)l(s^t), \quad a(s^t) > 0.$$

A representative individual is endowed with 1 unit of leisure.

He supplies labor and demands the consumption good and derives utility according to the cardinal utility index

$$u(c(s^t), 1 - l(s^t)).$$

that satisfies standard monotonicity, curvature and boundary conditions.

The preferences of the individual over consumption-employment paths commencing then are described by the separable, von Neumann-Morgenstern utility function

$$u(c(s^t, 1 - l(s^t)) + \mathrm{E}_{s^t} \sum_{k>0} \beta^k u(c(s^{t+k}, 1 - l(s^{t+k})), \quad ,0 < \beta < 1.$$

Balances, $m(s^t)$, provide liquidity services.

Elementary securities, $\theta(s^{t+1}|s^t)$, serve to transfer wealth to and from immediate successor date-events.

The price level is $p(s^t)$, and the wage rate is

$$w(s^t) = a(s_t)p(s^t),$$

as profit maximization requires.

The nominal, risk-free rate of interest is $r(s^t)$.

The cash-in-advance constraint is

$$a(s^t)p(s^t)l(s^t) \leq m(s^t).$$

Price of elementary securities are

$$q(s^{t+1}|s^t) = \frac{\mu(s^{t+1}|s^t)}{1 + r(s^t)},$$

with $\mu(\cdot|s^t)$ a "nominal pricing measure," which guarantees the non-arbitrage relation

$$\sum_{s^{t+1}} q(s^{t+1}|s^t) = \frac{1}{1 + r(s^t)}.$$

Inductively,

$$\mu(s^{t+k}|s^t) = \mu(s^{t+k}|s^{t+k-1})\mu(s^{t+k-1}|s^t),$$

and the implicit price of revenue at successor date-events is

$$q(s^{t+k}|s^t) = \frac{\mu(s^{t+k}|s^t)}{1 + r(s^{t+k-1})} q(s^{t+k-1}|s^t).$$

The individual has initial wealth $\tau(s^t)$.

Initial wealth constitutes a claim against a monetary-fiscal authority.

Alternatively, it can be interpreted as outside money.

The flow budget constraint is

$$p(s^t)z(s^t) + m(s^t) + \sum_{s^{t+1}} q(s^{t+1}|s^t)\theta(s^{t+1}|s^t) \leq \tau(s^t),$$

where

$$z(s^t) = c(s^t) - a(s^t)l(s^t)$$

is the effective excess demand for consumption.

Debt limit constraints are

$$-\tau(s^t) \leq -\sum_k \sum_{s^{t+k}} q(s^{t+k}|s^t) \frac{1}{1 + r(s^t)} a(s^{t+k})$$

or, equivalently

$$\lim_{k \to \infty} \sum_{s^{t+k}} q(s^{t+k}|s^t)\tau(s^{t+k}|s^t) \leq 0.$$

Wealth at successor date-events is

$$\tau(s^{t+1}, s^t) = \theta(s^{t+1}|s^t) + m(s^t),$$

and the flow budget constraint reduces to

$$p(s^t)z(s^t) + \frac{r(s^t)}{1+r(s^t)}a(s^t)l(s^t) + \sum_{s_{t+1}} q(s_{t+1}|s^t)\tau(s_{t+1}|s^t) \leq \tau(s^t).$$

Alternatively,

- $\tilde{m}(s^t) = \frac{1}{p(s^t)}m(s^t)$ are real balances,
- $\tilde{\tau}(s^t) = \frac{1}{p(s^t)}\tau(s^t)$ is real wealth,
- $\pi(s^{t+1}|s^t) = \frac{p(s^{t+1})}{p(s^t)} - 1$ is the rate of inflation, and
- $\tilde{q}(s^{t+1}|s^t) = \frac{\mu(s^{t+1}|s^t)(1+\pi(s^{t+1}|s^t))}{1+r(s^t))}$ are prices of indexed securities.

Real wealth at successor date-events is

$$\tilde{\tau}(s^{t+1}|s^t) = \left(\frac{\theta(s^{t+1}|s^t) + m(s^t)}{p(s^t)}\right)\frac{1}{1+\pi(s^{t+1}|s^t)},$$

and the flow budget constraint reduces to

$$z(s^t) + \frac{r(s^t)}{1+r(s^t)}a(s^t)l(s^t) + \sum_{s_{t+1}} \tilde{q}(s_{t+1}|s^t)\tilde{\tau}(s_{t+1}|s^t) \leq \tilde{\tau}(s^t).$$

First order conditions for an optimum are

$$\frac{\partial u(c(s^t),1-l(s^t))}{\partial c(s^t)} = \frac{\partial u(c(s^t),1-l(s^t))}{\partial l(s^t)}\left(\frac{a(s^t)}{1+r(s^t)}\right)^{-1},$$

$$\beta f(s^{t+1}|s^t)\frac{\partial u(c(s^{t+1}),1-l(s^{t+1}))}{\partial c(s^{t+1})}\tilde{q}(s^{t+1}|s^t)^{-1} = \frac{\partial u(c(s^t),1-l(s^t))}{\partial c(s^t)},$$

and the transversality condition is

$$\lim_{k\to\infty}\sum_{s^{t+k}} q(s^{t+k}|s^t)\tau(s^{t+k}|s^t) = 0.$$

The monetary-fiscal authority sets rates of interest and accommodates the demand for balances.

Equilibrium requires that the excess demand for output vanishes:

$$z(s^t) = c(s^t) - a(s^t)l(s^t) = 0,$$

which determines the path of employment and consumption:

$$\frac{\partial u(c(s^t), 1 - l(s^t))}{\partial c(s^t)} = \frac{\partial u(c(s^t), 1 - l(s^t))}{\partial l(s^t)} \left(\frac{a(s^t)}{1 + r(s^t)} \right)^{-1};$$

in turn, this determines the prices of indexed elementary securities:

$$\beta f(s^{t+1}|s^t) \frac{\partial u(c(s^{t+1}), 1 - l(s^{t+1}))}{\partial c(s^{t+1})} \tilde{q}(s^{t+1}|s^t)^{-1} = \frac{\partial u(c(s^t), 1 - l(s^t))}{\partial c(s^t)}.$$

The initial price level remains indeterminate as well.

More importantly, the decomposition of equilibrium asset prices into an inflation process, $\pi(\cdot|s^t)$, and a nominal pricing measure, $\mu(\cdot|s^t)$, remains indeterminate:

$$\tilde{q}(s^{t+1}|s^t) = \frac{\mu(s^{t+1}|s^t)(1 + \pi(s^{t+1}|s^t))}{1 + r(s^t))}.$$

1.3 A stationary economy

The resolution of uncertainty follows a stationary stochastic process.

Conditional on an elementary state of the world, transition probabilities are $f(s'|s)$.

Rates of interest, $r(s)$, determine the path of consumption, $c(s)$, and employment, $l(s)$, at equilibrium, which, in turn, determine the prices of indexed elementary securities:

$$\beta f(s'|s) \frac{\partial u(c(s'), 1 - l(s'))}{\partial c(s')} \tilde{q}(s'|s)^{-1} = \frac{\partial u(c(s), 1 - l(s))}{\partial c(s)}$$

or

$$\tilde{Q} = \beta Du(s)^{-1} F D(s').$$

Here,

$$Du(s) = diag(\ldots, \frac{\partial u(c(s), 1 - l(s))}{\partial c(s)}, \ldots)$$

is the diagonal matrix of marginal utilities of consumption, and

$$F = (f(s'|s)) \quad \text{and} \quad \tilde{Q} = (\tilde{q}(s'|s))$$

are, respectively, the matrices of transition probabilities and of prices of indexed elementary securities.

With

$$\tilde{m} = (\dots \frac{r(s)}{1+r(s)} l(s) \dots)$$

the vector of real balances at equilibrium, real wealth at the steady state,

$$\tilde{\tau} = (\dots \tau(s), \dots),$$

is determined by by the equation

$$\tilde{m} + \tilde{Q}\tilde{\tau} = \tilde{\tau} \quad \text{or} \quad \tilde{\tau} = (I - \tilde{Q})^{-1}\tilde{m}.$$

The initial price level remains indeterminate.

More importantly, the decomposition of equilibrium asset prices into an inflation process, $\pi(\cdot|s)$, and a nominal pricing measure, $\mu(\cdot|s)$, remain indeterminate:

$$\tilde{Q} = R^{-1} M \otimes \Pi.$$

Here,

$$R = diag(\dots, (1 + r(s)), \dots)$$

is the diagonal matrix of interest factors, and

$$M = (m(s'|s)) \quad \text{and} \quad \Pi = ((1 + \pi(s'|s)))$$

are, respectively, the matrices of "nominal pricing transition probabilities" and inflation factors.

Determinacy obtains if the inflation process, which is endogenous, is restricted to take the form

$$(1 + \pi(s'|s)) = h(s)a(s'),$$

where $a(s)$ and are known function of the fundamentals of the economy and, as a consequence,

$$M \otimes \Pi = HMA.$$

Here,

$$a = (\dots, a(s), \dots), \quad \text{and} \quad h = (\dots, h(s), \dots),$$

and A and H are the associated diagonal matrices.

Then,

$$\tilde{Q} = R^{-1} M \otimes \Pi \quad \Leftrightarrow \quad R\tilde{Q}A^{-1} = HM,$$

which determines the inflation process as as well as nominal pricing probabilities, since

$$M\mathbf{1}_S = \mathbf{1}_S \quad \Leftrightarrow \quad M = \left(diag(RQA^{-1})\right)^{-1} RQA^{-1}.$$

This is indeed the case under conventional monetary policy.

Real wealth at successor date-events is (by a slight abuse of notation)

$$\tilde{\tau}(s') = \left(\frac{\theta(s'|s) + m(s)}{p(s)}\right) \frac{1}{1 + \pi(s'|s)},$$

and conventional monetary policy requires that

$$\theta(s'|s) = \theta(s).$$

The argument fails if, alternatively,

$$(1 + \pi(s'|s)) = h(s')a(s),$$

and, as a consequence,

$$M \otimes \Pi = AMH.$$

In this case,

$$\tilde{Q} = R^{-1} M \otimes P \quad \Leftrightarrow \quad A^{-1} R \tilde{Q} = MH,$$

and

$$M\mathbf{1}_S = \mathbf{1}_S \quad \Leftrightarrow \quad H^{-1}\mathbf{1}_S = Q^{-1}R^{-1}A\mathbf{1}_S$$

that need not be positive.

Non-ricardian monetary-fiscal policy determines the initial price level by setting exogenously the level of initial nominal claims.

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