Risk-Taking, Rent-Seeking, and CEO Incentives when Financial Markets are Noisy*

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October 21, 2013

Preliminary and Incomplete

Abstract

We analyze investment incentives and risk-taking by firms when equity markets aggregate information with noise. Noisy information aggregation drives a wedge between the expected social value and the market value of investments, causing initial shareholders to engage in inefficient rent-seeking behavior and generating corporate short-termism. Excessive risk taking is particularly severe if upside risks are coupled with near constant returns to scale, in which case even small market frictions lead to negative social value, but large shareholder rents. In our model, public news can trigger exaggerated market responses, i.e. investment panics on the downside or irrational exuberance on the upside, and that news disclosures can have negative welfare effects by expanding rent-seeking opportunities. Adding information feedback from share prices to investment decisions generates an endogenous element of upside risk, leading to excessive investment volatility, and low correlation of investment with fundamentals. When CEO compensation is unrestricted, the optimal contract design incentivizes risk-taking through the grant of stock options, or discourages it through the use of compensation ceilings. Finally we compare different regulatory or policy interventions and argue that limiting executive incentives to restricted equity contracts may be the most effective way to eliminate investment distortions.

* This paper subsumes our earlier paper titled "information aggregation, investment and managerial incentives" (Albagli, Hellwig and Tsyvinski, 2011). We thank Bruno Biais, John Campbell, V.V. Chari, Amil Dasgupta, Zhiguo He, Richard Kihlstrom, Peter Kondor, Markus Brunnermeier, Stephen Morris, Alp Simsek, Jeremy Stein, Jean Tirole, and Pietro Veronesi and numerous seminar and conference audiences for comments. Hellwig gratefully acknowledges financial support from the European Research Council (starting grant agreement 263790). Tsyvinski is grateful to NSF for support and EIEF for hospitality.

1 Introduction

Asset prices play a central role in aggregating information about the value of firms. By pooling together the dispersed knowledge of individual actors, prices provide information that shapes investor expectations about a firms' future earnings. If financial markets are efficient (in the sense that the firm's share price equals expected future dividends conditional on all available information) and there are no direct externalities from firm decisions, the share price then aligns the shareholder's private returns with social returns to the investment choices, even if the firms' shareholders do not hold on to their shares to see the returns from such investments realize. What's more, in publicly traded companies where decision-making is delegated to managers, shareholders can align the managers' preferences with their own, and by extension, society's, through appropriately linking managerial compensation to share prices. Under this worldview, unregulated financial markets will induce socially efficient firm decisions and channel available financial resources to their most productive uses. Any attempt to regulate the firms' investment behavior, restrict contracting with managers, or intervene in financial markets can only have negative welfare effects.

Critics of this view argue that if left unregulated, firms and managers engage in rent-seeking behavior, sacrificing long-term fundamental values to maximize short-term market value. Moreover, corporate short-termism is seen as socially harmful, some times resulting in dramatic failures of major corporations, coupled with costly interventions by governments to mitigate the economic consequences of such failures. This poses important questions regarding the optimal regulation of firm behavior. What are the departures from market efficiency that induce corporate short-termism and inefficient risk-taking? If corporate decisions expose taxpayers and society to excessive financial or technological risks, then is there scope for society to intervene and regulate corporate decision-making? If so, what form should such regulation take? How does it depend on the firm's characteristics, the market environment, and the risks to which it exposes society? And how robust are the lessons of the laisser-faire approach to small departures from market efficiency?

To answer these questions, we propose a theory of corporate investment and risk-taking when financial markets aggregate information with noise. Consider a setting in which the incumbent shareholders of a firm take an investment decision (or delegate this decision to a manager), before selling a fraction of their shares in a financial market populated by informed and noise traders. The share price then emerges as a noisy signal pooling the dispersed information investors have about the firm's value. We explore how the information aggregation friction in the financial market affects the initial shareholder's ability to capture the returns from their investment, under what conditions the firm's decisions depart from socially efficient levels, and how this is reflected in managerial incentive contracts. Finally, we return to the regulatory questions and argue that even small frictions in financial markets provide an important rationale for regulating firm behavior.[1]

Our analysis treats the firm's investment as the outcome of an optimal contracting problem between initial shareholders and the firms managers, which internalizes the effects of investment on the subsequent share price. If the manager is risk-neutral it is always possible to implement the investment level that maximizes the incumbent shareholders' surplus. Inefficiencies therefore stem solely from a misalignment between incumbent shareholder preferences and social surplus that is caused by market frictions, and not from agency frictions with managers.

Our results all from a simple but central observation: with noisy information aggregation and limits to arbitrage, the price is not just a noisy but also a biased estimate of the firm's dividends. This bias generates a wedge between the expected market returns to investment and its social value, which induces rent-seeking by initial shareholders.

More specifically, market-clearing implies that the price must adjust to stochastic shifts in demand and supply of shares, whenever there are limits to arbitrage, i.e. whenever informed traders have limited ability or willingness to adjust asset holdings to absorb shocks. These shifts appear on top of the information conveyed through the price, and thus imply that the share price generically reacts too much to the information that is aggregated through the market, relative to the shares' expected dividend value. What's more, the magnitude of this distortion is linked to uncertainty about firm dividends and scales with the firms' investment choice.

From an ex ante perspective, the incumbent shareholders' marginal return from investing is no longer aligned with the firm's expected dividend value. This introduces a rent-seeking motive to their incumbent shareholder's incentives, i.e. the expected gap between the share price and the dividend value is a pure transfer from final investors to initial shareholders, whose sign and magnitude depend on the characteristics of the underlying dividend return and on the investment scale. If the firm's investment is characterized by upside risk, the expected market return exceeds expected dividend return, and the firm invests more than the efficient level. If instead the investment is characterized by downside risk, the expected market return falls short of dividend values and the firm invests too little. These inefficiencies become particularly pronounced if the firm's technology

[1] Although our model does not focus on financial risks and financial corporations per se, our results suggest that the incentive problems we highlight are particularly pronounced in this sector, whose activity is motivated by the very frictions that are at the heart of our analysis. In fact, excessive risk-taking appears to be a leading cause in the recurrent costly failures of large financial corporations.

has nearly constant returns to scale, in which case (i) the surplus from the investment is small, and (ii) the firm's investment choice is highly sensitive to return perceptions, so that the scope for rent-seeking becomes very large. If near constant returns are coupled with upside risks, even small frictions in financial markets can have very large efficiency consequences – so large in fact that the firm generates negative expected surplus, while incumbent shareholders take excessive risks purely in an attempt to maximize the financial market rent they capture from selling their shares.

In our baseline model, the firm chooses an unconditional investment scale without access to further information about returns. We next consider two variations of our model to highlight potential distortions in response to additional information. First, we incorporate a public signal of returns on which both the firm's investment and the market price can be conditioned. We show that the firm's response to this signal is distorted in anticipation of a distorted response of market returns. Because of excess volatility and excess uncertainty about market returns, a signal that is highly informative about fundamental returns provides far less information about expected market returns, and reduces uncertainty about the market far less than it reduces uncertainty about fundamentals. This has two effects. First, we show that certain types of public news may trigger market and investment panics ("deleveraging") on the downside, and irrational exuberance on the upside. Consider an investment with an upside risk, such as the development of a new technology for which there is a small chance of success. There exist ranges of mildly positive signals, which trigger optimism about future market returns, even though they shouldn't really be cause for optimism about the likely success of the technology. This induces shareholders to over-invest in the hopes of subsequently selling shares at inflated market prices. On the downside, certain signals that ought to reassure incumbent shareholders may instead trigger investment panics if they lead to fears about low market returns. Second, we show that information disclosures may exacerbate investment distortions and reduce welfare, in particular, when returns to scale are nearly constant, where the distortions are especially severe. Better information merely expands the incumbent shareholders' opportunities for rent-seeking which further reduces the firm's fundamental value.

Second, we consider a variant in which investment is implemented after the market has opened. This allows for an informational feedback from share prices to the investment decision. This feedback introduces an endogenous element of upside risk: the firm will invest more when the information conveyed through the price is more positive. However, the social value of this feedback need not be positive, for it once again enriches rent-seeking opportunities by initial shareholders. As the market price is excessively sensitive to the market information, the chosen investment level also becomes excessively volatile, to the point that when noise is sufficiently important, investment decisions are disconnected from fundamental returns, yet highly sensitive to stock prices.

We then turn to the optimal design of managerial contracts. Following standard arguments, agency costs disappear and any investment level is implementable, if the manager is risk-neutral. Giving incumbent shareholders full discretion to design performance contracts then merely gives them all the tools they need to optimize their rents, thus turning the laisser-faire argument against regulation of performance pay on its head. In particular if incumbent shareholders benefit from excessive risk-taking (i.e. the case of upside risks or informational feedbacks), the implemented contract will take the form of stock option compensation. Generally speaking, the incumbent shareholders will try to use any discretion in contract design that they may have to distort incentives to their advantage. As a result, implementing the socially efficient investment level requires very severe restrictions on the set of allowable performance contracts, de facto ruling out anything other than untraded, restricted equity contracts.

Finally, we discuss regulatory and policy interventions. We compare four types of interventions: regulation of executive pay, investment caps or direct regulation of risk-taking, financial transactions taxes, and market interventions in the form of asset purchasing programs. All these interventions can play a positive role in correcting investment inefficiencies. However, they also all come with important distributional consequences: initial shareholders are made strictly worse off under any regulatory intervention, unless the policy includes a direct shareholder subsidy. Of the proposed policies, by far the simplest and most effective is to rule out any form of performancy pay other than untraded restricted equity contracts – this regulation does not require any additional information on the firm's technologies, the market environment, or the information on which the firm bases its decisions, and fully restores incentives to implement efficient investment levels. The other policies can achieve a similar objective but their implementation require that the regulator has far more information about the firm and its equity market. In addition, asset purchases suffer from a winner's curse problem and must be complemented with tax policies in order to be revenue neutral – without such taxes, they amount to direct shareholder subsidies.

Related Literature. Our modeling of the financial market builds on the literature on noisy information aggregation through stock prices (Grossman and Stiglitz, 1980, Hellwig, 1980, Diamond and Verrecchia, 1981). In particular, we make use of the formulation in our earlier paper (Albagli, Hellwig and Tsyvinski, 2011) which enables us to offer return implications for arbitrary securities in a non-linear noisy REE model.[2] Here this model serves as the baseline ingredient for analyzing

[2] In Albagli, Hellwig, and Tsyvinski (2011 and 2013), we further argue that our model can account for important stylized facts on asset returns such as a negative relation between skewness and asset returns, or the so-called credit

the link between investment and market returns. For us, non-linear, non-normal returns are crucial for defining the notions of upside and downside risks, and the corresponding investment distortions.

An extensive literature has studied the link between managerial incentives and financial markets. The classic paper by Holmstrom and Tirole (1993) emphasizes the positive aspects of tying compensation to market performance. This literature emphasizes the important role of prices in aligning private and social costs and benefits from investing under the condition that prices accurately reflect expected future dividend values. In our analysis instead, market signals contain valuable information, but they are not unbiased. What's more, these biases in turn feed back into ex ante incentives to invest, thus resulting in investment distortions.

Some papers emphasize the limitations of market-based incentive schemes. Closest to our work are Bolton, Scheinkman, and Xiong (2006). As in our paper, corporate short-termism is in the interest of incumbent shareholders who design contracts that induce inefficient managerial decisions aimed at boosting short-term prices. Different from us, they model information frictions by considering traders that "agree to disagree" about the informativeness of profit signals, which creates a speculative component in prices that can be exploited by managerial decisions. In our model instead mispricing depends on the distribution of cash-flow risks, allowing the possibility of both systematic over- and under-valuation of securities, and over- and under- investment. Moreover, our model features a common prior and imposes no restrictions on contracts which enables us to go much further in discussing contract design, welfare and optimal regulatory interventions.

Other literature instead emphasizes agency conflicts between shareholders and managers as a source of distortions. This literature discusses how asymmetric information between managers and shareholders may induce the former to take costly actions to manipulate the firms' earnings. Stein (1988) argues that earnings manipulation may result even if there is no conflict of interest, if the asymmetry leads to temporary price dips that invite takeovers at disadvantageous prices. Stein (1989) argues that earnings manipulation is pervasive if price-based compensation induces short-termism by managers, even when in equilibrium shareholders are not fooled on a systematic basis. Benmelech, Kandel, and Veronesi (2010) take a longer-term view by studying managerial compensation throughout the lifecycle of firms. While early on share price compensation leads to high effort by managers, it creates the potential for earnings manipulation when firms mature and growth opportunities decline. While these papers study the link between stock markets and managerial decisions through priced-based compensation, the analysis differs from ours by placing the emphasis on the managers' ability to manipulate information. In our model instead, the

spread puzzle for corporate bonds. Further quantitative investigations are planned or in progress.

information structure cannot be manipulated by managers or initial shareholders, and managerial short-termism is wanted by the incumbent shareholders.

Our extension to public information contributes to the literature on the social value of public information. Morris and Shin (2002) and Angeletos and Pavan (2007) show that public imformation disclosures may be welfare-reducing, if strategic complementarities distort agents' responses to information. Amador and Weil (2010) obtain a similar result but focus solely on informational externalities. In our model, market forces lead to a similar distortion in the response of share prices to public information. The welfare consequences of this distortion however result from the firm's rent-seeking behavior, not coordination failures or informational externalities in the financial market. To the extent that additional information is exploited by initial shareholders for rent-seeking purposes, transparency may backfire, yet it will always be in the interest of incumbent shareholders who benefit from the information.

Our extension to price-contingent investment contributes to the literature on information feedback between stock prices and real investment. Subrahmanyam and Titman (1999) consider a one-way feedback effect where investment depends on, but does not affect, share prices. Dow and Rahi (2003) study risk-sharing and welfare in a setting with endogenous investment in a CARA-Normal setup, while Goldstein and Guembel (2008) show how manipulative short-selling strategies that distort firm's investment decision can be profitable to some traders but detrimental to overall firm value. Goldstein, Ozdendoren, and Yuan (2010) discuss efficiency and trading frenzies arising from the socially suboptimal weighting of private and exogenous public signals. Angeletos, Lorenzoni, and Pavan (2010) model the interaction between early investment choices by entrepreneurs and the later transfer of firm property to traders, but focus on how the dispersed nature of entrepreneurial information causes excess non-fundamental volatility in real investment and asset prices. Our analysis instead focuses on dispersed information in the financial market ex post and rent-seeking by incumbent shareholders ex ante, and the results in our model do not depend on strategic interactions between traders or entrepreneurs.

Finally, several recent empirical studies provide evidence consistent with some of our central predictions. In our model, upside cash-flow risk will systematically lead to excessive investment, the more so the larger the information frictions and the bias towards short-term results by incumbent shareholder. Gilchrist, Himmelberg and Huberman (2005) find that an increase in belief heterogeneity (proxied by analyst forecast dispersion) leads to an increase in new equity issuance, Tobin's Q, and real investment. Polk and Sapienza (2008) find a positive relation between abnormal investment and stock overpricing (proxied by discretionary accruals), a pattern that is more prevalent in firms they identify as being more opaque (higher R&D intensity), as well as in firms with shorter shareholders' horizons (proxied by share turnover). Moreover, excessive investment is typically followed by abnormally low returns, especially in firms with the aforementioned characteristics.

Section 2 introduces our general model and derives the equilibrium characterization in the financial market. Section 3 analyzes rent-seeking behavior by incumbent shareholders, investment distortions and the associated welfare losses. Section 5 extends the analysis to public news, section 6 to informational feedback from share prices to investment. Section 7 discusses optimal contract design, section 8 discusses regulatory interventions. Section 9 offers discussion of further extensions.

2 The model

Our model has three stages. In the first stage, a firm with a unit measure of shareholders contracts with a manager. The manager takes an unobservable investment decision $k \geq 0$ for the firm. In the second stage, the initial shareholders sell a fraction $\alpha \in (0, 1]$ of the shares to outside investors. At the final stage, the firm's cash flow is realized as a function of k and a stochastic fundamental $\theta \in \mathbb{R}$, and paid to the final shareholders. This cash flow takes the form $\Pi(\theta, k) = R(\theta) k - C(k)$, where $R(\cdot)$ is a positive, increasing function of the firm's fundamental (interpreted as the return to the investment k), and $C(\cdot)$ is an increasing, convex function of k, interpreted as the firm's cost of investment. For normative results, we use the functional form $C(k) = \frac{1}{1+\gamma} k^{1+\gamma}$, where γ indexes the firm's degree of return to scale. The fundamental θ is distributed according to $\theta \sim \mathcal{N}(0, \lambda^{-1})$. The manager's compensation $W(\cdot)$ is a function of the final dividend Π.

We define efficiency from the perspective of the final shareholders, who have to live with the consequences of the decisions taken in stage 1. The *ex post efficient investment* equals $k^{FB}(\theta) = \psi(R(\theta))$, where $\psi(\cdot) = (C')^{-1}(\cdot)$ is the inverse marginal cost function. The *ex ante efficient investment* $k^* = \psi(\mathbb{E}(R(\theta)))$ maximizes $\mathbb{E}(\Pi(\theta, k))$, given the information available at stage 1.

Our analysis focuses on the impact of frictions in the financial market at stage 2 on managerial incentives, investment, and contract design at stage 1. Our next subsection introduces the financial market environment and characterizes the market equilibrium, taking as given the outcome of the first stage contract and investment decision. Our financial market model is a variant of the non-linear noisy rational expectations model we introduced in AHT, so this sub-section will closely follow our earlier work. Afterwards, we will turn to the initial contract design problem.

2.1 Stage 2 – Financial Market

Description of the Financial Market. As a convention, the manager is paid by initial shareholders, so outside investors care only about $\Pi(\theta, k)$. Moreover, investors can infer the equilibrium choice of k from the contract design, so only θ remains uncertain. There are two types of outside investors: a unit measure of risk-neutral informed traders, who are indexed by i, and noise traders.

Informed traders observe a private signal $x_i \sim \mathcal{N}(\theta, \beta^{-1})$, which is i.i.d. across traders (conditional on θ). After observing x_i, an informed trader submits a price-contingent demand schedule $d_i(\cdot) : \mathbb{R} \to [0, \alpha]$, to maximize expected wealth $w_i = d_i \cdot (\Pi(\theta, k) - P)$. That is, informed traders cannot short-sell, and can buy at most α units of the shares. Individual trading strategies are then a mapping d from triplets (x_i, P) into $[0, \alpha]$. Aggregating traders' decisions leads to the aggregate demand $D(\theta, P) = \int d(x, P) d\Phi(\sqrt{\beta}(x - \theta))$, where $\Phi(\sqrt{\beta}(x - \theta))$ represents the cross-sectional distribution of private signals x_i conditional on the realization of θ, and $\Phi(\cdot)$ denotes the cdf of a standard normal distribution.[3]

Noise traders place an order to purchase a random quantity $\alpha\Phi(u)$ of shares, where $u \sim \mathcal{N}(0, \delta^{-1})$ is independent of θ, and δ^{-1} measures the informativeness of the share price.

Once informed and noise traders have submitted their orders, these orders are executed at a price P that satisfies the market-clearing condition $\alpha = D(\theta, P) + \alpha\Phi(u)$ at which the total demand of shares equals the supply α.

Let $H(\cdot|x, P)$ denote the traders' posterior cdf of θ, conditional on observing a private signal x, and a market-clearing price P. A *noisy Rational Expectations Equilibrium* at stage 2 consists of a demand function $d(x, P)$, a price function $P(\theta, u; k)$, and posterior beliefs $H(\cdot|x, P)$, such that $d(x, P)$ is optimal given the shareholder's beliefs $H(\cdot|x, P)$; $P(\theta, u; k)$ clears the market for all (θ, u) and k; and $H(\cdot|x, P)$ satisfies Bayes' Rule whenever applicable.

Equilibrium Characterization. Suppose that the prior at stage 2 is that $\theta \sim \mathcal{N}(\mu, \hat{\lambda}^{-1})$, and traders expect an investment level k.[4] Our first result characterizes the equilibrium share price in a noisy Rational Expectations Equilibrium. Moreover this is the unique equilibrium in which demand schedules are non-increasing in P. Monotonicity restrictions arise naturally if trading takes place through limit orders.

[3] We assume that the Law of Large Numbers applies to the continuum of traders, so that conditional on θ the cross-sectional distribution of signal realizations ex post is the same as the ex ante distribution of traders' signals.

[4] In our basic model, no new information emerges between the contracting and the financial market stages, so this is the same as the stage 1 prior ($\theta \sim \mathcal{N}(0, \lambda^{-1})$). By keeping a more flexible prior, our equilibrium characterization will serve as a reference for later sections in which we depart from this benchmark.

Proposition 1 ***Equilibrium Characterization and Uniqueness***

Define $z \equiv \theta + 1/\sqrt{\beta} \cdot u$. *In the unique equilibrium in which the informed traders' demand* $d(x, P; k)$ *is non-increasing in* P*, the market-clearing price function is*

$$P(z,k) = \mathbb{E}\left(R(\theta)\,|x=z,z\right)\cdot k - C(k)\,. \tag{1}$$

The variable z is normally distributed with mean θ and precision $\beta\delta$, and serves as a sufficient statistic for the information conveyed through the share price. This characterization of $P(z,k)$ gains its significance from the comparison with the share's expected dividend value $V(z,k)$:

$$V(z,k) = \mathbb{E}\left(R(\theta)\,|z\right)\cdot k - C(k)\,. \tag{2}$$

The equilibrium share price differs systematically from the expected dividend value: Both are charecterized as expected dividends conditional on the information contained in z, but the share price treats the signal z as if it had precision $\beta + \beta\delta$ (equal to the sum of the private and the price signal precision), when in reality its precision is only equal to $\beta\delta$. Hence the market price is based on an expectation of the marginal return to the investment level k that conditions too much on the market signal z, relative to its objective information content.

We label the difference between $P(z,k)$ and $V(z,k)$ the *information aggregation wedge* $\Omega(z,k)$:

$$\Omega(z,k) = k\cdot\left\{\mathbb{E}\left(R(\theta)\,|x=z,z\right) - \mathbb{E}\left(R(\theta)\,|z\right)\right\}. \tag{3}$$

The wedge results from market clearing with heterogeneous information. Both the price and the expected dividend reflect the public information z that is conveyed through the price. However in addition, the price must shift to equate demand and supply: if θ increases, all trader become more optimistic through the observation of private signals, and hence demand more. Of course demand also increases when u rises, due to the order by noise traders. To compensate for this increase in demand that arises with a shift in z, the market price must respond to these shocks by more than what is consistent merely with the information provided through the price. That is why $P(z,k)$ reacts more strongly to the realization of z than $V(z,k)$.

We discussed properties of this wedge at length in our preceding paper.[5][6] Here we highlight only the interaction of $\Omega(z,k)$ with k: since the wedge results from how traders in the market treat

[5] In AHT, we also show that the equilibrium characterization and the core properties of this wedge are robust to general, non-normal distributional assumptions, risk aversion and arbitrary position limits. The functional form assumptions only invite convenient closed-form solutions, but are not important for the economic forces in play.

[6] The only (minor) modification relative to AHT is the normalization of demand by α instead of 1. This insures that α only isolates changes in the incentive problem, not in the market setting. We could have equivalently normalized the total amount of shares for sale to 1 (along with the position limits and demand by noise traders), along with a total supply of shares to $\alpha^{-1} > 1$.

uncertainty regarding cash-flows, the magnitude of the wedge scales with the firm's expected risk exposure. Hence the inferred investment level implemented at the first stage influences the extent to which share prices depart from expected dividend values later on. This is the channel through which the initial contract design imposes externalities on final shareholders.

Proof. *Characterization.* Let $\hat{x}(P)$ denote the private signal of a trader who is just indifferent between buying and not buying the share at a given price P, so that $P = \int \Pi(\theta, k)\, dH(\theta|\hat{x}(P), P)$. Since $R(\cdot)$ is increasing in θ, $\int \Pi(\theta, k)\, dH(\theta|x, P)$ must be monotone in x, and any trader whose private signal exceeds $\hat{x}(P)$ must strictly prefer to purchase a share, while any trader whose signal is less than $\hat{x}(P)$ prefers not to buy. Thus, the total demand by the informed traders is $\alpha\left(1 - \Phi(\sqrt{\beta}\,(\hat{x}(P) - \theta))\right)$. Equating demand and supply, a price P clears the market in state (θ, u) if and only if $\hat{x}(P) = \theta + 1/\sqrt{\beta} \cdot u \equiv z$. Therefore, in any equilibrium, it must be the case that $\hat{x}(P(\theta, u; k)) = z$, and if P is a function of z only, then it must be invertible. But if $P(\cdot)$ is invertible, observing P is informationally equivalent to observing $\hat{x}(P) = z \sim \mathcal{N}(\theta, (\beta\delta)^{-1})$. Along the equilibrium path, the traders thus treat the signals $\hat{x}(P) \sim \mathcal{N}(\theta, (\beta\delta)^{-1})$ and $x \sim \mathcal{N}(\theta, \beta^{-1})$ as mutually independent normal signals and their posterior beliefs $H(\cdot|x, P)$ are given by

$$\theta|x, P \sim \mathcal{N}\left(\frac{\hat{\lambda}\mu + \beta x + \beta\delta\hat{x}(P)}{\hat{\lambda} + \beta + \beta\delta}, \left(\hat{\lambda} + \beta + \beta\delta\right)^{-1}\right).$$

Substitute $\hat{x}(P) = z$, we restate the informed traders' indifference condition in terms of z: $P(z, k) = \mathbb{E}(\Pi(\theta, k)\,|x = z, z)$. The expression for $V(z, k) = \mathbb{E}(\Pi(\theta, k)\,|z)$ is derived analogously using only the information from the market signal, $\theta|P \sim \mathcal{N}(\frac{\hat{\lambda}\mu + \beta\delta\hat{x}(P)}{\hat{\lambda} + \beta\delta}, \left(\hat{\lambda} + \beta\delta\right)^{-1})$.

Uniqueness. If demand is restricted to be non-increasing in P, $\hat{x}(P)$ must be non-decreasing. If $\hat{x}(P)$ is strictly monotone everwhere, then it is invertible, P is informationally equivalent to $\hat{x}(P) = z$, and we arrive at the equilibrium characterized above. Suppose therefore that $\hat{x}(P)$ is flat over some range, i.e. $\hat{x}(P) = \hat{x}$ for $P \in (P', P'')$. Suppose further that for sufficiently low $\varepsilon > 0$, $\hat{x}(P)$ is strictly increasing over $(P' - \varepsilon, P')$ and $(P'', P'' + \varepsilon)$, and hence uniquely invertible.[7] But then for $z \in (\hat{x}(P' - \varepsilon), \hat{x})$ and $z \in (\hat{x}, \hat{x}(P'' + \varepsilon))$, $P(z)$ is uniquely defined, and must be characterized as above, from the indifference condition for $\hat{x}(P) = z$. But since the function $P(z, k)$ defined above is continuous and strictly monotonic in z, it must be the case that $P' = P''$, contradicting the existence of an interval for which $\hat{x}(P)$ is flat.

[7] It cannot be flat everywhere, because then informed demand would be completely inelastic, and there would be no way to absorb noise trader shocks.

2.2 Stage 1 – Contract Design

Problem Statement. At the first stage, the incumbent shareholders and the manager agree on a contract, which specifies a targeted investment level $\hat{k}$ and a compensation scheme $W(\cdot)$ that is defined as a function of Π. The incumbent shareholders and the manager are all risk neutral. The contract has to offer the manager an expected wage payment at least as high as his outside offer $\overline{w}$, and give the manager the incentives to implement the targeted investment level $\hat{k}$. In addition, incumbent shareholders and managers take as given the market-clearing price function $P(\theta, u; k)$ that will result at the financial market stage, when designing the initial contract. We start from the case in which the manager has no additional information when choosing k, and will later discuss how the optimal contract design makes use of additional information.

A triplet $(k, W(\cdot), P(\theta, u; k))$ is *implementable*, if (i) $P(\theta, u; k)$ is the market-clearing price function of a noisy rational expectations equilibrium in the financial market, and (ii) the manager's individual rationality and incentive compatibility constraints hold: $\mathbb{E}\{W(\Pi(\theta, k))\} \geq \overline{w}$ and $k \in \arg\max_{k \geq 0} \mathbb{E}\{W(\Pi(\theta, k))\}$. Since it's always possible to hold the manager to $\overline{w}$ by rescaling the contract, we ignore the former from now on. The initial shareholders maximize the expected proceeds from the shares (both from the sale at stage 1, and from the dividends at stage 2), net of the wage payments to the manager

$$\mathbb{E}\{\alpha P(\theta, u; k) + (1-\alpha)\Pi(\theta, k) - W(\Pi(\theta, k))\} \tag{4}$$

with respect to the set of triplets $(k, W(\Pi), P(\theta, u; k))$ that are implementable. Proposition 1 greatly simplifies our task. Treating $z \sim \mathcal{N}(\theta, (\beta\delta)^{-1})$ as our state variable at the financial market stage, we replace the implementability condition that $P(\cdot)$ be a market-clearing price function with the unique equilibrium characterization given by 1. We determine optimal contract design in two steps. First, we determine the investment level that maximizes the initial shareholders' expected payoffs before wage payments, disregarding the incentive compatibility constraint. Then we characterize the compensation scheme that implements the optimal investment level. The next three sections discuss optimal investment when there is no additional information on which investment can be conditioned (Section 3), when investment can be conditioned on additional public signals (Section 4), and when there are informational feedbacks from share prices to investment (Section 5). Section 6 discusses the design of compensation schedules.

Our efficiency benchmark. Any discussion of welfare and efficiency is complicated by potential distributional effects between initial shareholders, informed traders and noise traders, and remains necessarily incomplete without fully accounting for the noise traders' motives for buying the shares. Here we are primarily interested in the externalities that the contract design imposes on final shareholders who have no say in the investment decision, but fully suffer its consequences. Hence we have chosen an efficiency benchmark, in which expected dividends are maximized.

Departures from this benchmark are caused by the combination of two elements. First, the initial shareholder's plan to sell part of their shares. If initial shareholders hold on to all their shares, i.e. in the limiting case where $\alpha \to 0$, they have no interest in deviating from efficiency.

The second element is the wedge between share prices and expected dividends. Consider the alternative *efficient markets* scenario in which the share price is automatically equated to expected dividend value, i.e. $P(z;k) = V(z;k)$.[8] Efficient markets eliminate the conflict of interest between initial shareholders and managers on the one hand, and final shareholders on the other. The share price motivates initial shareholders to implement the efficient investment level, even though they plan to sell a fraction of their shares. Moreover, the efficient investment level is implemented if W is linear in Π, i.e. managers are given a participation in the dividend value of the firm. Thus, when market prices align initial and final shareholder incentives, executive compensation that is tied to share price performance aligns shareholder and manager incentives, thereby aligning privately optimal and socially efficient investment choices.

Once prices depart from expected dividend values, initial and final shareholder incentives are no longer aligned. While our analysis emphasizes the role of noisy information aggregation and limits to arbitrage, these are by no means the only potential sources of mis-pricing. Yet, noisy information aggregation has two properties that are key for distorting incentives: First, the information aggregation wedge doesn't just add noise to stock prices, which would average out from an ex ante perspective, but it responds systematically to the price realization (in other words, P and $P - V$ are not orthogonal to each other). Second, the investment decision taken at the first stage influence its magnitude. Initial shareholders can therefore use the investment decision to influence the rents they extract through mis-pricing of their shares in the market.

[8] This could result for example with free entry of uninformed arbitrageurs as in Kyle (198?). Alternatively, consider the case where (i) there was no private information, and (ii) z simply represented an exogenous public signal with precision $\beta\delta$. Because in this scenario, all informed traders are identical, and their collective wealth exceeds the available supply, it must be that these traders are indifferent between purchasing and not purchasing the shares, which requires that at equilibrium $P(z,k) = V(z,k)$. Efficient markets also emerge as the limiting case of our private information model, when $\beta \to 0$, while $\beta\delta$ is held constant.

3 Investment with Financial Market Frictions

Assume for now that $C'(0) = 0$ so that the optimal investment is always positive, and that the market prior is the same as the initial prior, $\mu = 0$, and $\hat{\lambda} = \lambda$. From the initial shareholders' perspective, the optimal value $\hat{k}$ maximizes $\mathbb{E}\{\Pi(\theta, k) + \alpha\Omega(z, k)\}$. The optimal contract implements an investment level $\hat{k}$ that satisfies

$$C'\left(\hat{k}\right) = \alpha\mathbb{E}\{\mathbb{E}(R(\theta)|x = z, z)\} + (1 - \alpha)\mathbb{E}(R(\theta)).$$

Proposition 2 ***Market frictions cause investment distortions.***
$\hat{k} \gtreqless k^*$ whenever $\mathbb{E}\{\mathbb{E}(R(\theta)|x = z, z)\} \gtreqless \mathbb{E}(R(\theta))$. $|\hat{k}/k^* - 1|$ is increasing in α.

The optimal contract equates the marginal cost of investment to a weighted average of the expected market return, $\mathbb{E}\{\mathbb{E}(R(\theta)|x = z, z)\}$ and the expected dividend return $\mathbb{E}(R(\theta))$. The optimal investment level $\hat{k}$ departs from the efficient investment level k^* if the expected market return iffers from the expected dividend return. The difference between these two expected marginal values corresponds to the marginal impact of k on the expected information aggregation wedge, $\mathbb{E}\{\mathbb{E}(R(\theta)|x = z, z)\} - \mathbb{E}(R(\theta))$. Whenever the expected wedge is positive, the expected market price exceeds the expected dividend value, hence on average the shares are over-valued. Moreover, since the expected market return to investment exceeds the dividend return the initial shareholders find it optimal to over-invest, to enhance the over-valuation of their shares. When instead the expected wedge is negative, the initial shareholders want to under-invest in order to limit the under-valuation of their shares. Thus through the investment level, initial shareholders influence the expected magnitude of the information aggregation wedge. This expected wedge enters the initial shareholders' private returns to investing, but not the social returns, because the share price is a pure transfer from the final to the initial shareholders.

Information aggregation and return characteristics. Our next result links the extent of over- or under-investment to the return distribution $R(\cdot)$ and the parameters determining how much noisy information aggregation affects average prices. Drawing on AHT (Section 3), we define

$$\gamma_P = \frac{\beta(1+\delta)}{\lambda + \beta + \beta\delta} \text{ and } \gamma_V = \frac{\beta\delta}{\lambda + \beta\delta}.$$

The market's posterior of $\theta|z$ is normal with mean $\gamma_P z$ and variance $(1 - \gamma_P) \cdot \lambda^{-1}$, while the prior over z is normal with mean 0 and variance $\lambda^{-1} + (\beta\delta)^{-1} = 1/\gamma_V \cdot \lambda^{-1}$. Compounding the two distributions, we compute $\mathbb{E}\{\mathbb{E}(R(\theta)|x = z, z)\}$ as a "market-implied" prior expectation of $R(\theta)$, i.e. as the expectation of $R(\theta)$ with respect to a distribution over θ that is normal with mean zero, and variance λ_P^{-1}, where $\lambda_P^{-1} = \gamma_P^2/\gamma_V \cdot \lambda^{-1} + (1-\gamma_P) \cdot \lambda^{-1} > \lambda^{-1}$. From an ex ante perspective the market thus attributes too much weight to the tail realizations of the θ, relative to the objective prior distribution. The ratio $\lambda_P^{-1}/\lambda^{-1}$ governs by how much the market overweighs the tail realizations, and thereby governs the impact of information aggregation frictions on equilibrium share prices and investment levels. The direction of the distortion then depends on which of the two tail risks is more important. This is formalized by the following partial ordering of return risks $R(\cdot)$ and the subsequent proposition.

Definition 1 ***Upside and Downside Risk***

(i) A return $R(\cdot)$ has symmetric risks if $R'(\theta) = R'(-\theta)$ for all $\theta > 0$.

(ii) A return $R(\cdot)$ is dominated by upside risks, if $R'(\theta) \geq R'(-\theta)$ for all $\theta > 0$. A return $R(\cdot)$ is dominated by downside risks, if $R'(\theta) \leq R'(-\theta)$ for all $\theta > 0$.

(iii) Consider two returns R_1 and R_2 such that $\mathbb{E}(R_1(\theta)) = \mathbb{E}(R_2(\theta))$. Then R_1 has more upside (less downside) risk than R_2 if $R_1'(\theta) - R_1'(-\theta) \geq R_2'(\theta) - R_2'(-\theta)$ for all $\theta > 0$.

This definition classifies returns by comparing marginal gains and losses at fixed distances from the prior mean to determine whether returns are steeper on the upside or on the downside. The following result is a direct corollary of Theorem 2 in AHT.

Proposition 3 ***Upside (downside) risk induces over- (under-)investment***

(i) ***Sign:*** *If R has symmetric risk, then $\hat{k} = k^*$. If R is dominated by upside risk, then $\hat{k} > k^*$. If R is dominated by downside risk, then $\hat{k} < k^*$.*

(ii) ***Comparative Statics w.r.t.*** λ_P^{-1}***:*** *If R is dominated by upside or downside risk, then $|\hat{k}/k^* - 1|$ is increasing in λ_P^{-1}.*

(iii) ***Comparative Statics w.r.t.*** *R****:*** *If two returns R_1 and R_2 have the same expectation, but R_1 has more upside risk than R_2, then the investment level $\hat{k}_1$ that is associated with R_1 is strictly larger than the investment level $\hat{k}_2$ that is associated with R_2.*

This result classifies firms according to their dividend risk. Returns that have a significant upside component (for example, if $R(\theta)$ is increasing and convex) will lead to over-investment, while returns that are dominated by the downside risk (for example, if $R(\theta)$ incorporates mainly the risk of a failure if the fundamentals are low) will lead to under-investment. From an ex ante perspective, the market expectations place too much weight on tail realizations of θ, relative to their objective probabilities, so if the tail events are dominated by the upside (as in the first scenario), the firms optimal investment is distorted up, while when the tail events are dominated by the downside (as in the second scenario), the firms optimal investment is distorted down. Moreover, for a given expected dividend return, the distortion are larger if the returns are more asymmetric.

Rent Extraction and Efficiency Losses. The initial shareholders' rents from market frictions scale with the investment choice, are positive when $\mathbb{E}\left\{\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right\} > \mathbb{E}\left(R\left(\theta\right)\right)$, and negative when $\mathbb{E}\left\{\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right\} < \mathbb{E}\left(R\left(\theta\right)\right)$. Therefore, $\hat{k}/k^*$ measures the ratio between the rents obtained at the optimal contract, and the rents consistent with efficient investment decisions, and $|\hat{k}/k^* - 1|$ measures the amount of rent manipulation. We measure the resulting efficiency loss by the percentage loss in expected dividends, relative to the ex ante efficient benchmark. If $V^* = \mathbb{E}\left(R\left(\theta\right)\right) \cdot k^* - C\left(k^*\right)$ denotes the maximal expected dividend, and $\hat{V} = \mathbb{E}\left(R\left(\theta\right)\right) \cdot \hat{k} - C\left(\hat{k}\right)$ the firm dividend expected under the optimal contract, then the efficiency loss is denoted by $\Delta = 1 - \hat{V}/V^*$. If Δ exceeds 1, then the distortion is so severe that in expectation the firm actually makes losses, i.e. the investment cost exceeds the expected dividends. After some algebra, Δ takes the following form:

$$\Delta = 1 - \left(1 + \alpha\left(\frac{\mathbb{E}\left\{\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right\}}{\mathbb{E}\left(R\left(\theta\right)\right)} - 1\right)\right)^{1/\gamma} \cdot \left\{1 - \frac{\alpha}{\gamma}\left(\frac{\mathbb{E}\left\{\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right\}}{\mathbb{E}\left(R\left(\theta\right)\right)} - 1\right)\right\}$$

Our next result provides comparative statics of $|\hat{k}/k^* - 1|$ and Δ with respect to the returns to scale γ and the ratio $\mathbb{E}\left\{\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right\}/\mathbb{E}\left(R\left(\theta\right)\right)$ of expected market to dividend returns.

Proposition 4 ***: Rent manipulation and efficiency losses increase with market frictions and returns to scale.***

(i) ***Comparative Statics:*** *$|\hat{k}/k^* - 1| = \Delta = 0$ only if $\mathbb{E}\left\{\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right\} = \mathbb{E}\left(R\left(\theta\right)\right)$ or $\gamma \to \infty$. $|\hat{k}/k^* - 1|$ and Δ are decreasing in γ and increasing in $|\mathbb{E}\left\{\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right\}/\mathbb{E}\left(R\left(\theta\right)\right) - 1|$.*

(ii) ***Bounded Distortions on the Downside:*** *If $\mathbb{E}\left\{\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right\} < \mathbb{E}\left(R\left(\theta\right)\right)$, then $\lim_{\gamma\to 0} \hat{k}/k^* = 0$ and $\lim_{\gamma\to 0} \Delta = 1$.*

(iii) ***Unbounded Distortions on the Upside:*** *$|\hat{k}/k^* - 1|$ and Δ become infinitely large, if either $\mathbb{E}\left\{\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right\}/\mathbb{E}\left(R\left(\theta\right)\right) \to \infty$, or $\gamma \to 0$ and $\mathbb{E}\left\{\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right\} > \mathbb{E}\left(R\left(\theta\right)\right)$.*

(iv) ***Negative Expected Dividends:*** *The implemented investment level leads to negative expected dividends whenever*

$$\alpha\left(\frac{\mathbb{E}\left\{\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right\}}{\mathbb{E}\left(R\left(\theta\right)\right)} - 1\right) > \gamma.$$

The ratio of expected market to dividend returns governs the initial shareholders' incentive to distort their marginal costs. The returns to scale parameter γ in turn translates these marginal cost distortions into investment distortions. When γ is very low, the firm operates close to constant returns (γ is close to 0), the optimal investment level is very sensitive to changes in the marginal return expectations, or to incentives that put more weight on market values. In that case, the incentive to manipulate the rent, as well as the associated efficiency losses can become very large. At the other extreme, investment distortions and welfare losses are small if marginal costs are very sensitive to the investment level.

In extreme cases, welfare losses exceed 100% of the first-best welfare level, i.e. the firm's investment ends up with negative expected cash flows, and thus destroys value. This occurs, whenever the elasticity of marginal costs γ is less than the return distortion, which is given by the distance of the return ratio from 1, multiplied by the fraction of shares sold. Even a small departure from the efficient markets benchmark (in terms of $\mathbb{E}\left\{\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right\}/\mathbb{E}\left(R\left(\theta\right)\right)$) can thus have very large efficiency consequences for firms that operate near constant returns, and with investments that are characterized by upside risk. On the other hand, with under-investment the firm's expected dividends always remain positive.

4 Distorting the response to public news

Here we enrich our model to consider the response of market prices, expected market returns and investment to public information disclosures. The same forces that bias the weight the market price attributes to the endogenous signal z also reduce the weight attributed to external news. As a result, signals that offer accurate forecasts of fundamental returns may be far less accurate predictors of future market returns, and in some cases even move expectations of market returns and investment in an opposite direction of expectations about future dividends. These distortions in the response of investment to new information may be severe enough that it may be better from a welfare standpoint to withhold certain information.

Formally, suppose that the manager can condition the investment on a public signal $y \sim \mathcal{N}\left(\theta,\kappa^{-1}\right)$, which is also available to investors in the market. To simplify, we further assume that $\alpha=1$, i.e. initial shareholders sell their entire share and only care about expected market returns. The contract design problem implements an investment function $k\left(y\right)$ instead of just a single investment level, and the share price is conditioned on y as well as z. At the efficient markets benchmark, the price equals the expected dividend value $V\left(y,z;k\right)$. Incentivizing the manager with shares yields an investment decision $k^{*}\left(y\right)=\psi\left(\mathbb{E}\left(R\left(\theta\right)|y\right)\right)$ that maximizes $\mathbb{E}\left(V\left(y,z;k\right)\right)$ and incorporates the information contained in y optimally according to Bayes' Rule. As the signal becomes infinitely precise, $k^{*}\left(y\right)$ converges in probability to the first-best decision rule $k^{FB}\left(\theta\right)$.

Conditional on the realization of y, the contracting problem and investment decisions are characterized exactly as in the previous section, with the minor modification that the prior at the market stage is adjusted to reflect the information contained in the public signal. That is, we now set $\mu = \frac{\kappa}{\lambda+\kappa} y$ (instead of 0) for the expectation and $\hat{\lambda} = \lambda + \kappa$ (instead of λ) for the precision at the market stage. With these adjustments, the $P(y, z; k)$ and $V(y, z; k)$ take the form

$$\begin{aligned} P(y,z;k) &= \int R(\theta)\, d\Phi\left(\sqrt{\lambda+\kappa+\beta+\beta\delta}\left(\theta - \frac{\kappa y + (\beta+\beta\delta) z}{\lambda+\kappa+\beta+\beta\delta}\right)\right) \cdot k - C(k) \\ V(y,z;k) &= \int R(\theta)\, d\Phi\left(\sqrt{\lambda+\kappa+\beta\delta}\left(\theta - \frac{\kappa y + \beta\delta z}{\lambda+\kappa+\beta\delta}\right)\right) \cdot k - C(k). \end{aligned}$$

The market price thus overweighs the market information z, but reduces the weight attached to the public signal y. The optimal investment is obtained from the same first-order condition after conditioning the expected market returns on y, $C'\left(\hat{k}(y)\right) = \mathbb{E}\left\{\mathbb{E}\left(R(\theta) | x = z, z\right) | y\right\}$. Conditional on y, the market-implied uncertainty about θ is $1/\lambda_P(\kappa) = (\lambda+\kappa)^{-1}\left(1 + \gamma_P^2/\gamma_V - \gamma_P\right)$, where

$$\gamma_P(\kappa) = \frac{\beta+\beta\delta}{\lambda+\kappa+\beta+\beta\delta} \text{ and } \gamma_V(\kappa) = \frac{\beta\delta}{\lambda+\kappa+\beta\delta}$$

are adjusted for κ. Our next result discusses how λ_P changes with κ. Recall that under Bayes' Rule, the posterior precision increases one-for-one with κ.

Lemma 1 ***Disclosures reduce uncertainty about market returns by less than uncertainty about future dividends.***

(i) $\lambda_P(\kappa)$ *is increasing in* κ. *However,* $\lambda_P'(\kappa) < 1$. $\lim_{\kappa\to\infty} \lambda + \kappa - \lambda_P(\kappa) = (\beta+\beta\delta)/\delta$.

(ii) $\lambda_P(\kappa)$ *and* $\lambda_P'(\kappa)$ *may be arbitrarily low, if* δ *is low and* $(\lambda+\kappa)/\beta$ *is not too large.*

(iii) $\lambda_P(\kappa)/(\lambda+\kappa)$ *reaches a minimum when* $\lambda+\kappa = \beta+\beta\delta$.

These comparative statics are illustrated in the following figure **(to be added)**. Disclosures reduce market uncertainty less than one-for-one, and therefore have the potential of widening the gap between the uncertainty about market returns and uncertainty about dividends, which in turn affects investment decisions. At the contracting stage, the news are used to forecast the likely market reaction, i.e. y serves as a signal of z. However, since the market puts excessive weight on z, the noise in z coming from u (noise traders) limits how much information disclosures can reduce the uncertainty about market returns that is measured by λ_P. That's why at low levels of κ, the information disclosure less effective at reducing market uncertainty than objective uncertainty (and in some cases far less effective). The relative size of the gap between $\lambda_P(\kappa)$ and $\lambda+\kappa$ then reaches a maximal level when $\lambda+\kappa = \beta+\beta\delta$. Only once the public signal is sufficiently precise (i.e. in the limit as $\kappa \to \infty$) does the public information disclosure lead to a reduction in market uncertainty, relative to the fundamental uncertainty.

The observation that public news may be particularly ineffective at reducing market uncertainty is at the heart of how the information distorts investment. The impact of the news comes on the one hand through its impact on market frictions (i.e. the effect of κ on λ_P), and on the other hand through how the realization of y shifts perceptions of upside or downside risk. We illustrate these two channels through two examples.

Exponential dividends. Suppose that $R(\theta) = e^{\theta}$. In this case, we have

$$\begin{aligned} C'(\hat{k}(y)) &= \hat{k}(y)^{\gamma} = \mathbb{E}\{\mathbb{E}(R(\theta)|x=z,z)|y\} = e^{\frac{\kappa}{\lambda+\kappa}y+\frac{1}{2}\lambda_P^{-1}}, \text{ and} \\ C'(k^*(y)) &= k^*(y)^{\gamma} = \mathbb{E}(R(\theta)|y) = e^{\frac{\kappa}{\lambda+\kappa}y+\frac{1}{2}(\lambda+\kappa)^{-1}}. \end{aligned}$$

Therefore, the investment distortion is independent of y: $\hat{k}(y)/k^*(y) = e^{\frac{1}{2\gamma}(\lambda_P^{-1}-(\lambda+\kappa)^{-1})}$ is constant and depends only on the gap between market uncertainty λ_P^{-1} and objective uncertainty $(\lambda+\kappa)^{-1}$. Although $\lambda_P^{-1} - (\lambda+\kappa)^{-1}$ is decreasing in κ (i.e. the public signal reduces the distortion), our next proposition shows that if the frictions are sufficiently severe to cause expected dividends to be negative, provisions of noisy public information may lead to further reduction in welfare.

Proposition 5 ***Noisy public news may reduce welfare.***

For any λ, β, and δ, if γ is sufficiently small, then there exists $\hat{\kappa}$ such that expected welfare is negative and decreasing in κ for $\kappa \in [0, \hat{\kappa}]$.

This result illustrates that, in the exponential example, information disclosures can have potentially adverse effects. Using the same notation as in the previous section, let $\hat{V}(y)$ the expected dividend at the equilibrium investment, $V^*(y)$ the welfare level at the efficient investment, and $\Delta(y) = 1 - \hat{V}(y)/V^*(y)$ the reduction in welfare at the equilibrium, relative to the efficient welfare level, all conditional on y. Then the ex ante welfare can be written as

$$\mathbb{E}\left(\hat{V}(y)\right) = \mathbb{E}(V^*(y)) - \mathbb{E}(V^*(y)\cdot\Delta(y)).$$

The first term here measures the welfare level at the efficient benchmark, while the second term measures the welfare loss due to investment distortions. An improvement in information unambiguously increases the first term, but this is not sufficient to guarantee an overall welfare increase if at the same time the distortions get worse. In fact, this will typically be the case if the public information offers the initial shareholders an additional dimension along which they are able to influence the rents.

This is exactly what happens in the exponential example. Notice that here, $\Delta(y)$ is independent of y and only depends on κ. Taking derivatives w.r.t. κ, we have

$$\frac{\partial \mathbb{E}\left(\hat{V}(y)\right)}{\partial \kappa} = \frac{\partial \mathbb{E}\left(V^*(y)\right)}{\partial \kappa}(1-\Delta) - \frac{\partial \Delta}{\partial \kappa}\mathbb{E}\left(V^*(y)\right).$$

It's straight-forward to check that in the exponential example $\frac{\partial \Delta}{\partial \kappa} < 0$, so the second term is positive, but if $1-\Delta < 0$, the first term will be negative. Moreover, as $\gamma \to 0$, $\mathbb{E}\left(V^*(y)\right) \to 0$ and $\Delta \to \infty$, so the increase in distortion ends up unambiguously dominating the welfare effect of information provision. The end result is that public information worsens rent extraction and reduces welfare. Information is welfare-improving only once it is sufficiently precise to crowd out the rent-seeking motive, so that the firm returns to positive expected dividends.

Proposition 5 resembles Morris and Shin (2002)'s result that noisy public information reduces welfare in beauty contest games. While both results build on a similar distortion in the use of public information, our set-up and intuition are different. Here, the frictions in financial markets are key in generating the rent-seeking activities that cause information processing to be distorted – information provides initial shareholders with one further dimension of flexibility along which they can optimize their rent. In Morris and Shin instead, a rent-dissipation game motivated the zero-sum reduced-form coordination motives, but financial markets were not modeled explicitly.

Threshold returns. Next, we explore how the realization of y influences investment $\hat{k}(y)$. The impact of y can be decomposed into an effect on return expectations $\mathbb{E}\left(R(\theta)|y\right)$, which raises efficient as well as implemented investment levels, and its impact on the perception of upside vs. downside risk and the magnitude of the distortions. To analyze this channel, we assume that $R(\cdot)$ takes a value of 1, if and only if θ exceeds some threshold $\bar{\theta}$, and takes a value of 0 if $\theta \leq \bar{\theta}$. With the public signal, the investment distortion is then characterized by

$$\frac{C'\left(\hat{k}(y)\right)}{C'\left(k^*(y)\right)} = \frac{\mathbb{E}\left\{\mathbb{E}\left(R(\theta)|x=z,z\right)|y\right\}}{\mathbb{E}\left(R(\theta)|y\right)} = \frac{\Phi\left(\sqrt{\lambda_P}\left(\frac{\kappa}{\lambda+\kappa}y-\bar{\theta}\right)\right)}{\Phi\left(\sqrt{\lambda+\kappa}\left(\frac{\kappa}{\lambda+\kappa}y-\bar{\theta}\right)\right)}.$$

There is over-investment if the return is viewed as an upside risk ($\frac{\kappa}{\lambda+\kappa}y < \bar{\theta}$), and under-investment otherwise. This distortion however is non-monotone in y: it is unbounded on the upside, as $\frac{\kappa}{\lambda+\kappa}y-\bar{\theta} \to -\infty$, but bounded away from 0 as $\frac{\kappa}{\lambda+\kappa}y-\bar{\theta} \to \infty$, and reaches a minimum at a value of y, such that $\frac{\kappa}{\lambda+\kappa}y > \bar{\theta}$. Intuitively, with sufficiently good news ($\frac{\kappa}{\lambda+\kappa}y >> \bar{\theta}$), the market expectation that $R(\theta)=1$ is close to the objective one, and both are sufficiently close to 1 that the resulting wedge has only very minor effects on investment. With terribly bad news ($\frac{\kappa}{\lambda+\kappa}y << \bar{\theta}$), the wedge may be large in relative terms, but remains small in absolute terms, since the market and objective probabilities of a success are close to 0. The distortion is therefore largest at a positive, intermediate level of $\frac{\kappa}{\lambda+\kappa}y - \bar{\theta}$.

If the distribution of $R(\cdot)$ is symmetric ex ante ($\bar{\theta} = 0$), there is no investment distortion when $\kappa = 0$ ($\hat{k} = k^* = \psi(1/2)$). The same remains true when $\kappa > 0$ and $y = 0$. However, for $y \neq 0$, it follows from $\lambda_P < \lambda + \kappa$ that $k^*(y) > \hat{k}(y) > \psi(1/2)$ for $y > 0$, and $\psi(1/2) > \hat{k}(y) > k^*(y)$ for $y < 0$, i.e. the contracted investment choice *under-reacts* to the arrival of public news, compared to what the efficient investment level would prescribe. While the ex ante risk is symmetric, a positive realization of y shifts the expectation up, but the remaining uncertainty more to the downside - hence the implemented investment will be below the efficient level. A negative realization of y on the other hand shifts the expectation down, but the remaining uncertainty is shifted to the upside, so the implemented investment level is above the efficient one.

By far the most interesting effects arise at intermediate realizations of y and if the ex ante return distribution is asymmetric ($\bar{\theta} \neq 0$). Consider the case where $\bar{\theta} < 0$, i.e. ex ante the return is a downside risk.[9] Consider the realization of "moderately bad" signals, in which the news are not good, but they are also not so terrible that a failure seems likely. If the news suggest that the prior was too optimistic about θ, then the resulting shift in posterior expectations will have a far bigger effect on the market's assessment of the tail probability of failure than on the objective assessment. Since the market will turn pessimistic much faster than the objective risks, seemingly harmless signals can trigger exaggerated shifts in prices. Anticipating this, the news cause investment to drop, even though the objective risk of a failure remains low.

Proposition 6 ***Moderately bad news about downside risks cause unjustified market fears and investment panics.***

(i) If $\bar{\theta} < 0$, then the provision of news amplifies investment distortions on the downside.

(ii) For some interval of signal realizations, $\hat{k}(y) < \hat{k} < k^* < k^*(y)$.

The proposition is a direct consequence of the observation in lemma 1 that public news provision is not effective at "calming the market", i.e. at reducing the market uncertainty about fundamentals. Consider for example a signal realization of y, for which the objective default probability remains small. Because $\sqrt{(\lambda + \kappa)/\lambda_P} > 1$, the same signal realization may turn out to be much more worrisome from the perspective of market expectations. For example, if $\sqrt{\lambda+\kappa}\left(\frac{\kappa}{\lambda+\kappa}y - \bar{\theta}\right) = 3$, the default threshold is more than 3 standard deviations away from the posterior expectation

[9] This corresponds, for example to a bond, with a risk of default if the bond fundamental falls below a certain threshold $\bar{\theta}$.

of fundamentals, and hence the perceived default risk is less than 0.15%. At the same time, if $\sqrt{(\lambda+\kappa)/\lambda_P} = 1.5$, the market-implied standard deviation is 50% higher than the objective one, and the market's perception of the default risk is ca. 2.3%, i.e. roughly 15 times as high as the objective risk. Once signals move into this range, the expected market returns quickly deteriorate on the fears that a low realization of z will lead to a far lower market return.

In some cases, news that is objectively reassuring and increases the efficient investment level instead lowers the chosen investment level. Since $\lambda_P(\kappa) < \lambda + \kappa$ and $\frac{d\lambda_P(\kappa)}{d\kappa} < 1$, there exist realizations of $y < 0$ for which

$$\sqrt{\frac{\lambda}{\lambda+\kappa}} < \frac{\frac{\kappa}{\lambda+\kappa}y - \bar{\theta}}{-\bar{\theta}} < \sqrt{\frac{\lambda_P(0)}{\lambda_P(\kappa)}}$$

i.e. news that actually lower the default risk relative to the prior (inequality on the left) cause the market-implied default risk to go up (inequality on the right, where $\lambda_P(\kappa)$ and $\lambda_P(0)$ denote the market-implied precisions with and without the signal).

Here, the excess uncertainty about the market price causes shareholders to overreact in their investment choices. It is precisely when news about the returns suggest that "there are some concerns, but objectively the risks are very minor" that the initial shareholders will start to worry about the market reaction. Even if they fully agree with the news, they worry that the market doesn't recognize this and instead reacts strongly to the downside. The initial shareholders thus face the risk of a lower market return, to which they respond by lowering their investment. More often than not the fears about the market turn out to be unjustified ex post, but this is of little help to the initial shareholders when selling their shares. With downside fears about the market price, the shareholder's optimal investment strategy is "better be safe than sorry".

With upside risk ($\bar{\theta} > 0$), investment overreacts to moderately good news. In other words, *moderately good news about upside returns cause investment booms and irrational exuberance.* Think for example about news regarding a potential technological breakthrough that is unlikely yet not impossible. In this case there exists a range of moderately positive news, i.e. news that improve the prior of θ but ultimately have little effect on the likelihood of success, which trigger overly optimistic market expectations about returns and excessive investment by the firm. The formal argument is the flipside of the previous example so we will not repeat it here.

With upside risk, mildly positive news can have magnified effects on shareholder expectations of market returns. In the same way as moderately bad news triggered excessive fears about downside risk, mildly positive news generate excessive hopes about the realization of upside risk. Initial shareholders knowingly over-invest in the hope that the market will become even more exuberant and they are able to sell their shares at inflated prices. In the end, more often than not these expectations about the market are too good to be true.

5 Distorting the response to market signals

In this section, we consider how investment decisions distort the use of information aggregated through share prices. We modify our benchmark model by assuming that the investment decision is implemented after the price is realized, allowing for informational feedback effects. The initial shareholders incentivize the manager to implement a price-contingent investment rule $k(\cdot)$ that internalizes the impact of its decisions on the share price. The wage contract is then designed to align the manager's ex post incentives with the shareholders' ex ante objective, so that the managers have no incentive to alter their behavior ex post (what's more, if $\alpha > 0$, such commitment is valuable to the initial shareholders). Effectively the model implies that the firm pre-commits to an investment rule before the market opens, and the market can perfectly anticipate the investment level that will realize given the price.[10]

We depart from the analysis in section 3 by assuming that the investment rule is a price-contingent, or equivalently z-contingent function $k(z)$. Since the market can still perfectly anticipate the investment level that will realize at a given price, nothing changes from the characterization in proposition 1, except that k is replaced by $k(z)$. We further assume that there exists a unique $\hat{z}$ s.t. $\mathbb{E}(R(\theta)|x=z,z) \gtreqless \mathbb{E}\{R(\theta)|z\}$ for $z \gtreqless \hat{z}$.[11] For a given $k(z)$, the equilibrium share price, is

$$P(z) = P(z, k(z)) = \mathbb{E}(R(\theta)|x=z,z) \cdot k(z) - C(k(z)),$$

The expected dividend value takes the form

$$V(z) = V(z, k(z)) = \mathbb{E}(R(\theta)|z) \cdot k(z) - C(k(z)).$$

To constitute an equilibrium, it must be the case that $P(z)$ is strictly monotonic in z, a condition that is no longer automatically satisfied due to the endogenous feedback from z to k.

[10] Instead of thinking of the pre-commitment to a rule as the result of the manager's incentive contract, we can also interpret this as a model of explicit agreement to a rule before the market opens, and the internal reporting and decision procedures in place generate the commitment that make it difficult to revisit the decision afterwards. The important feature here is that final shareholders are unable to renegotiate the contract terms with the manager once they have taken control, and before the investment is implemented.

[11] This condition is satisfied for many return distributions, and is completely unrelated to our concepts of upside vs. downside risks: For any bounded return $R_1(\theta)$ with upside risk that satisfies the condition, the same condition also holds for $R_2(\theta) = R - R_1(-\theta)$.

Lemma 2 *Suppose that*

$$1 \geq \frac{k'(z)/k(z)}{\frac{\partial \mathbb{E}(R(\theta)|x=z,z)}{\partial z} / \mathbb{E}(R(\theta)|x=z,z)} \left(\frac{C'(k(z))}{\mathbb{E}(R(\theta)|x=z,z)} - 1 \right).$$

Then, $P(z,k(z))$ is strictly increasing in z.

The condition in this lemma highlights that monotonicity of $P(z)$ requires the investment rule to be not too responsive to z in states where the firm invests more than what the market would like to see. Below, we will assume that monotonicity is alway satisfied. Due to an envelope condition, this holds automatically for $\alpha = 1$, i.e. the chosen investment rule maximizes the share price conditional on z. For $\alpha < 1$, monotonicity is not automatic unless one imposes additional restrictions on the shape of returns, since monotonicity may fail when the return $\mathbb{E}(R(\theta)|x=z,z)$ that is expected by the market is significantly below the objective return, $\mathbb{E}(R(\theta)|z)$, and the initial shareholders put sufficient weight on the latter (i.e. α is close to 0).[12]

The optimal investment rule $\hat{k}(z)$ maximizes the initial shareholder's objective function conditional on z, $(1-\alpha)V(z,k) + \alpha P(z,k)$. With market frictions, $\hat{k}(z)$ satisfies

$$C'\left(\hat{k}(z)\right) = \alpha \mathbb{E}(R(\theta)|x=z,z) + (1-\alpha)\mathbb{E}(R(\theta)|z).$$

If markets were efficient ($V(z,k) = P(z,k)$), the implemented decision rule $k^*(z) = \psi(\mathbb{E}\{R(\theta)|z\})$ incorporates the information contained in the price optimally according to Bayes' Rule. Relative to $k^*(z)$, the implemented investment tilts investment in the direction dictated by the market expectations of returns. It aligns marginal costs of investment more closely with the market's expected returns, conditional on the realization of z.

Information Feedback. Our first result shows that the informational feedback from share prices to investment creates an endogenous element of upside risk. This effect comes from the value of aligning investment with the information contained in the share price and is present even with the efficient investment rule.

Proposition 7 ***Informational Feedback creates endogenous upside risk and increases initial shareholder rents.***

(*i*) ***Increased Shareholder Rents:*** *For any strictly increasing investment function $k(z)$, $\mathbb{E}(\Omega(z,k(z))) > \mathbb{E}(\Omega(z,k(\hat{z})))$.*

[12] For example, it suffices to assume that $\lim_{\theta \to -\infty} R(\theta) \geq \mathbb{E}\{R(\theta)|\hat{z}\}/(1+\gamma)$, or to assume that the firms' dividend includes an additional component that is increasing in θ but not affected by investment.

(*ii*) ***Endogenous Upside Risk:*** $\mathbb{E}(\Omega(z, k(z))) > 0$ *if either* $\mathbb{E}(\mathbb{E}(R(\theta)|x = z, z)) \geq \mathbb{E}(R(\theta))$, *or* $\mathbb{E}(\mathbb{E}(R(\theta)|x = z, z)) < \mathbb{E}(R(\theta))$ *and* $\inf_z k'(z)/k(z)$ *is sufficiently large.*

(*iii*) ***Unbounded Rents:*** *If* $\inf_z k'(z)/k(z) \to \infty$, *then* $\mathbb{E}(\Omega(z, k(z))) \to \infty$, *for any* $R(\cdot)$.

Therefore, any investment strategy that responds positively to the news in z also tilts the initial shareholder's expected rents to the upside. If the underlying risk was (weakly) upside dominated, the feedback effect strengthens the upside bias and increases initial shareholder rents. If instead the underlying risk was dominated by the downside, then the endogenous investment response mitigates the downside exposure, and if investment is sufficiently responsive it overturns it. In the limit where investment becomes infinitely responsive, the initial shareholder rents are positive and arbitrarily large regardless of the underlying $R(\cdot)$.

We can write the expected wedge as

$$\mathbb{E}(\Omega(z, k(z))) = \mathbb{E}(\Omega(z, \mathbb{E}(k(z)))) + cov(k(z); \mathbb{E}(R(\theta)|x = z, z) - \mathbb{E}\{R(\theta)|z\})$$

The expected wedge consists of two terms. The first term, $\mathbb{E}(\Omega(z, \mathbb{E}(k(z))))$, corresponds to the expected wedge when investment is fixed at its unconditional expectation $\mathbb{E}(k(z))$. The second term, $cov(\mathbb{E}(R(\theta)|x = z, z) - \mathbb{E}\{R(\theta)|z\}; k(z))$, captures the fact that investment responds to the information contained in z. Therefore, the feedback from prices to investment creates endogenous upside risk, and increases rents to initial shareholders. The response of investment leads to more risk-taking on the upside, when the realization of z is positive, and to less risk-taking on the downside. Thus, endogenously the amount of exposure increases with expected returns. This result arises whenever investment responds positively to the news in z.

Investment Distortions with Informational Feedback. Next, we discuss how the implemented investment rule distorts the information coming from z. The implemented investment rule $\hat{k}(z)$ equates marginal cost to a weighted average of the market's expected return and the expected dividend return, conditional on z. The signal z thus receives more weight in forming posterior expectations about θ than would be justified from an objective point of view.

At signal realization $\hat{z}$, $\mathbb{E}(R(\theta)|x = z, z) = \mathbb{E}(R(\theta)|z)$, so regardless of α, the implemented investment level will be the efficient one, $\hat{k}(z) = k^*(z)$. Away from $\hat{z}$, the response to market prices is distorted in the direction of market expectations, and the distortions become larger the larger is α. Therefore, for $z > \hat{z}$, $\hat{k}(z) > k^*(z)$, while for $z < \hat{z}$, $\hat{k}(z) < k^*(z)$. Thus, by shifting the weight given to z in the direction of the market's expectations, the implemented investment rule magnifies the response of investment to the information contained in the price. These results are summarized in the next proposition:

Proposition 8 ***Market noise causes excess investment volatility***

(i) ***Excess investment sensitivity:*** *The investment distortion* $\left|\hat{k}\left(z\right)/k^{*}\left(z\right)-1\right|$ *is increasing in* α*, and increasing in* z *whenever* $\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)/\mathbb{E}\left(R\left(\theta\right)|z\right)$ *is increasing in* z*.*

(ii) ***Fundamentals vs. Market Noise:*** *If market noise is sufficiently important, then volatility of investment is high, but the correlation of investment with future returns is low.*

(iii) ***Unbounded rents and welfare losses:*** *If the market friction is sufficiently important or* γ *sufficiently close to* 0*,* $\mathbb{E}(\Omega(z,\hat{k}\left(z\right)))$ *becomes arbitrarily large, and* $\mathbb{E}(V(z,\hat{k}\left(z\right)))$ *lower than if investment was determined before the market opens.*

The rent manipulation logic of the two previous sections extends to the setting in which investment is implemented after the market opens. Here, the initial shareholders can manipulate the rents not just through the average level of investment, but also through its response to the information z. Effectively, by conditioning investment on the share price, initial shareholders have access to an additional marginal along which they can maximize their rents. Since initial shareholder rents are increasing in the sensitivity of investment to z, shareholders take advantage by incentivizing an investment rule that is excessively responsive to market information: the signal z receives too much weight relative to the information it conveys. This causes excess volatility in investment: on the upside, shareholders encourage too much investment in order to maximize the positive rents they extract from inflated share prices. On the downside, they encourage under-investment in order to limit the losses they incur from the market price being below the fundamental value. If $\alpha=1$, the investment equates marginal costs to the expected market return conditional on z, but the latter can be far more volatile than the fundamental return expectation, if information frictions are sufficiently important.

This channel not only increases the volatility of investment, but it also reduces the connection of investment decision from information about fundamentals. A high price translates into a higher investment level, but the increase in investment and marginal costs exceeds the increase in fundamental return expectation. If the market is sufficiently noisy, i.e. $\beta\delta$ is small relative to λ and β, it may be that prices are almost exclusively driven by market noise, and carry little information about fundamental returns, yet investment responds aggressively in order to capture rents on the upside or limit losses on the downside. In the limiting case where investment is orthogonal to fundamentals, the feedback from prices to investment leads to strictly lower efficiency than a fixed ex ante investment decision. In this case, investment volatility purely serves to increase upside risk and extract rents form final shareholders, and is strictly welfare reducing, even though an efficient use of the same information could entail large welfare gains.

6 Designing the compensation scheme

Next we consider the design of the manager's compensation. Clearly, the efficient investment k^* is implemented if $W^*(\Pi) = \omega\Pi$, i.e. if the manager's compensation is linear in the final dividend. We will call this a "restricted equity" contract, since the manager cannot sell the equity share in the market. Compensation floors (i.e. stock options) and ceilings can be used to modify incentives, relative to this baseline. Let $\underline{R} = \lim_{\theta\to-\infty} R(\theta)$ and $\bar{R} = \lim_{\theta\to\infty} R(\theta)$. The interval $\left(\psi(\underline{R}), \psi(\bar{R})\right)$ contains all investment levels that are efficient for some θ; investment levels outside cannot be optimal for the initial or final shareholders under any circumstances. We show that any $k \in \left(\psi(\underline{R}), \psi(\bar{R})\right)$ is implementable by a contract that adds either a floor or a ceiling to one of these two benchmark contracts.

Proposition 9 ***(Almost) anything is implementable with equity, options, and caps.***

(i) Any $k \in \left(k^*, \psi(\bar{R})\right)$ can be implemented with a contract of the form $W(\Pi) = \max\{\underline{W}, \omega\Pi\}$. Any $k \in (\psi(\underline{R}), k^*)$ can be implemented with a contract of the form $W(\Pi) = \min\{\bar{W}, \omega\Pi\}$.

(ii) Any $k \notin \left(\psi(\underline{R}), \psi(\bar{R})\right)$ cannot be implemented through a Π-contingent contract $W(\cdot)$.

Thus, introducing a minimal compensation level into the benchmark contract $W^*(\Pi)$ strengthens incentives by making manager payoffs more convex, thus increasing investment from a benchmark level of k^*. Introducing a cap on total compensation on the other hand concavifies payoffs and weakens incentives, thus lowering investment from the benchmark levels. Thus stock options or compensation floors, or the use of total compensation ceilings can be used to distort investment decisions in any direction and to any level within the range $\left(\psi(\underline{R}), \psi(\bar{R})\right)$ that is desired by initial shareholders, with contracts just conditioned on final dividends.

When $\alpha > 0$, two possibilities arise. If $\hat{k} > k^*$, $W^*(\Pi)$ too little investment relative to the optimal level. An optimal contract will add a compensation floor to $W^*(\Pi)$. If instead, $\hat{k} < k^*$, the incentives provided by $W^*(\Pi)$ are too strong and need to be modified with a cap on total compensation. In our model, α governs not only the extent to which there is a conflict of interest between initial and final shareholders, but also the misalignment between initial shareholders and manager incentives at the two baseline contracts. Under $W^*(\Pi)$, initial shareholders are more focused on the short-term share price than managers, so wage floors and ceilings are used to strengthen incentives and increase distortions.

The proposed contracts have all assumed that k is not conditioned on any additional information and therefore apply directly to the problem studied in section 3. They also apply to the problem with additional public information, provided that the contract terms (i.e. the floors and ceilings) can adjust to the additional signal. Finally, they also implement the optimal investment rule of the model with feedback in section 5, provided that the terms are conditioned on the price. That is, in section 5, the implementation problem is fully solved through the provision of a restricted share contract that includes either a floor or a ceiling, whose exact value is determined by P. Here, price-contingent options and ceilings are crucial to generate managerial commitment to investment strategies that maximize initial shareholder payoffs even beyond the horizon at which these shareholders are actually owning the firm. This commitment is valuable to initial shareholders because the rents generated from market frictions also create time inconsistency in managerial incentives.

Thus, to summarize, the investment distortions do not result from an artificial restriction on the contract space, since any targeted investment level can be implemented without agency costs through a Π-contingent contract. Even if the contract design could condition arbitrarily on other variables such as the share price, the investment level or even the realized θ, initial shareholders would still want to distort investment to influence the market price. The implementation possibilities only expand with a richer contract space.[13]

Our next result generalizes these observations to show that k^* can only be implemented if initial shareholders are not given any flexibility to distort incentives in their desired direction. Suppose that the initial shareholders can use $N+1$ "securities" or "instruments" to compensate the manager. Each security $n = 0, ..., N$ is defined by a an expected transfer $T_n(k)$ to the manager as a function of the chosen k. Each $T_n(\cdot)$ is assumed to be bounded and continuously differentiable, and that $T_0(k^*) > T_0(k)$ for all $k \neq k^*$, so that k^* is implemented by $T_0(\cdot)$.

Proposition 10 ***Efficient investment requires sharp restrictions on incentive pay.***
The initial shareholders will implement $k \neq k^*$ *whenever* $\left(\hat{k} - k^*\right) T_n'(k^*) > 0$ *for some* n.

In other words, the efficient investment level will be implemented if and only if there exists a security for which the implemented investment choice is k^*, and all other securities would lead to an investment distortion that is opposite of what the initial shareholders would like to do. As soon as there is one security that can be used to push the incentives in the desired direction, the initial shareholders will find it optimal to do so. Even the minimal qualification above can be dispensed with, if negative positions are allowed by the contract (i.e. a contract that leverages

[13] Notice however that in our model, price contingent contracts would have no effects on incentives, unless the investment was publicly and perfectly observable by the market. With noise in the observation, the market would always assume that an observation different from the equilibrium level was caused by noise. Likewise, in the model considered in section 5, where investment takes place after the market opens, price-contingent compensation would have no impact on incentives, since the price is already determined by the time investment is set.

different securities against each other), so that any security can be used to distort in any direction. In practical terms, investment distortions are unavoidable, unless there is an almost complete ban on the use of any compensation contract other than a restricted equity contract.

7 Regulation and Policy Interventions

In this section, we use our model to discuss policy or regulatory interventions that have either been proposed in the ongoing discussion on financial reform, or actively implemented by policy makers in some form or another in the current crisis. These policies can be grouped into two categories: policies that take the market outcome as given and target firm behavior to reduce the initial shareholders' ability to extract rents, such as regulation of executive compensation, caps on investment size, or other direct regulation of risk-taking behavior by firms, and policies that target the market outcome and affect incentives by modifying investment returns, such as financial transaction taxes or direct market interventions. At the efficient markets benchmark, any intervention of either kind only leads to distortions in investment behavior and thus reduces welfare. Our model instead offers a clear efficiency rationale for each of these interventions, and provides some additional insights into their respective practical advantages and drawbacks.[14]

Regulation of Executive Pay. Proposition 10 suggests that regulation of executive pay – if done correctly – offers a simple solution to rent-seeking by incumbent shareholders: just ban any form of performance pay other than restricted equity shares. This eliminates the incumbent shareholders' discretion and fully implements the efficient investment levels throughout all three models we considered. This proposal has several important advantages. First, this policy does not require any knowledge on the severity of market frictions, firm technologies, or return distributions. It is thus a simple "one-size-fits-all" policy that is not easily manipulated by incumbent shareholders, and doesn't require fine-tuning or direct regulatory oversight. This is particularly relevant in cases where the return characteristics are either not directly observable, or where they can be tailored by the initial shareholders or the manager. In it's simplest form our analysis abstracts from CEO risk aversion or additional incentive problems which may cause mis-alignment between the manager's and final shareholder preferences. If these additional elements are important, they offer a rationale

[14]These proposals are typically discussed from the perspective of systemic exposures and aims mainly at large financial institution. Our focus is complementary to this discussion since it is more micro-economic in nature and based on partial equilibrium analysis, and we do not target large financial institutions specifically, although the financial sector certainly matches the characteristics of upside risk, large risk-taking, and near constant returns for which our results are particularly relevant.

to allow for some additional discretion in the design of executive compensation. However, as long as there is potential for rent manipulation through investment distortions, our analysis suggests that policy makers should impose some restrictions on executive contract design by limiting how much the executive contract can depart from a pure restricted equity contract. For upside risks, this is equivalent to limiting the grant of stock options relative to restricted equity.[15]

Our analysis also suggests that a partial approach to regulation that focuses on specific instruments (for example, a ban on stock option compensation, or participation in share sales) will not have any impact as long as incumbent shareholders can create the same incentive schemes through other means, for which the possibilities are endless. A successful regulation must thus take a wholesale view and tightly specify the margins along which initial shareholders are given discretion to select the contracting terms.

Size Caps. Certain proposals suggest to limit the size of financial institutions' balance sheets, or to break them up into smaller units. Introducing a cap on investments in our model, $\bar{k}$, such that only $\hat{k} \leq \bar{k}$ can be implemented, has advantages and disadvantages. Because it is a one-sided policy, it only works to limit over-investment, and can never be optimal to correct under-investment. In our baseline model without additional information, the optimal size cap sets $\bar{k} = k^*$, simply preventing firms from over-investing. Notice however that such a cap depends a great deal on the firm's return characteristics and technologies – an important limitation if these firm characteristics are not directly observable to the regulator, or manipulable by the firm. If investment conditions on additional information, a size cap is a particularly blunt tool – once it is binding, new information no longer enters the investment choice, while as long as it is not binding, the cap has no bite on the externalities. The optimal design of a size cap will trade off capping size distortions against losing information. Size caps therefore appear to be most useful as a crude but effective way of eliminating upside distortions, when the resulting information loss is not too costly.[16]

Financial Transaction Tax. A tax on share sales modifies the incentives to distort investments, because it shifts a part of the shareholder rents to the tax authorities. With an uncontingent tax τ on the proceeds of share sales, the initial shareholders maximize $\mathbb{E}\left((1-\tau)\,\alpha P\left(z;k\right)+(1-\alpha)\,V\left(z,k\right)\right)$.

[15] We defer more complete discussion of the incentive problems with additional moral hazard and CEO risk aversion to the next section.

[16] A less crude strategy to accomplish the same objective may be through direct monitoring of risk exposures. This would allow the policy maker to condition the size cap on additional information that the firm would also want to use. But it does not address the issue of asymmetric information about firm characteristics between the firm and the regulator, and may even worsen the resulting agency costs if the additional information increases the scope for rent manipulation.

A tax on share sales thus reduces the relative weight on the share price from α to $(1-\tau)\,\alpha/\,(1-\alpha\tau)$. This reduces the externality, but unless the tax is completely confiscatory, such a tax can never fully correct for the externality. Small transactions taxes have little effect on shareholder incentives.[17]

Taxes have to be state-contingent in order to be effective. Consider a tax scheme $\tau(z)$ that is contingent on the market price. If $\tau(z) = 1 - V(z,k)/P(z;k) = \Omega(z,k)/P(z;k)$, the tax implements the efficient benchmark by directly compensating for the mispricing. The expected tax revenue $\alpha\mathbb{E}(\Omega(z,k))$ captures the entire expected rent from incumbent shareholders. The optimal intervention thus taxes on the upside to subsidize on the downside, and it requires a great deal of information about the firms' returns, market frictions and technologies.[18]

Direct Market Interventions. Market interventions, i.e. policies to purchase or short-sell assets to influence market returns, are another channel through which a policy maker can influence investment incentives. Consider a program such as the Fed's TARP, or the ECB's OMT, in which the policy maker commits to buy shares if the share price falls below a threshold level. Since a price support increases expected market returns, such a policy reduces investment distortions, only if the return distribution represents a downside risk.[19]

It's possible to analyze the effects of simple interventions within the context of our model. Suppose that $R(\cdot)$ is a downside risk which yields ex ante under-investment. Suppose that a policy maker announces a share purchasing program at a guaranteed price $\bar{P}(k) = \bar{R}k - C(k)$, i.e. the policy maker is willing to support a market return $\bar{R}$ for the securities. For any z such that $P(z,k) > \bar{P}(k)$, this intervention has no effect. However, if $P(z,k) < \bar{P}(k)$, the policy maker buys a positive level of shares. This occurs whenever z falls below some threshold $\hat{z}$. This policy automatically limits the investment's downside risk to $\bar{R}$, and the investment first-order condition now takes the form

$$C'\left(\hat{k}\right) = \alpha\mathbb{E}\left(\max\left\{\bar{R}, \mathbb{E}\left(R(\theta)\,|x=z,z\right)\right\}\right) + (1-\alpha)\,\mathbb{E}\left(R(\theta)\right).$$

The price support policy thus strengthens investment incentives, which reduces the downside risk distortions: $\hat{k}$ is strictly increasing in the level of the price support $\bar{R}$. Our next proposition however

[17] All this would change if costs $C(k)$ didn't come out of dividends, but were paid upfront by initial shareholders. In that case a fixed tax on the share price changes the distortion by shifting the ratio of marginal costs to expected returns. The same would apply with an investment tax or a subsidy on investment costs.

[18] We can also discuss dividend taxes. Uncontingent taxes on final dividends have no effect on incentives, since the tax is fully passed through into the share price, so that it doesn't affect the initial shareholders weights attached to share price vs. final dividends. Contingent dividend taxes on the other hand can be used just like contingent transaction taxes to modify expected returns to investment to align initial and final shareholder incentives.

[19] For upside risks, one would have to consider short-selling policies that seek to limit over-pricing.

shows that such policy have a fiscal cost, i.e. the expected revenues from the purchased shares are less than the cost of purchasing them, and therefore the policy provides an implicit subsidy to the initial shareholders. A revenue-neutral form of interventions requires to offset the fiscal cost of the intervention through a transactions or dividend tax, with important distributional consequences on the gains and losses from the intervention.

Proposition 11 ***With downside risk, direct market interventions improve efficiency but come at a cost.***

(i) *Any direct market intervention generates negative expected revenues for the policy maker and increases rents to initial (and sometimes final) shareholders.*

(ii) *The policy maker can achieve the same efficiency gains in a revenue-neutral fashion by combining TARP with a tax on transactions or final dividends, but this unambiguously reduces the welfare of initial shareholders.*

Thus, a pure TARP-like price-support intervention is impossible without subsidizing the initial shareholders. The policy maker's revenues trade off gains from arbitraging the under-pricing against exposure to a winner's curse problem when the informed traders offload their shares. Unfortunately, from the policy makers' perspective the winner's curse always dominates. The second part of the proposition states that it is possible to offset the fiscal burden of the market intervention through a transaction or dividend tax. But this has important distributional effects, because the tax eliminates the subsidy to initial shareholders and in addition correcting the investment distortion shifts the rents from initial to final shareholders (due to downside risk). As a result, initial shareholders unambiguously prefer to avoid revenue-neutral market interventions.[20]

To summarize, regulation of executive pay appears to be by far the most robust and effective means to limiting adverse incentive effects. In our model, the optimal regulation is admittedly stark, i.e. requiring a complete ban on anything other than untraded equity shares, and in particular, stock options or bonuses tied to market performance. While the optimal scheme may differ from untraded equity with risk aversion or other agency frictions, our more general message should be robust: initial shareholders will try to exploit the flexibility they are given to extract rents. Size caps, transaction taxes or market interventions can also help to reduce distortions, but they either require far more complex regulatory oversight (in terms of the firm-specific information about return

[20] In fact, exactly the same arguments can be made about interventions that amount to guarantees on ex post returns, which would for example shift returns from $R(\theta)$ to $\max\{\bar{R}, R(\theta)\}$ but require funding either from outside or through dividend taxation.

distributions, costs, market frictions etc.), or they suffer from other drawbacks, such as limiting the extent to which investment adjusts to valuable information. What's more, incumbent shareholders have a clear preference against regulation if this reduces the rents they can capture by distorting investments, and doesn't offer a compensating subsidy. In fact, among the proposals discussed here, direct interventions without tax offsets are the only policy that incumbent shareholders will potentially support, since this policy directly subsidizes their shareholdings, while other policies just alter incentives without offering compensation for losing shareholder rents.

8 Discussion and Extensions

Financial Market structure.

Manager Risk Aversion and Agency Frictions.

Market Discipline.

Disagreement among initial shareholders.

Dynamics.

9 Conclusion

10 Appendix: Proofs

Proof of Propositions 1- 3. The proof for prop. 1 and 2 is given in the text. Prop. 3 is a direct corollary of Theorem 2 in AHT. ■

Proof of Proposition 4. To simplify notation, let $\Upsilon = \alpha\left(\mathbb{E}\left\{\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right\}/\mathbb{E}\left(R\left(\theta\right)\right)-1\right)$. We begin with the results concerning $\hat{k}/k^*$. Since $\hat{k}/k^* = (1+\Upsilon)^{1/\gamma}$, it is immediate that $\hat{k}/k^*$ is increasing in Υ, equal to 1 if and only if $\Upsilon = 0$, and unbounded as $\Upsilon \to \infty$. Moreover, $\partial\left(\hat{k}/k^*\right)/\partial\gamma^{-1} = \log\left(1+\Upsilon\right)\left(1+\Upsilon\right)^{1/\gamma}$, which is positive if and only if $\Upsilon > 0$. Hence $\hat{k}/k^*$ is decreasing in γ, if $\Upsilon < 0$ and increasing in γ, if $\Upsilon > 0$, which proves that investment distortions are worse, the lower is γ. Finally, if $\Upsilon > 0$, then clearly $\hat{k}/k^*$ is unbounded as $\gamma \to 0$, while if $\Upsilon < 0$, $\hat{k}/k^* \geq (1-\alpha)^{1/\gamma} > 0$.

Next we consider comparative statics w.r.t. Δ. Since $\Delta = 1 + (1+\Upsilon)^{1/\gamma}(\Upsilon/\gamma - 1)$, we have

$$\begin{aligned}
\frac{\partial \Delta}{\partial \Upsilon} &= \frac{1}{\gamma}\frac{1+\gamma}{\gamma}(1+\Upsilon)^{1/\gamma}\frac{\Upsilon}{1+\Upsilon} \text{ and} \\
\frac{\partial \Delta}{\partial \gamma^{-1}} &= (1+\Upsilon)^{1/\gamma}\left(\Upsilon - \log(1+\Upsilon)(1-\Upsilon/\gamma)\right),
\end{aligned}$$

and one therefore obtains that $\Delta = 0$ iff $\Upsilon = 0$, Δ is increasing in Υ (and therefore positive) if $\Upsilon > 0$, and Δ is decreasing in Υ (and therefore again positive) if $\Upsilon < 0$. Furthermore, if $\Upsilon \geq \gamma > 0$, it is clear that $\frac{\partial\Delta}{\partial\gamma^{-1}} > 0$, while if $\Upsilon < \gamma$, $\Upsilon - \log(1+\Upsilon)(1-\Upsilon/\gamma) > \Upsilon - \Upsilon(1-\Upsilon/\gamma) > \Upsilon^2/\gamma$ and so once again $\frac{\partial\Delta}{\partial\gamma^{-1}} > 0$. The limiting behavior, the bounds, and the result that $\Delta > 1$ if $\Upsilon > \gamma$ also follow immediately. ■

Proof of Lemma 1.

Since

$$\frac{\gamma_P(\kappa) - \gamma_V(\kappa)}{\gamma_V(\kappa)} = \frac{\frac{\lambda+\kappa}{\lambda+\kappa+\beta\delta} - \frac{\lambda+\kappa}{\lambda+\kappa+\beta+\beta\delta}}{\frac{\beta\delta}{\lambda+\kappa+\beta\delta}} = \frac{\lambda+\kappa}{\beta\delta}\frac{\beta}{\lambda+\kappa+\beta+\beta\delta} = \frac{1}{\delta}(1-\gamma_P(\kappa)),$$

$\lambda_P(\kappa)$ can be rewritten as

$$\lambda_P(\kappa) = (\lambda+\kappa)\frac{1}{1+\frac{\gamma_P(\kappa)}{\gamma_V(\kappa)}(\gamma_P(\kappa)-\gamma_V(\kappa))} = (\lambda+\kappa)\frac{\delta}{\delta+\gamma_P(\kappa)(1-\gamma_P(\kappa))}.$$

Parts (ii) and (iii) then follow immediately. For part (i), note that $(\lambda+\kappa)\gamma_P'(\kappa) = -\gamma_P(\kappa)(1-\gamma_P(\kappa))$, and therefore

$$\begin{aligned}
\lambda_P'(\kappa) &= \frac{\lambda_P(\kappa)}{\lambda+\kappa}\left(1-(\lambda+\kappa)\gamma_P'(\kappa)\frac{\delta(1-2\gamma_P(\kappa))}{\delta+\gamma_P(\kappa)(1-\gamma_P(\kappa))}\right) \\
&= \frac{\lambda_P(\kappa)}{\lambda+\kappa}\left(1+\frac{\delta\gamma_P(\kappa)(1-\gamma_P(\kappa))(1-2\gamma_P(\kappa))}{\delta+\gamma_P(\kappa)(1-\gamma_P(\kappa))}\right) \\
&= \frac{\delta}{\delta+\gamma_P(\kappa)(1-\gamma_P(\kappa))}\frac{\delta+2\gamma_P(\kappa)(1-\gamma_P(\kappa))^2}{\delta+\gamma_P(\kappa)(1-\gamma_P(\kappa))} \\
&= 1-\left(\frac{\gamma_P(\kappa)}{\delta+\gamma_P(\kappa)(1-\gamma_P(\kappa))}\right)^2\left((\delta+1-\gamma_P(\kappa))^2-\delta^2\right) < 1
\end{aligned}$$

The limiting properties also follow immediately. ■

Proof of Proposition 7. The ex ante expectation of $\Omega(z, k(z))$ is

$$\begin{aligned}
\mathbb{E}\{\Omega(z,k^*(z))\} &= k(\hat{z})\int(\mathbb{E}(R(\theta)\,|x=z,z)-\mathbb{E}(R(\theta)\,|z))\frac{k(z)}{k(\hat{z})}d\Phi\left(\sqrt{\lambda\frac{\beta\delta}{\lambda+\beta\delta}}z\right) \\
&> k(\hat{z})(\mathbb{E}(\mathbb{E}(R(\theta)\,|x=z,z))-\mathbb{E}(R(\theta))) = \mathbb{E}\{\Omega(z,k^*(\hat{z}))\}
\end{aligned}$$

Moreover, if $k'(z)/k(z) \geq \chi$, then $k(z) \geq k(\hat{z})e^{\chi(z-\hat{z})}$ if $z > \hat{z}$ and $k(z) \leq k(\hat{z})e^{\chi(z-\hat{z})}$ if $z < \hat{z}$. Thus, if $\chi \to \infty$, $\mathbb{E}\{\Omega(z, k^*(z))\} \to \infty$, and $\mathbb{E}\{\Omega(z, k^*(z))\} > 0$ for χ sufficiently large. ■

Proof of Proposition 9. A continuous, differentiable contract $W(\Pi)$ implements investment level k if and only if it satisfies the manager's first-order condition:

$$\mathbb{E}\left\{W'(\Pi(\theta,k))\left(R(\theta) - C'(k)\right)\right\} = 0 \text{ or } C'(k) = \frac{\mathbb{E}\{W'(\Pi(\theta,k))R(\theta)\}}{\mathbb{E}\{W'(\Pi(\theta,k))\}}.$$

Clearly, if $W'(\Pi)$ is constant, the implemented investment level is k^*. Now, for each $\hat{\theta}$, there exists a unique $k\left(\hat{\theta}\right) > k^*$ such that $C'(k) = \mathbb{E}\left(R(\theta)|\theta > \hat{\theta}\right)$, and a unique $\underline{W}\left(\hat{\theta}\right) = R\left(\hat{\theta}\right)k\left(\hat{\theta}\right) - C\left(k\left(\hat{\theta}\right)\right)$. Therefore, by construction the contract $W(\Pi) = \max\left\{\underline{W}\left(\hat{\theta}\right), \Pi\right\}$ implements effort level $k\left(\hat{\theta}\right)$. As $\hat{\theta} \to \infty$, $k\left(\hat{\theta}\right) \to \psi\left(\bar{R}\right)$, while as $\hat{\theta} \to -\infty$, $k\left(\hat{\theta}\right) \to k^*$. Likewise, for each $\widetilde{\theta}$, there exists a unique $k\left(\widetilde{\theta}\right) < k^*$ such that $C'(k) = \mathbb{E}\left(R(\theta)|\theta < \widetilde{\theta}\right)$, and a unique $\bar{W}\left(\widetilde{\theta}\right) = R\left(\widetilde{\theta}\right)k\left(\widetilde{\theta}\right) - C\left(k\left(\widetilde{\theta}\right)\right)$. Therefore, by construction the contract $W(\Pi) = \min\left\{\Pi, \bar{W}\left(\widetilde{\theta}\right)\right\}$ implements effort level $k\left(\widetilde{\theta}\right)$. As $\widetilde{\theta} \to \infty$, $k\left(\widetilde{\theta}\right) \to k^*$, while as $\widetilde{\theta} \to \infty$, $k\left(\widetilde{\theta}\right) \to \psi\left(\underline{R}\right)$.

Finally, $\Pi(\theta, k)$ is strictly increasing in k for $k < \psi(\underline{R})$, for all θ, so that for any non-decreasing contract $W(\cdot)$, $\mathbb{E}\{W(\Pi(\theta,k))\} \leq \mathbb{E}\{W(\Pi(\theta, \psi(\underline{R})))\}$ for $k < \psi(\underline{R})$, and the inequality is strict if $W(\cdot)$ is strictly increasing for some Π. Likewise since $\Pi(\theta,k)$ is strictly decreasing in k for $k > \psi\left(\bar{R}\right)$, for all θ, it follows that $\mathbb{E}\{W(\Pi(\theta,k))\} \leq \mathbb{E}\left\{W\left(\Pi\left(\theta, \psi\left(\bar{R}\right)\right)\right)\right\}$ for $k > \psi\left(\bar{R}\right)$, for any non-decreasing contract $W(\cdot)$, that is strictly increasing for some θ. ■

Proof of Proposition 10. We need to show that whenever $\left(\hat{k} - k^*\right)T_n'(k^*) > 0$ for some $T_n(\cdot)$, the initial shareholders can use $T_n(\cdot)$ to distort investment in their desired direction.

By construction, for any $\eta > 0$, there exist $\varepsilon_1 > 0$, and $\varepsilon_2 > 0$, such that $T_0(k^*) - \eta = T_0(k^* - \varepsilon_1) = T_0(k^* + \varepsilon_2)$, and $T_0(k^*) - \eta \leq T_0(k)$ iff $k \in [k^* - \varepsilon_1, k^* + \varepsilon_2]$, and $T_0'(k) > 0$ for $k \in [k^* - \varepsilon_1, k^*]$, and $T_0'(k) < 0$ for $k \in [k^*, k^* + \varepsilon_2]$.

Suppose now that $\hat{k} > k^*$ and there exists $T_n(\cdot)$, such that $T_n'(k^*) > 0$. Choose η sufficiently small, so that $T_n'(k) > 0$ for all $k \in [k^* - \varepsilon_1, k^* + \varepsilon_2]$, and $T_n(\cdot)$ is bounded, so we can choose $\xi \in (0, \eta/(2\max_k \|T_n(k)\|))$. Then, for $k \notin [k^* - \varepsilon_1, k^* + \varepsilon_2]$, $T_0(k) + \xi T_n(k) - T_0(k^*) - \xi T_n(k^*) < -\eta + 2\xi \max_k \|T_n(k)\| \leq 0$, so a contract paying $T_0(k) + \xi T_n(k)$ must implement an investment level $k \in [k^* - \varepsilon_1, k^* + \varepsilon_2]$. Moreover, $T_0'(k) + \xi T_n'(k) > 0$ for $k \in k \in [k^* - \varepsilon_1, k^*]$, implying that the contract $T_0(k) + \xi T_n(k)$ is maximized at $k \in (k^*, k^* + \varepsilon_2)$.

By the same argument, if $\hat{k} < k^*$ and there exists $T_n(\cdot)$, such that $T_n'(k^*) < 0$, then the contract $T_0(k) + \xi T_n(k)$ implements $k < k^*$, for ξ sufficiently small. ■

Proof of Proposition 11. Let k denote the implemented investment level. For each share bought, the policy maker earn a realized return $\Pi(\theta,k) - \bar{R}k - C(k) = \left(R(\theta) - \bar{R}\right)k$. Let $\hat{x}\left(\bar{R}\right)$ denote the investor threshold that prevails when the support price is active. Then the total number of units purchased, given a realization of θ and u, is $\Phi\left(\sqrt{\beta}\left(\hat{x}\left(\bar{R}\right)-\theta\right)\right) - \Phi(u) = \Phi\left(\sqrt{\beta}\left(\hat{x}\left(\bar{R}\right)-\theta\right)\right) - \Phi\left(\sqrt{\beta}(z-\theta)\right) = \Pr\left(x \in \left[z, \hat{x}\left(\bar{R}\right)\right] |\theta\right)$. The support price in turn is active, whenever $z < \hat{z}$, where $\bar{R} = \mathbb{E}\left(R(\theta)|x = \hat{z}, \hat{z}\right)$. The threshold $\hat{x}\left(\bar{R}\right)$ is larger than $\hat{z}$ and satisfies $\bar{R} = \mathbb{E}\left(R(\theta)|\hat{x}\left(\bar{R}\right), z \leq \hat{z}\right)$. The expected revenue from the policy is then

$$
\begin{aligned}
& k\int_{-\infty}^{\infty}\int_{-\infty}^{\hat{z}}\left(R(\theta)-\bar{R}\right)\left(\Phi\left(\sqrt{\beta}\left(\hat{x}\left(\bar{R}\right)-\theta\right)\right)-\Phi\left(\sqrt{\beta}(z-\theta)\right)\right)d\Phi\left(\sqrt{\beta\delta}(z-\theta)\right)d\Phi\left(\sqrt{\lambda}\theta\right) \\
= \;& k\left(\mathbb{E}\left(R(\theta)|x\in\left[z,\hat{x}\left(\bar{R}\right)\right], z\leq\hat{z}\right)-\bar{R}\right)\cdot\Pr\left(x\in\left[z,\hat{x}\left(\bar{R}\right)\right], z\leq\hat{z}\right) \\
= \;& k\left(\mathbb{E}\left(R(\theta)|x\in\left[z,\hat{x}\left(\bar{R}\right)\right], z\leq\hat{z}\right)-\mathbb{E}\left(R(\theta)|\hat{x}\left(\bar{R}\right), z\leq\hat{z}\right)\right)\cdot\Pr\left(x\in\left[z,\hat{x}\left(\bar{R}\right)\right], z\leq\hat{z}\right) \\
< \;& 0.
\end{aligned}
$$

■