# Information Aggregation in Dynamic Markets Under Ambiguity*

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**Abstract**

This paper studies information aggregation in a dynamic trading model with partially informed and ambiguity averse traders. We show that separable securities, introduced by Ostrovsky (2012) in the context of expected utility, no longer aggregate information if some traders are ambiguity averse. More importantly, this failure comes not because traders are unable to reach a consensus about the price of the security, but because they eventually agree on the "wrong" price. As a result, it is impossible for an outside observer to know whether the equilibrium price reveals all information or just false information, unless she is confident that no trader has imprecise probabilities. We define a class of securities which strengthens separability and is robust to these considerations. In particular, we show that strongly separable securities characterize information aggregation in both strategic and non-strategic environments.

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# 1 Introduction

When do financial markets aggregate the information which is dispersed among individual traders? The mechanism, through prices, is intuitive. If the price is low (high) and some traders have private information that the real value of the security is high (low), they will increase (decrease) their demand and the price. Moreover, these price movements could

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reveal to a trader information that others might have, prompting her to update her beliefs and either buy or sell, thus further revealing some of her own private information.

This intuition is largely correct. Starting from Hayek (1945), there is an extensive literature showing that under various settings, information gets aggregated. For instance, Ostrovsky (2012) shows that even if there are few large and strategic traders, information gets aggregated for a large class of securities, called *separable*, which includes the Arrow-Debreu securities.

In recent years, many firms and institutions have leveraged this property by designing *prediction* markets, as a forecasting tool for several issues, such as political events, the release of new products and box office success (O'Leary (2011)). Google, Microsoft, Ford, General Electric and HP, among others, run internal prediction markets as a corporate governance and predictions tool, whereas Cultivate Labs, Inkling Markets, Consensus Point, Crowdcast and Iowa Electronic Markets are examples of Internet based prediction markets. These markets usually implement some form of a market scoring rule (Hanson (2003)), which is based on proper scoring rules (Brier (1950)).[1]

In several cases, prediction markets perform significantly better than other conventional forecasting methods, such as polls. Berg et al. (2008) compared the predictions, for the five presidential elections between 1988 and 2004, of the Iowa Electronic Markets and those of 964 polls. They found that 74% of the time the prediction market was closer to the truth, whereas for forecasts 100 days in advance it outperformed the polls at every election.

In other cases, however, where the questions are less common, prediction markets do not perform as well. Dreber et al. (2015) show that a prediction market was better at predicting the reproducibility of 44 studies published in prominent psychological journals, as compared to the pre-trade average of the market participants' individual forecasts. However, Camerer et al. (2016) show that the two methods are equally capable of predicting the reproducibility of economics studies.

Interestingly, in the case of a "once in a lifetime" event, prediction markets fared significantly worse. Cultivate Labs designed a prediction market on the outcome of the Brexit referendum. It run for 10 days prior to the polling day but failed spectacularly, as the closing prediction was a 20% probability of voting for Brexit.[2] On the other hand, an average of all polls, reported by the Financial Times on the day of the referendum, found 48% in favour of remain and 46% in favour of leave. The actual result was 51.9% and 48.1%, respectively.[3]

Our motivation for this paper stems from trying to understand when prediction (and more generally financial) markets are less efficient at aggregating information relative to other methods, and in particular for events that are rare or uncommon, and for which beliefs are imprecise. Up to our awareness, the literature has so far only focused on traders who have precise probabilities about events and expected utility preferences. But is this always a valid assumption, especially for events, like the outcome of the Brexit referendum, which occur once in a generation? Empirically, such an assumption has an impact. For instance, Kotronis (2016) shows experimentally that introducing imprecise probabilities leads to significantly

[1] Such an implementation is described at https://www.cultivatelabs.com/prediction-markets-guide/how-does-logarithmic-market-scoring-rule-lmsr-work.

[2] The market can be found in https://alphacast.cultivateforecasts.com/questions/1311-will-the-uk-vote-to-leave-the-eu-in-the-june-2016-referendum.

[3] The details can be found at https://ig.ft.com/sites/brexit-polling/.

worse information aggregation.

We first explore an example where imprecise probabilities imply that information does not get aggregated. We propose a strengthening of separability which Ostrovsky (2012) showed is necessary and sufficient for aggregation of information in the case of a unique prior and expected utility. We then show that in both strategic and non-strategic environments, strongly separable securities aggregate information.

The following example with a unique common prior shows how prices aggregate information in the simple case of myopic, or non-strategic, participants. Consider the Brexit referendum in the UK and suppose there are three possible states: the referendum takes place and the result is in favour of Brexit, it takes place but the outcome is against Brexit, or the referendum is cancelled. Let Arrow-Debreu security $X$ which pays 1 if Brexit occurs and 0 otherwise.

| | **Expert 1** | **Expert 2** |
|---|---|---|
| **Brexit** | Referendum not cancelled | Either Brexit or cancelled |
| **No Brexit** | Referendum not cancelled | No Brexit |
| **Referendum cancelled** | Referendum cancelled | Either Brexit or cancelled |

Figure 1: Private signals

Two experts participate in the market, announcing sequentially their expected value of $X$, with private signals as shown in Figure 1. In particular, Expert 1 is informed whether the referendum is cancelled or not, whereas if the referendum takes place Expert 2 knows its outcome. As a result, their pooled information always reveals the true state.

Suppose that Brexit is the true state and that the two experts have a common prior which assigns positive probability to that state. Then, by announcing sequentially their expectations about the value of security $X$, information gets aggregated. In the first round, the announcement of Expert 1 about $X$ is positive, hence Expert 2 realises that Expert 1 put probability 1 to the state that the referendum is cancelled, as this would imply an announcement of 0. The public information that is revealed is that the referendum will take place. In the second round, Expert 2, by combining the extra information with his own private signal, realises that Brexit will happen and announces 1. In the third round, Expert 1 realises, as well, that Brexit is the realised state, hence he also announces 1 and the prediction market aggregates all information.

This result of information aggregation relies heavily on the assumption that each expert's belief is a unique prior.[4] However, Brexit is a once in a lifetime event, for which no historical data for similar events exist. How can we be sure that the experts have precise probabilities for such a hard to quantify event?

If we cannot maintain the hypothesis of a unique prior and expected utility, it is no longer the case that markets aggregate information, even if the experts' (multiple) priors are *common*. More importantly, even a slight departure from a unique prior could result in the experts agreeing on a value of the security that is very far from the true one.

[4]More generally, the assumption we need is that of a common prior.

To show this, consider the ambiguity aversion model of Gilboa and Schmeidler (1989), where a decision maker acts as if having multiple priors over the states, and chooses the prediction that maximizes the minimum expected utility over these priors. The formal treatment is presented in Example 1. It turns out, as we show in Lemma 1, that even with multiple priors, the announcement of a myopic trader is the expectation of security $X$ according to some prior.

Assuming that at least one prior (but not all) assigns zero probability to Brexit, the expected value of the security with respect to that prior is 0. Therefore, the initial announcement of Expert 1 is 0. No public information is revealed by this announcement, because Expert 1 would make the same announcement at all states. As a result, Expert 2 does not learn anything from 1's announcement and his announcement is, for similar reasons, 0. In turn, Expert 1 also announces 0. As a result, the market fails to aggregate information, because no one learns that Brexit will happen, hence no one learns that the true value of the security is 1. The experts agree on their announcements but on a value for the security which is very far from the truth.

More importantly, an outside observer can never know whether the consensus value of 0 means that Brexit will not happen, unless he can safely exclude the possibility that some participants have imprecise probabilities. This problem is even more pronounced for markets which are created exactly for the reason of revealing information to their designers.

To accommodate the case of imprecise probabilities, consider security $Y$ whose value is 3 if Brexit happens, 2 if No Brexit happens and 1 if the referendum is canceled. Expert 1's announcement about $Y$ now depends on whether the referendum takes place, because 1 is not a convex combination of 2 and 3. Since the possible announcements are different, in the next round Expert 2 can infer that Expert 1's signal cannot be that the referendum is canceled. Combining this piece of information with his own signal, Expert 2 concludes that Brexit is the true state, hence announcing 1. In the next round, Expert 1 can infer as well that Expert 2 knows that Brexit will happen, therefore information gets aggregated.

Note that security $Y$ aggregates information irrespective of whether market participants have precise probabilities or they are ambiguity averse and have multiple priors. Hence, it is more robust as compared to the separable securities of Ostrovsky (2012). We call such securities strongly separable and show that they are always separable, but the converse is not true. Theorem 1 characterizes information aggregation in terms of strongly separable securities, for the case of myopic or non-strategic players. For the strategic case, we introduce a solution concept which is closely related to the Revision-proof equilibrium of Asheim (1997), Ales and Sleet (2014) and characterize information aggregation in Theorem 2.

We conclude by making a few observations. First, prediction markets are usually implemented using a Market Scoring Rule (MSR), introduced by McKelvey and Page (1990) and Hanson (2003, 2007). These markets are fairly general as they can be reinterpreted in order to correspond to the classic approach with an inventory based market maker who continuously adjusts the price of the securities, depending on the orders he receives.[5] Second, there is nothing particular about having a prior which assigns probability 0 to the true state, in order to show that separable securities do not aggregate information in the case of ambiguity aversion. In Appendix C we provide a more general example with full support priors and

[5] See Appendix C for more details.

more than two traders.

Finally, introducing ambiguity aversion changes the model significantly, as compared to the expected utility model of Ostrovsky (2012). The reason is that even when traders are non-strategic, their predictions depend not only on the acquired information, but also on the previous prediction. In the strategic case, this implies that a new equilibrium concept should be defined in order to accommodate the fact that traders may be dynamically inconsistent.

## 1.1 Market Microstructure

Dynamic markets are studied using several different approaches. The *no trade theorems* (Aumann (1976), Milgrom and Stokey (1982), Sebenius and Geanakoplos (1983), Geanakoplos and Polemarchakis (1982)) imply that in dynamic markets, in which a common prior is assumed, a source to subsidize trading should exist. This is the reason why the existence of noise traders or heterogeneous priors is assumed in models of financial markets. In this paper the dynamic market follows the MSR of Hanson (2003) and it is similar to Ostrovsky (2012), Chen et al. (2012) and Dimitrov and Sami (2008). Whereas the assumption of common prior holds, instead of noise traders there is an automated market maker who admits to have bounded losses. A crucial difference of our setup is that instead of a single *common prior*, traders hold a *common set of multiple priors* according to Gilboa and Schmeidler (1989).

The dynamic trading mechanism in our model starts with an initial public announcement about the value of the security by the market maker, which is made in order to open the market, and with nature choosing, ambiguously, the true state. Then, each trader sequentially announces, in public, their predictions and after every announcement all traders refine their information and beliefs accordingly. A score of each prediction, interpreted as the first part of trader's per period utility, is calculated after the whole uncertainty about the true state is revealed and it is based on a strictly proper scoring rule. In particular, the per period utility of a trader is calculated by subtracting from the score of her prediction the score of the prediction made by the previous trader. This can be interpreted as if each time traders make a prediction, they "buy out" the previous one. The scores are derived using a strictly proper scoring rule.

In fact, according to the market mechanism described, and assuming that traders are myopic, it is implied that a trader announces the price of the security that maximises his t-period payoff which is the expected value of the security according to some posterior belief, out of the whole set of beliefs, conditioned to the, possibly refined, information partition.[6] Therefore, we can conclude that the trading procedure generalises the communication process of Geanakoplos and Polemarchakis (1982). However, in the MSR model, ambiguity aversion complicates this process considerably, as it implies that each prediction depends on the previous one. Hence, the decision function of each trader is different round by round.

When traders are strategic, the trading procedure is an infinite horizon game with incomplete information. Given that the traders are ambiguity averse, they might potentially be dynamically inconsistent: their optimal continuation strategy at time $t$ might not be optimal at a later time. This feature of our model does not exist in Ostrovsky (2012).

[6] That is because the market scoring rule is incentive compatible as a static mechanism for myopic traders.

A widely used equilibrium concept for dynamically inconsistent players is called Consistent Planning, axiomatised in Siniscalchi (2011). At each time $t$, the corresponding player optimises the continuation payoff over deviations from the current round's equilibrium action subject to following the equilibrium afterwards. We use a stronger equilibrium concept than that, called *Revision-proof Equilibrium*. It was defined in Asheim (1997), and similar concepts can be found in Strotz (1955), Peleg and Yaari (1973), Goldman (1980) and Ales and Sleet (2014), among others, but here we generalise it for incomplete information. The stronger part of this concept is that it allows the equilibrium strategy to be checked over multiple stage deviations. Intuitively, Revision-proof equilibrium requires that each player $i$, when considering deviating at time $t_0$, cannot find an alternative strategy that will make her and all of her future selves (at times $t_0 + n, t_0 + 2n, \ldots$) weakly better off, and at least one of them strictly better off. Note that because of ambiguity and (possibly) dynamic inconsistency, an optimal plan for $i$ at $t_0$ may not be optimal at $t_0 + n$ for $i$.

## 1.2 Related literature

There is a large literature related to information aggregation and information revelation in dynamic markets, starting with Hayek (1945). Grossman (1976) proved that in equilibrium the price aggregates information. Radner (1979) introduced the concept of Rational Expectations Equilibrium (REE) and proved that generically the prices aggregate information dispersed among traders (fully revealing REE). Several results have been proven regarding the convergence in REE in dynamic settings: Hellwig (1982), Dubey et al. (1987), Wolinsky (1990), Golosov et al. (2014), McKelvey and Page (1986), Nielsen et al. (1990), Nielsen (1984) among others.

Aumann (1976), Geanakoplos and Polemarchakis (1982), Cave (1983), Sebenius and Geanakoplos (1983), Nielsen (1984) and Nielsen et al. (1990) study information communication in an opinion game using posterior beliefs or other aggregate statistics. These papers however do not fully characterise under what conditions the consensus yields the true posterior or expectation of the security. DeMarzo and Skiadas (1998, 1999) go beyond the consensus result of the former papers finding necessary and sufficient condition for information aggregation.

Furthermore, Chen et al. (2012), Ostrovsky (2012) study information aggregation in a market with either myopic or strategic traders, under a similar assumption for the securities as DeMarzo and Skiadas (1998, 1999) and prove that this assumption is both necessary and sufficient for information aggregation. In Chen et al. (2012), Ostrovsky (2012) the models are based on MSR and hence their results are particularly related to prediction markets as well. Similar approaches can be found in Chen et al. (2010), Dimitrov and Sami (2008) where the focus is on considering whether information gets aggregated when traders are strategic, under various assumption regarding the signal structure.

In addition, there are several results attempting to study the effect of ambiguity on information revelation. In particular, Dominiak and Lefort (2013, 2015), Carvajal and Correia-da Silva (2010) and Kajii and Ui (2005, 2009), among others, are related mostly to the seminal *cannot agree to disagree* type result of Aumann (1976). In fact, they study a consensus result in the presence of ambiguity averse traders (Kajii and Ui (2005, 2009), Carvajal and Correia-da Silva (2010)) or with CEU and neo-additive capacities (Dominiak and Lefort

(2013, 2015)). Finally, within a REE setting expanded in order to include preferences that display ambiguity aversion, the existence and robustness of partially-revealing rational expectations equilibria (REE) is proved in Condie and Ganguli (2011).

Our contribution to the literature is, therefore, twofold. First, our result for myopic traders contributes to the literature on the effect of ambiguity on information revelation. In fact, we prove that myopic, partially informed ambiguity averse traders with a common set of priors *cannot disagree forever* in an MSR market. Second, we contribute to the literature on information aggregation by defining a class of securities which characterise information aggregation, for both myopic and strategic traders, in the presence of ambiguity.

### 1.3 Overview

The paper is organized as follows. Section 2 describes the model, whose components are the ambiguity averse preferences, the MSR trading environment, the decision function for the myopic case and the equilibrium notion for the strategic case. In Section 3, we characterise information aggregation for the case of myopic traders, whereas in Section 4 we examine the strategic case. We conclude in Section 5. All proofs are included in the appendices.

## 2 Model

In this section we first describe the ambiguity averse preferences of the traders and the market scoring rule (MSR) trading environment. We then distinguish between two cases. First, all traders are myopic, so that they only care about the current period's payoff. Second, all traders act strategically and care about the future. We formulate the game and define our solution concept, which is related to the Revision-proof equilibrium of Asheim (1997) and Ales and Sleet (2014).

### 2.1 Preferences and updating

Consider a finite state space $\Omega = \{\omega_1, ..., \omega_l\}$ and let the powerset $\mathcal{P}(\Omega)$ be the sigma-algebra over $\Omega$. Traders are ambiguity averse and have multiple priors (MP) preferences (Gilboa and Schmeidler (1989)). In particular, each trader evaluates act $f : \Omega \to \mathbb{R}$ as

$$V(f) = \min_{p \in \mathcal{P}} \int u(f(s))dp(s),$$

where $\mathcal{P}$ is a convex and closed subset of $\Delta(\Omega)$, endowed with the weak* topology. We assume that $\mathcal{P}$ is common among all traders and, without loss of generality, $\bigcup_{p \in \mathcal{P}} Supp(p) = \Omega$, so that each state is considered possible by some $p \in \mathcal{P}$.

The set of traders is $I = \{1, \dots, n\}$. Trader $i$'s initial private information is represented by partition $\Pi_i$ of $\Omega$. Without loss of generality, we assume that the join (the coarsest common refinement) of partitions $\Pi = \{\Pi_1, \dots \Pi_n\}$ consists of singleton sets. This implies that, for any two states $\omega_1 \neq \omega_2$, there exists trader $i$ such that $\Pi_i(\omega_1) \neq \Pi_i(\omega_2)$, so that the traders' pooled information always reveals the true state.

When a trader learns event $E$, her beliefs are $\mathcal{P}_E$, the prior by prior updating of $\mathcal{P}$.[7] This rule is well defined as long as each prior assigns positive probability to $E$. We say that measures $p_1, p_2 \in \mathcal{P}$ are mutually absolutely continuous with respect to a collection of events $\mathcal{E}$ if, for all $E \in \mathcal{E}$, $p_1(E) = 0$ if and only if $p_2(E) = 0$. Compact and convex set $\mathcal{P} \subseteq \Delta(\Omega)$ is *regular* with respect to $\mathcal{E}$ if all $p_1, p_2 \in \mathcal{P}$ are mutually absolutely continuous with respect to $\mathcal{E}$.

## 2.2 Trading environment

Trading is organized as follows. At $t_0 = 0$, nature selects a state $\omega^* \in \Omega$ and the uninformed market maker makes a prediction $y_0$ about the value of security $X : \Omega \to \mathbb{R}$. At time $t_1 > t_0$, trader 1 makes a revised prediction $y_1$, at $t_2 > t_1$ trader $t_2$ makes her prediction, and so on. At time $t_{n+1} > t_n$ trader 1 makes another prediction $y_{n+1}$. Let $a(t)$ be the player that makes a prediction at time $t$. All predictions are observed by all traders. Each prediction $y_k$ is required to be within the set $Y = [\min_{\omega \in \Omega} X(\omega), \max_{\omega \in \Omega} X(\omega)] = [\underline{y}, \overline{y}]$.

Let the one-to-one and onto function $\mathcal{O} : I \longrightarrow \{1, ..., n\}$ describe the order with respect to which the finitely many traders $I$ participate in the market. From here and onwards we will use the set $\{1, ..., n\}$ to refer to the traders.

At time $t^* > 1$ the true value $x^* = X(\omega^*)$ is revealed. The traders' payoffs are computed using a market scoring rule (MSR), $s(y, x^*)$, where $x^*$ is the true value of the security and $y$ is a prediction. A scoring rule is *proper* if, for any probability measure $p$ and every random variable $X$, the expectation of $s$ is maximised at $y = E_p[X]$. It is *strictly proper* if $y$ is unique. We focus on continuous strictly proper scoring rules. Examples are the quadratic, where $s(y, x) = -(x - y)^2$, and the logarithmic, where $s(y, x) = (x - a)ln(y - a) + (b - x)ln(b - y)$ with $a < \min_{\omega \in \Omega} X(\omega), b > \max_{\omega \in \Omega} X(\omega)$.

Under the basic MSR (McKelvey and Page (1990), Hanson (2003, 2007)), each trader is paid at each time $t$ where she makes a prediction. Her payoff from announcing $y_t$ at $t$, when the previous announcement was $y_{t-1}$, is $s(y_t, x^*) - s(y_{t-1}, x^*)$. We then say that the trader "buys out" the previous trader's prediction.

In the expected utility framework, the optimal choice of $y_t$ that maximises $E_p[s(y_t, x^*) - s(y_{t-1}, x^*)]$ does not depend on the previous announcement $y_{t-1}$, because $p$ is fixed. This is no longer the case with MP preferences, further complicating our analysis.[8] However, the following Lemma shows that the choice of the announcement is still unique for continuous strictly proper scoring rules.[9]

**Lemma 1** *Let $s$ be a continuous strictly proper scoring rule on $Y = [a, b]$, $a, b \in \mathbb{R}$, and $z \in Y$ be an announcement. Then, $y^* = \underset{y \in Y}{argmax} \ \underset{p \in \mathcal{P}}{min} E_p\big[s(y, X) - s(z, X)\big]$ exists, is a singleton and $y^* = E_p[X]$ for some (not necessarily unique) $p \in \underset{p \in \mathcal{P}}{argmin} \ \underset{y \in Y}{max} E_p\big[s(y, X) - s(z, X)\big]$.*

[7] This rule is axiomatised by Pires (2002).

[8] As we explain later in our analysis, this makes the myopic behavior similar to properties of equilibrium concepts. In particular, the myopic behavior of the previous player affects the one of the next player, as it holds, in principle, in a Perfect Bayesian Equilibrium for example.

[9] Lemma 1 is related to a result in Chambers (2008). The proofs are closely related, too.

We examine trading in two settings. The first, analyzed in Section 3, is the myopic, or non-strategic, case where each trader does not care about future payoffs when making an announcement. We denote this setting by $\Gamma^M(\Omega, \Pi, X, \mathcal{O}, \mathcal{P}, y_0, \underline{y}, \overline{y}, s)$.

The second setting is strategic, studied in Section 4. Following Dimitrov and Sami (2008), we focus on the discounted MSR, which specifies that the payment at $t$ is $\beta^t(s(y_t, x^*) - s(y_{t-1}, x^*))$, where $0 \leq \beta \leq 1$. The total payoff of each trader is the sum of all payments for revisions. Denote this setting by $\Gamma^S(\Omega, \Pi, X, \mathcal{O}, \mathcal{P}, y_0, \underline{y}, \overline{y}, s, \beta)$. In both settings the traders are risk-neutral.

## 2.3 Information aggregation

We say that information gets aggregated if the traders' predictions converge to the true value of the security, $X(\omega^*)$.

**Definition 1** *Under a profile of pure strategies in $\Gamma^M$ or $\Gamma^S$, information gets aggregated if sequence $y_k$ converges in probability to random variable $X(\omega^*)$, for all $\omega^* \in \Omega$.*

Ostrovsky (2012) provides a similar definition, with the only exception that he allows for mixed strategies. However, he also assumes expected utility.

## 2.4 Strong separability

Ostrovsky (2012) introduced the notion of separable securities, which are sufficient for aggregating information in an environment with expected utility.

**Definition 2** *A security $X$ is called non-separable under partition structure $\Pi$ if there exists probability $p$ and value $v \in \mathbb{R}$ such that:*

*(i) $X(\omega) \neq v$ for some $\omega \in Supp(p)$,*

*(ii) $E_p[X|\Pi_i(\omega)] = v$ for all $i = 1, ..., n$ and $\omega \in Supp(p)$.*

*Otherwise, it is called separable.*

A security $X$ is non-separable if, for some belief $p$ that assigns positive probability to a state where $X$ does not pay $v$, all traders agree on its conditional expected value to be $v$, irrespective of which private signal they have received. In such a case, even if all traders truthfully and repeatedly announce $v$, no new information is revealed. However, their pooled information reveals the state, hence information aggregation fails.[10] To avoid this, the security must be separable. The most common example is the Arrow-Debreu security, which pays 1 at some state and 0 otherwise. Unfortunately, with ambiguity aversion even separable securities may not aggregate information, as the following example shows.

[10] An example of a non-separable security is the following, provided by Ostrovsky (2012). Let $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ and suppose $X(\omega_1) = X(\omega_4) = 1$, $X(\omega_2) = X(\omega_3) = -1$. Partitions are $\Pi_1 = \{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\}$ and $\Pi_2 = \{\{\omega_1, \omega_3\}, \{\omega_2, \omega_4\}\}$. For $p$ that assigns 1/4 at each state, both players always have an expectation of 0, although their pooled information always reveals the true value of $X$, which is never 0.

**Example 1** *Consider two traders with a common set of priors $\mathcal{P}$, which is the convex hull of $p^1 = (0, \frac{1}{2}, \frac{1}{2})$ and $p^2 = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. Trader 1's information partition is $\Pi_1 = \{\{\omega_1, \omega_2\}, \{\omega_3\}\}$, whereas 2's is $\Pi_2 = \{\{\omega_1, \omega_3\}, \{\omega_2\}\}$. They trade an Arrow-Debreu security $X$ that pays 1 at $\omega_1$ and 0 otherwise. Payoffs are determined using the quadratic scoring rule $s(y, x^*) = -(y - x^*)^2$, where $y$ is the prediction and $x^*$ is the true value. The true state is $\omega_1$, so that the correct price to be inferred is $X(\omega_1) = 1$.*

*Suppose that the initial price of the security is $y_0 = 0$, set by the market maker. Trader 1 is informed that $E_1 = \{\omega_1, \omega_2\}$ has occurred and maximises her utility myopically. Using Lemma 1 and letting $p_{E_1}$ be the conditional of $p$ given $E_1$, she solves $\min_{p\in\mathcal{P}} E_{p_{E_1}}[s(E_{p_{E_1}}[X], X(\omega)) - s(0, X(\omega))] = \min_{p\in\mathcal{P}}[p_{E_1}(\omega_1)^2(2 - p_{E_1}(\omega_1) - p_{E_1}(\omega_2))] = \min_{p\in\mathcal{P}} p_{E_1}(\omega_1)^2$. We conclude that the solution is $p^1$ with $p^1(\omega_1) = 0$ and her prediction is $y_1 = 0$. Similar arguments establish that her prediction would still be $y_1 = 0$ if her private signal was $\{\omega_3\}$.*[11]

*The above imply that trader 2 cannot learn anything from 1's announcement, hence can only rely on her private signal $E_2 = \{\omega_1, \omega_3\}$. Maximising myopically her utility she solves $\min_{p\in\mathcal{P}} E_{p_{E_2}}[s(E_{p_{E_2}}[X], X(\omega)) - s(0, X(\omega))] = \min_{p\in\mathcal{P}}[p_{E_2}(\omega_1)^2(2 - p_{E_2}(\omega_1) - p_{E_2}(\omega_3)] = \min_{p\in\mathcal{P}} p_{E_2}(\omega_1)^2$. The solution is again $p^1$, with $p^1(\omega_1) = 0$, and her prediction is $y_2 = 0$.*

*Each trader learns nothing from the other's announcement, which is always 0. Because the true value of the security is 1, there is no information aggregation through the announcements, even though their pooled information would reveal state $\omega_1$ and the true value.*

The result of no information aggregation does not rely on $p^1$ assigning 0 to the true state. Example 3 in Appendix C obtains the same result by assuming that all priors have full support.

In order to maintain information aggregation in an environment with ambiguity aversion, we need to strengthen the notion of separability. Let

$$d_{\mathcal{P}}(E, v) = \underset{y\in Y}{argmax} \min_{p\in\mathcal{P}_E} E_p\big[s(y, X) - s(v, X)\big]$$

be the announcements that maximise the trader's utility if her beliefs are $\mathcal{P}_E$, the previous announcement was $v$ and she is myopic, so that she only cares about her current period payoff.[12] Note that if $\mathcal{P} = \{p\}$ is a singleton, so we are back to the expected utility case, $d_{\mathcal{P}}(E, v) = E_p[X|E]$ for any $v$ and proper scoring rule $s$. Hence, it is a direct generalization. Below, we generalise the notion of separability.

**Definition 3** *A security $X$ is called not strongly separable under partition structure $\Pi$ and proper scoring rule $s$ if there exist a regular $\mathcal{P} \subseteq \Delta(\Omega)$ with respect to each $\Pi_i$, $i = 1, ..., n$, and $v \in \mathbb{R}$ such that:*

(i) *$X(\omega) \neq v$ for some $\omega \in \bigcup_{p\in\mathcal{P}} Supp(p)$,*

[11] The argument is based on Lemma 1, which ensures that if we find a solution $p^*$ to the "minmax", then we get the solution of the "maxmin" optimisation problem as $y^* = E_{p^*}[X]$.

[12] Lemma 1 shows that this function is well defined.

(ii) $d_{\mathcal{P}}(\Pi_i(\omega), v) = v$ *for all* $i = 1, ..., n$ *and* $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$.

*Otherwise, it is called strongly separable.*

The interpretation of a not strongly separable security is similar to that of a non-separable security. The only difference is that $\mathcal{P}$ is not a singleton and, as a result, truthful announcement $E_p[X|\Pi_i(\omega)] = v$ is generalised to $d_{\mathcal{P}}(\Pi_i(\omega), v) = v$. However, in both definitions, each trader announces $v$, given that the previous announcement was $v$ and irrespective of the private signal that she has received.

In Example 1, the Arrow-Debreu security is not strongly separable under the given information partition structure and quadratic proper scoring rule. Condition $(ii)$ in the definition is satisfied for all states with $v = 0$. Since some priors put positive probability to $\omega_1$ and $X(\omega_1) = 1 \neq v$, condition $(i)$ is also satisfied.

Observe that if a security is non-separable (for some prior $p$), then it is not strongly separable as well (for $\mathcal{P} = \{p\}$). This means that the notion of not strong separability leaves out more securities, hence strong separability implies separability. Moreover, the converse is not true, as shown in Example 1. Finally, notice that the class of strongly separable securities is not empty, in general. For example, consider state space $\Omega = \{\omega_1, \omega_2, \omega_3\}$ and security $X$ with $X(\omega_1) = X(\omega_2) = 1, X(\omega_3) = 0$. Under the partition structure $\Pi_1 = \{\{\omega_1, \omega_2\}, \{\omega_3, \}\}, \Pi_2 = \{\{\omega_1, \omega_3\}, \{\omega_2\}\}$ and any continuous proper scoring rule, $X$ is strongly separable.

Ostrovsky (2012) proposes a useful characterization of separable securities. It says that $X$ is separable if and only if for any possible announcement $v$, we can find numbers $\lambda_i(\Pi_i(\omega))$, for each $i$ and $\omega$, such that the sum over all traders has the same sign as the difference of $X(\omega) - v$. Intuitively, for any $v$ and at each $\omega$, all traders "vote" and the sign of the sum of the votes has to agree with the difference between the value of the security and $v$.

**Proposition 1 (Ostrovsky (2012))** *Security $X$ is separable under partition structure if and only if, for every $v \in \mathbb{R}$, there exist functions $\lambda_i : \Pi_i \to \mathbb{R}$ for $i = 1, \ldots, n$ such that, for every state $\omega$ with $X(\omega) \neq v$,*

$$(X(\omega) - v) \sum_{i \in I} \lambda_i(\Pi_i(\omega)) > 0.$$

We provide a similar but stronger condition, which ensures that the security is strongly separable. We first define a weakening of the notion of a partition of $\Omega$, taken from Geanakoplos (1989).

**Definition 4** *Possibility correspondence $Q : \Omega \to 2^\Omega \setminus \emptyset$ is nested if for all $\omega_1, \omega_2 \in \Omega$, either $Q(\omega_1) \cap Q(\omega_2) = \emptyset$, or $Q(\omega_1) \subseteq Q(\omega_2)$, or $Q(\omega_2) \subseteq Q(\omega_1)$. It satisfies Knowing that you know (KTYK) if, for all $\omega, \omega_1 \in \Omega$, $\omega_1 \in Q(\omega)$ implies $Q(\omega_1) \subseteq Q(\omega)$.*

**Lemma 2** *Suppose that for every $v \in \mathbb{R}$, where $\Omega' = \{\omega \in \Omega : X(\omega) \neq v\}$, there exist $Q : \Omega' \to 2^{\Omega'} \setminus \emptyset$, with $Q(\omega) = \Pi_{i_\omega}(\omega) \cap \Omega'$ for some $i_\omega \in I$ and $Q(\omega) = Q(\omega')$ implies $i_\omega = i_{\omega'}$, and function $\lambda : Q \to \mathbb{R}$, such that $Q$ is nested, satisfies KTYK and for all $\omega \in \Omega'$,*

$$(X(\omega) - v)\lambda(Q(\omega)) > 0.$$

*Then, security $X$ is strongly separable under partition structure $\Pi$.*

The condition requires that for each $v$, for each $\omega$ where $X(\omega) - v \neq 0$, only one trader $i$ "votes" with $\lambda(\Pi_i(\omega))$. Moreover, the generated possibility correspondence satisfies nested and KTYK.

# 3 Myopic traders

Let $\Gamma^M(\Omega, \Pi, X, \mathcal{O}, \mathcal{P}, y_0, \underline{y}, \overline{y}, s)$ be a myopic environment, where any trader making an announcement at $t$ only cares about her $t$ period payoff. Suppose $\omega^*$ is the true state and $y_0$ is the market maker's initial announcement. Then, trader 1 announces her prediction $y_1 \in Y$, where $y_1 \in d_{\mathcal{P}}(\Pi_1(\omega^*), y_0) = arg\max_{y \in Y} \min_{p \in \mathcal{P}_{\Pi_1(\omega^*)}} E_p\big[s(y, X) - s(y_0, X)\big]$.[13] As mentioned above, $y_1$ depends on the market maker's announcement $y_0$, which is not the case with expected utility.

The prediction of any trader is public, therefore the new information revealed refines the information partitions of all other traders. In particular, the initial public information is $\mathcal{F}^0(\omega^*) = \Omega$ and the updated is $\mathcal{F}^1(\omega^*) = \{\omega' \in \mathcal{F}^0(\omega^*) : d_{\mathcal{P}}(\mathcal{F}^0(\omega^* \cap \Pi_1(\omega')), y_0) = y_1\}$. Note that from Lemma 1, the announcement is unique. Trader $i$'s new private information is $\mathcal{F}^1(\omega^*) \cap \Pi_i(\omega^*)$.

Trader 2 is next to make a public announcement and her private information is $\mathcal{F}^1(\omega^*) \cap \Pi_2(\omega^*)$. She announces $y_2 = d_{\mathcal{P}}(\mathcal{F}^1(\omega^*) \cap \Pi_2(\omega^*), y_1)$ and the updated public information is $\mathcal{F}^2(\omega^*) = \{\omega' \in \mathcal{F}^1(\omega^*) : d_{\mathcal{P}}(\mathcal{F}^1(\omega^*) \cap \Pi_2(\omega'), y_1) = y_2\}$. Trader 3 updates her private information to $\mathcal{F}^2(\omega^*) \cap \Pi_3(\omega^*)$, makes an announcement and and the process goes on. More generally, player $a(t) = i$ at time $t$ has private information $F = \mathcal{F}^{t-1}(\omega^*) \cap \Pi_i(\omega^*)$ and announces $y_t = d_{\mathcal{P}}(F, y_{t-1}) = arg\max_{y \in Y} \min_{p \in \mathcal{P}_F} E_p\big[s(y, X) - s(y_{t-1}, X)\big]$.

Let $\mathcal{E} = \{\mathcal{F}^t(\omega) \cap \Pi_{a(t)}(\omega)\}_{t \geq 0, \omega \in \Omega}$ be the collection of all events on which the traders update their beliefs, given that it is their turn to make an announcement. We say that $\Gamma^M$ is *regular* if $\mathcal{P}$ is regular with respect to $\mathcal{E}$.

## 3.1 Information aggregation

Our first main result is to fully characterize information aggregation in terms of strongly separable securities.

**Theorem 1** *Fix security $X$, information structure $\Pi$ and continuous strictly proper scoring rule $s$. Information gets aggregated for any regular $\Gamma^M(\Omega, \Pi, X, \mathcal{O}, \mathcal{P}, y_0, \underline{y}, \overline{y}, s)$ if and only if $X$ is strongly separable.*

To provide some intuition, consider Lemma 3, which adapts the results of Geanakoplos and Polemarchakis (1982) to the setting with ambiguity averse and myopic traders.[14]

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[13] In general, we denote $E_{p_{\Pi_i(\omega^*)}}[s(z', X) - s(z, X)] = \frac{\sum_{\omega' \in \Pi_i(\omega^*)} p(\omega')[s(z', X(\omega')) - s(z, X(\omega'))]}{p(\Pi_i(\omega^*))}$ and $E_p[X] = \sum_{\omega \in \Omega} p(\omega) X(\omega)$.

[14] We use this Lemma also in the proof of Theorem 2, which deals with the strategic case.

**Lemma 3** *Let regular* $\Gamma^M(\Omega, \Pi, X, \mathcal{O}, \mathcal{P}, y_0, \underline{y}, \overline{y}, s)$*. At any state* $\omega$*, there exists time* $t_0$ *such that*

*(i) Public information is no longer updated,* $\mathcal{F}^t(\omega) = \mathcal{F}^{t_0}(\omega)$ *for every* $t \geq t_0$*,*

*(ii) No trader changes her prediction* $v_i$ *after time* $t_0 + 2n$*,*

*(iii) The traders reach an agreement, so that* $v = v_i$ *for all* $i \in I$*.*

This result is essentially a generalisation of reaching a consensus, studied in Geanakoplos and Polemarchakis (1982), Cave (1983), Bacharach (1985) and in particular related to the theorem about negation of asymmetric information in Geanakoplos (1994). It states that under any true state and in some finite steps the true state's information cell of the agents will reach its finest form. However, even though the traders can arrive at the common knowledge event, this does not imply that the traders, directly, agree on their predictions. They might even change their own prediction as well, depending on what is the previous prediction made. However, finally they will reach a consensus.

Intuitively, the difference between sequential announcements, among ambiguity averse traders, and the classic results in Geanakoplos and Polemarchakis (1982), Geanakoplos (1994) and Sebenius and Geanakoplos (1983) is implied by the differences of their decision functions: in our setting decision functions are not just different across traders (as in Geanakoplos (1994)) but they also depend both on private information and on the announcement of the previous trader.

Using Lemma 3, the proof for Theorem 1 proceeds in three steps. First, in finitely many rounds the information communication stops and agreement is reached. Second, as a result of no further information revelation, a specific structure of the beliefs is implied. Subsequently, combining that structure with properties of the security, the step is concluded by stating that the security is constant in all the remaining states. Finally, using properties of the decision rule, and in particular of Lemma 1, we show that information gets aggregated.

# 4 Strategic traders

Consider a game $\Gamma^S(\Omega, \Pi, X, \mathcal{O}, \mathcal{P}, y_0, \underline{y}, \overline{y}, s, \beta)$, where $s$ is a strictly proper scoring rule, $y_0$ is the market maker's initial announcement at time $t_0$ and $\beta$ is the common discount rate. Let $H^t = (y_1, \ldots, y_t)$ be a history of predictions up to time $t$ and $Y^t$ be the set of all histories up to time $t$. Given any two histories $H^k = (y_1, ..., y_n)$ and $H^l = (y'_1, ..., y'_m)$, let $(H^k, H^l)$ be their concatenation.

Let $a(t) = i$ denote the player who makes an announcement at time $t$. If $\omega$ is the true state, her initial private information is $\Pi_{a(t)}(\omega)$. If $H^{t-1}$ is the history of announcements up to $t-1$, she is at decision node $(H^{t-1}, \omega)$. Player $i$'s information set at $(H^{t-1}, \omega)$ is denoted $\mathcal{I}(H^{t-1}, \omega) = \{(H^{t-1}, \omega') : \omega' \in \Pi_i(\omega)\}$. Let $\mathscr{I}$ be the set of all information sets.

Player $i$ trades at periods $t_{i+nk}$, $k \in \mathbb{N}$, hence $a(t_{i+nk}) = i$. A strategy $y = \{y_t\}_{t=1}^{t=\infty}$, where $y_t : \Omega \times Y^{t-1} \longrightarrow Y$ is a measurable function, so that $\omega' \in \Pi_{a(t)}(\omega)$ implies $y_t(\omega', \cdot) = y_t(\omega, \cdot)$. It describes player behavior at each $t$, given every history and state. Therefore, player $i$'s strategy at $t_{i+nk}$ depends on the element of his partition and on the history of predictions until time $t_{i-1+nk}$. Let $(y(\omega)|H^t) = \{y_{t+r}(\omega, H^t, \cdot)\}_{r=1}^{\infty}$ denote the continuation of strategy $y$ given true state $\omega$ and after some history $H^t$. Let $\Phi^t(y(\omega))$ denote the history up to time $t$ generated by strategy $y$ at $\omega$. Let $\Phi^t(y(\omega)|H^k)$, $k < t$, be the history of announcements, up to time $t$, generated by strategy $y$ at state $\omega$ and given history $H^k$.

Let $\mathcal{F}_t^{t+k}(y(\omega)|H^{t-1})$ be the public information that is revealed between times $t$ and $t+k$, given state $\omega$, history $H^{t-1}$ and strategy $y$. We define this recursively. For $k=0$, $\mathcal{F}_t^t(y(\omega)|H^{t-1}) = \{\omega' \in \Omega : y(\omega', H^{t-1}) = y(\omega, H^{t-1})\}$. For $k > 0$, $\mathcal{F}_t^{t+k}(y(\omega)|H^{t-1}) = \{\omega' \in \mathcal{F}_t^{t+k-1}(y(\omega)|H^{t-1}) : y(\omega', \Phi^{t+k-1}(y(\omega)|H^{t-1})) = y(\omega, \Phi^{t+k-1}(y(\omega))|H^{t-1})\}$.

A system of beliefs is a collection of compact and convex sets of beliefs, one for each information set.

**Definition 5** *A system of beliefs is a tuple $\mathscr{P} = \{\mathcal{P}(\mathcal{I})\}_{\mathcal{I}\in\mathscr{I}}$ such that each $\mathcal{P}(\mathcal{I})$ is compact and convex.*

**Definition 6** *The continuation payoff of player $a(t) = i$ at time $t > 0$ and state $\omega$, given strategy profile $y$, history $H^{t-1}$ and system of beliefs $\mathscr{P}$ is*

$$V_t(H^{t-1}, \omega, y, \mathscr{P}) = \min_{p\in\mathcal{P}(H^{t-1},\omega)} E_p\Bigg[\sum_{k=0}^{\infty} \beta^{nk}\Big(s\Big(y_{t+nk}\big(\omega, \Phi^{t+nk-1}(y(\omega')|H^{t-1})\big), X\Big)$$
$$-s\Big(y_{t+nk-1}\big(\omega', \Phi^{t+nk-2}(y(\omega')|H^{t-2})\big), X\Big)\Big)\Bigg].[15]$$

## 4.1 Equilibrium

Before defining our equilibrium notion, it is important to note that due to ambiguity aversion the off-equilibrium path behavior is important, unlike the expected utility case of Ostrovsky (2012). There are two reasons for this. First, the decision function depends on the previous announcement, as it was highlighted in Section 2.4. Second, dynamic consistency is violated, whereas Ostrovsky (2012) uses dynamic consistency to check for deviations with all strategies in every on-path continuation game (i.e. sequential rationality). As a result, on-equilibrium path arguments used in Ostrovsky (2012) cannot be applied here.

**Definition 7** *Given game $\Gamma^S(\Omega, \Pi, X, \mathcal{O}, \mathcal{P}, y_0, \underline{y}, \overline{y}, s, \beta)$, assessment $(\mathscr{P}, y)$ is consistent if, at any decision node $(H^t, \omega)$, where $F = \mathcal{F}_{t+1}^{t+n-1}(y(\omega)|H^t) \bigcap \Pi_{a(t)}(\omega) \neq \emptyset$,*

*(i) $\mathcal{P}(H^{t-1}, \omega)$ is regular with respect to $F$,*

*(ii) If $\bigcup_{p\in\mathcal{P}(H^{t-1},\omega)} Supp(p) \bigcap F \neq \emptyset$, then $\mathcal{P}(\Phi^{t+n-1}(y(\omega)|H^t), \omega) = \mathcal{P}(H^{t-1}, \omega)_F$.*

Fix decision node $(H^{t-1}, \omega)$. At $t$, player $a(t) = i$ has beliefs $\mathcal{P}(H^{t-1}, \omega)$ and makes an announcement $y_t$, which may or may not be according to strategy $y$. From $t+1$ onwards

[15] When referring to the continuation payoff at time $i + nt$, we mean the continuation payoff divided by $\beta^{i+nt}$.

and up to $t+n-1$, every other player makes an announcement, playing according to $y$. The public information revealed from these announcements is $\mathcal{F}_{t+1}^{t+n-1}(y(\omega)|H^t)$. The first condition requires that $i$'s beliefs at decision node $(H^{t+n-1}, \omega)$ are regular with respect to her private information, $\mathcal{F}_{t+1}^{t+n-1}(y(\omega)|H^t) \bigcap \Pi_{a(t)}(\omega)$. The second condition specifies that her beliefs are updated prior by prior, using this private information.

We now provide our definition of a Revision-proof equilibrium.

**Definition 8** *Given game $\Gamma^S(\Omega, \Pi, X, \mathcal{O}, \mathcal{P}, y_0, \underline{y}, \overline{y}, s, \beta)$, consistent assessment $(y^*, \mathscr{P})$ is a pure Revision-proof equilibrium if there is no decision node $(H^{t-1}, \omega)$, player $a(t) = i$ and alternative strategy $y = (y^i, y^{*-i})$ such that for all $r \in \mathbb{N}$ and $H^{nr}$:*

$$V_{t+nr}((H^{t-1}, H^{nr}), \omega, y, \mathscr{P}) \geq V_{t+nr}((H^{t-1}, H^{nr}), \omega, y^*, \mathscr{P}),$$

*with the inequality strict for at least one $H^{nr}$.*

The equilibrium concept is closely related to the Revision-proof equilibrium of Asheim (1997) and Ales and Sleet (2014). There are some differences. First, they only consider complete information games, hence they do not consider how beliefs are updated. Essentially, we generalise their concept to incomplete information games. Second, they consider deviations from any set of subsequent players, whereas we only consider deviations of a single player at multiple subsequent periods.

Following the idea of Perfect Bayesian Equilibrium, the Revision-proof equilibrium is an assessment $(y^*, \mathscr{P})$, where $y^*$ is the strategy profile and $\mathscr{P} = \{\mathcal{P}(\mathcal{I})\}_{\mathcal{I} \in \mathscr{I}}$ is a system of beliefs for each information set. In short, instead of the idea of "sequential rationality" here we have the Revision-proof concept, with the expectation of the continuation games being based on MP preferences. In addition, we assume that there is prior by prior updating whenever possible. In particular, the definition of the equilibrium concept is given below.

Due to time inconsistency of agents' preferences (which applies to our setting because of the potential failure of dynamic consistency), a usual approach in the literature is to use the consistent planning approach (Strotz (1955), Peleg and Yaari (1973) and Goldman (1980)). The idea behind the Revision-proof equilibrium is to strengthen the concept of consistent planning in order to exclude some implausible equilibria. For example, it is plausible the consistent planning equilibrium to be such that there is no player, realised state, an alternative strategy for the player and a history such that every player's "self" after that history has weakly better payoff and strictly better at least one time. This is essentially defined as the Revision-proof property.

Our equilibrium concept is stronger than the consistent planning equilibrium which is often met in the literature. To provide a comparison, we provide below a definition of consistent planning in our framework.

**Definition 9** *Given game $\Gamma^S(\Omega, \Pi, X, \mathcal{O}, \mathcal{P}, y_0, \underline{y}, \overline{y}, s, \beta)$, consistent assessment $(y^*, \mathscr{P})$ is a pure Consistent-planning equilibrium if there is no decision node $(H^{t-1}, \omega)$, player $a(t) = i$ and alternative strategy $y = (y^i, y^{*-i})$, with $y^i_k = y^{*i}_k$ for all $k \neq t$, such that*

$$V_t(H^{t-1}, \omega, y, \mathscr{P}) > V_t(H^{t-1}, \omega, y^*, \mathscr{P}).$$

There are two differences from a Revision-proof equilibrium. First, $i$ is allowed to deviate only at some period $t$. Second, what matters for whether the deviation is successful is only whether $i$ at $t$ strictly prefers it, not whether subsequent selves would also feel the same way.

For the results about strategic traders we restrict attention to pure Revision-proof equilibria that also converge. In particular, convergence requires that on path the predictions of each trader converge to some real number (not necessarily the same across traders).

**Definition 10 (Convergence)** *Strategy profile $y^* = (y^{*1}, ..., y^{*n})$ is converging if, for all $i \in I$ and $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$, there exists measurable $y^{*i}_{\infty} : \Omega \times Y^{\mathbb{N}} \longrightarrow Y$ such that $y^{*i}_{\infty}(\omega_1, H) = y^{*i}_{\infty}(\omega, H)$ if $\omega_1 \in \Pi_i(\omega)$ and*

$$\lim_{k \to \infty} y^*_{i+nk}\big(\omega, \Phi^{i-1+nk}(y^*(\omega))\big) = y^{*i}_{\infty}\big(\omega, \Phi^{\infty}(y^*(\omega))\big).$$

*We say that $(y^*, \mathcal{P})$ is converging if $y^*$ is converging.*

We can think of two classes of potential equilibria: the first one is when the strategic prediction of the traders are converging, on path, to a real number (the limit is, potentially, different among traders) and the second one is when the predictions are not converging for at least one trader and state. In this paper we want to restrict attention to the first class, as in the second class of equilibria, obviously, information cannot get aggregated.

In Ostrovsky (2012), there was no separation between those two classes. In that way, the information aggregation result for separable securities that was proven, essentially implies a necessary condition for the existence of equilibrium in MSR games under subjective expected utility. In fact, the necessary condition is that those equilibria not only belong to the first class but also the real number that strategic predictions are converging are the same for all traders (and, in fact, the true value of the security).

In our case, we will restrict to the first class of equilibria. The reason for making such an assumption is the lack of properties as compared to the subjective expected utility setting. In particular, the arguments about instant opportunity and arbitrage in Ostrovsky (2012) use heavily properties inherent to subjective expected utility, allowing to bypass the issue whether in equilibrium the predictions for each trader converge to a real number. In fact, these arguments are based on the existence of a unique prior. In a MP environment, a similar approach would require more assumptions on the structure of the common set of beliefs, but this would imply a less practically meaningful result.

## 4.2 Information aggregation

We now provide our second main result, which shows that strong separability is necessary and sufficient for information aggregation in strategic environments.

**Theorem 2** *Fix information structure $\Pi$, scoring rule $s$ and bounds $\underline{y}, \overline{y}$.*

*(i) If security $X$ is strongly separable under $\Pi$ and $s$, then for any $\Gamma^S$ and any pure converging Revision-proof equilibrium, information gets aggregated.*

(*ii*) *If security $X$ is not strongly separable under $\Pi$ and $s$, then there exist game $\Gamma^S$ and a pure converging Revision-proof equilibrium such that information does not get aggregated.*

The proof of Theorem 2 is more involved and proceeds in three steps. First, we argue that in a Revision-proof equilibrium the continuation games for each player $i$, at every realised true state and after any history that the player trades should be a non negative real number. Second, since strategies converge and using properties of the MSR we show that the continuation payoffs should converge to zero. Finally, we conclude that in the limit the Revision-proof predictions are the myopic ones and then by strong separability we show that information gets aggregated.

### 4.3 Myopic behavior can be Revision-proof

In this section we provide two results, relating the myopic behavior with a pure Revision-proof equilibrium, when a strongly separable security is traded and the discounting factor is $\beta \in (0, (\frac{1}{3})^{1/n})$, where $n$ is the number of players. Suppose that all public information is revealed at round $R$ given a Revision-proof equilibrium. Then, from the next round each trader plays the myopic strategy. In addition, if all public information is always revealed at the first round, then each trader always plays the myopic strategy.

**Theorem 3** *Suppose that $X$ is a strongly separable security and $\beta \in (0, (\frac{1}{3})^{1/n})$. Then, in any pure converging Revision-proof equilibrium where all public information is revealed at round $R$, each trader plays the myopic strategy from round $R+1$ onwards. Moreover, if $R = 1$ at all states, then traders play the myopic strategy from the beginning.*

## 5 Concluding remarks

The main purpose of the paper is to study the information aggregation in a dynamic market using the market scoring rule (MSR) that is mainly used in prediction markets, with ambiguity averse and partially informed traders. Our results indicate that when either myopic or strategic traders participate in the market, information does not always get aggregated, unless the security is strongly separable. Compared to similar results under subjective expected utility , this result is new and indicates that ambiguity aversion creates an extra layer that precludes information aggregation.

Finally, the paper leaves open some interesting questions. For example, can the assumption of converging strategies be avoided, hence more technical arguments should be used in the proof for strategic traders? What happens, in terms of information aggregation, when the players do not play in fixed turn but randomly? Finally, under which condition there is an equilibrium?

## A Appendix

Where convenient, we use the notation $s(y)(.) \equiv s(y, X(.))$.

**Proof of Lemma 1.** We first show that $arg\max_{y\in Y}\min_{p\in\mathcal{P}} E_p\big[s(y)-s(y_{-1})\big]$ does, in fact, exist. This is true because $s$ is continuous function, therefore $\min_{p\in\mathcal{P}} E_p\big[s(y)-s(y_{-1})\big]$ is upper semi continuous (as infimum of continuous functions) as a function of $y$. Since $Y$ is compact, a maximum exists and the set $arg\max_{y\in Y}\min_{p\in\mathcal{P}} E_p\big[s(y)-s(y_{-1})\big]$ is not empty.

Next, we define $Z$ to be the convex hull of $\{s(y)\}_{y\in Y}$. The set $\{s(y)\}_{y\in Y}$ is compact in $\mathbb{R}^l$ because $s$ is continuous in $y$ and $Y$ is compact, hence $Z$ is compact. Consider the function $G:\mathcal{P}\times Z\longrightarrow\mathbb{R}$ defined by $G(p,z)=E_p[z-s(y_{-1})]$. The function is linear in $p$ and affine in $z$. Moreover, it is continuous both in $p$ and in $z$. The first is because of the definition of weak$^*$ convergence and the second applying Lebesgue's dominated convergence theorem.

By Sion's minimax Theorem (Berge (1963), p. 210), there exists $p^*\in\mathcal{P}$ and $z^*\in Z$ such that for all $(p,z)\in\mathcal{P}\times Z$ it is $E_{p^*}\big[z-s(y_{-1})\big]\le E_{p^*}\big[z^*-s(y_{-1})\big]\le E_p\big[z^*-s(y_{-1})\big]$. Then we get that $\min_{p\in\mathcal{P}}\max_{z\in Z} E_p\big[z-s(y_{-1})\big]=\max_{z\in Z}\min_{p\in\mathcal{P}} E_p\big[z-s(y_{-1})\big]$ and it is achieved at $p=p^*$, $z=z^*$.

For a fixed $p$, and because $G(p,z)$ is affine in $z$, the unique maximiser of $E_p\big[z-s(y_{-1})\big]$ over $z$ is $s(E_p[X])$ (since $s$ is a proper scoring rule, by definition of Z), so that $z^*=s(E_{p^*}[X])$. Hence we may conclude $\min_{p\in\mathcal{P}}\max_{y\in Y} E_p\big[s(y)-s(y_{-1})\big]=\max_{y\in Y}\min_{p\in\mathcal{P}} E_p\big[s(y)-s(y_{-1})\big]$ and it is achieved at $p=p^*$, $y=E_{p^*}[X]$.

We claim that $y=E_{p^*}[X]$ is a unique element of $arg\max_{y\in Y}\min_{p\in\mathcal{P}} E_p\big[s(y,X(\omega))-s(y_{-1},X(\omega))\big]$. To see that, let $y'\ne E_{p^*}[X]$. Then,

$$\min_{p\in\mathcal{P}} E_p\big[s(y',X(\omega))-s(y_{-1},X(\omega))\big]\le E_{p^*}\big[s(y^{'},X(\omega))-s(y_{-1},X(\omega))big]<$$

$$E_{p^*}\big[s(E_{p^*}[X],X(\omega))-s(y_{-1},X(\omega))\big]=\max_{y\in Y}\min_{p\in\mathcal{P}} E_p\big[s(y,X(\omega))-s(y_{-1},X(\omega))\big].$$

Hence, the maximiser is unique. ■

**Proof of Lemma 2.** Suppose the conditions of the Lemma are true but $X$ is not strongly separable for $\mathcal{P}$ and $v$. Then, for each $\omega\in\bigcup_{p\in\mathcal{P}} Supp(p)=E$, from Lemma 1, for all $i\in I$, we have $E_p[X(\omega)-v|\Pi_i(\omega)]=0$, for some $p\in\mathcal{P}$. This implies that for all $\omega\in\Omega'$, where $Q(\omega)=\Pi_{i_\omega}(\omega)\cap\Omega'$ for some $i_\omega\in I$, we have $\lambda(Q(\omega))E_{p_\omega}[X(\omega)-v|Q(\omega)]=0$, for some $p_\omega\in\mathcal{P}$. Moreover, if $Q$ satisfies nested and KTYK, then $Q':\Omega'\cap E\to 2^{\Omega'\cap E}\setminus\emptyset$ defined as $Q'(\omega)=Q(\omega)\cap E$ also satisfies these two properties. So without loss of generality we denote $Q'$ as $Q$.

We will show that if, for all $\omega\in E$, $\lambda(Q(\omega))E_{p_\omega}[X(\omega)-v|Q(\omega)]=0$, for some $p_\omega\in\mathcal{P}$, then $\lambda(Q(\omega))E_p[X(\omega)-v]=0$, for some $p\in\Delta(E)$. We prove by induction on the cardinality of $E$. If $E$ has only one element, the claim is true. Assume that whenever $E$ has $k$ elements the claim is true, so that (equivalently) if, for all $\omega\in E$, $\lambda(Q(\omega))E_{p_\omega}[X(\omega)-v]=0$, for some $p_\omega$, then $\lambda(Q(\omega))E_p[X(\omega)-v]=0$ for some $p\in\Delta(E)$.

Suppose that $E$ consists of $k+1$ elements. Let $E_1=\{\omega\in\Omega:Q(\omega)=E\}$. If $E_1=E$ then the claim is true. If $E_1\ne E$, define $E_2=Q(\omega)$, where $Q(\omega)$ has the largest number of states less than $k+1$. Let $E_3=E\setminus(E_1\cup E_2)$. Applying arguments similar to Geanakoplos (1989), since $Q$ satisfies nondelusion ($\omega\in Q(\omega)$ for all $\omega\in E$), nested and KTYK, $E_1,E_2,E_3$

are disjoint and their union is $E$.

We apply the induction hypothesis on $E_2$ and $E_3$ separately, with probabilities $p_2$ and $p_3$. Since they are disjoint, the claim is true also on $E_2 \cup E_3$, by adding over the two sets and normalising to a new $p$. By setting $p(\omega) = 0$ to all $\omega \in E_1$, the claim is true in $E$ for $p \in \Delta E$, because $E_1 \subsetneq E$. Since the support of $p$ is in $E$, there exists $\omega \in E$ with $X(\omega) \neq v$, $E \subseteq \{\omega \in \Omega : X(\omega) \neq v\}$ and $\lambda(Q(\omega))E_p[X(\omega) - v] = 0$. But then, it cannot be that $(X(\omega) - v)\lambda(Q(\omega)) > 0$ for all $\omega$ with $X(\omega) \neq v$, a contradiction.

■

**Proof of Lemma 3.**

(i) At $t = 1$, trader 1 makes a myopic prediction, hence her announcement is $y_1 = d_{\mathcal{P}}(\Pi_1(\omega), y_0)$. The public information at state $\omega$ was initially $\mathcal{F}^0(\omega) = \{\omega' \in \Omega : p(\omega') > 0 \textit{ for some } p \in \mathcal{P}\} = \Omega$, but after the first prediction it is $\mathcal{F}^1(\omega) = \{\omega' \in \mathcal{F}^0(\omega) : d_{\mathcal{P}}(\Pi_1(\omega'), y_0) = y_1\}$. Each trader refines her own information as well, hence at time $t$ the prediction of player $a(t)$ is $y_t = d_{\mathcal{P}}(\Pi_{a(t)}(\omega) \cap \mathcal{F}^{t-1}(\omega), y_{t-1})$ with $\mathcal{F}^t(\omega) = \{\omega' \in \mathcal{F}^{t-1}(\omega) : d_{\mathcal{P}}(\Pi_{a(t)}(\omega') \cap \mathcal{F}^{t-1}(\omega), y_{t-1}) = y_t\}$.

By construction, $\mathcal{F}^0(\omega) \supseteq \mathcal{F}^1(\omega) \supseteq ... \supseteq \mathcal{F}^t(\omega)$. Because $\Omega$ is finite, there exists $t_0$ such that $\mathcal{F}^t(\omega) = \mathcal{F}^{t_0}(\omega)$ for every $t \geq t_0$. Without loss of generality, suppose that $t_0$ is the time of the first player in round $T$.

(ii) Firstly, we can observe that the function $\Phi(p) = E_p[s(E_p[X], X) - s(z, X)]$, with $p \in \Delta(\Omega)$ is convex in $p$, for any $z \in [a, b]$, with $a = \min\{E_p[X] : p \in \Delta(\Omega)\}$ and $b = \max\{E_p[X] : p \in \Delta(\Omega)\}$. Define the function $g(E_p[X]) = \Phi(p)$. Note that g is convex in $\{E_p[X] : p \in \Delta(\Omega)\}$ and because its unique minimiser is at $z$ we get that $g$ is decreasing at $[a, z]$ and increasing at $[z, b]$.[16] From Lemma 1, the myopic announcement of trader $i$ at time $t$, when the previous announcement is $z$, is given by $d_{\mathcal{P}}(\mathcal{F}^{t-1}(\omega) \cap \Pi_i(\omega), z) = E_{p^*}[X]$ for some $p^* \in \mathcal{P}_{\mathcal{F}^{t-1}(\omega) \cap \Pi_i(\omega)}$, hence $d_{\mathcal{P}}(\mathcal{F}^{t-1}(\omega) \cap \Pi_i(\omega), z) = \underset{x \in \{E_p[X] : p \in \mathcal{P}_{\mathcal{F}^{t-1}(\omega) \cap (\Pi_i(\omega))}\}}{\arg\min} g(x)$.

In fact, if $z$ (the unique minimiser in $\Delta(\Omega)$) is on the left hand side of $\{E_p[X] : p \in \mathcal{P}_{\mathcal{F}^{t-1}(\omega) \cap \Pi_i(\omega)}\}$ then the left hand side extreme point of the interval $\{E_p[X] : p \in \mathcal{P}_{\mathcal{F}^{t-1}(\omega) \cap \Pi_i(\omega)}\}$ is the minimising value, and the right hand side extreme point in case $z$ in on the right hand side (because of the convexity of $\Phi$, hence of $g$ and the fact that $z$ is global minimum).

**Step 1**: Define $A^i_{\omega'} = \{E_p[X|\Pi_i(\omega')] : p \in \mathcal{P}_{\mathcal{F}^{t_0}(\omega)}\}$ for every $i = 1, ..., n$ and $\omega' \in \mathcal{F}^{t_0}(\omega) = \{\omega_1, \ldots, \omega_k\}$. Because there is no information revelation after $t_0$, we show that $A^i = \bigcap_{\omega' \in \mathcal{F}^{t_0}(\omega)} A^i_{\omega'} \neq \emptyset$ for every $i = 1, ..., n$.

To prove that claim, suppose that, for some $i$, $A^i = \emptyset$. Define $m = \min\{k : \bigcap_{\omega' \in \{\omega_1, \ldots, \omega_k\}} A^i_{\omega'} = \emptyset\}$. Because $A^i_{\omega_m}$ and $\bigcap_{\omega' \in \{\omega_1, \ldots, \omega_{m-1}\}} A^i_{\omega'}$ have empty intersection, by definition of $m$, and because both are intervals let's assume, without loss of generality, that interval $A^i_{\omega_\lambda}$ is

[16] We can observe that there exists $p \in \Delta(\Omega)$ such that $E_p[X] = z$. In addition, the set $\{E_p[X] : p \in \mathcal{P}\}$ is an interval, as a convex and closed set of the real numbers.

to the left of interval $\bigcap_{\omega' \in \{\omega_1,...,\omega_{m-1}\}} A^i_{\omega'}$. We can observe, as well, that the left extreme point of $\bigcap_{\omega' \in \{\omega_1,...,\omega_{m-1}\}} A^i_{\omega'}$ is the left extreme point of $A^i_{\omega_k}$ for some $\omega_k \in \{\omega_1, ..., \omega_{m-1}\}$. Therefore, for any announcement of $(i-1)$-th trader, the candidate predictions, of trader $i$, at state $\omega_m$ and $\omega_k$ are different, hence there should be information revelation, a contradiction.

Similarly, we can conclude that for no information to be revealed, the announcement of each trader $i$ (which is the same for every state in $\mathcal{F}^{t_0}(\omega)$) should lie in the interval $A^i$ (which we know it is non empty by the previous claim). To prove it, assume again that the announcement $y \notin A^i$ for some trader $i$, and in particular, without loss of generality we assume it is on the left hand side of it. Hence, as before, there is $\omega_m \in \mathcal{F}^{t_0}(\omega)$ such that the left hand side extreme point of $A^i_{\omega_m}$ is the same as the left hand side extreme point of $A^i$, and therefore the announcement of $i$ at $\omega_m$ is different than $y$ (for any announcement of the $(i-1)$-th trader). Hence we have revelation of information, a contradiction.

**Step 2**: We now show that no trader changes her prediction after $t_0 + 2n$. There are two cases, either $\bigcap_{j \in \{1,...,n\}} A^j = \emptyset$ or $\bigcap_{j \in \{1,...,n\}} A^j \neq \emptyset$.

First, suppose that $\bigcap_{j \in \{1,...,n\}} A^j = \emptyset$. Define $i_0 = \min\{i : \bigcap_{j \in \{1,...,i\}} A^j = \emptyset\}$. Therefore, $A^{i_0}$ has empty intersection with $\bigcap_{j \in \{1,...,i_0-1\}} A^j$, and without loss of generality we assume that $A^{i_0}$ is on the left hand side of $\bigcap_{j \in \{1,...,i_0-1\}} A^j$. Because $\bigcap_{j \in \{1,...,i_0-1\}} A^j$ is an interval we can conclude that there are $A^{i_1}$ and $A^{i_2}$ such that one of them define the left hand side extreme point of the interval and the other one the right hand side extreme point.

We can observe that for any value that $y_{t_0-1}$ might have, either trader $i_1$ or $i_2$, and in particular trader $i_3 = \max\{i_1, i_2\}$, gives a prediction belonging in the set $\bigcap_{j \in \{1,...,i_0-1\}} A^j$.[17] By the definition of the function $g$ we get that from trader $\max\{i_1, i_2\}$ until trader $i_0-1$ the corresponding prediction belongs, as well, to $\bigcap_{j \in \{1,...,i_0-1\}} A^j$. Hence, always the prediction of $i_0$ is the right hand side extreme point of $A^{i_0}$, and let's denote it by $v_{i_0}$. We denote the announcements of traders $j = i_0 + 1, ..., n$ with $v_j$.

For the next round, the $v_n$ potentially triggers different announcements for traders $j = 1, ..., i_0$ (compared to their announcements of the previous period) and denote them by $v_j$. But, with exactly the same argument as before, the announcement of $i_0 - 1$ should belong to $\bigcap_{j \in \{1,...,i_0-1\}} A^j$. Hence, the announcements of traders $i_0, ..., n$ remain $v_{i_0}, ..., v_n$ and therefore the $v_1, ..., v_n$ remain the same for any later round.

[17] This is because either $i_1$ or $i_2$ (or both) will bring the announcement to the left or right hand side of $\bigcap_{j \in \{1,...,i+1\}} A^j$. Hence for the trader $\max\{i_1, i_2\}$ until $t_0 - 1$ the announcement will be the same (because the minimiser of $g$ is achieved within their candidate predictions set).

Second, suppose $\bigcap_{j\in\{1,...,n\}} A^j \neq \emptyset$. There are $A^{i_1}$ and $A^{i_2}$ such that one defines the left hand side extreme point of the interval and the other defines the right hand side extreme point. Using similar arguments as before, for any value of the $y_{t_0-1}$ either trader $i_1$ or $i_2$ (and in particular trader $i_3 = \max\{i_1, i_2\}$) gives a prediction belonging in the set $\bigcap_{j\in\{1,...,n\}} A^j$. We denote the corresponding announcement with $v_{i_3}$. By the definition of $g$ we conclude that for $j = i_3, ..., n$ their announcements are $v_{i_3}$. Because $v_{i_3} \in \bigcap_{j\in\{1,...,n\}} A^j$ we conclude that at the next round the announcement of each trader $1, ..., i_3 - 1$ is $v_{i_3}$, too. Hence we get $v_1 = ... = v_n = v_{i_3}$. Note here that the agreement only applies to this second case.

(iii) Denote, for simplicity, $\mathcal{F}^{t_0}(\omega) = \mathcal{F}^T$. Let $v_i$ be trader $i$'s permanent (from $(ii)$) prediction. From $(ii)$ we get that $\min_{p\in\mathcal{P}} E_{p|\mathcal{F}^T\cap\Pi_i(\omega')}\big[s(v_i, X) - s(v_{i-1}, X)\big] \geq 0$ for all $\omega' \in \mathcal{F}^T$ and $i = 1, ..., n$.[18] Therefore, for every $p \in \mathcal{P}$ it is $p(\mathcal{F}^T \cap \Pi_i(\omega'))E_{p|\mathcal{F}^T\cap\Pi_i(\omega')}\big[s(v_i, X) - s(v_{i-1}, X)\big] \geq 0$ and $p(\mathcal{F}^T \cap \Pi_i(\omega')) > 0$, for every $\omega' \in \mathcal{F}^T$.[19] Summing over $\mathcal{C}_i = \{\Pi_i(\omega) : \omega \in \mathcal{F}^T\}$ we get $p(\mathcal{F}^T)E_{p|\mathcal{F}^T}\big[s(v_i, X) - s(v_{i-1}, X)\big] \geq 0$, for all $i = 1, ..., n$. By summing over $i$ and ignoring $p(\mathcal{F}^T)$, we have

$$E_{p|\mathcal{F}^T}\big[s(v_1, X) - s(v_n, X)\big] + E_{p|\mathcal{F}^T}\big[s(v_2, X) - s(v_1, X)\big] + \ldots +$$
$$+E_{p|\mathcal{F}^T}\big[s(v_n, X) - s(v_{n-1}, X)\big] = 0.$$

Because each term is non negative, for every $p \in \mathcal{P}$ we have $E_{p|\mathcal{F}^T}\big[s(v_i, X) - s(v_{i-1}, X)\big] = 0$, $i = 1, ..., n$. For the same reason, $E_{p|\mathcal{F}^T\cap\Pi_i(\omega')}\big[s(v_1, X) - s(v_n, X)\big] = 0$ for all $\omega' \in \mathcal{F}^T$ and $p \in \mathcal{P}$.

Therefore, for every $i = 1, ..., n$ it is $\min_{p\in\mathcal{P}} E_{p|\mathcal{F}^T\cap\Pi_i(\omega')}\big[s(v_i, X) - s(v_{i-1}, X)\big] = 0$ for every $\omega' \in \mathcal{F}^T$. Hence, by $(ii)$, for every $\omega' \in \mathcal{F}^T$ it is $E_{p^*|\mathcal{F}^T\cap\Pi_i(\omega')}\big[s(E_{p^*|\mathcal{F}^T\cap\Pi_i(\omega')}[X], X) - s(v_{i-1}, X)\big] = 0$, where $E_{p^*|\mathcal{F}^T\cap\Pi_i(\omega')}[X] = d_{\mathcal{P}}(\mathcal{F}^T \cap \Pi_i(\omega'), v_{i-1}) = v_i$ (note here that $p^*$ depends on all arguments of $d_{\mathcal{P}}$).

Because $s$ is a strictly proper scoring rule, we get $v_{i-1} = E_{p^*|\mathcal{F}^T\cap\Pi_i(\omega')}[X] = d_{\mathcal{P}_{\mathcal{F}^T}}(\Pi_i(\omega'), v_{i-1}) = v_i$ for every $\omega' \in \mathcal{F}^T$ and every $i = 1, ..., n$. Hence $v_i = v_j$ for all $i, j$ and agreement is reached.

■

**Proof of Theorem 1.**

($\Leftarrow$) Suppose $X$ is strongly separable. By Lemma 3 $(i)$, we know that there exists time $t_0$ such that $\mathcal{F}^t(\omega) = \mathcal{F}^{t_0}(\omega)$ for every $t \geq t_0$. We denote this set by $\mathcal{F}^T$.

From Lemma 3, traders reach an agreement (at most within three rounds after $t_0$), hence there exists $v \in \mathbb{R}$ such that for every $i = 1, ..., n$ it is $d_{\mathcal{P}}(\Pi_i(\omega) \cap \mathcal{F}^T, v) = v$ for every $\omega \in \mathcal{F}^T$, with $p(\omega|\mathcal{F}^T) > 0$ for some $p \in \mathcal{P}$ (this last property is trivially satisfied by construction of

[18] By $v_0$ we denote, when appropriate, the $v_n$.

[19] It is $p(\mathcal{F}^T \cap \Pi_i(\omega')) > 0$ for every $p \in \mathcal{P}$. This is because for every $\omega' \in \mathcal{F}^T$ there exists $p \in \mathcal{P}$ with $p(\omega') > 0$, by its definition. Regularity then implies $p(\mathcal{F}^T \cap \Pi_i(\omega')) > 0$ for every $p \in \mathcal{P}$.

$\mathcal{F}^t(\omega)$). By defining $\mathcal{P}_{\mathcal{F}^T} = \{p(\cdot|\mathcal{F}^T) : p \in \mathcal{P}\}$, we can observe that for every $i = 1, ..., n$ it is $d_{\mathcal{P}_{\mathcal{F}^T}}(\Pi_i(\omega), v) = v$ for every $\omega \in \Omega$, with $q(\omega) > 0$ for some $q \in \mathcal{P}_{\mathcal{F}^T}$.

By Definition 3 of strong separability, the set of priors $\mathcal{P}_{\mathcal{F}^T}$, which is convex, compact and mutually absolute continuous with respect to $\Pi_i$ for every $i = 1, ..., n$, and the $v$, we observe that $(ii)$ is satisfied hence $(i)$ should be violated. Therefore, we get that $X(\omega) = k$ for some $k \in \mathbb{R}$ for every $\omega \in \Omega$, with $q(\omega) > 0$ for some $q \in \mathcal{P}_{\mathcal{F}^T}$.[20] Hence, information gets aggregated, because as soon as the security is constant for all states, for which at least one (updated) belief gives it strictly positive probability, and because by Lemma 1 the myopic prediction is an expectation using the appropriate (updated) belief, we conclude that the myopic predictions, which are all $v$, are just the constant value of the security.

($\Rightarrow$) Suppose information gets aggregated, so that $y_t(\omega) = d_{\mathcal{P}}(\Pi_{a(t)}(\omega) \cap \mathcal{F}^{t-1}(\omega), y_{t-1}) \longrightarrow X(\omega)$, for every $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$. We show that, for any regular $\mathcal{P}$ and $v \in \mathbb{R}$, if $(ii)$ in Definition 3 is satisfied, then $(i)$ is violated.

Suppose there exists $v \in \mathbb{R}$ such that $d_{\mathcal{P}}(\Pi_i(\omega), v) = v$ for all $i = 1, ..., n$ and $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$. Consider $\Gamma^M$ with initial announcement $y_0 = v$. Then, the predictions $y_t(\omega)$, $t = 0, 1, ...$, are constant for all $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$, and in particular will be $v$ always.

Because no information is revealed, $\mathcal{F}^T = \mathcal{F}^t(\omega) = \{\omega \in \Omega : p(\omega) > 0 \textit{ for some } p \in \mathcal{P}\}$ for every $t = 1, 2, ....$ Hence, from the information aggregation assumption, for the selections made before and for $v$ to be the initial announcement and observing that $\mathcal{P}$ is regular, we have that $y_t = v = d_{\mathcal{P}}(\mathcal{F}^{t-1}(\omega) \cap \Pi_{a(t)}(\omega), v) = d_{\mathcal{P}}(\Pi_{a(t)}(\omega), v) \longrightarrow X(\omega)$ for every $\omega \in \Omega$, with $p(\omega) > 0$ for some $p \in \mathcal{P}$.[21] But as we observed, we have that for every $t = 1, 2, ...$ the prediction $y_t$ is the same for all $\omega \in \mathcal{F}^T$. From the uniqueness of the limit, $X(\omega) = k$ for every $\omega \in \Omega$ (for some real number $k$), with $p(\omega) > 0$ for some $p \in \mathcal{P}$. Hence, the security is strongly separable. ■

# B Appendix

Before we proceed with the proof of Theorem 2, we state a useful result.

**Lemma 4** *Consider a convex and compact set of priors $\mathcal{P} \subseteq \Delta(\Omega)$, trader $i \in I$, discount factor $0 < \beta < 1$ and state $\omega^*$ with $p(\omega^*) > 0$ for some $p \in \mathcal{P}$. Then, the function $f : Y^{\mathbb{N}} \times Y^{\mathbb{N}} \longrightarrow \mathbb{R}$ with $f\big((x_n)_{\mathbb{N}}, (y_n)_{\mathbb{N}}\big) = \min_{p \in \mathcal{P}} \sum_{k=0}^{\infty} \beta^k E_{p|\Pi_i(\omega^*)}\Big[s\Big(x_k, X\Big) - s\Big(y_k, X\Big)\Big]$ is continuous in $\big((x_n)_{\mathbb{N}}, (y_n)_{\mathbb{N}}\big)$.*

[20] The absolute continuity is true because for an arbitrary $\omega \in \Omega$ and $i \in \{1, ..., n\}$ the $\Pi_i(\omega) \cap \mathcal{F}^T$ is either an empty set or not. If it is empty, by definition of the set of priors, we get that all priors in $\mathcal{P}_{\mathcal{F}^T}$ give zero probability. If it is not empty and if $\omega' \in \Pi_i(\omega) \cap \mathcal{F}^T$, then (by the definition of $\mathcal{F}^T$) it can be realised as true state and because $\omega' \in \mathcal{F}^T$ every prediction would be the same hence it is $\mathcal{F}^T = \mathcal{F}^T(\omega', y_{T-1})$ and hence by the regularity assumption we get that the finest information set of the trader, $\mathcal{F}^T \cap \Pi_i(\omega')$, has strictly positive probability for every prior, and hence the likelihood of $\Pi_i(\omega) = \Pi_i(\omega')$ is strictly positive for every prior.

[21] Information aggregation is (by definition) allowed to be violated only for a set of states that is of zero probability for all $p \in \mathcal{P}$. This implies that information should be aggregated at least for those states for which there exists a prior which gives the state strictly positive probability.

**Proof.** As a first step, consider the function $f_1 : Y^{\mathbb{N}}\times Y^{\mathbb{N}} \longrightarrow Y^{\mathbb{N}}\times Y^{\mathbb{N}}$ with $h\big((x_n)_{\mathbb{N}},(y_n)_{\mathbb{N}}\big) = \Big(\big(s(x_n, X(\omega))\big)_{\mathbb{N}}, \big(s(y_n, X(\omega))\big)_{\mathbb{N}}\Big)$, which is continuous with the product topology for every $\omega \in \Omega$, because $s$ is continuous.[22]

In addition, consider the function $f_2 : Y^{\mathbb{N}}\times Y^{\mathbb{N}} \longrightarrow \mathbb{R}$ with $f_2\big((x_n)_{\mathbb{N}},(y_n)_{\mathbb{N}}\big) = \sum_{k=0}^{\infty}\beta^k\big[x_k - y_k\big]$. We prove below that this function is continuous in $\big((x_n)_{\mathbb{N}},(y_n)_{\mathbb{N}}\big)$. Given that, the function $f_2 \circ f_1$ is, then, continuous for every $\omega \in \Omega$.

In order to prove the continuity consider $\big((x_n)_{\mathbb{N}}\big)^m \longrightarrow_m (x_n)_{\mathbb{N}}$ and $\big((y_n)_{\mathbb{N}}\big)^m \longrightarrow_m (y_n)_{\mathbb{N}}$. Then we can observe the following:

$$|\sum_{k=0}^{\infty}\beta^k(x_k^m - y_k^m) - \sum_{k=0}^{\infty}\beta^k(x_k - y_k)| =$$

$$|\sum_{k=0}^{\infty}\beta^k\big((x_k^m - x_k) + (y_k - y_k^m)\big)| \leq \sum_{k=0}^{\infty}\beta^k\big[|x_k^m - x_k| + |y_k - y_k^m|\big] =$$

$$\sum_{k=0}^{\infty}\beta^k|x_k^m - x_k| + \sum_{k=0}^{\infty}\beta^k|y_k - y_k^m| =$$

$$d\Big(\big((x_n)_{\mathbb{N}}\big)^m, (x_n)_{\mathbb{N}}\Big) + d\Big(\big((y_n)_{\mathbb{N}}\big)^m, (y_n)_{\mathbb{N}}\Big).$$

Hence we get the continuity.

Next, consider the set $\mathcal{P} \subseteq \Delta(\Omega)$ with the weak* topology and finally the set $Y^{\mathbb{N}} \times Y^{\mathbb{N}} \times \mathcal{P}$ with the product topology. Consider the function $h : Y^{\mathbb{N}} \times Y^{\mathbb{N}} \times \mathcal{P} \longrightarrow \mathbb{R}$, with $h\big((x_n)_{\mathbb{N}},(y_n)_{\mathbb{N}},p\big) = \sum_{k=0}^{\infty}\beta^k E_{p|\Pi_i(\omega^*)}\big[s(x_k,X) - s(y_k,X)\big]$. We will prove that this function is continuous in $\big((x_n)_{\mathbb{N}},(y_n)_{\mathbb{N}},p\big)$.

Firstly, we can observe that $h\big((x_n)_{\mathbb{N}},(y_n)_{\mathbb{N}},p\big) = \sum_{k=0}^{\infty}\beta^k E_{p|\Pi_i(\omega^*)}\Big[s(x_k,X) - s(y_k,X)\Big] = E_{p|\Pi_i(\omega^*)}\Big[\sum_{k=0}^{\infty}\beta^k\big(s(x_k,X) - s(y_k,X)\big)\Big] = \sum_{\omega\in\Pi_i(\omega^*)}p(\omega)\Big[\sum_{k=0}^{\infty}\beta^k\big(s(x_k,X) - s(y_k,X)\big)\Big]$, where the first equality is because of the *Beppo-Levi* theorem.[23]

Take a sequence $\Big(\big((x_n)_{\mathbb{N}}\big)^j, \big((y_n)_{\mathbb{N}}\big)^j, p_j\Big)_{j\in\mathbb{N}} \in Y^{\mathbb{N}}\times Y^{\mathbb{N}}\times\mathcal{P}$ such that $\Big(\big((x_n)_{\mathbb{N}}\big)^j, \big((y_n)_{\mathbb{N}}\big)^j, p_j\Big) \longrightarrow_j \big((x_n)_{\mathbb{N}},(y_n)_{\mathbb{N}},p\big)$. Then, $h\Big(\big((x_n)_{\mathbb{N}}\big)^j, \big((y_n)_{\mathbb{N}}\big)^j, p_j\Big) = \sum_{\omega\in\Pi_i(\omega^*)}p_j(\omega)\Big[\sum_{k=0}^{\infty}\beta^k\big(s(x_k^j,X(\omega)) - s(y_k^j,X(\omega))\big)\Big]$.

For every $\omega \in \Pi_i(\omega^*)$ we have that $\sum_{k=0}^{\infty}\beta^k\big[s(x_k,X(\omega)) - s(y_k,X(\omega))\big]$ is continuous in $\big((x_n)_{\mathbb{N}},(y_n)_{\mathbb{N}}\big)$, as long as it is the function $f_2 \circ f_1$. In addition, $p_j \longrightarrow p$ in the weak* topology if and only if $p_j(\omega) \longrightarrow_j p(\omega)$, for every $\omega \in \Omega$. Therefore, we conclude

[22]We assume the metric $d\big((x_n)_{\mathbb{N}},(y_n)_{\mathbb{N}}\big) = \sum_{k=0}^{\infty}\beta^k\big[d_k(x_n,y_n)\big]$, where $\beta \in (0,1)$ is the discounting factor and $d_k$ is the euclidean distance at each coordinate. The topology implied by this metric is equivalent to the product topology in $Y^{\mathbb{N}}$.

[23]This is a standard result of measure theory. It states that if $\{g_n\}$ is a sequence of non-negative measurable functions, then $\int\sum_{n=1}^{\infty}g_n dm = \sum_{n=1}^{\infty}\int g_n dm$.

$$h\Big(\big((x_n)_{\mathbb{N}}\big)^j, \big((y_n)_{\mathbb{N}}\big)^j, p_j\Big) = \sum_{\omega\in\Pi_i(\omega^*)} \frac{p_j(\omega)}{p_j(\Pi_i(\omega^*))}\Big[\sum_{k=0}^{\infty}\beta^k\big(s(x_k^j, X) - s(y_k^j, X)\big)\Big] \longrightarrow_j$$

$$\sum_{\omega\in\Pi_i(\omega^*)} \frac{p(\omega)}{p(\Pi_i(\omega^*))}\Big[\sum_{k=0}^{\infty}\beta^k\big(s(x_k, X) - s(y_k, X)\big)\Big] = h\big((x_n)_{\mathbb{N}}, (y_n)_{\mathbb{N}}, p\big).$$

Hence we can conclude that the function $h$ is continuous in $\big((x_n)_{\mathbb{N}}, (y_n)_{\mathbb{N}}, p\big)$.

The next step is to apply appropriately the *maximum* theorem. First observe that we can see $\mathcal{P}$ as a metric space, because $\Omega$ is separable. In addition, consider the compact valued correspondence $\mathcal{C} : Y^{\mathbb{N}} \times Y^{\mathbb{N}} \longrightarrow \mathcal{P}$ with $\mathcal{C}\big((x_n)_{\mathbb{N}}, (y_n)_{\mathbb{N}}\big) = \mathcal{P}$ for every $\big((x_n)_{\mathbb{N}}, (y_n)_{\mathbb{N}}\big) \in Y^{\mathbb{N}} \times Y^{\mathbb{N}}$, which is both upper and lower hemicontinuous, as it is constant correspondence. Therefore, by the *maximum* theorem we get that $\min_{p\in\mathcal{P}} h\big((x_n)_{\mathbb{N}}, (y_n)_{\mathbb{N}}, p\big)$ is continuous in $\big((x_n)_{\mathbb{N}}, (y_n)_{\mathbb{N}}\big)$, hence we get the result of the theorem. ■

**Proof of Theorem 2 $(i)$.**

The proof proceeds in three steps. First, we show that on equilibrium path the continuation payoff, for each player $i$ at every realised true state and at every time that she trades, converges to some non negative real number. Second, using the fact that the strategies converge and properties of the MSR we conclude that the continuation payoffs converge to zero. Finally, we show that in the limit the Revision-proof announcements are the myopic ones which, by strong separability, aggregate information.

**Step 1: On equilibrium path the continuation payoff at any time $t$ converges to a non negative real number.**

Because the state space is finite, public information stops updating after some period $T$, so that $\mathcal{F}^t(y(\omega^*)) = \mathcal{F}^T(y(\omega^*))$ for all $t \geq T$. Define $\mathcal{F}(\omega^*) = \mathcal{F}^T(y(\omega^*))$ and $\mathcal{F}_i(\omega^*) = \mathcal{F}(\omega^*)\bigcap \Pi_i(\omega^*)$.

Suppose that for some player $i$ and state $\omega^* \in \bigcup_{p\in\mathcal{P}} Supp(p)$, the continuation payoff $V_{i+nt}(\Phi^{i-1+nt}(y^*(\omega^*)), \omega^*, y^*, \mathscr{P})$ converges (in $t$) to some $c^{\Pi_i(\omega^*)} < 0$. The limit exists because

$$V_{i+nt}(\Phi^{i-1+nt}(y^*(\omega^*)), \omega^*, y^*, \mathscr{P}) =$$

$$\min_{p\in\mathcal{P}_{\mathcal{F}_i(\omega^*)}} \sum_{k=0}^{\infty}\beta^{nk}E_p\Big[s\Big(y^*_{i+nk+nt}\big(\omega^*, \Phi^{i-1+nk+nt}(y^*(\omega'))\big), X\Big) -$$

$$s\Big(y^*_{i-1+nk+nt}\big(\omega', \Phi^{i-2+nk+nt}(y^*(\omega'))\big), X\Big)\Big].$$

Because $\mathcal{F}(\omega^*)$ is the finest public information that is revealed if no-one deviates, we have that for all $t > T$, $i = 1, ..., n$ and $k \in \mathbb{N}$,

$$y^*_{i+nk+nt}\big(\omega^*, \Phi^{i-1+nk+nt}(y^*(\omega^*))\big) = y^*_{i+nk+nt}\big(\omega, \Phi^{i-1+nk+nt}(y^*(\omega))\big)$$

for every $\omega \in \mathcal{F}(\omega^*)$, for which there exists $p \in \mathcal{P}$ with $p(\omega) > 0$ (otherwise more information would be revealed). This in turn implies that for $t > T$, for every $i = 1, ..., n$ and $k \in \mathbb{N}$ it is $\Phi^{i-1+nk+nt}(y^*(\omega^*)) = \Phi^{i-1+nk+nt}(y^*(\omega))$ for every $\omega \in \mathcal{F}(\omega^*)$, for which there exists $p \in \mathcal{P}$ with $p(\omega) > 0$.

Hence, considering the sequences $\big((x_k)_{k\in\mathbb{N}}\big)^t = \Bigg(\Big(y^*_{i+nk+nt}\big(\omega^*, \Phi^{i-1+nk+nt}(y^*(\omega^*))\big)\Big)_{k\in\mathbb{N}}\Bigg)^t$ and $\Big((y_k)_{k\in\mathbb{N}}\Big)^t = \Bigg(\Big(y^*_{i-1+nk+nt}\big(\omega^*, \Phi^{i-2+nk+nt}(y^*(\omega^*))\big)\Big)_{k\in\mathbb{N}}\Bigg)^t$, we conclude that for every $i = 1, ..., n$:

$$\Bigg(\Big(y^*_{i+nk+nt}\big(\omega^*, \Phi^{i-1+nk+nt}(y^*(\omega^*))\big)\Big)_{k\in\mathbb{N}}\Bigg)^t \longrightarrow_t \Big(y^*_{i,\infty}\big(\omega^*, \Phi^{\infty}(y^*(\omega^*))\big)\Big)_{k\in\mathbb{N}}$$

and

$$\Bigg(\Big(y^*_{i-1+nk+nt}\big(\omega^*, \Phi^{i-2+nk+nt}(y^*(\omega^*))\big)_{k\in\mathbb{N}}\Big)\Bigg)^t \longrightarrow_t \Big(y^*_{i-1,\infty}\big(\omega^*, \Phi^{\infty}(y^*(\omega^*))\big)\Big)_{k\in\mathbb{N}}.$$ [24]

Note that, since we have defined that each strategy $y$ assigns the same action given some history and two states $\omega_1, \omega_2 \in \Pi_(\omega)$, the same is true for the limit.

This is because the convergence of the sequences is in the product topology and for every $k \in \mathbb{N}$ it is $y^*_{i+nk+nt}\big(\omega^*, \Phi^{i-1+nk+nt}(y^*(\omega))\big) \longrightarrow_t y^*_{i,\infty}\big(\omega^*, \Phi^{\infty}(y^*(\omega^*))\big)$ for every $i$ (because $(y^*, \mathscr{P})$ is converging). Using Lemma 4 we conclude that the limit of the tail exists and it is

$$\min_{p\in\mathcal{P}_{\mathcal{F}_i(\omega^*)}} \frac{E_p\Big[s\Big(y^*_{i,\infty}\big(\omega^*, \Phi^{\infty}(y^*(\omega'))\big), X\Big) - s\Big(y^*_{i-1,\infty}\big(\omega', \Phi^{\infty}(y^*(\omega'))\big), X\Big)\Big]}{1-\beta^n}.$$

Denote the limit $c^{\Pi_i(\omega^*)}$. If $c^{\Pi_i(\omega^*)} < 0$ then there exists $t_0 \in \mathbb{N}$ such that for every $t \geq t_0 > T$,

$$V_{i+nt}(\Phi^{i-1+nt}(y^*(\omega^*)), y^*, \omega^*, \mathscr{P}) =$$
$$\min_{p\in\mathcal{P}_{\mathcal{F}_i(\omega^*)}} \sum_{k=t}^{\infty} \beta^{nk-nt} E_p\Big[s\Big(y^*_{i+nk}\big(\omega^*, \Phi^{i-1+nk}(y^*(\omega'))\big), X\Big) -$$
$$s\Big(y^*_{i-1+nk}\big(\omega', \Phi^{i-2+nk}(y^*(\omega'))\big), X\Big)\Big] < 0.$$ [25]

Consider equilibrium history $\Phi^{i+nt_0-1}(y^*(\omega^*))$ and the implied public information $\mathcal{F}^{i+nt_0-1}(y^*(\omega^*))$. The set of states that player $i$ thinks possible is $\mathcal{F}^{i+nt_0-1}(y^*(\omega^*)) \cap \Pi_i(\omega^*)$.[26] According to the assumption made in this step, if player $i$ follows her equilibrium strategy then the continuation payoff is negative. However, we can define the following deviation strategy $y' = (y'^i, y^{*-i})$, so that $-i$ follow $y^*$ and $y'^i$ is identical to $y^{*i}$ up to time $i + nt_0 - 1$. We now

[24]Because we are after the time the finest information for every player is achieved the states $\omega \in \mathcal{F}_i(\omega^*)$ do not reveal information, as the strategic predictions over that set of states are constant. Hence, we select to represent that constant value by using the realised state $\omega^*$. However, when we need to have an expectation we turn to the normal notation.

[25]Notice here that we can interchange the sum with the expectation using the Beppo-Levi theorem.

[26]Although $\mathcal{F}^{i+nt_0-1}(y^*(\omega^*)) = \mathcal{F}(\omega^*)$, we use the more analytic notation in order to make clear the information set after the deviation, in which case it is not always true, in general, that no more information revelation occurs.

define how $y'^i$ differs from $y^{*i}$ after $i + nt_0 - 1$.

$(i)$ Define $y'_{i+nt_0}\big(\omega^*, \Phi^{i+nt_0-1}(y^*(\omega^*))\big)$ to be whatever the previous player predicted. Given that $i$ deviates and $-i$ stick to the equilibrium strategy $y^*$, let $H^1, ..., H^m$ be the possible paths of announcements by $-i$, from $i + nt_0$ to $i + nt_0 + n - 1$.

$(ii)$ For any continuation path $H^1, \dots, H^m$, where $(\Phi^{i+nt_0}(y'(\omega^*)), H^m)$ denotes the path where $i$ first deviates with $y'$ at $i + nt_0$ and the continuation from the other players according to $y^*$ is $H^m$,

- $(a)$ if $V_{i+nt_0+n}((\Phi^{i+nt_0}(y'(\omega^*)), H^m), y^*, \omega^*, \mathscr{P}) \geq 0$, then the deviation strategy $y'$ coincides with $y^*$ at every succeeding information set (including the one at round $t_0 + 1$. Note that this the continuation payoff where $i$ plays $y^*$ at $i + nt_0 + n$.
- $(b)$ If not, then define the deviation action $y'_{i+nt_0+n}(\omega^*, (\Phi(y'(\omega^*)), H^m))$ to be equal to the previous player's prediction.

If $(b)$ happens, then for the corresponding information set and the history until it is reached, we repeat the same reasoning as in $(i)$ and $(ii)$.

We define the deviation strategy $y'$ to be the same with $y^*$ everywhere else that is not defined by the previous procedure.

$(iii)$ The continuation payoff at time $i + nt_0$ from $y' = (y'^i, y^{*-1})$ is

$$V_{i+nt_0}(\Phi^{i+nt_0-1}(y^*(\omega^*)), y', \omega^*, \mathscr{P}) =$$

$$\min_{p \in \mathcal{P}_{\mathcal{F}^{i+nt_0-1}(y^*(\omega^*)) \cap \Pi_i(\omega^*)}} \sum_{k=t_0+1}^{\infty} \beta^{nk-nt_0} E_p \Big[ s\Big( y'_{i+nk}\big(\omega^*, \Phi^{i-1+nk}(y'(\omega')|\Phi^{i+nt_0}(y'(\omega^*))), X\Big) - s\Big( y^*_{i-1+nk}\big(\omega', \Phi^{i-2+nk}(y'(\omega')|\Phi^{i+nt_0}(y'(\omega^*)))), X\Big) \Big].$$ [27]

For $t \geq t_0$, define $i$'s continuation payoff at round $t$ if the true state is $\omega$ to be

$$f_t(\omega) = \Big[ \sum_{k=t}^{\infty} \beta^{nk-nt_0} \Big[ s\Big( y'_{i+nk}\big(\omega, \Phi^{i-1+nk}(y'(\omega')|\Phi^{i+nt_0}(y'(\omega^*))), X\Big) - s\Big( y^*_{i-1+nk}\big(\omega, \Phi^{i-2+nk}(y'(\omega)|\Phi^{i+nt_0}(y'(\omega^*)))), X\Big) \Big].$$

We therefore have that $V_{i+nt_0}(\Phi^{i+nt_0-1}(y^*(\omega^*)), y', \omega^*, \mathscr{P}) = \min_{p \in \mathcal{P}_{\mathcal{F}^{i+nt_0-1}(y^*(\omega^*)) \cap \Pi_i(\omega^*)}} E_p[f_{t_0}]$.

Define the set $A$, through its complement as following: $A^C = \{\omega \in \mathcal{F}^{i+nt_0-1}(y^*(\omega^*)) \cap \Pi_i(\omega^*) : V_{i+nt_0+nk}(\Phi^{i+nt_0+nk-1}(y'(\omega)|\Phi^{i+nt_0-1}(y^*(\omega^*))), y^*, \omega^*, \mathscr{P}) < 0 \textit{ for every } k \geq 0\}$. Then for every $\omega \in A^C$ we have $f_{t_0}(\omega) = 0$, by the definition of $y'^i$.

[27] Note that at $i + nt_0$ her payoff is 0 because the player deviates and repeats the previous announcement.

For every $\omega \in A$ take:

$$k_\omega = \min\{k \geq 1 : V_{i+nt_0+nk}(\Phi^{i+nt_0+nk-1}(y'(\omega)|\Phi^{i+nt_0-1}(y^*(\omega^*))), y^*, \omega^*, \mathscr{P}) \geq 0\}.$$

We know that the minimum is well defined by the construction of $A^C$. By the definition of $y'^i$ we have that for all $\omega \in \mathcal{F}(\omega^*) \cap \Pi_i(\omega^*)$, if at some $t$ the continuation payoff is positive, $i$ no longer deviates, so that $y'^i = y^*$ after that time and payoffs from the two strategies are the same. Formally,

$$V_{i+nt_0+nk_\omega}(\Phi^{i+nt_0+nk_\omega-1}(y'(\omega)|\Phi^{i+nt_0-1}(y^*(\omega^*))), y', \omega^*, \mathscr{P}) =$$

$$V_{i+nt_0+nk_\omega}(\Phi^{i+nt_0+nk_\omega-1}(y'(\omega)|\Phi^{i+nt_0-1}(y^*(\omega^*))), y^*, \omega^*, \mathscr{P}) \geq 0,$$

and for the preceding information sets at rounds $t_0, ..., t_0 + k_\omega$ the deviation action of $i$ is the previous trader's equilibrium announcement.

For every $\omega \in A$ define

$$A_\omega = \{\omega' \in A : \Phi^{i+nt_0+nk_\omega-1}(y'(\omega')|\Phi^{i+nt_0-1}(y^*(\omega^*))) =$$

$$\Phi^{i+nt_0+nk_\omega-1}(y'(\omega)|\Phi^{i+nt_0-1}(y^*(\omega^*))).$$

Intuitively, this is the set of states that $i$ thinks are possible, when the realised state is $\omega^*$, the round is $(t_0 + k_\omega)$ and the history is $\Phi^{i+nt_0+nk_\omega-1}(y'(\omega)|\Phi^{i+nt_0-1}(y^*(\omega^*)))$.

In fact, these sets form a partition of $A$. Name the partition $A_{\omega_1}, ..., A_{\omega_l}$.

Essentially, it is the joint information between the public information created by the corresponding history and $i$'s private signal.

Next, we can observe that

$$V_{i+nt_0}(\Phi^{i+nt_0-1}(y^*(\omega^*)), (y'^i, y^{*-i}), \omega^*) = E_{p^*}[f_{t_0}] = \sum_{\omega \in A} f_{t_0}(\omega)p^*(\omega),$$

for some $p^* \in \mathcal{P}_{\mathcal{F}^{i+nt_0-1}(y^*(\omega^*)) \cap \Pi_i(\omega^*)}$. However, we also have

$$\sum_{\omega \in A} f_{t_0}(\omega)p^*(\omega) =$$

$$p^*(A_{\omega_1}) \sum_{\omega \in A_{\omega_1}} \beta^{nk_{\omega_1}} f_{t_0+k_{\omega_1}}(\omega) \frac{p^*(\omega)}{p^*(A_{\omega_1})} + ... + p^*(A_{\omega_l}) \sum_{\omega \in A_{\omega_l}} \beta^{nk_{\omega_l}} f_{t_0+k_{\omega_l}}(\omega) \frac{p^*(\omega)}{p^*(A_{\omega_l})}.$$

In addition, for every $\omega \in \{\omega_1, ..., \omega_l\}$,

$$\sum_{\omega' \in A_\omega} \beta^{nk_\omega} f_{t_0+k_\omega}(\omega') \frac{p^*(\omega')}{p^*(A_\omega)} \geq$$

$$\beta^{nk_\omega} V_{i+nt_0+nk_\omega}(\Phi^{i+nt_0+nk_\omega-1}(y'(\omega)|\Phi^{i+nt_0-1}(y^*(\omega^*))), y', \omega^*, \mathscr{P}) =$$

$$\beta^{nk_\omega} V_{i+nt_0+nk_\omega}(\Phi^{i+nt_0+nk_\omega-1}(y'(\omega)|\Phi^{i+nt_0-1}(y^*(\omega^*))), y^*, \omega^*, \mathscr{P}) \geq 0.$$[28]

Hence, $V_{i+nt_0}(\Phi^{i+nt_0-1}(y^*(\omega^*)), (y'^i, y^{*-i}), \omega^*) \geq 0$.

Assume now that we are on deviation strategy's path at round, say $t_0 + \lambda$. If for every $\omega \in \mathcal{F}^{i+n(t_0+\lambda)-1}(y'(\omega^*)) \cap \Pi_i(\omega^*)$ for which it is $k_\omega \leq t_0 + \lambda$ then the deviation strategy in the continuation is the same as the equilibrium's one.[29] Otherwise, we can apply the previous reasoning for the new information set (essentially the only change in the arguments above is that instead of having history $\Phi^{i+nt_0-1}(y^*(\omega^*))$ we would have $\Phi^{i+nt_0+(k+\lambda)n-1}(y'(\omega)|\Phi^{i+nt_0-1}(y^*(\omega^*)))$.

In addition, outside the deviation path we have that the payoffs are the same, by definition. Hence, we get a contradiction, because $y^*$ is Revision-proof. We conclude that $c^{\Pi_i(\omega^*)} \geq 0$, for every $i$ and $\omega^*$, for which $p(\omega^*) > 0$ for some $p \in \mathcal{P}$.

**Step 2: On equilibrium path, the continuation payoff at any time $t$ converges to zero.**

Because of Step 1, in the limit we have that for every $\omega^*$, such that there exists $p \in \mathcal{P}$ with $p(\omega^*) > 0$, where public information stops updating after some period $T$ and $\mathcal{F}(\omega^*) = \mathcal{F}^T(y(\omega^*))$, $\mathcal{F}_i(\omega^*) = \mathcal{F}(\omega^*) \bigcap \Pi_i(\omega^*)$, we have

$$\min_{p \in \mathcal{F}_i(\omega^*)} \frac{E_p\Big[s\Big(y^*_{i,\infty}\big(\omega^*, \Phi^\infty(y^*(\omega'))\big), X\Big) - s\Big(y^*_{i-1,\infty}\big(\omega', \Phi^\infty(y^*(\omega'))\big), X\Big)\Big]}{1-\beta^n} \geq 0.$$

Therefore, for every $p \in \mathcal{P}$ it is

$$E_{p_{\mathcal{F}_i(\omega^*)}}\Big[s\Big(y^*_{i,\infty}\big(\omega^*, \Phi^\infty(y^*(\omega'))\big), X\Big) - s\Big(y^*_{i-1,\infty}\big(\omega', \Phi^\infty(y^*(\omega'))\big), X\Big)\Big] \geq 0$$

and hence

$$p\big(\mathcal{F}_i(\omega^*)\big)E_{p_{\mathcal{F}_i(\omega^*)}}\Big[s\Big(y^*_{i,\infty}\big(\omega^*, \Phi^\infty(y^*(\omega'))\big), X\Big) - s\Big(y^*_{i-1,\infty}\big(\omega', \Phi^\infty(y^*(\omega'))\big), X\Big)\Big] \geq 0.$$[30]

Note that $\omega \in \mathcal{F}(\omega')$ implies $\mathcal{F}(\omega) = \mathcal{F}(\omega')$. Summing over $\mathcal{C}_i = \{\Pi_i(\omega) : \omega \in \mathcal{F}(\omega^*)\}$ and for every $p \in \mathcal{P}$ we have

$$\sum_{\Pi_i(\omega) \in \mathcal{C}_i} p\big(\Pi_i(\omega) \cap \mathcal{F}(\omega^*)\big) E_{p_{\Pi_i(\omega)\cap\mathcal{F}(\omega^*)}}\Big[s\Big(y^*_{i,\infty}\big(\omega, \Phi^\infty(y^*(\omega'))\big), X\Big)$$

[28] The first inequality is true because trader $i$ updates prior by prior.

[29] Notice here that if there exists $\omega \in \mathcal{F}^{i+n(t_0+\lambda)-1}(y'(\omega^*)) \cap \Pi_i(\omega^*)$ $\omega \in \mathcal{F}^{y^*}_{i+n(t_0+\lambda)-1}(\omega^*) \cap \Pi_i(\omega^*)$ it is $k_\omega \leq t_0+\lambda$ then the same applies for every $\omega \in \mathcal{F}^{i+n(t_0+\lambda)-1}(y'(\omega^*)) \cap \Pi_i(\omega^*)$ $\omega \in \mathcal{F}^{y^*}_{i+n(t_0+\lambda)-1}(\omega^*) \cap \Pi_i(\omega^*)$.

[30] Note that there exists $p_0 \in \mathcal{P}$ with $p_0(\omega^*) > 0$, hence the regularity assumption of Definition 7 implies $p(\mathcal{F}_i(\omega^*)) > 0$ for all $p \in \mathcal{P}$.

$$-s\Big(y^*_{i-1,\infty}\big(\omega', \Phi^{\infty}(y^*(\omega'))\big), X\Big)\Big] =$$

$$\sum_{\omega \in \mathcal{F}(\omega^*)} p(\omega)\Big[s\Big(y^*_{i,\infty}\big(\omega', \Phi^{\infty}(y^*(\omega'))\big), X\Big) - s\Big(y^*_{i-1,\infty}\big(\omega', \Phi^{\infty}(y^*(\omega'))\big), X\Big)\Big] =$$

$$= p\big(\mathcal{F}(\omega^*)\big)E_{p_{\mathcal{F}(\omega^*)}}\Big[s\Big(y^*_{i,\infty}\big(\omega', \Phi^{\infty}(y^*(\omega'))\big), X\Big) - s\Big(y^*_{i-1,\infty}\big(\omega', \Phi^{\infty}(y^*(\omega'))\big), X\Big)\Big] \geq 0.$$

Summing over all $i$, we get that the sum is zero. Since each term of the summation is more than or equal to zero, it must be that every term is also zero. Using similar arguments, we conclude that $c_1^{\Pi_1(\omega^*)} = ... = c_n^{\Pi_n(\omega^*)} = 0$ for every $\omega^*$, such that there exists $p \in \mathcal{P}$ with $p(\omega) > 0$.

**Step 3: On equilibrium path the announcements are the myopic ones which, by strong separability, aggregate information.**

Consider $i$ and $\omega^* \in \bigcup_{p \in \mathcal{P}} Supp(p)$ such that

$$\min_{p \in \mathcal{P}_{\mathcal{F}_i(\omega^*)}} E_p\Big[s\Big(d_{\mathcal{P}}(\mathcal{F}_i(\omega^*), y^*_{i-1,\infty}\big(\omega', \Phi^{\infty}(y^*(\omega'))\big)), X\Big) -$$

$$s\Big(y^*_{i-1,\infty}\big(\omega', \Phi^{\infty}(y^*(\omega'))\big), X\Big)\Big] > \eta > 0.$$

Then for all $t > T$,

$$\min_{p \in \mathcal{P}_{\mathcal{F}_i(\omega^*)}} E_p[s(d_{\mathcal{P}}\big(\mathcal{F}_i(\omega^*), y^*_{i-1+nt}(\omega', \Phi^{i-2+nt}(y^*(\omega')))\big), X) -$$

$$s(y^*_{i-1+nt}\big(\omega', \Phi^{i-2+nt}(y^*(\omega'))\big), X)] > \eta.$$

Fix such $t_0$, with the added property that all continuation payoffs are less than $\eta > 0$. This is possible because the continuation payoffs converge to zero, as we proved in Step 2. Using the same reasoning as in Step 1, consider alternative strategy $y'^i$ which is the same as in Step 1, except at time $i + nt_0$ where trader $i$ announces $d_{\mathcal{P}}(\mathcal{F}_i(\omega^*), y^*_{i+nt_0-1}\big(\omega^*, \Phi^{i+nt_0-2}(y^*(\omega^*)))\big)$ and earns at least $\eta$. Let $y' = (y'^i, y^{*-i})$.

Let $A^C = \{\omega \in \mathcal{F}_i(\omega^*) : V_{i+nt_0+nk}(\Phi^{i+nt_0+kn-1}(y'(\omega)|\Phi^{i+nt_0-1}(y^*(\omega^*))), y^*, \omega^*, \mathscr{P}) < 0 \textit{ for every } k \geq 1\}$. Then for every $\omega \in A^C$ we have $f_{t_0+1}(\omega) = 0$, by definition of $y'^i$, because at times where her continuation payoff is negative, she deviates and announces the the previous player's announcement, thus ensuring a payoff of zero.

For every $\omega \in A$, let

$$k_\omega = \min\{k \geq 1 : V_{i+nt_0+nk}(\Phi^{i+nt_0+kn-1}(y'(\omega)|\Phi^{i+nt_0-1}(y^*(\omega^*))), y^*, \omega^*, \mathscr{P}) \geq 0\}.$$

We know that the minimum exists by the definition of $A$. By the definition of $y'^i$ we have that, for every $\omega \in \mathcal{F}_i(\omega^*)$,

$$V_{i+nt_0+nk_\omega}(\Phi^{i+nt_0+k_\omega n-1}(y'(\omega)|\Phi^{i+nt_0-1}(y^*(\omega^*))), y', \omega^*, \mathscr{P}) =$$

$$V_{i+nt_0+nk_\omega}(\Phi^{i+nt_0+k_\omega n-1}(y'(\omega)|\Phi^{i+nt_0-1}(y^*(\omega^*))), y^*, \omega^*, \mathscr{P}) \geq 0.$$

Moreover, for the preceding information sets at rounds $t_0+1, ..., t_0+k_\omega$ the deviation action of $i$ is the same as the previous trader's equilibrium action, but for round $t_0$ where the trader announces $d_{\mathcal{P}}\big(\mathcal{F}_i(\omega^*), y^*_{i+nt_0-1}\big(\omega^*, \Phi^{i+nt_0-2}(y^*(\omega^*))\big)\big)$.

For every $\omega \in A$, let

$$A_\omega = \{\omega' \in A : \Phi^{i+nt_0+k_\omega n-1}(y'(\omega')|\Phi^{i+nt_0-1}(y^*(\omega^*))) =$$
$$\Phi^{i+nt_0+k_\omega n-1}(y'(\omega)|\Phi^{i+nt_0-1}(y^*(\omega^*)))\}.$$

Intuitively, this is the set of states that $i$ considers possible, when the realised state is $\omega^*$, the round is $(t_0+k_\omega)$ and the history is $\Phi^{i+nt_0+k_\omega n-1}(y'(\omega)|\Phi^{i+nt_0-1}(y^*(\omega^*)))$. As in Step 1, these sets form a partition $\{A_{\omega_1}, ..., A_{\omega_m}\}$ of $A$. Essentially, it is the joint information between the public information created by the corresponding history and $i$'s private signal.

Next, we can observe that:

$$V_{i+nt_0}(\Phi^{i+nt_0-1}(y^*(\omega^*)), y', \omega^*, \mathscr{P}) =$$
$$E_{p^*}[f_{t_0}] = E_{p^*}\Big[s\Big(d_{\mathcal{P}}\big(\mathcal{F}_i(\omega^*), y^*_{i+nt_0-1}\big(\omega^*, \Phi^{i+nt_0-2}(y^*(\omega^*))\big)\big), X\Big) -$$
$$s\Big(y^*_{i-1+nt}(\omega', \Phi^{i-2+nt}(y^*(\omega'))), X\Big)\Big] + \sum_{\omega\in A} \beta^n f_{t_0+1}(\omega) p^*(\omega),$$

for some $p^* \in \mathcal{P}_{\mathcal{F}_i(\omega^*)}$.[31]

However, we have

$$\sum_{\omega\in A} \beta^n f_{t_0+1}(\omega) p^*(\omega) =$$
$$p^*(A_{\omega_1}) \sum_{\omega\in A_{\omega_1}} \beta^{nk_{\omega_1}} f_{t_0+k\omega_1}(\omega) \frac{p^*(\omega)}{p^*(A_{\omega_1})} + ... + p^*(A_{\omega_l}) \sum_{\omega\in A_{\omega_m}} \beta^{nk_{\omega_l}} f_{t_0+k\omega_l}(\omega) \frac{p^*(\omega)}{p^*(A_{\omega_l})}.$$

In addition, for every $\omega_j \in \{\omega_1, ..., \omega_l\}$,

$$\sum_{\omega\in A_{\omega_j}} \beta^{nk_{\omega_j}} f_{t_0+k\omega_j}(\omega) \frac{p^*(\omega)}{p^*(A_{\omega_j})} \geq$$
$$\beta^{nk_{\omega_j}} V_{i+nt_0+nk_{\omega_j}}(\Phi^{i+nt_0+k_{\omega_j} n-1}(y'(\omega_j)|\Phi^{i+nt_0-1}(y^*(\omega^*))), y', \omega^*, \mathscr{P}) =$$
$$\beta^{nk_{\omega_j}} V_{i+nt_0+nk_{\omega_j}}(\Phi^{i+nt_0+k_{\omega_j} n-1}(y'(\omega_j)|\Phi^{i+nt_0-1}(y^*(\omega^*))), y^*, \omega^*, \mathscr{P}) \geq 0.$$

Hence $V_{i+nt_0}(\Phi^{i+nt_0-1}(y^*(\omega^*)), y', \omega^*, \mathscr{P}) \geq \eta$.[32]

[31] We write $E_{p^*}\Big[s\Big(d_{\mathcal{P}}\big(\mathcal{F}_i(\omega^*), y^*_{i+nt_0-1}\big(\omega^*, \Phi^{i+nt_0-2}(y^*(\omega^*))\big)\big), X\Big) - s\Big(y^*_{i-1+nt}(\omega', \Phi^{i-2+nt}(y^*(\omega^*))), X\Big)\Big]$, instead of $E_{p^*}\Big[s\Big(d_{\mathcal{P}}\big(\mathcal{F}_i(\omega^*), y^*_{i+nt_0-1}\big(\omega^*, \Phi^{i+nt_0-2}(y^*(\omega^*))\big)\big), X\Big) - s\Big(y^*_{i-1+nt}(\omega', \Phi^{i-2+nt}(y^*(\omega'))), X\Big)\Big]$, because $\Phi^{i-2+nt}(y^*(\omega')) = \Phi^{i-2+nt}(y^*(\omega^*))$ for every $\omega' \in \mathcal{F}_i(\omega^*)$ and $t > T$.

[32] In this argument we use consistency from Definition 7: the deviator (i.e. $i$ trader) updates prior by

Assume now that we are on deviation strategy's path at round, say $t_0+\lambda$, with $\lambda \geq 1$. If for every $\omega \in \mathcal{F}^{i+n(t_0+\lambda)-1}(y'(\omega^*)) \cap \Pi_i(\omega^*)$ it is $k_\omega \leq t_0+\lambda$ then the deviation strategy in the continuation is the same as the equilibrium's one.[33] Otherwise, we can apply exactly the reasoning of Step 1 for the corresponding information set.

In addition, outside the deviation path we have that the payoffs are the same, by its definition, too. Hence, we get a contradiction, because $(y^*, \mathscr{P})$ is a Revision-proof equilibrium.

Therefore, it should be the case that for all $i$ and $\omega^* \in \bigcup_{p\in\mathcal{P}} Supp(p)$,

$$\min_{p\in\mathcal{P}_{\mathcal{F}_i(\omega^*)}} E_p\Big[s\Big(d_{\mathcal{P}}(\mathcal{F}_i(\omega^*), y^*_{i-1,\infty}\big(\omega', \Phi^\infty(y^*(\omega'))\big)), X\Big) - s\Big(y^*_{i-1,\infty}\big(\omega', \Phi^\infty(y^*(\omega'))\big), X\Big)\Big] = 0.$$

For every $i$ the limit strategies are constant over $\mathcal{F}(\omega^*)$ and hence $y^*_{i,\infty}\big(\omega^*, \Phi^\infty(y^*(\omega^*))\big) = y^*_{i,\infty}\big(\omega', \Phi^\infty(y^*(\omega'))\big)$ for every $\omega' \in \mathcal{F}(\omega^*)$ with $p(\omega') > 0$ for some $p \in \mathcal{P}$. Therefore, for every $\omega^* \in \bigcup_{p\in\mathcal{P}} Supp(p)$,

1. $d_{\mathcal{P}}\Big(\mathcal{F}_i(\omega^*), y^*_{i-1,\infty}\big(\omega^*, \Phi^\infty(y^*(\omega^*))\big)\Big) = y^*_{i-1,\infty}\big(\omega^*, \Phi^\infty(y^*(\omega^*))\big)$,
2. $c_i^{\Pi_i(\omega^*)} = 0$.

From the uniqueness (using arguments from Lemma 1) of

$$arg\max_{y\in Y} \min_{p\in\mathcal{P}_{\mathcal{F}_i(\omega^*)}} E_p\Big[s\Big(y, X(\omega)\Big) - s\Big(y^*_{i-1,\infty}(\omega', \Phi^\infty\big(y^*(\omega')\big)), X(\omega)\Big)\Big]$$

and $c_i^{\Pi_i(\omega^*)} = 0$, we have that $y^*_{i,\infty}\big(\omega^*, \Phi^\infty(y^*(\omega^*))\big) = d_{\mathcal{P}}(\mathcal{F}_i(\omega^*), y^*_{i-1,\infty}\big(\omega^*, \Phi^\infty(y^*(\omega^*))\big))$, for all $i$.

Hence $y^*_{i,\infty}\big(\omega^*, \Phi^\infty(y^*(\omega^*))\big) = y^*_{i-1,\infty}\big(\omega^*, \Phi^\infty(y^*(\omega^*))\big)$ for every $i \in \{1, ..., n\}$. Then, defining $v = \{y^*_{i,\infty}(\omega^*, \Phi^\infty(y^*(\omega^*)))\}$, for some $i \in \{1, ..., n\}$, and using that $X$ is strongly separable, for $\mathcal{P}_{\mathcal{F}(\omega^*)}$, we conclude that information gets aggregated. This is because we have shown that condition (ii) of Definition 3 is satisfied, hence (i) should be violated and the security must be constant over $\mathcal{F}(\omega^*)$.[34]

We have $d_{\mathcal{P}}\Big(\mathcal{F}_i(\omega^*), y^*_{i-1,\infty}\big(\omega^*, \Phi^\infty(y^*(\omega^*))\big)\Big) = y^*_{i-1,\infty}\big(\omega^*, \Phi^\infty(y^*(\omega^*))\big)$. From Lemma 1, $d_{\mathcal{P}}\Big(\mathcal{F}_i(\omega^*), y^*_{i-1,\infty}\big(\omega^*, \Phi^\infty(y^*(\omega^*))\big)\Big)$ is an expectation of the security using the appropriate (updated) belief, and hence we conclude that the constant value of the security is $y^*_{i,\infty}\big(\omega^*, \Phi^\infty(y^*(\omega^*))\big)$ (which is equal to $y^*_{i,\infty}\big(\omega', \Phi^\infty(y^*(\omega'))\big)$ for every $\omega' \in \mathcal{F}(\omega^*)$ with $p(\omega') > 0$ for some $p \in \mathcal{P}$). Therefore, information gets aggregated. ■

prior.

[33] Notice here that if there exists $\omega \in \mathcal{F}^{i+n(t_0+\lambda)-1}(y'(\omega^*)) \cap \Pi_i(\omega^*)$ it is $k_\omega \leq t_0+\lambda$ then the same applies for every $\omega \in \mathcal{F}^{i+n(t_0+\lambda)-1}(y'(\omega^*)) \cap \Pi_i(\omega^*)$.

[34] By Definition 8, the priors in $\mathcal{P}_{\mathcal{F}(\omega^*)}$ are mutually absolute continuous with respect to each $\Pi_i$, $i = 1, ..., n$.

**Lemma 5** *Let security $X$, proper scoring rule $s$ and consider partition structure $\Pi$, a regular $\mathcal{P} \subseteq \Delta(\Omega)$ with respect to each $\Pi_i$ and $v \in \mathbb{R}$. If $d_{\mathcal{P}}(\Pi_i(\omega), v) = v$ for all $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$ then there exists a probability measure $q_i$ over $\Omega$ such that $E_{q_i}[X|\Pi_i(\omega)] = v$ for all $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$.*

**Proof.** From Lemma 1 there exist probability measures $p^*{}_{\Pi_i(\omega)}$ over $\Pi_i(\omega)$ such that $E_{p^*{}_{\Pi_i(\omega)}}[X] = v$ for all $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$. Without loss in generality assume that $\Omega = \bigcup_{\omega \in \{\omega_{1_i}, \ldots, \omega_{l_i}\}} \Pi_i(\omega)$. The probability measure $q_i$ can be defined for $\omega \in \Pi_i(\omega_k)$ as $q_i(\omega) = \frac{1}{l_i} \times p^*{}_{\Pi_i}(\omega)$. We can observe that for any $k$ and $\omega \in \Pi_i(\omega_k)$ it is $q_{i|\Pi_{\omega_k}}(\omega) = \frac{q_i(\omega)}{q_i(\Pi_i(\omega_k))} = \frac{\frac{1}{l_i} \times p^*{}_{\Pi_i}(\omega)}{\frac{1}{l_i} \times 1} = p^*{}_{\Pi_i(\omega_k)}(\omega)$. Subsequently, for any $k$ it is $E_{q_i}[X|\Pi_i(\omega_k)] = E_{p^*{}_{\Pi_i(\omega_k)}}[X] = v$. ■

**Proof of Theorem 2** *(ii)***.**

Suppose $X$ is not strongly separable under $\Pi$ and $s$. Then, there exist $\mathcal{P} \subseteq \Delta(\Omega)$, regular with respect to each $\Pi_i$, and $v \in \mathbb{R}$, such that (i) $X(\omega) \neq v$ for some $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$ and (ii) $d_{\mathcal{P}}(\Pi_i(\omega), v) = v$ for all $i = 1, ..., n$ and $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$.

Consider game $\Gamma^S(\Omega, \Pi, X, \mathcal{O}, \mathcal{P}, y_0, \underline{y}, \overline{y}, s, \beta)$, where the initial announcement of the market maker is $y_0 = v$. We will show that there exists a converging Revision-proof equilibrium in which information does not get aggregated.

Define assessment $(y^*, \mathscr{P})$, where $y^*$ specifies that each trader $i$ announces $v$ after any history. Observe that $y^*$ is converging. At each information set $\mathcal{I}$ of trader $i$, set $\mathscr{P}(\mathcal{I}) = \mathcal{P}_{\Pi_i(\omega)}$ when the previous announcement is $v$ and otherwise $\mathscr{P}(\mathcal{I}) = \{q_i\}$, where $q_i$ is derived by Lemma 5. Given $y^*$, no public information is revealed, hence $(y^*, \mathscr{P})$ is consistent.

Given that the initial announcement is $y_0 = v$, the strategic traders are essentially playing their myopic strategy on equilibrium path because for every $i$ and for every $\omega$, such that there exists $p \in \mathcal{P}$ with $p(\omega) > 0$, it is $d_{\mathcal{P}}(\Pi_i(\omega), v) = v$ or in other words their strategic actions solve $\max_{y \in Y} \min_{p \in \mathcal{P}} E_{p|\Pi_1(\omega^*)}\big[s(y, X(\omega)) - s(v, X(\omega))\big]$. In case the previous announcement is not $v$ the following trader, say $i$, is essentially an expected utility player with beliefs $q_i$ and as a result irrespective of the previous announcement her myopic action is to announce $v$ (Lemma 5).

In general, at any information set $\mathcal{I}(H^t, \omega^*)$ which follows an announcement of $v$, and $i$ makes an announcement, i's beliefs are $\mathcal{P}_{\Pi_i(\omega^*)}$. Setting $\mathcal{P}_i = \mathcal{P}_{\Pi_i(\omega^*)}$, her maximum continuation payoff, given that $-i$ follow $y^*$, is:

$$\min_{p \in \mathcal{P}_i} E_p\Big[\sum_{k=t}^{\infty} \beta^{nk-nt}\Big(s(E_p[X], X(\omega)) - s(v, X(\omega))\Big)\Big] =$$

$$= \min_{p \in \mathcal{P}_i} \frac{E_p\big[s(E_p[X], X(\omega)) - s(v, X(\omega))\big]}{1 - \beta^n} =$$

$$= \min_{p\in\mathcal{P}_i}\max_{y\in Y} \frac{E_p\big[s(y, X(\omega)) - s(v, X(\omega))\big]}{1-\beta^n}.$$ [35]

However, by the arguments of Lemma 1, we have that $\min_{p\in\mathcal{P}_i}\max_{y\in Y} E_p\big[s(y, X(\omega)) - s(v, X(\omega))\big] = \max_{y\in Y}\min_{p\in\mathcal{P}_i} E_p\big[s(y, X(\omega)) - s(v, X(\omega))\big]$. Therefore no alternative strategy profile gives strictly better payoff for such an information set.

Similarly, at any information set $\mathcal{I}(H^t, \omega^*)$ which follows an announcement of $v_{t-1} = v' \neq v$, and $i$ makes an announcement, i's beliefs are defined to be the singleton $\{q_i\}$. Indeed, because the equilibrium strategy for all traders is to announce $v$ condition $(ii)$ of Definition 8 is not met therefore there is the flexibility for $i$ to assign random beliefs for that information set. Her maximum continuation payoff, given that $-i$ follow $y^*$ ($v = v_k$ for $k > t$), is:

$$E_{q_i}\Big[\sum_{k=t}^{\infty} \beta^{nk-nt}\Big(s(E_{q_i}[X], X(\omega)) - s(v_k, X(\omega))\Big)\Big] =$$

$$= \frac{E_{q_i}\big[s(E_{q_i}[X], X(\omega)) - s(v_k, X(\omega))\big]}{1-\beta^n} =$$

$$= \max \frac{E_{q_i}\big[s(y, X(\omega)) - s(v_k, X(\omega))\big]}{1-\beta^n}$$

.

Therefore no alternative strategy profile gives strictly better payoff for such an information set.

Therefore, $(y^*, \mathscr{P})$ is Revision-proof equilibrium. ■

[35] In order to understand the maximum continuation payoff for agent $i$ we need firstly to observe that because the equilibrium actions of the other agents are $v$ whatever the history or the state is, then the strategy for agent $i$ can be written as, for every $k \geq t$ and $\omega' \in \Pi_i(\omega)$, $y_{i+nk}\Big(\omega', \Phi^{i-1+nk}\big((y_i, y^*_{-i})(\omega')\big|H^{i-1+nt}\big)\Big) = y_{i+nk}\Big(\omega, \Phi^{i-1+nk}\big((y_i, y^*_{-i})(\omega)\big|H^{i-1+nt}\big)\Big)$ which we can denote for simplicity by $y_{i+nk}$ because $\omega$ can be thought as given. Secondly, we claim that $\max_{(y_{i+nk})_k\in Y^{\mathbb{N}}}\min_{p\in\mathcal{P}_i} E_p\Big[\sum_{k=t}^{\infty}\beta^{nk-nt}\Big(s(y_{i+nk}, X) - s(v, X)\Big)\Big] \leq \max_{y\in Y}\min_{p\in\mathcal{P}_i} E_p\Big[\sum_{k=t}^{\infty}\beta^{nk-nt}\Big(s(y, X) - s(v, X)\Big)\Big]$. Indeed, because of properties of geometric series and by Lemma 1, we can see that $\max_{y\in Y}\min_{p\in\mathcal{P}_i} E_p\Big[\sum_{k=t}^{\infty}\beta^{nk-nt}\Big(s(y, X) - s(v, X)\Big)\Big] = E_{p^*}\Big[\sum_{k=t}^{\infty}\beta^{nk-nt}\Big(s(E_{p^*}[X], X) - s(v, X)\Big)\Big]$ for some $p^* \in \mathcal{P}_i$. Denote by $(y_{i+nk})_k$ a solution of the left hand side "maxmin" problem. Then $\min_{p\in\mathcal{P}_i} E_p\Big[\sum_{k=t}^{\infty}\beta^{nk-nt}\Big(s(y_{i+nk}, X) - s(v, X)\Big)\Big] \leq E_{p^*}\Big[\sum_{k=t}^{\infty}\beta^{nk-nt}\Big(s(y_{i+nk}, X) - s(v, X)\Big)\Big] \leq E_{p^*}\Big[\sum_{k=t}^{\infty}\beta^{nk-nt}\Big(s(E_{p^*}[X], X) - s(v, X)\Big)\Big]$. In other words, what it has just been proved is that in order to maximise your continuation expected payoff, given that the other players are playing at any case $v$, you need to consider only those continuation profiles that assign the same action for every future information set.

# C Appendix

In this section we give a number of examples in order to illustrate the robustness of issues that are encountered when a MSR market is populated by ambiguity averse traders and therefore the extent to which our results are useful.

The first example illustrates how the MSR model can be interpreted as an inventory based market maker. In addition, we show that, in the inventory based interpretation, information does not get aggregated always in the presence of ambiguity averse traders. The example is interesting because the interface of real markets might not be a sequential market as MSR, but rather designed with an interface of selling and buying securities (inventory based MM e.g. Inkling Markets) and therefore it is crucial to see if these markets do not aggregate information for some separable securities when the traders are ambiguity averse.

The second example illustrates a particular case of a separable security which does not aggregate information when the announcement is in the middle of the value range and for a particular set of priors. The novelty here is that for every prior, belonging in set of priors, its support is the whole state space.

**Example 2** *Consider the state space* $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$*, the price function to be the* $q(z) = e^{-z}$ *where* $z$ *is the market maker's net inventory. The security is given by* $X(\omega_1) = 2$*,* $X(\omega_2) = X(\omega_3) = X(\omega_4) = 1$ *and the information structure is* $\Pi_1 = \{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\}$ *and* $\Pi_2 = \{\{\omega_1, \omega_3\}, \{\omega_2, \omega_4\}\}$*. The set of priors is the* $\mathcal{P} = conv\{(0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}), (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})\}$*. Consider that initially the market maker holds zero inventory of the security (i.e. z=0).*

*Firstly, trader 1 makes a myopic decision about how much shares of the security to buy or sell. We assume, for consistency, that the amount of shares belong to* $Z = p^{-1}(Y)$*, which is compact. Thus it is implied that trader solves (for the true state to be either* $\omega_1$ *or* $\omega_2$*)* $\max_{z\in Z}\min_{p\in\mathcal{P}} E_p[\int_0^z q(\overline{z}) - X(\omega)d\overline{z}] = \min_{p\in\mathcal{P}}\max_{z\in Z} E_p[\int_0^z q(\overline{z}) - X(\omega)d\overline{z}]$*. We have the equality by applying the same argument as in the proof of Lemma 1.*[36]

*As in Ostrovsky (2012), given the price function we can define the strictly proper scoring rule* $s(X(\omega), y) = \int_0^{q^{-1}(y)} q(z) - X(\omega)dz$*. We have that the price function p is 1-1 continuous with continuous inverse function. Therefore we can conclude that in the MSR market, based on that strictly proper scoring rule, the trader solves* $\max_{y\in Y}\min_{p\in\mathcal{P}} E_p[\int_0^{q^{-1}(y)} q(\overline{z}) - X(\omega)d\overline{z}] = \min_{p\in\mathcal{P}}\max_{y\in Y} E_p[\int_0^{q^{-1}(y)} q(\overline{z}) - X(\omega)d\overline{z}]$*.*[37] *We shall show that if* $z^*$ *solves the first optimisation problem and* $y^*$ *the second one, then it is* $p(z^*) = y^*$ *and that the revenue or losses are the same, i.e.* $\max_{z\in Z}\min_{p\in\mathcal{P}} E_p[\int_0^z q(\overline{z}) - X(\omega)d\overline{z}] = \max_{y\in Y}\min_{p\in\mathcal{P}} E_p[\int_0^{q^{-1}(y)} q(\overline{z}) - X(\omega)d\overline{z}]$*. The conclusion is that the purchase of the optimal amount of shares and the announcement of the myopic prediction are related with a one to one relation using the pricing function and that the two markets are equivalent in terms of revenues and losses.*

*We can observe that for every* $p \in \mathcal{P}$ *the amount* $z'_p$ *that solves the* $\max_{z\in Z} E_p[\int_0^z q(\overline{z}) - X(\omega)d\overline{z}]$ *is unique and such that* $p(z'_p) = E_p[X]$*. Similarly, for every* $p \in \mathcal{P}$ *the prediction*

[36]We use that $F(z) = \int_0^z q(\overline{z}) - X(\omega)d\overline{z}$ is continuous and we follow the arguments of Lemma 1.

[37]Similarly, we follow the arguments of Lemma 1 with the continuous function $F(y) = \int_0^{q^{-1}(y)} q(\overline{z}) - X(\omega)d\overline{z}$.

*$y'_p$ that solves the $\max_{y\in Y} E_p[\int_0^{q^{-1}(y)} q(\overline{z}) - X(\omega)d\overline{z}]$ is the $y'_p = E_p[X]$, hence $q^{-1}(y'_p) = z'_p$.*

*Therefore, for every $p \in \mathcal{P}$ we have that $E_p[\int_0^{z'_p} q(\overline{z}) - X(\omega)d\overline{z}] = E_p[\int_0^{q^{-1}(y'_p)} q(\overline{z}) - X(\omega)d\overline{z}]$. We can conclude that $\min_{p\in\mathcal{P}} E_p[\int_0^{z'_p} q(\overline{z}) - X(\omega)d\overline{z}] = \min_{p\in\mathcal{P}} E_p[\int_0^{q^{-1}(y'_p)} q(\overline{z}) - X(\omega)d\overline{z}]$ and it is achieved in the same $p^*$.*

*We conclude that the optimal quantity of shares $z^*$ for the ambiguity averse trader is such that $q(z^*) = E_{p^*}[X]$ and the optimal prediction $y^*$ is such that $y^* = E_{p^*}[X]$ and thus we get the conclusion.*[38]

*Finally, the first trader finds the belief that achieves the minimum gives at state $\omega_1$ zero probability. From the previous paragraph we conclude that the optimal amount to purchase, $z^*$, is such that $p(z^*) = 0*2+1*1 = 1$ or equivalently (as long as p is 1-1) $z^* = 0$. Hence she neither buy or sell any shares (equivalently she would have announced 1 as her prediction, i.e. the price). It is easy to see that the same would happen for every state in the partition $\{\omega_3, \omega_4\}$ and for the trader 2 for symmetry reasons. The conclusion is that both traders does not purchase shares from the market maker and no one can infer the true state, even if that would be the case if they pooled their information.*

**Example 3** *Consider the state space $\Omega = \{\omega_1, ..., \omega_6\}$ and the partition structure $\Pi_1 = \{\{\omega_1, \omega_3\}, \{\omega_2, \omega_4\}, \{\omega_5, \omega_6\}\}$, $\Pi_2 = \{\{\omega_1, \omega_2, \omega_6\}, \{\omega_3, \omega_4, \omega_5\}\}$, $\Pi_3 = \{\{\omega_1, \omega_2\}, \{\omega_3, \omega_5\}, \{\omega_4, \omega_6\}\}$. The security is $X(\omega_1) = X(\omega_5) = 0, X(\omega_2) = X(\omega_6) = 2, X(\omega_3) = 1$ and $X(\omega_4) = -1$.*

*We first prove that this security is separable.*

***Claim 1*** *For every $v \in \mathbb{R}$ and every prior $p$ over the state space, if for every $i = 1, ..., n$ and every $\omega$, with $p(\omega) > 0$, it is $E_p[X|\Pi_i(\omega)] = v$ then for every $\omega$, with $p(\omega) > 0$, it is $X(\omega) = v$.*

***Proof.*** *Let $v \in \mathbb{R}$ and $p$ a prior over the state space $\Omega$. There are two cases.*

*Case 1: There exists $\omega \in \Omega$ and $i \in \{1, ..., n\}$ such that $p(\Pi_i(\omega)) = 0$. For every $i \in \{1, ..., n\}$ define $A_i$ to the the maximal set of i's information cells such that $p(\pi) > 0$ for every $\pi \in A_i$.*

*If the prior is such that, for every $i \in \{1, ..., n\}$ it is $E_p[X|\Pi_i(\omega)] = v$ for every $\omega \in \Omega$, with $p(\omega) > 0$, equivalently the prior is such that for every $i \in \{1, ..., n\}$ it is $\sum_{\omega'\in\Pi_i(\omega)} X(\omega')p(\omega') = v \cdot p(\Pi_i(\omega))$ for every $\omega \in \Omega$, with $p(\omega) > 0$. Name this set of equations $(*)$.*

*Then we can conclude that the prior is such that for every $i \in \{1, ..., n\}$ it is $\sum_{\omega'\in\Pi_i(\omega)} X(\omega')p(\omega') = v \cdot p(\Pi_i(\omega))$ for every $\omega \in \Omega$, name this set of equations $(**)$. This is true because if there exists $\omega$ such that $\Pi_i(\omega) \notin A_i$ then $p(\Pi_i(\omega)) = 0$ therefore $p(\omega') = 0$ for every $\omega' \in \Pi_i(\omega)$ and hence $\sum_{\omega'\in\Pi_i(\omega)} X(\omega')p(\omega') = v \cdot p(\Pi_i(\omega)) = 0$. Therefore, if $p$ solves $(*)$ then should solve $(**)$, too. We will prove that $(**)$ has solutions that all of them satisfy the claim.*

*Analytically, the equations $(**)$, denoting $p = (p_1, ..., p_6)$, are the following:*

[38] By using the saddle point inequality and the uniqueness of the optimal quantity and prediction (given the belief $p^*$).

$$2p_2 + 2p_6 = vp_1 + vp_2 + vp_6 \tag{1}$$

$$p_3 - p_4 = vp_3 + vp_4 + vp_5 \tag{2}$$

$$p_3 = vp_1 + vp_3 \tag{3}$$

$$2p_2 - p_4 = vp_2 + vp_4 \tag{4}$$

$$2p_6 = vp_5 + vp_6 \tag{5}$$

$$2p_2 = vp_1 + vp_2 \tag{6}$$

$$p_3 = vp_3 + vp_5 \tag{7}$$

$$2p_6 - p_4 = vp_4 + vp_6 \tag{8}$$

*If $v \neq 0, 1, -1, 2$ then the system of equations has a unique solution, the $p = (0, ..., 0)$. Because we want p to be a probability distribution we conclude that there is not such solution in our setting.*

*If $v = 0$ then the solutions are infinite and of the form $p = (a, 0, 0, 0, b, 0)$ with $a + b = 1$. We have that $X(\omega_1) = X(\omega_5) = 0$ and hence the claim is satisfied.*

*If $v = 1$ then the system has s unique solution the $p = (0, 0, 1, 0, 0, 0)$. The claim is satisfied for this prior (because $X(\omega_3) = 1$).*

*If $v = -1$ then the system has a unique solution the $p = (0, 0, 0, 1, 0, 0)$. The claim is satisfied for this prior (because $X(\omega_4) = -1$).*

*If $v = 2$ then the system has infinite solutions of the form $p = (0, a, 0, 0, 0, b)$ with $a + b = 1$. Again the definition of separability is satisfied for this prior (because $X(\omega_2) = X(\omega_6) = 2$).*

*Case 2: For every $\omega \in \Omega$ and for every $i \in \{1, ..., n\}$ it is $p(\Pi_i(\omega)) > 0$. We can proceed exactly as before, concluding that there is not a prior satisfying Case 2 and the hypothesis of the claim.*

*Hence we conclude that the security is separable.* ■

*However, the security is not strongly separable. To see that, suppose that the MM's initial announcement is $v = 0.5$, the middle of the price range, and consider any strictly proper scoring rule, such as the quadratic. Given $y_0$, consider any compact and convex set of priors that includes the priors $p = (\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{3}{8}, \frac{1}{8})$, $p' = (\frac{6}{18}, \frac{1}{18}, \frac{7}{18}, \frac{2}{18}, \frac{1}{18}, \frac{1}{18})$ and $p'' = (\frac{3}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8})$. Note that both conditions of Definition 3 are satisfied, hence the security is not strongly separable (which implies that there is no state in which information gets aggregated).*

# D Appendix

**Proof of Theorem 3.**

For the first claim, suppose $X$ is a strongly separable security and $(y^*, \mathscr{P})$ is a Revision-proof equilibrium. By Theorem 2 we know that for every state $\omega^*$ that can be realised there exists some time $T$ such that $\mathcal{F}^T = \mathcal{F}^T(y^*(\omega^*))$ is the finest public information and for every $\omega \in \mathcal{F}^T$ it is $X(\omega) = X(\omega^*)$. Therefore, we can conclude that if traders are myopic, for a round later than $T$, they should announce $X(\omega^*)$. From Theorem 2, again, we know that information gets aggregated and thus $y_t^* \longrightarrow_{t>T} X(\omega^*)$, where $y_t^* = y^*_{a(t)+n(t-1)}\big(\omega^*, \Phi^{t-1}(y^*(\omega^*))\big)$. In addition, $s_t^* = s(y_t^*, X(\omega^*)) \longrightarrow s^* = s(X(\omega^*), X(\omega^*))$.

Assume that there exists $y_t^*$, for $t > T$, such that $y_t^* \neq X(\omega^*)$. Hence it is $s_t^* \neq s^*$. Then we can define $t_0 = \min\{m : \textit{ for every } k \geq m \textit{ it is } |s_k^* - s^*| \leq \epsilon\}$, with $0 < \epsilon < |s^* - s_t^*| = s^* - s_t^*$.[39] We have that $t_0 - 1 \geq t > T$.

Without loss of generality, assume that $y^*_{t_0-1}$ is actually the $y^*_{i+nt_1}$ which in turn corresponds to $s^*_{i+nt_1}$. By its definition $|s^* - s^*_{t_0-1}| = |s^* - s^*_{i+nt_1}| = s^* - s^*_{i+nt_1} > \epsilon$.

By definition of $t_0$ we have that for every $k > t_1$ it is $|s^*_{i+nk} - s^*_{i-1+nk}| \leq 2\epsilon$. Hence for $\beta^n < \frac{1}{3}$ we get that $\beta^n \sum_{k=t_1+1}^{\infty} \beta^{n(k-t_1-1)}(s^*_{i+nk} - s^*_{i-1+nk}) \leq \beta^n \sum_{k=t_1+1}^{\infty} \beta^{n(k-t_1-1)}|s^*_{i+nk} - s^*_{i-1+nk}| \leq \frac{\beta^n}{1-\beta^n}2\epsilon \leq \epsilon$. Therefore, $s^*_{i+nt_1} - s^*_{i-1+nt_1} + \beta^n \sum_{k=t_1+1}^{\infty} \beta^{n(k-t_1-1)}(s^*_{i+nk} - s^*_{i-1+nk}) \leq s^*_{i+nt_1} - s^*_{i-1+nt_1} + \epsilon < s^* - s^*_{i+nt_1} + s^*_{i+nt_1} - s^*_{i-1+nt_1} = s^* - s^*_{i-1+nt_1}$.

By definition of $\mathcal{F}^T$ and because $X(\omega) = X(\omega^*)$ for every $\omega \in \mathcal{F}^T$ we have that $V_{i+nt_1}(\Phi^{i-1+nt_1}(y^*(\omega^*)), y^*, \omega^*, \mathscr{P}) = s^*_{i+nt_1} - s^*_{i-1+nt_1} + \beta^n \sum_{k=t_1+1}^{\infty} \beta^{n(k-t_1-1)}(s^*_{i+nk} - s^*_{i-1+nk})$ and

$$\min_{p \in \mathcal{P}_{\mathcal{F}^T \cap \Pi_i(\omega^*)}} E_p[s(d_{\mathcal{P}_{\mathcal{F}^T}}\left(\Pi_i(\omega^*), y^*_{i-1+nt_1}(\omega', \Phi^{i-2+nt_2}(y^*(\omega')))\right), X) -$$

$$s(y^*_{i-1+nt_1}\big(\omega', \Phi^{i-2+nt_2}(y^*(\omega'))\big), X)] = s^* - s^*_{i-1+nt_1}.$$

By the first argument in Step 3 of Theorem 2, we get a contradiction because $(y^*, \mathscr{P})$ is Revision-proof.

For the second claim, by Theorem 2 we know that for every state $\omega^*$ that can be realised it is $X(\omega) = X(\omega^*)$ for every $\omega \in \mathcal{F}^n(y^*(\omega^*))$. From Theorem **??** we know that after $n+1$-th announcement (including $n+1$) every trader plays myopically, hence for every state $\omega^*$ that can be realised and every $i = 1, ..., n$ they predict $y^*_{i+kn}\big(\Pi_i(\omega^*), \Phi^{i-1+kn}(y^*(\omega^*))\big) = X(\omega^*)$, for every $k = 1, 2, ....$

The utility of trader $i = 1, ..., n$, divided by $\beta^i$, at their information set when $k = 0$ is as follows:

$$\min_{p \in \mathcal{P}_{\mathcal{F}^{i-1}(y^*(\omega^*)) \cap \Pi_i(\omega)}} E_p\Bigg(s\Big(y_i^*\big(\Pi_i(\omega^*), \Phi^{i-1}(y^*(\omega'))\big), X\Big) - s\Big(y_{i-1}^*\big(\Pi_{i-1}(\omega'), \Phi^{i-2}(y^*(\omega'))\big), X\Big)\Bigg) +$$

$$\sum_{k=1}^{\infty} \beta^{nk}\Bigg(s\Big(y^*_{i+nk}\big(\Pi_i(\omega^*), \Phi^{i-1+nk}(y^*(\omega'))\big), X\Big) - s\Big(y^*_{i-1+nk}\big(\Pi_{i-1}(\omega'), \Phi^{i-2+nk}(y^*(\omega'))\big), X\Big)\Bigg).$$

Therefore, for every $i = 2, ..., n$ the utility turns out to be:

$$\min_{p \in \mathcal{P}_{\mathcal{F}^{i-1}(y^*(\omega^*)) \cap \Pi_i(\omega)}} E_p\Bigg(s\Big(y_i^*\big(\Pi_i(\omega^*), \Phi^{i-1}(y^*(\omega'))\big), X\Big) - s\Big(y_{i-1}^*\big(\Pi_{i-1}(\omega'), \Phi^{i-2}(y^*(\omega'))\big), X\Big)\Bigg).$$

This is because for every state $\omega^*$ that can be realised it is $y^*_{i+kn}\big(\Pi_i(\omega^*), \Phi^{i-1+kn}(y^*(\omega^*))\big) = X(\omega^*)$ for every $i = 1, ..., n$, for every $k = 1, 2, ....$

If there exists a trader $i \in \{2, ..., n\}$ that in his first round action does not play myopically, then if she would have played myopically her utility when $k = 0$ would be higher than the utility from her continuation game (on equilibrium path). Hence, with the same argument

[39] This is because $s^*$ is the score for the true state when the prediction is the true state, and $X(\omega) \neq y_t^*$.

as in Step 3 of Theorem 2 we get a contradiction with $y^*$ being Revision-proof. Hence, every $i \in \{2, ..., n\}$ plays myopically for every state that can be realised.

The same applies for $i = 1$ as well. However we need the result of the previous paragraph and the following argument to conclude it: for every $\omega^*$, with $p(\omega^*) > 0$ for some $p \in \mathcal{P}$, we have that for every $\omega \in \mathcal{F}^{y^*}_{n+1}(\omega^*)$ it is $X(\omega) = X(\omega^*)$ and that $\mathcal{F}^n(y^*(\omega^*)) \supseteq \Pi_n(\omega^*) \cap \mathcal{F}^{n-1}(y^*(\omega^*))$ therefore $X(\omega) = X(\omega^*)$ for every $\omega \in \Pi_n(\omega^*) \cap \mathcal{F}^{n-1}(y^*(\omega^*))$. Hence, the $n$-th trader's myopic prediction is essentially $X(\omega^*)$, and trader's 1 continuation value is zero. ■

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