Indeterminacy and Learning: An Analysis of Monetary Policy in the Great Inflation*

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PRELIMINARY AND INCOMPLETE

Abstract

We argue in this paper that the Great Inflation of the 1970s can be understood as the result of equilibrium indeterminacy in which loose monetary policy engendered excess volatility in macroeconomic aggregates and prices. We show, however, that the Federal Reserve inadvertently pursued policies that were not anti-inflationary enough because it did not fully understand the economic environment it was operating in. Specifically, it had imperfect knowledge about the structure of the U.S. economy and it was subject to data misperceptions since the real-time data flow did not capture the true state of the economy, as large subsequent revisions showed. It is the combination of learning about the economy and, more importantly, signal extraction to filter out measurement noise that resulted in policies that the Federal Reserve believed to be optimal, but when implemented led to equilibrium indeterminacy in the private sector.

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1 Introduction

There are three strands of narratives about the Great Inflation and the Great Moderation in the academic literature. At opposite ends of the spectrum are the good/bad luck and good/bad policy stories. The 1970s was a time of economic upheaval with strong and persistent exogenous shocks that occurred with high frequency. It was simply bad luck to have been a central banker at that time since despite best intentions the incidence of shocks was too much for the central banker’s arsenal to handle. When the 1980s came around, however, the reduced incidence and persistence of shocks rang in the Great Moderation. This view is exemplified by Sims and Zha (2006). An almost orthogonal narrative argues that the Federal Reserve conducted bad policy in the 1970s in that was not aggressive enough in fighting inflation. It is only through Volcker’s disinflation engineered through a high-interest rate policy that the Great inflation was reigned in. This bad policy view has been advocated by Clarida, Gali, and Gertler (2000) and subsequently Lubik and Schorfheide (2004). A third narrative, typically associated with Orphanides (2001), relies on the idea that the Federal Reserve did not perceive the economic scenario of the 1970s correctly. Data misperceptions led it to implement policies that delivered bad outcomes and that were only abated in the 1980s with a better understanding of the state of the world.

Our paper attempts to integrate the bad policy narrative with the data misperception narrative. More specifically, we provide an explanation why the Federal Reserve, almost unwillingly, engaged at first in policy that led to bad outcomes (the Great Inflation), but subsequently pursued monetary policy resulting in good outcomes (the Great Moderation). We show that what appears in the data as invariably good and bad outcomes is the result of an optimal policy problem under imperfect information. In doing so, we also combine various recent contributions to the empirical and theoretical macroeconomic literature on learning.

We take as a starting point Orphanides (2001) observation that the Federal Reserve did not perceive the productivity slowdown as it was occurring during the 1970s. We capture data misperception by assuming that the Federal Reserve observes all data with error. We then follow Primiceri (2006) and assume additionally that the central bank does not know the true data-generating process. It thus gathers information by estimating a restricted reduced-form VAR and updates its beliefs about the state of the world and the underlying economic model using least-squares learning. The linear-quadratic optimal policy problem and its solution for a time-consistent policy is taken from Primiceri (2006).
Private sector behavior is captured by a typical New Keynesian framework that is close to that in Lubik and Schorfheide (2004) for reference purposes. The private sector knows the current period monetary policy rule, forms rational expectations conditional on that rule, and takes the central bank’s policy rule as given. The optimal policy rule derived from the central bank’s policy problem thus completes the private sector system, which together results in a rational expectations model. The original source for indeterminacy, that is, for multiple solutions, possibly involving sunspots, that arise from the rational expectations systems is the same as in Bullard and Mitra (2002), Woodford (2003), and Lubik and Schorfheide (2004); to wit, a violation of the Taylor principle. In the face of inflationary pressures the central bank is not aggressive enough in raising the real rate of interest through its control of the nominal interest rate. As shown in these papers, the violation of the Taylor principle can be tied to the value of the policy coefficients in a (linear) interest-rate rule.

In this paper, we thus provide a rationale for why the central bank may choose policy coefficients that inadvertently induce indeterminate outcomes. Given the learning mechanism and the misperception of the data due to measurement issues, the estimated coefficients of the central bank’s reduced-form model and thus the optimal policy coefficients change period-by-period. The rational expectations equilibrium that arises each period is either unique or indeterminate given the policy rule in place that period. It is the endogenous shifts of the policy rule for fixed private sector parameters that move the economy across the threshold between the determinate and indeterminate regions of the parameter space. ‘Bad policy’, that is, indeterminacy, arises not because of intent but because of data mis-measurement and incomplete knowledge of the economy on behalf of the central bank.

We estimate the model using Bayesian methods on real-time and final data. Our findings confirm the pattern of indeterminacy and determinacy during, respectively, the Great Inflation and the Great Moderation as identified by Lubik and Schorfheide (2004). Yet, it is rationalized by data misperception as argued by Orphanides (2001) and by learning as argued by Primiceri (2006). Federal Reserve policy led to indeterminate outcomes especially during the second half of the 1970s and before Volcker’s disinflation took hold. Afterwards, during the Volcker-Greenspan period, endogenous policy under learning with measurement error led to determinate outcomes in the Great Moderation. The driver for these results is the extent of data revisions, and thus, the ex-post implied data misperception. We identify two especially prominent turning points when the initially observed output decline turned out to be much less dramatic following the revision. In other words, the Federal Reserve
was confronted with a situation where a decline in growth implied a lessening of inflationary pressures and a commensurately softer policy. Since the output decline was perceived and the real economy in much better shape than believed, the Federal Reserve unwittingly violated the Taylor principle.

The paper is structured as follows. The next section presents a simple example of the mechanism that we see at work. We first discuss determinate and indeterminate equilibria in a simple rational expectation model and then show how a least-squares learning mechanism can shift the coefficient that determines outcomes across the determinacy boundaries. We present our theoretical model in section 3 and discuss in detail the timing and information assumptions that we bring to bear. Section 4 discusses data and estimation issues and how we choose indeterminate equilibria in the empirical model. Section 5 presents the baseline estimation results, while section contains a bevy of robustness checks. Section 6 concludes and lays out a path for future research.

2 A Primer on Indeterminacy and Learning

The argument in our paper rests on two methodological areas in dynamic macroeconomics, namely the determinacy properties of linear rational expectations models and the dynamic properties of least-square learning mechanisms. In this section, we discuss equilibrium determinacy and least-squares learning by means of a simple example. The key points that we want to emphasize are, first, whether a rational expectations equilibrium is determinate or indeterminate rests on the values of structural parameters; and second, that in a learning environment the inferred values of the underlying parameters in a structural model are time varying. By connecting these two concepts we can develop a rationale for the behavior of the Federal Reserve in the 1970s. The discussion of equilibrium determinacy borrows a simple framework from Lubik and Surico (2010), while the exposition of least-squares learning rests on.

2.1 Determinate and Indeterminate Equilibria

We consider a simple expectational difference equation:

$$x_t = aE_t x_{t+1} + \varepsilon_t,$$  \hspace{1cm} (1)

where $a$ is a structural parameter, $\varepsilon_t$ is a white noise process with mean zero and variance $\sigma^2$, and $E_t$ is the rational expectations operator conditional on information at time $t$. A solution to this equation is an expression that does not contain any endogenous variables and
that depends only on exogenous shocks and lagged values of the variables in the information set. The type of such a reduced-form solution depends on the value of the parameter $a$.

If $|a| < 1$ there is a unique ('determinate') solution which is simply:

$$x_t = \varepsilon_t.$$  \hfill (2)

This solution can be found by iterating the equation (1) forward. Imposing covariance stationarity as an equilibrium concept for rational expectations models and utilizing transversality arguments results in this expression. Substituting the determinate (unique) solution into the original expectational difference equation verifies that it is, in fact, a solution.

On the other hand, if $|a| > 1$, there are multiple solutions and the rational expectations equilibrium is indeterminate. In order to derive the entire set of solutions we follow the approach developed by Lubik and Schorfheide (2003). We rewrite the model by introducing endogenous forecast errors $\eta_t = x_t - E_{t-1}x_t$, which by definition have the property that $E_{t-1}\eta_t = 0$ and thereby impose restrictions on the set of admissible solutions. Define $\xi_t = E_t x_{t+1}$ so that equation (1) can be rewritten as:

$$\xi_t = \frac{1}{a} \xi_{t-1} - \frac{1}{a} \varepsilon_t + \frac{1}{a} \eta_t.$$  \hfill (3)

We note that under the restriction that $|a| > 1$ this is a stable difference equation in its deterministic part, where the process for $\xi_t$ is driven by the exogenous shock $\varepsilon_t$ and the endogenous error $\eta_t$. Any covariance-stationary stochastic process for $\eta_t$ is a solution for this model since there are no further restrictions on the evolution of the endogenous forecast error $\eta_t$.\footnote{In the case of determinacy, the restriction imposed is that $\xi_t = 0$, $\forall$ $t$, which implies $\eta_t = \varepsilon_t$.}

In general, the forecast error can be expressed as a linear combination of the model’s fundamental disturbances and extraneous sources of uncertainty, typically labeled ‘sunspots’. We can therefore write:

$$\eta_t = m \varepsilon_t + \zeta_t,$$  \hfill (4)

where the sunspot $\zeta_t$ is a martingale-difference sequence, and $m$ is an unrestricted parameter.\footnote{There is a technical subtlety in that $\zeta_t$ is actually, in the terminology of Lubik and Schorfheide (2003), a reduced-form sunspot shock, with $\zeta_t = m_\zeta \zeta_t^*$. Setting $m_\zeta = 0$ would therefore result in a sunspot equilibrium without sunspots. Furthermore, in less simple models, there would be additional restrictions on the coefficients which depend on other structural parameters.} Substituting this into Eq. (3) yields the full solution under indeterminacy:

$$x_t = \frac{1}{a} x_{t-1} + m \varepsilon_t - \frac{1}{a} \varepsilon_{t-1} + \zeta_t.$$  \hfill (5)
The evolution of $x_t$ now depends on an additional (structural) parameter $m$ which indexes specific rational expectations equilibria.

Indeterminacy affects the behavior of the model in three main ways. First, indeterminate solutions exhibit a richer lag structure and more persistence than the determinate solution for the same underlying model. This feature can be exploited for distinguishing between the two types of rational expectations equilibria for a given model. In the simple example, this is fairly obvious: under determinacy the solution for $x_t$ is white noise, while under indeterminacy the solution is described by an ARMA(1,1) process. Specifically, the (composite) error term exhibits both serial correlation and a different variance when compared to the determinate solution. Second, under indeterminacy sunspot shocks can affect equilibrium dynamics. Other things being equal, data generated by sunspot equilibria are inherently more volatile than their determinate counterparts. The third implication, is that indeterminacy affects the response of the model to fundamental shocks, whereas the response to sunspot shocks is uniquely determined. In the example, innovations to $\varepsilon_t$ could either increase or decrease $x_t$ depending on the sign of $m$.

What is important for the purposes of our paper is that the nature and properties of the equilibrium can change when the parameter $a$ changes and moves across the boundary between determinacy and indeterminacy, $|a| = 1$. The simple example assumes that the parameter $a$ is fixed. Our argument about indeterminacy being caused by the central bank’s data misperceptions relies on the idea that parameters which affect the type of equilibrium, such as coefficients in a monetary policy rule, move around. We can capture this rationale by a learning mechanism. The next section therefore fixes ideas by discussing a simple example of the general mechanism in our paper.\(^3\)

### 2.2 Indeterminacy through Learning

We illustrate the basic mechanism at work by means of a simple example. The true data-generating process is equation (1), where we assume for illustration purposes that $a = 0.01$. The solution under rational expectations is therefore $x_t = \varepsilon_t$, and thus determinate. In the

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\(^3\)There is a subtlety here that we abstract from in this paper. We assume that the private sector operates under rational expectations in an environment where structural and policy parameter are believed to be fixed forever. The private sector is myopic in the sense that it does not realize that the policy coefficients are time-varying and therefore change period by period. Moreover, the private sector does not take into account that the central bank solves a learning problem. These assumptions considerably simplify our computational work since, with respect to the latter assumptions, we do no have to solve a second private sector learning problem. The first assumption allows us not having to solve what amounts to a regime-switching rational expectations model under indeterminacy. Research into this area is still in its infancy (see Davig and Leeper, 2007, and Farmer et al., 2009).
environment with learning we assume that the agents have the perceived law of motion:

\[ x_t = bx_{t-1} + \nu_t, \quad (6) \]

which they estimate by least squares to gain knowledge about the underlying structural model. In our full model, this would be equivalent to the VAR that the central bank estimates for the economy. In the determinate case \( b = 0 \), while under indeterminacy \( b = 1/a \). The least-squares estimate of the lag coefficient in the perceived law of motion, \( \hat{b}_t \), is varying over time as the information changes under constant-gain learning and thereby can introduce persistence in the actual evolution of the economy. We note that in this special case any deviation from \( \hat{b}_t = 0 \) would indicate an indeterminate equilibrium.

We can derive the actual law of motion, that is, the evolution of the data-generating process under the application of the perceived law of motion by substituting the latter into (1). In our full model framework, this is equivalent to the Federal Reserve announcing the policy rule to the private sector each period. Since \( E_t (\hat{b}_t x_t + \nu_{t+1}) = \hat{b}_t x_t \) for given \( \hat{b}_t \), we find:

\[ x_t = (1 - ab_t)^{-1} \epsilon_t. \quad (7) \]

Although the rational expectations solution is i.i.d. the learning mechanism by itself introduces persistence into the actual path of \( x_t \) which would indicate an indeterminate equilibrium.

We present results from a simulation exercise in Figure (3). We draw i.i.d. shocks for 180 periods and have the agent estimate the lag coefficient in the perceived law of motion. Panel A of the figure shows the estimate and the 5th and 95th percentile bands for \( \hat{b}_t \) in the case when there is no measurement error. The estimates are centered at zero. In a second simulation for the same draws of the shocks we add measurement. From period 80 to 100 we force the learning agent to observe the actual data with error which we assume to be equal to 2 standard deviations of the innovations in the model. After period 100, the measurement error disappears.

As Panel B of Figure (3) shows, agents believe that there is substantial persistence in the economy as there would be under indeterminacy. The estimate of the perceived autoregressive reduced-form parameter \( b \) reaches values as high as 0.4, which would indicate a structural parameter of \( a = 2.5 \) and therefore an indeterminate solution to (1). Given the time series from Panel A, an econometrician tasked with deciding between a determinate and an indeterminate equilibrium would likely favor the latter because of the higher

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\(^4\)We abstract from the subtlety that under indeterminacy the rational expectations solution is an ARMA(1,1) process, which could be reflected in the actual law of motion.
observed persistence.\textsuperscript{5} We want to emphasize that in our simulation the true value of $a = 0.01$. The incorrect inference stems from the combination of least-squares learning and, more importantly, the introduction of measurement error. The simple example simulation thus shows that an economy can inadvertently drift into the indeterminacy region of the parameter space. We now turn to our full modelling framework, where we add an optimal policy problem to capture the idea of inadvertent indeterminacy.

3 The Model

3.1 Overview and Timing Assumptions

Our model consists of the following agents. There is a central bank which is learning about the state of the economy. It only has access to imperfect measurements of economic data. It is also unaware of the imperfect nature of the measurement. The central bank does not know the structure of the data-generating process (DGP). Instead, it uses a reduced-form specification to conduct inference. The central bank’s policy is guided by an ad-hoc quadratic loss function. The private sector knows the central bank’s current period policy rule and determines inflation and output accordingly. It is aware of the mismeasurement problem that the central bank faces. At the same time, the private sector is myopic in that it accepts the period-by-period varying central bank policy as fixed indefinitely.\textsuperscript{6}

The timing of the model is such that the central bank estimates its model of the economy at the beginning of period $t$ using data up to and including period $t - 1$. The central bank then minimizes its loss function subject to the estimated law of motion, treating parameter estimates as fixed. This dynamic programming problem results in a time-consistent solution for the optimal solution in terms of policy coefficients, which are then communicated to the public. The private sector observes the true state of the world and the policy coefficients. Shocks are then realized and equilibrium outcomes are formed. The central bank’s policy rule, as viewed through the lens of private sector information, and the structural equations of the private sector form a linear rational expectations model that can have a determinate or an indeterminate solution. The central bank observes these new outcomes and updates its estimates at the beginning of the next period.

\textsuperscript{5}This intuition is discussed in more detail in Lubik and Schorfheide (2004). Figure 1 on p.196 shows the likelihood functions for both cases.

\textsuperscript{6}We will discuss this “anticipated utility” assumption in more detail below.
3.2 The Central Bank

The central bank deviates from the rational expectations norm in two critical aspects. First, it does not know the structure of the economy. Hence, it conducts inference based on a reduced-form model, typically a VAR. We restrict the VAR in such a way that it resembles the equations estimated by the central bank in Primiceri (2006). In contrast to that paper, we make it explicit that the central bank’s policy instrument is the nominal interest rate.

The central bank then employs a learning mechanism, least-squares learning with constant gain in our benchmark case, to infer the exact reduced-form representation of the structural model. Second, the central bank observes the actual data with error. This is designed to mimic the problems central banks face when data arrive in real time, but are subject to frequent revisions.

We assume that the central bank observes $X_t$, a noisy measurement of the true state $X_t^{true}$:

$$X_t^{true} = X_t + \eta_t,$$

where $\eta_t$ is a measurement error independent of the true outcome $X_t^{true}$. We assume that the error is serially correlated of order one:

$$\eta_t = \rho_\eta \eta_{t-1} + \varepsilon_\eta^t,$$

whereby the Gaussian innovation $\varepsilon_\eta^t$ has zero mean and is independent of $X_t^{true}$. While it may be difficult to justify autocorrelated measurement errors on a priori grounds, we note that it is a key finding in Orphanides’ (20xx) analysis of the Great Inflation. Perhaps more importantly, we also assume that the central bank does not learn about the measurement error, which therefore persists during the estimation period. We consider the alternative in a robustness exercise below.

The central bank sets the interest rate target:

$$i_t^{CB} = i_t + \varepsilon_i^t,$$

based on a policy rule of the form $i_t^{CB} = \alpha_t X_t$, where $\varepsilon_i^t$ is a zero-mean monetary policy implementation error. The policy coefficients $\alpha_t$ are chosen from an optimal policy problem. Time variation in $\alpha_t$ arises from the learning problem described below. We follow Sargent, Williams, and Zha (2006) and Primiceri (2006) in assuming that the central bank chooses the coefficients $\alpha_t$ in the linear rule $i_t^{CB} = \alpha_t X_t$ to minimize a quadratic loss function:

$$W_t = E_t \sum_{j=t}^{\infty} \beta^{j-t} \left[ (\pi_j - \pi_{target})^2 + \lambda_y (\Delta y_j - \Delta y_{target})^2 + \lambda_i (i_t - i_{t-1})^2 \right],$$

(11)
subject to estimated laws of motion for the relationship between the state variables, inflation \( \pi_t \) and output \( y_t \), and the policy variable \( i_t^{CB} \) and the definition of the policy instrument. 

\[ 0 < \beta < 1 \] is the constant discount factor, \( \lambda_y, \lambda_i \geq 0 \) are weights in the loss function that we treat as structural parameters.\footnote{A loss function of this kind can be derived from a representative household’s utility function within a New Keynesian framework, in which case \( \lambda_y \) and \( \lambda_i \) would be functions of underlying structural parameters. While it is conceptually possible to derive a loss function within our learning framework, it is beyond the scope of our paper. Nevertheless, using a welfare-based loss function with a reduced form model of the economy might be problematic since that raises the question how the central bank can calculate the welfare-based loss function without knowledge of the structure of the economy.} \( \pi_{\text{target}} \) and \( \Delta y_{\text{target}} \) are fixed target values for inflation and output growth, respectively.

We follow Primiceri (2006) in specifying the empirical model that the central bank uses to conduct inference about the state of the economy. In general, the central bank estimates a (restricted) VAR of the type:

\[ X_j = \sum_{k=1}^{n} A_{t,k} X_{j-k} + \sum_{l=0}^{m} B_{t,l} i_{j-l} + u_j \]  
\[(12)\]

The set of matrices \( A \) and \( B \) carry \( t \)-subscripts since they are re-estimated every period. However, they are taken as fixed by the central bank when it minimizes its loss function. This leads to a standard linear-quadratic decision problem that we need to solve every period for a varying set of coefficient matrices. We also restrict the matrices in the central bank’s model further so that we have one equation that resembles a backward-looking Phillips curve, and another that resembles a dynamic IS-equation. Specifically, the central bank estimates the two-equation model:

\[ \pi_j = c_\pi + a(L) \pi_{j-1} + b(L) y_{j-1} + u_\pi, \]
\[ \Delta y_j = c_y + d(L) \Delta y_{j-1} + \gamma i_{t-1} + u_y. \]
\[(13)-(14)\]

All coefficients in the lag-polynomials \( a(L), b(L), \) and \( d(L) \), and the interest-rate coefficient \( \gamma \) are potentially changing over time as are the intercepts \( c_\pi \) and \( c_y \).

Given the estimates of the empirical model, the central bank needs to update its beliefs about the state of the economy. We assume that it uses least squares learning. Suppose the central bank wants to estimate an equation of the following form:

\[ q_t = p_{t-1}^t \phi_t + \xi_t \]

where \( q_t \) is the dependent variable or a vector of dependent variables, \( p_{t-1} \) a vector or matrix of regressors, \( \xi_t \) the residual(s) and \( \phi_t \) the vector of parameters of interest. Using
this notation, the least squares learning algorithm can be written as:

\[ R_t = R_{t-1} + g_t (p_{t-1}p'_{t-1} - R_{t-1}), \]
\[ \phi_t = \phi_{t-1} + g_t R_t^{-1} p_{t-1} (q_t - p'_{t-1} \phi_{t-1}), \]

which are the updating formulas for recursive least squares estimation. A key parameter is the gain \( g_t \). The standard assumption in the literature, as in Primiceri (2006) and Sargent, Williams, and Zha (2006) is to use a constant gain \( g_t = g \). This amounts to assuming that the agents who estimate using constant gain think that parameters drift over time.\(^8\)

### 3.3 The Private Sector

The behavior of the private sector is described by a New Keynesian Phillips curve that captures inflation dynamics using both forward- and backward-looking elements:

\[ \pi_t - \pi_t = \beta [\alpha \pi E_t \pi_{t+1} + (1 - \alpha) \pi_{t-1} - \pi_t] + \kappa y_t - z_t. \tag{15} \]

\( 0 \leq \alpha \pi \leq 1 \) is the coefficient determining the degree of inflation indexation, while \( \kappa > 0 \) is a coefficient determining the slope of the Phillips curve. \( z_t \) is a serially correlated shock with law of motion \( z_t = \rho z_{t-1} + \varepsilon^z_t \). Output dynamics is governed by an Euler-equation in terms of output:

\[ y_t = -\sigma^{-1} (i_t - \bar{T}_t - E_t(\pi_{t+1} - \pi_t)) + E_t y_{t+1} + g_t, \tag{16} \]

where \( \sigma > 0 \) is the coefficient of relative risk aversion. \( g_t \) is a serially correlated shock with law of motion \( g_t = \rho g_{t-1} + \varepsilon^g_t \). The innovations to both AR(1) processes are Gaussian.

The private sector equations share the same structure as in Lubik and Schorfheide (2004). The equations can be derived from an underlying utility and profit maximization problem of, respectively, a household and a firm. Since these steps are well known we do not do these derivations explicitly. We deviate from the standard specification in that we include the time-varying inflation target \( \pi_t \) separately in these equations because the views about the steady state level of inflation change as the central bank changes its policy rule.

We make the crucial assumption that the private sector knows the time \( t \) policy rule of the central bank when making its decision at time \( t \), but it assumes that this policy rule will not change in the future. This is akin to the anticipated utility assumption that the central bank is making above and that is more generally often made in the learning literature.\(^8\)

\(^8\)An alternative is to use a decreasing gain. For instance, a recursive version of OLS would set the gain equal to a decreasing function of \( t \).
More specifically, the private sector realizes that the central bank makes a mistake in terms of basing the policy rule decision on mismeasured data. Yet, it is myopic in the sense that it does not assign any positive probability to changes in that policy rule when making decisions.

4 Data and Estimation

4.1 Data

In our model, there are two data concepts. The key assumption we make is that the central bank only has access to real-time data. That is, its decisions are based on data releases as they first become available. These are then subject to data revisions. We therefore use real-time data from the Federal Reserve Bank of Philadelphia for the estimation problem of the central bank. Our sample period starts in 1965:Q2 due to the availability of real-time data. The last data point is 2012:Q2. We use the first 6 years of data for a pre-sample analysis to initialize the prior. The effective sample period over which the model is estimated starts therefore in 1971. The data are collected at the quarterly frequency.

A key assumption in our framework is that the private sector serves as data-generating process for the final data. Our estimation then combines real-time and final observations on output growth and the inflation rate in addition to the nominal interest rate which is observed without error (since it is the policy instrument of the central bank). We use as policy rate the Federal Funds rate, while output is measured as real GDP, and inflation is the percentage change in the CPI. Figures (1) and (2) depict, respectively, the real time and the final data for the growth rate in real GDP and CPI inflation. The Appendix contains further details on the construction of the data series.

In our estimation exercise, we find it convenient to calibrate some parameters. Table 1 lists the calibrated parameters and their source. We set the inflation target $\pi^{\text{target}}$ in the central bank’s loss function to an annual rate of 2%. While for much of the sample period, the Federal Reserve did not have an official inflation target, we take it to be commonly understood, and even mandated by the (revision to the) Federal Reserve Act of 1977, that it pursued stable prices, a proxy for which we consider a CPI inflation rate of 2%. The output growth target $\Delta y^{\text{target}}$ is set to a quarter-over-quarter rate of 0.75%, which is simply the sample average. We fix the discount factor at $\beta = 0.99$. The model estimation turned out to be sensitive to the specification of the backward-looking NKPC and Euler equations. For instance, Sargent and Surico (2011) find almost purely backward-looking dynamics in their rational-expectations model. We therefore experimented with various specifications of the
lag terms in these equations. The best-fitting specification was one with a backward-looking coefficient of 0.5 in the NKPC and no backward-looking dynamics for the output gap in the Euler-equation. We thus fix the respective coefficients to these values in our estimation.

We assume that the lag length in all central bank regressions is 3. Based on preliminary investigation, we found that for shorter lag lengths most of the draws would have implied indeterminacy throughout the sample, which we did not find plausible. We fix the gain for the regressions at 0.01, which is at the lower end of the values used in the learning literature. When we estimated this parameter (while restricting it to be no smaller than 0.01) all estimates clustered around this value. As in Primiceri (2006) we therefore chose to calibrate it.

4.2 Likelihood Function and Bayesian Inference

We use the Kalman filter to calculate the likelihood function. Let $Y_t$ denote the observables used to calculate the likelihood function. Our solution method for solving linear rational expectations models, the Gensys algorithm from Sims (2002) and adapted by Lubik and Schorfheide (2003) for the case of indeterminacy, delivers a law of motion for each time period for the following vector of variables as a solution to the system of equations:

$$Z_t = \begin{pmatrix} X_{t+1}^{true} \\ \eta_t \\ z_t \\ g_t \\ i_t \end{pmatrix} \quad (17)$$

The state space system to calculate the likelihood function is then given by:

$$Y_t = RZ_t + \epsilon^y_t, \quad (18)$$

$$Z_t = S_tZ_{t-1} + \epsilon^z_i, \quad (19)$$

where $S_t$ will be given by the time $t$ solution to the above equation system.

A key element of the learning algorithm is the specification of the initial beliefs held by the central bank. We follow Primiceri (2006) and use real-time data from a training sample, together with gain parameter from the current parameter draw in the MCMC algorithm. The training sample only includes the information available to the central bank at the end of the training sample, not the final data releases. We prefer this approach since otherwise the number of parameters to estimate becomes very large. An alternative approach, namely to

\footnote{Recall that $X_t$ only contains observables up to and including date $t - 1$.}
fix all standard parameters in the model and then estimate initial beliefs within the learning algorithm, is pursued as a robustness exercise.

In our benchmark specification, we also assume that the central bank never has access to updated data and never learns the true values of the variables. This assumption is made for convenience, but also parsimony since we do not have to model the process by which data gets updated over time. In order to avoid stochastic singularity when we use the full set of real time and final data we add a monetary policy shock to the model. The central bank’s decision problem is unaffected by this because of certainty equivalence. Furthermore, we assume that the measurement errors in the central bank’s observation of output growth and inflation are AR(1) processes, the parameters of which we estimate along with the model’s other structural parameters. In any case, the private sector knows the structure of the measurement errors and understands the central bank’s informational shortcomings.

We use a standard Metropolis-Hastings algorithm to take 300,000 draws from which we discard the first 50,000 as burn-in. The estimation problem is computationally reasonably straightforward but time-consuming since we have to solve a linear-quadratic dynamic programming problem and a linear rational expectations model every period for every draw. We also estimated the model using the adaptive Metropolis-hastings algorithm of Haario et al. (2001) to safeguard against any pathologies. The results remain unchanged.

4.3 Indeterminacy and Equilibrium Selection

In order to compute a model solution when the equation solver indicates non-existence of equilibrium, we use a projection facility. That is, if a policy rule in a certain period implies non-existence of a stationary equilibrium, the policy rule is discarded and last year’s policy rule is carried out. If the policy rule implies an indeterminate equilibrium, we pick the equilibrium chosen by rational expectations solver as in Sims (2001).

5 Estimation Results

Figure (4) shows the marginal posterior distributions for each parameter that we estimate, while Table 2 shows their median estimates and the 5th and 95th percentile. The estimation algorithm seems to capture the behavior around the posterior mode reasonably well, with parameters being tightly estimated. The “supply” and “demand” shocks, $z_t$ and $g_t$, respectively, show a high degree of persistence at $\hat{\rho}_z = 0.91$ and $\hat{\rho}_g = 0.67$. These numbers are very close to those found by Lubik and Schorfheide (2004) and other papers in the literature for this sample period. While the measurement error in the inflation rate is small, not very
volatile, and especially not very persistent ($\hat{\rho}_\pi = 0.075$), the picture is different for output growth. Its median AR(1) coefficient is estimated to be $\hat{\rho}_y = 0.49$, which appears considerable. This observation appears to confirm the notion that the Federal Reserve missed the productivity slowdown in the 1970s and thus misperceived the state of the business cycle in their real-time observations of output growth. Finally, the structural parameter estimates a low weight on output growth and a considerably stronger emphasis on interest rate smoothing in the central bank’s loss function, the latter of which generates the observed persistence in interest rate data.

Figure (5) contains the key result in the paper. It shows the determinacy indicator over the estimated sample period. A value of ‘1’ indicates a unique equilibrium, while a value of ‘0’ means indeterminacy. The indicator is computed by drawing from the posterior distribution of the estimated model at each data point, whereby each draw results in either a determinate or an indeterminate equilibrium. We then average over all draws, so that the indicator can be interpreted as a probability similar to the regime-switching literature. As it turns out, our estimation results are very unequivocal as far as equilibrium determinacy is concerned since the indicator attains either zero or one.

Two observations stand out from Figure (5). First, the U.S. economy has been in a unique equilibrium since the Volcker disinflation of 1982:3 which implemented a tough anti-inflationary stance through sharp interest rate increases. In the literature, these are commonly interpreted as a shift to a policy rule with a much higher feedback coefficient on the inflation term (see Clarida et al., 2000). The second observation is that before the Volcker disinflation the economy alternated between a determinate and an indeterminate equilibrium. The longest indeterminate stretch was from 1977:1 until 1980:4 which covers the end of Burns’ chairmanship of the Federal Reserve, Miller’s short tenure, and the early Volcker period of a policy of non-borrowed reserve targeting. This was preceded by a short determinacy period starting at the end of 1974. The U.S. economy was operating under an indeterminate equilibrium at the beginning of our effective sample period.

What drives the switches between determinacy and indeterminacy is the implied policy reaction chosen by the central bank which can change period by period depending on the incoming data. While we do not describe policy, as in much of the literature, via a linear rule, we can gain some insight by contrasting the optimal Federal Funds rate from the model with the actual rate in Figure (6). We note that for much of the sample period optimal policy is tighter than actual policy. This obviously explains the determinate outcomes after the Volcker disinflation, as the Fed wanted to implement a tighter policy than was eventually
realized.

At the same time, we also note that during the indeterminacy period during the second half of the 1970s the perceived optimal Federal Funds rate path was considerably above the realized path. This illustrates the main point of our paper. The Federal Reserve desired to implement a policy that would have resulted in a unique equilibrium, but because of its imperfect understanding of the structure of the economy and measurement issues in the real-time data flow, the chosen policy led to indeterminacy. Figure (6) also shows a quite dramatic interest rate hike in late 1974, where the Federal Reserve intended to raise the FF rate to almost 30%. As Figure (5) shows this generated a switch from indeterminacy to determinacy which persisted for a year despite a sharp reversal almost immediately.

In Figures (7) and (8) we plot the measurement errors in, respectively, inflation and output growth against the determinacy indicator. This gives insight into the underlying determinants of the optimal policy choice. We define the measurement error as the difference between the real-time data and the final data. A positive measurement error thus means that the data are coming in stronger than they actually are. Consider the inflation picture in 1974:3. The Federal Reserve observes inflation two percentage points higher than the final revision. We note that the true data, i.e., the final data, are generated by the private sector equations. The seemingly high inflation thus prompts the Fed to jack up the policy rate, as shown in Figure (6). Note that this is quickly reversed as the next data points indicate a negative measurement error, but because of the persistence in the learning process and the sluggishness of the Fed’s backward-looking model, determinacy switches tend to last for several periods.

Sharp changes in policy, and thus the potential for a determinacy switch, thus occur when there is a large discrepancy between the real-time observations and the true underlying data. In the last quarter of 1974, inflation came in at 2 percentage points above its true value, yet at the same time the measurement error in output growth was a negative 8 percentage points. The combination of high inflation and low growth prompted the Fed to keep policy roughly unchanged, continuing in the indeterminate equilibrium.

Figures (7) and (8) also show that the volatility and extent of the measurement errors declined after the Volcker disinflation, which is the main reason that the Great Moderation period is, in fact, one of equilibrium determinacy. Moreover, over the course of the sample, the learning central bank has developed a better understanding of the underlying structural model and the nature of the measurement error simply because of longer available data series as time goes by. Nevertheless, data misperception issues can still arise as evidenced
by the spike in late 2008 in Figure (8) and the seemingly increased volatility of the inflation error during the Great Recession. Consequently, these periods can lean towards a switch to indeterminacy as we will see in our robustness analysis below.

Whether an equilibrium is determinate or indeterminate is determined by the private sector equations once the central bank has communicated the policy rule for this period.\footnote{This is where the assumption of promised utility bears most weight since we can solve the linear rational expectations model in the usual manner (Sims, 2001) and do not have to account for the potential future switches in policy in every period.} The switches should therefore be evident from changes in the chosen policy parameters. We can back out time series for the policy coefficients from the estimated model. These are reported in Figure (9). Since the chosen form of the policy rule contains more lags, namely three, than is usual for the simple New Keynesian framework upon which most of our intuition is built, we also report the long-run coefficients (as defined by Woodford, 2003) to gauge the effective stance of policy in Figure (10).

At the beginning of the sample, the inflation coefficients are essentially zero. With only mild support from positive output coefficients, the resulting private sector equilibrium is indeterminate. The switch to a determinate equilibrium is in 1974:3 is evident from the sharp rise in the inflation coefficients at all lags and also in the long-run coefficient. This is accompanied by an increase in the output coefficients. The switch back to an indeterminate equilibrium during the late Burns-Miller period seems a knife-edge case as both inflation and output coefficients come down, but to levels that might not be considered a priori inconsistent with a determinate equilibrium. The behavior of the coefficient on the lagged interest rate is interesting in this respect. It is well known (Woodford, 2003) that highly inertial policy rules support determinate equilibria even if the inflation coefficients are not large. The rule becomes less inertial as the early 1970s progress, reaching almost zero in 1976. It then climbs only gradually which is consistent with the indeterminate equilibrium occurring in the late 1970s.

After 1980 the policy coefficients settle at their long-run values. There is virtually no variation afterwards. What is striking from the graphs is that the Volcker disinflation is essentially absent from the output and inflation coefficients. It appears only as the endpoint of the gradual rise in the lagged interest-rate coefficient. The Volcker disinflation can therefore be seen not as an abrupt change in the Federal Reserve's responsiveness to inflation, but rather as the culmination of a policy move towards a super-inertial policy.\footnote{Coibion and Gorodnichenko (2011) offer a similar interpretation.}

Finally, we can also contrast the Federal Reserve's and the private sector's one-period
ahead inflation expectations. These are reported in Figure (11). The private sectors expectations are the rational expectations from within the structural model given the policy rule, while the Fed’s expectations are computed from its reduced-from model as a one-period ahead forecast. The latter are noticeably less volatile and smoother than the former, which reflects the different nature of the expectation formation. Moreover, the Fed’s expectations were consistently higher than the private sector’s expectations during the Great Inflation, whereas in the Great Moderation the respective expectations line up more closely and fluctuate around a 2% inflation target. This is therefore further evidence of data misperceptions as the underlying source for indeterminate outcomes. The Federal Reserve consistently expected higher inflation than actually materialized and chose policy accordingly.

Our results thus confirm those of Clarida et al. (2000), Lubik and Schorfheide (2004), and others that have argued that the Great Moderation was kick-started by a shift to a more anti-inflationary policy under Volcker; whereas the Great Inflation was largely the outcome of weak policy. Much of this literature rests, however, on sub-sample estimation with exogenous break dates\textsuperscript{13}; moreover, it also often lacks a rationale as to why a central bank would pursue ostensibly sub-optimal policies. Our approach instead is not subject to the former concern, and we provide an answer to the latter question. Our model is estimated over the whole sample period, while the shifts between determinacy occur endogenously as the central bank changes its behavior in the light of new data. The seemingly sub-optimal behavior is rationalized by the signal extraction problem the policymakers face. From their perspective policy is chosen optimally. Where our results deviate from the previous literature is that they show that the 1970s also exhibited determinate equilibria, especially for an extended period in the middle of the decade.

6 Robustness

It is well known that models under learning are quite sensitive to specification assumptions. We therefore conduct a broad range of robustness checks to study the validity of our interpretation of the Great Inflation in the benchmark model. Broadly speaking, our results are robust. We begin by assessing the sensitivity of the baseline results to changes in individual parameters based on the posterior mean estimates. This gives us an idea how significant in a statistical sense our determinacy results are. The second exercise changing the central bank’s forecasting model to make it closer to the underlying structural model of the private sector. Both exercises confirm the robustness of our benchmark results. These are sensitive,

\textsuperscript{13}Notable exceptions are [references needed]
however, to a change in how capture the central bank’s initial beliefs at the beginning of the sample. We show how alternative, but arguably equally plausible assumptions change the determinacy pattern over the full sample period considerably.\footnote{We also intend to assess the benchmark’s robustness by considering (i) the benchmark model but without measurement error, (ii) alternative indeterminate and sunspot equilibria in the private sector system, (iii) alternative learning mechanism for the central bank such as decreasing gain learning or full information revelation after some time.}

6.1 Sensitivity to Parameters

Our results in terms of the determinacy indicators are fairly unequivocal which equilibrium obtained at each data point over the entire sample period. Probabilities of a determinate equilibrium are either zero or one. As we pointed out above, the determinacy indicator is an average over the draws from the posterior distribution at each point, which appears highly concentrated in either the determinacy or the indeterminacy region of the parameter space. A traditional coverage region to describe the degree of uncertainty surrounding the determinacy indicator would therefore be not very informative.

To give a sense of the robustness of the indicator with respect to variations in the parameters, we perform the following exercise. We fix all parameters at their estimated posterior means. We then vary each parameter one by one for each data point and each imputed realization of the underlying shocks and measurement errors, and record whether the resulting equilibrium is determinate or indeterminate. As the results of, for instance, Bullard and Mitra (2002) indicate the boundary between determinacy and indeterminacy typically depends on all parameters of the model, specifically on the NKPC parameter $\kappa$ and the indexation parameter $\alpha_\pi$. While this certainly is the case in our framework as well\footnote{New analytical results by Bhattarai et al. (2013) in a New Keynesian model with a rich lag structure support this conjecture.}, we find, however, that the determinacy indicator is not sensitive to almost all parameters in the model, the exception being the two weights in the central bank’s loss function, $\lambda_y$ and $\lambda_i$.\footnote{This finding is reminiscent of the results in Dennis (2006), who estimates these weights using likelihood-based methods in a similar model, albeit without learning and measurement error. He finds that the main determinant of fit and the location of the likelihood function in the parameter space are the central bank’s preference parameters.}

We report the simulation results for the two parameters in Figures (12) and (13), respectively. We vary each parameter over the range $[0, 1]$. Each point in the underlying grid in these figures is a combination of a quarterly calendar date and the value of the varying parameter over the range. We depict indeterminate equilibria in blue, determinate equilibria are red. The posterior mean of $\lambda_y$ is 0.065. The horizontal cross-section at this
value replicates Figure (5). Indeterminacy in the early 1970s was followed by a determinate period around 1975, after which another bout of indeterminacy towards the late 1970s was eradicated by the Volcker disinflation.

Figure (12) shows that a higher weight on output growth in the Federal Reserve’s loss function would generally tilt the economy towards indeterminacy, other things being equal. The reason is that a higher weight on output reduces the relative weight on inflation so that in the presence of inflation surprises, be they in the actual or in the real time data that are subject to measurement error, the central bank responds with less vigor in the implied policy rule. A second observation is that the indeterminacy and determinacy regimes in the early to mid 1970s are largely independent of the central bank’s preferences. Similarly, the pattern of determinate equilibria from the mid-1990s on appears unavoidable in the sense that even a strong preference for output growth could not have avoided it. The pattern for variations in the weight on interest-rate smoothing $\lambda_i$ is similar. At the posterior mean of 0.82 the determinacy indicator is not sensitive to large variations in this parameter.

6.2 Model Structure

Our results are obviously dependent on the specification of the model used by the private sector, namely the structural model that we regard as the data-generating process for the final data, and on the empirical model used by the central bank to learn about the private sector’s model. In our baseline specification we chose a restricted VAR for the central bank’s forecasting model following the by now standard specification of Primiceri (2006). It is restricted in the sense that we included a one-period lagged nominal interest rate in the output equation, but not lagged values for the inflation rate. Moreover, lag lengths were chosen by the usual criteria and not with reference to the ARMA-structure implied by the structural model.

We therefore consider an alternative specification that removes some of these restrictions. Specifically, we include lag polynomials for the inflation rate and the nominal interest rate in the empirical output equation. This brings the empirical model closer to the reduced-form structural model since output growth in the Euler-equation (16) depends on the real interest rate path, whereas the New Keynesian Phillips curve (15) only depends on output. The results from this specification (not reported, but available upon request) are generally the same as for our benchmark specification. The determinacy period during the mid-1970s last longer, and a determinate equilibrium obtains at the beginning of the effective sample

17 We are grateful to Alejandro Justiniano for suggesting this specification to us.
period. The latter is driven by a comparatively large measurement error in inflation at the beginning of the sample which prompted a sharp initial interest rate hike. As we have seen in the baseline specification, switching dates between determinacy and indeterminacy are associated with large measurement errors.

6.3 The Role of Initial Beliefs

A key determinant of the learning dynamics is the choice of initial beliefs held by the central bank. Since updating the parameter estimates in the face of new data can be quite slow, initial beliefs can induce persistence and therefore make switching less likely, everything else equal. There is no generally accepted way to choose initial beliefs. In our baseline specification we pursued the to us most plausible approach in that we estimate initial beliefs as part of the overall procedure. That is, we fix standard parameters and then estimate initial beliefs and the gain using flat priors. An alternative is to start with flat priors on initial beliefs. The determinacy indicator for this specification is depicted in Figure (14).

Indeterminacy lasts throughout the 1970s and well into the middle of the 1980s. Initial beliefs are such that policy is too accommodative an the data pattern in the 1970s is not strong enough to lead to different central bank policies. Moreover, the learning mechanism is slow moving so that initial beliefs need not be quickly dispersed. Although the Volcker disinflation happened it took a while for the Federal Reserve to catch up. For the rest of the Volcker-Greenspan period determinate equilibria obtained, which were only interrupted by the soft policy response to the recession in 1991 and the Great Recession. These results thus show monetary policy actions at the Bernanke Federal Reserve induced indeterminacy in the U.S. economy.

7 Conclusion

We argue in this paper that the Great Inflation of the 1970s can be understood as the result of equilibrium indeterminacy in which loose monetary policy engendered excess volatility in macroeconomic aggregates and prices. We show, however, that the Federal Reserve inadvertently pursued policies that were not anti-inflationary enough because it did not fully understand the economic environment it was operating in. Specifically, it had imperfect knowledge about the structure of the U.S. economy and it was subject to data misperceptions since the real-time data flow did not capture the true state of the economy, as large subsequent revisions showed. It is the combination of learning about the economy and, more importantly, signal extraction to filter out measurement noise that resulted in policies
that the Federal Reserve believed to be optimal, but when implemented led to equilibrium indeterminacy in the private sector.

In this sense, we combine the insights of Clarida et al. (2000) and Lubik and Schorfheide (2004) about the susceptibility of New Keynesian modelling frameworks to sub-optimal interest rate rules with the observation of Orphanides (2001) that monetary policy operates in a real-time environment that cannot be perfectly understood. It is only the passage of time that improves the central bank’s understanding of the economy through learning. Additionally, a reduction in measurement error, that is, a closer alignment of the real-time data with their final revisions, reduced the possibility of implementing monetary policies that imply indeterminacy. Consequently, and in this light, the Volcker disinflation and the ensuing Great Moderation can be understood as just the result of better data and improved Federal Reserve expertise.

The key contributions of our paper are therefore twofold. First, we offer an interpretation of the Great Inflation and the Great Moderation that combines and reconciles the good policy/bad policy viewpoint with the data misperception argument. The weakness of the former is that it offers no explanation why the Burns-Miller Federal Reserve behaved in this manner. We provide this explanation through introducing measurement error through data misperceptions into the methodological framework of Lubik and Schorfheide (2004). Interestingly, our results should also offer comfort to the good luck/bad luck viewpoint, as espoused by, for instance, Sims and Zha (2006) since we find that the switches between determinacy and indeterminacy are largely driven by the good or bad luck of obtaining real time data that are close, or not close to the final data. The second contribution, and one that follows from the previous observation, is that of a cautionary tale for policymakers. The possibility of slipping into an indeterminate equilibrium is reduced with better knowledge about the structure of the economy and the quality of the data. That this is a difficult issue is apparent from some of our robustness results which show that the Great Recession can be understood in this manner.

The main criticism to be levelled against our approach is that the private sector behaves in a myopic fashion despite forming expectations rationally. In order to implement our estimation algorithm we rely on the promised utility assumption of Sargent et al. (2006) which means that the private sector, despite all evidence to the contrary, maintains the belief that policy that is changing period by period will be fixed forever. A key extension of our paper would therefore be to model the private sector as incorporating the central bank’s learning problem by means of the approach in Farmer et al. (2009)
References


Appendix: Data Construction

All data we use is quarterly. The Federal Funds Rate is the average funds rate in a quarter obtained from the Board of Governors. For quarterly inflation and quarterly output growth data, we use the real time database at the Federal Reserve Bank of Philadelphia. The inflation data is constructed using a GDP deflator-based price index since this index gives us the longest available time series. The real-time output growth is constructed using the real output series. Both the output and price level series are seasonally adjusted. As a proxy for final data, we use the data of the most recent vintage we had access to when estimating the model (data up to 2012:Q1 available in 2012:Q2). The data starts in 1965:Q4.
### Table 1: Calibration

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<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
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<tr>
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<tr>
<td>Output Target $\Delta y_{\text{target}}$</td>
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<td>Q/Q Sample Average</td>
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<td>Habit Parameter $\delta_{y}$</td>
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### Table 2: Posterior Mean Estimates

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Figure 1: Real-Time and Final Data: Real GDP Growth Rate
Figure 2: Real-Time and Final Data: CPI Inflation Rate
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