

Sowing the Seeds of Financial Crises: Endogenous Asset Creation and Adverse Selection

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December 16, 2016

JOB MARKET PAPER

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Abstract

What sows the seeds of financial crises and what policies can help avoid them? To address these questions, I model the interaction between the ex-ante production of assets and ex-post adverse selection in financial markets. My results indicate that taking into account the endogenous asset supply is crucial. Positive shocks that increase market liquidity and prices exacerbate the production of low-quality assets. Indeed, I show that this can increase the likelihood of a financial market collapse. Government policies also have subtle effects. I show that an increase in government bonds increases total liquidity and reduces the incentives to produce bad assets, but can exacerbate adverse selection in private asset markets. Optimal policy balances these two effects, requiring more issuances when the liquidity premium is high. I also study transaction taxes and asset purchases, showing that policy should lean against the wind of market liquidity.

JEL Codes: E44, G01, G12, D82

*Department of Economics, Massachusetts Institute of Technology. Email: ncaramp@mit.edu. I am deeply indebted to my advisors Ivan Werning, Robert Townsend and Alp Simsek for their guidance and advice throughout this project. I also thank Daron Acemoglu, Rodrigo Adao, George-Marios Angeletos, Ricardo Caballero, David Colino, Daniel Greenwood, Sebastian Fanelli, Diego Feijer, Pablo Kurlat, Diana Moreira, Ameya Muley, Juan Passadore, Pascual Restrepo, Andres Sarto, Dejanir Silva, Mariano Spector, Ludwig Straub, Olivier Wang, and participants in the MIT Macro Seminar, MIT Macro Lunch, MIT Sloan Finance Lunch, and UTDT Seminar for their helpful comments and suggestions. I also thank the Macro Financial Modeling (MFM) group for financial support. All remaining errors are my own.

1 INTRODUCTION

It is widely believed that the recession that hit the US economy in 2008 originated in the financial sector. The years previous to the crisis were characterized by a rapid increase in the private production of assets that were considered safe, mostly through securitization. Many of the markets for these assets then collapsed, marking the starting point of the deepest recession in the post-war era. The extent to which this boom sowed the seeds of the posterior crisis is an important open question. Although many scholars have pointed to adverse selection to explain the observed collapse in these markets (e.g., Kurlat (2013), Chari, Shourideh and Zetlin-Jones (2014), Guerrieri and Shimer (2014a), Bigio (2015)), many important questions remain unanswered: where does the asset heterogeneity come from, how does it relate to the underlying state of the economy, and how does it interact with other sources of liquidity? These are the questions I seek to explore in this paper.

Safety refers to a characteristic of assets that are perceived as high quality, have an active (*liquid*) market, and facilitate financial transactions (as collateral or media of exchange more generally).¹ While traditionally this characteristic was mostly limited to government bonds and bank deposits, in the last 30 years there has been a large increase in the use of other privately produced assets, such as asset- and mortgage-backed securities.² Securitization was the instrument used by the private sector to provide the market with the safe assets it was demanding. This expanded the type of loans that were made, and riskier and more opaque borrowers were accepted. This process was particularly stark in the mortgage market, which saw an explosion of non-standard, low-documentation mortgages and low credit score borrowers.³ In fact, Bank for International Settlements (2001) articulated an early warning about the deterioration of the quality of assets used as collateral.

This paper presents a theory of asset quality determination in which the ex-ante production of assets interacts with ex-post adverse selection in financial markets. Assets in the economy derive their value from the dividends they pay and the liquidity services they provide. Better quality assets pay higher dividends, but because of adverse selection in markets, they sell at a pooling price with lower-quality assets. This cross-subsidization between high- and low-quality assets introduces a motive for agents to produce relatively more lemons when they expect prices to be high, since they expect to sell the assets rather than keep them until maturity. As a consequence, the theory predicts that the production of low-quality assets is more responsive to market conditions than that of high-quality assets. Therefore, shocks that improve the functioning of financial markets exacerbate the production of lemons and may even increase the exposure of the economy to a financial market collapse—a process that disrupts liquidity.

Moreover, the supplies of privately produced tradable assets and government bonds (private and public liquidity) interact through the liquidity premium. When the supply of public liquidity is

¹This has been recently emphasized, for instance, by Calvo (2013), Gorton, Lewellen and Metrick (2012) and Gorton (2016).

²See Gorton, Lewellen and Metrick (2012).

³See Ashcraft and Schuermann (2008). While origination of non-agency mortgages (subprime, Alt-A and Jumbo) were \$680 billion in 2001, they increased in 2006 to \$1,480 billion, a 118% growth. On the other hand, origination of agency (prime) mortgages decreased by 27%, from \$1443 billion in 2001 to \$1040 billion in 2006. Moreover, while only 35% of non-agency mortgages were securitized in 2001, that figure grew to 77% in 2006.

low, the private sector's incentives to produce close substitutes increase.⁴ But because low-quality assets are more sensitive to changes in the value of liquidity services, their production increases proportionally more, reducing the average quality composition in the economy. Indeed, my model predicts that the reductions in US government bonds in the late 90s due to sustained fiscal surpluses, as well as the increased foreign demand for US-produced safe assets in the early 2000s (a consequence of the so called "savings glut"), both generated perverse effects on the quality composition of privately produced assets.⁵

While the theory presented is silent about the specifics of the asset production process, I believe that the economic forces that it highlights are typical of the full process of transforming illiquid assets into liquid ones. In my interpretation, the production process constitutes both the origination process of loans (e.g., mortgages) and the posterior securitization process (e.g., AAA-rated private-label mortgage-backed securities).⁶ In both cases the "producers" know more than other market participants about the underlying quality of these products, either because they have collected information that cannot be credibly transmitted, or because they know how much effort they put into the process. Hence, the problem of quality production and adverse selection can be present in the whole intermediation chain.

The mechanics of the model hinge upon the behavior of the shadow valuation of different qualities. Suppose there are only two qualities. Agents with high-quality assets sell them only if their liquidity needs are high relative to the price discount they suffer in the market due to the adverse selection problem. In contrast, agents with low-quality assets always sell their holdings. Anticipating that this will be their strategy in the market, agents adjust their quality production decisions to the expected market conditions. If the market's expectations are high—in the sense that volume traded is high—agents anticipate that the probability they will sell their assets is relatively high, independent of the quality of those assets. In this case, more low-quality assets are produced because it is less attractive to exert effort to produce high-quality assets. That is, low-quality assets are produced for *speculative motives*: not for their fundamental value but for the profit the agent can make just from selling in the market. In this sense, good times can sow the seeds of a future crisis by providing incentives that lead to asset quality deterioration.

I consider two comparative statics that improve the functioning of financial markets: an improvement in the expected payoff of bad assets (or a reduction in their default probability) and an increase in liquidity needs (which can derive from increased productivity in the real economy or changes in the supply of public liquidity). I show that in both cases there is an increase in the production of assets and a deterioration of the asset quality composition, which can even lead to an increased exposure of the economy to a financial market collapse. While the direct effect of the exogenous shocks tends to *increase financial stability*, the endogenous response of the economy through

⁴This channel has been found empirically, for example, by Greenwood, Hanson and Stein (2015) and Krishnamurthy and Vissing-Jorgensen (2015).

⁵See Caballero (2006) and Caballero (2010) for a discussion on safe asset shortages.

⁶An important issue is the role of tranching in avoiding adverse selection. In my opinion there are two reasons why tranching can have a limited effect. First, if the balance sheets of financial intermediaries are difficult to monitor, then intermediaries can always go back to the market to sell any remaining fraction of assets. Second, certification by third parties (e.g., rating agencies) can have limited success if players learn how to game the rating models or if the incentives of the third party are compromised.

a worsening of the asset quality composition tends to *increase financial fragility*. To understand the importance of this result, consider what would happen if the asset quality distribution were exogenously given. A positive shock would improve market conditions, which would increase the volume traded and equilibrium prices. Since this quality composition would be fixed, the result would be an unambiguous reduction in the probability that the market would collapse. Hence, when asset quality is exogenous, positive shocks increase financial stability. However, when agents can react to the improved conditions of the market, the quality distribution deteriorates, which is a force that increases financial fragility. Which effect dominates depends on the relative size of each force. Moreover, I show that if the shock is transitory, financial fragility increases as the shock dies out, whatever its effect on impact. Hence, a boom can set the stage for a financial crisis.

I also consider the effects of reducing transaction costs. Financial innovation can reduce the cost of trading financial assets by facilitating the transformation of illiquid assets (e.g., mortgages) into liquid ones (e.g., MBS, ABS, CDOs). I show that if transaction costs are high, then the market for these assets remains inactive and agents produce only high-quality assets. As transaction costs decrease, agents who have sufficiently high liquidity needs find it optimal to sell their assets. Interestingly, while transaction costs remain relatively high, the presence of a secondary market is not enough to attract the production of low-quality assets. Therefore, while transaction costs remain at middle-range levels, the economy features a market for assets in which a low volume is traded and only high-quality assets are produced. Lastly, when transaction costs are sufficiently low, the production of low-quality assets becomes profitable and the economy can enter into a state in which high volumes are traded but with significant financial risk. These dynamics are consistent with developments of the last 30 years in the US economy, wherein the early stages of financial innovation could have improved the efficiency of the economy with no increased exposure to risk, but further innovation could have created perverse effects during the early 2000s, that culminated in a complete financial collapse in 2008.

On a more technical note, I show that a large amplification mechanism is present in the model. Due to the interaction between asset quality production and markets that suffer from adverse selection, prices might not be able to perform their role of clearing markets and guiding incentives. Suppose that the payoff of low-quality assets is distributed uniformly with bounds given by ϵ of distance around a mean. I show that there is a positive measure of parameter values such that as ϵ goes to zero (that is, the exogenous risk goes to zero), the endogenous risk of the economy remains positive and bounded away from zero. This is so because of the discontinuity of market prices to state variables in the presence of adverse selection. As the exogenous risk vanishes, the fundamentals of the economy in all states of nature become very similar. However, it can happen that similar prices in all states do not give the right incentives to agents during the production stage, when they make their investment decisions. If prices are low in all states, then agents have low incentives to produce low-quality assets, which is inconsistent with prices being low. On the other hand, if prices are high, the incentives to produce low-quality assets can be too high, which is inconsistent with prices being high. A fixed-point type of logic would argue for middle-range prices. However, these prices can be inconsistent with market clearing, because of the discontinuity of equilibrium market prices. Endogenous risk *convexifies* the expected prices, so that while prices clear the markets, risk

adjusts incentives during the production stage. As I demonstrate, the limit of an economy that has vanishing exogenous aggregate risk is the unique equilibrium of an economy that has no exogenous aggregate risk but does have sunspots.

Another important determinant of the dynamics of privately produced safe assets is the supply of public liquidity. A significant number of recent papers document that private production of safe assets increases when the supply of government bonds is low (and vice-versa). Gorton, Lewellen and Metrick (2012) and Krishnamurthy and Vissing-Jorgensen (2015) show that the supply of government bonds and the production of private substitutes in general are negatively correlated. Greenwood, Hanson and Stein (2015) find a negative correlation between the supply of US Treasuries and the supply of unsecured financial commercial papers, while Sunderam (2015) finds a similar result with respect to asset-backed commercial papers. Krishnamurthy and Vissing-Jorgensen (2012) shows that an increase in the supply of government bonds reduces the liquidity premium.

In my model, a higher volume of bonds increases the liquidity in the economy, which decreases the liquidity premium. As a consequence, government bonds crowd out private liquidity, which disproportionately reduces the incentives to produce low-quality assets. Hence, a shortage of safe assets induces a deterioration of private asset quality. This result seems to suggest that the government should provide all the liquidity the financial sector requires (a type of *Friedman Rule* applied to this setting). This appealing solution separates the liquidity value of assets from their dividend value so that assets are produced only for fundamental reasons. However, this policy might not be feasible for two reasons. First, the fiscal costs associated with it are likely to be large. Second, even if costs were low, there is no guarantee that the government bonds would end up in the hands of those who needed them the most, since agents with good investment opportunities would not purchase bonds. These two factors indicate why securitization can have social value: it allows investors to mitigate the trade-off they face between undertaking investment opportunities and keeping enough liquidity to satisfy future needs. Hence, any feasible intervention would tend to complement the private markets rather than replace them. In such a case, the government faces a subtle trade-off. On the one hand, it wants to provide the agents with the liquidity they need and reduce the production of bad assets. On the other hand, by crowding out the private markets, the government could exacerbate the adverse selection problem, since agents are less willing to sell their good assets at a discount to satisfy their liquidity needs, which are partly satisfied by government bonds. In the extreme case in which the quality distribution is exogenous, the presence of government bonds unambiguously increases the adverse selection problem and, consequently, fragility. That said, if the production elasticity of bad trees is high, government bonds can increase stability. Nonetheless, I find that the government should issue more bonds when the liquidity premium is high and less when the liquidity premium is low.

Ex-post policies could also be used. Tirole (2012) and Philippon and Skreta (2012) study how to restart a market that has collapsed because of adverse selection. In the optimal policy, the government buys from some agents assets that could be of the worst quality. From an ex-ante perspective, the anticipation of such policies exacerbates the production of lemons in the economy. To compensate for this, the government could tax financial transactions (and hence, lower market liquidity) in

high-liquidity states.⁷

Literature Review. This paper is most closely related to the literature that incorporates adverse selection in financial markets into macroeconomic models. Recently, adverse selection in financial markets has been invoked to explain certain phenomena experienced during the Great Recession, including the sudden collapse of the market of assets collateralized by mortgage related products. The work of Eisfeldt (2004), Kurlat (2013) and Bigio (2015) exemplifies this literature. They build dynamic general equilibrium models in which agents trade assets under asymmetric information in order to obtain the resources to satisfy some liquidity needs (fund investment projects in the case of Eisfeldt (2004) and Kurlat (2013), and obtaining working capital in Bigio (2015)). They show that adverse selection in financial markets can be an important source of amplification of exogenous shocks. In particular, Bigio (2015) demonstrates that adverse selection quantitatively explains the dynamics of the economy during the Great Recession. However, all of these papers take the distribution of asset quality as exogenously given. This paper builds on these insights but, taking a step back, it focuses on the endogenous determination of asset quality distribution. This extension is key to understanding the build-ups of risks emphasized in these papers. Also in this literature, Guerrieri and Shimer (2014a) and Chari, Shourideh and Zetlin-Jones (2014) study similar economies but under the assumption that markets are exclusive. However, they also assume that the quality distribution is exogenous.

Also relevant is Gorton and Ordoñez (2014), who study a dynamic model of credit booms and busts that emphasizes the information-insensitivity of assets that serve as collateral and, second, how changes in the incentives to produce information about the quality of the underlying assets can trigger a crisis. In contrast, I demonstrate that positive shocks play a role in reducing the incentives to produce good quality assets. Gorton and Ordonez (2013) also study the interaction between public and private liquidity, but their focus is in the production of information, whereas my model highlights the liquidity premium and the production of quality. In contrast to Gorton and Ordonez (2013), I find that government bonds have an ambiguous effect on the economy, and they can even increase financial fragility because they increase the adverse selection problem in private markets.

On the normative side, the focus has been on the problem of how to deal with markets that collapsed. Tirole (2012) and Philippon and Skreta (2012), who take an ex-post point of view,⁸ ask how markets that have suffered from adverse selection can be efficiently restored. My paper, which adopts an ex-ante perspective, studies two sets of policy instruments: government bonds and transaction taxes and subsidies (or asset purchase programs).

In addition, there is an empirical literature that tries to measure the extent of adverse selection in financial markets. Keys et al. (2010) use a regression discontinuity approach to ask whether the quality of loans that had a lower probability of being securitized was higher than those that had a higher probability, and they find that it was. Loans with a low probability of being securitized were about 10–25% less likely to default than similar loans that had a higher probability of being

⁷This *leaning against the wind* logic for policy is similar to Diamond and Rajan (2012) with respect to monetary policy.

⁸Tirole (2012) presents an ex-ante analysis but does not study the possibility of manipulating incentives through a combination of taxes and subsidies in different states of the economy.

securitized. This suggests that originators most carefully screened the loans they were most likely to keep. Other papers that show that asymmetric information could have been relevant in financial markets before the crisis include Demiroglu and James (2012), Downing, Jaffee and Wallace (2009), Krainer and Laderman (2014), and Piskorski, Seru and Witkin (2015).

Closest in theme and content to this paper is Neuhann (2016). In his independently developed model, bankers produce loans that are subject to aggregate risk. Because their funding ability is constrained by their net worth and their risk exposure, a secondary market for loans allows them to reduce their risk exposure and ultimately increase lending. The price in the market depends on the wealth in the hands of the buyers, so that when buyers' net worth is high, the market price is high enough to prompt some bankers to begin originating low-quality assets. Therefore, investment efficiency falls. When a negative shock hits the economy, low-quality assets default and buyers' wealth contracts, which makes the secondary market collapse. My paper takes a different approach. In my setup, the buyers' wealth channel is absent. I highlight the importance of the economy's fundamentals and the liquidity premium. I show that asset quality deteriorates after positive shocks, such as an increase in the fundamental value of low-quality assets, a reduction in trading costs, or an increase in the productivity in the real economy, and after a reduction in the supply of government bonds. This difference is important for our normative analysis. In contrast to Neuhann (2016), who argues that the growth of the buyer's net worth should be controlled, I study the optimal supply of government bond and transaction taxes (and subsidies).

This paper also contributes to the literature that emphasizes the role played by public liquidity in the facilitation of financial transactions. Woodford (1990) shows that when agents face binding borrowing constraints, a higher supply of government bonds can increase welfare. Government bonds supply the agents with the instruments necessary to respond to variations in income and spending opportunities through trade in secondary markets, which improves the allocation of resources. Holmström and Tirole (1998) also highlight the role of tradable instruments when agents cannot fully pledge their future income. They demonstrate that government bonds can complement private liquidity when the latter is not sufficient to satisfy all of the demand.

Gorton, Lewellen and Metrick (2012), Greenwood, Hanson and Stein (2015), Sunderam (2015), and Krishnamurthy and Vissing-Jorgensen (2015) document the negative relation between the private and public supply of money-like assets. Finally, Moreira and Savov (2016) emphasize the role of "shadow-banking" in supplying "money-like" assets. They show that "shadow-money" allows for higher growth but exposes the economy to aggregate risk. In this case, however, there is no asymmetric information problem in the economy.

Outline The rest of the paper is organized as follows. In section 2, I present a simple three-period model that features linear demand for liquidity to show the main forces of the model and study its positive implications. Section 3 extends the basic model to incorporate decreasing returns to liquidity, and it analyzes the interaction between the real economy and financial markets. Section 4 studies the effects of government bonds on the production of private assets. It also explores the role of transaction taxes and subsidies. In section 5, I extend the model to an infinite horizon setting. Section 6 concludes. All the proofs are presented in the appendix.

2 BASIC MODEL

In this section I present a simple three-period model that highlights the main forces of the economy. In the first period, agents choose the quality of the assets they produce anticipating that in the future they will face a "liquidity shock" that affects their intertemporal preferences for consumption, and a market for assets that suffers from adverse selection.

2.1 The Environment

Agents. There are three dates, 0, 1, and 2, and two types of goods: final consumption good, and Lucas (1978) trees. The economy is populated by a measure one of agents, $i \in [0, 1]$. Agents receive an endowment of final consumption good of W_0 in period 0, and W_1 in period 1.⁹ In period 0 they operate a technology that transforms final consumption goods into trees, which pay a dividend in period 2.

Agents' preferences are given by

$$U = d_0 + E_0 [\mu_1 d_1 + d_2],$$

where d_t is consumption in $t = 0, 1, 2$, μ_1 is a random idiosyncratic "liquidity shock" (uncorrelated across agents), which is private information of the agents, and the expectation is taken with respect to μ_1 and an aggregate state of the economy, described below. The liquidity shock affects the agent's marginal utility of consumption in period 1. From period 0 point of view, μ_1 is distributed according to the cumulative distribution function $G(\mu_1)$ in $[1, \mu^{\max}]$. I assume that G is such that with probability π , $\mu_1 = 1$, and with probability $1 - \pi$, μ_1 has a continuous cumulative distribution G_μ in $[1, \mu^{\max}]$. The mass of probability in $\mu_1 = 1$ simplifies the analysis of equilibrium prices below. In the extension of the model presented in the next section, π arises endogenously in equilibrium.

Technology. Agents have access to a technology to produce trees in period 0. This technology is idiosyncratic to each agent. There are two types of trees. An agent of type ξ can transform $q_G(\xi)$ units of the consumption good into 1 unit of high quality, "good", tree (denoted by G), and $q_B(\xi)$ units of the consumption good into 1 unit of low quality, "bad", tree (denoted by B), and ξ is distributed in the population uniformly in $[0, 1]$.¹⁰ The distribution of liquidity shocks in the population is independent of the types in period 0, ξ . I make the following assumptions.

⁹I assume that all agents receive the same endowment. As I show later, this assumption is without loss of generality.

¹⁰This assumption is WLOG since ξ affects the economy only through q_G and q_B . In particular, for any continuous cumulative distribution function $\Omega(\tilde{\xi})$ with support in $[0, 1]$ and associated density $\omega(\tilde{\xi})$, and differentiable functions \tilde{q}_G and \tilde{q}_B satisfying Assumption 1, it is possible to find differentiable functions q_G and q_B such that the distributions of \tilde{q}_G and \tilde{q}_B under $\tilde{\xi}$ coincide with the distribution of q_G and q_B under $\xi \sim U[0, 1]$:

$$Prob(\tilde{q}_j(\tilde{\xi}) \leq \bar{q}) = \int_1^{\tilde{q}_j^{-1}(\bar{q})} \omega(\tilde{\xi}) d\tilde{\xi} = \int_1^{q_j^{-1}(\bar{q})} d\xi = Prob(q_j(\xi) \leq \bar{q})$$

if and only if q_j satisfies

$$\frac{\omega(\tilde{q}_j^{-1}(\bar{q}))}{\tilde{q}_j'(\tilde{q}_j^{-1}(\bar{q}))} = \frac{1}{q_j'(\xi^{-1}(\bar{q}))},$$

for $j \in \{G, B\}$.

Assumption 1. The functions $q_G(\xi)$ and $q_B(\xi)$ are such that

1. $q_G(\xi)$ and $q_B(\xi)$ are continuous and increasing in ξ , with $q_G(0) = q_B(0) = 1$,
2. $q_G(\xi) \geq q_B(\xi)$ for all ξ ,
3. $\frac{q_G(\xi)}{q_B(\xi)}$ is increasing in ξ .

The first assumption implies that the cost of producing each type of tree is perfectly positively correlated, and that the agent with the lowest cost faces the same cost of producing good and bad trees (normalized to 1). The second assumption implies that producing bad trees is cheaper than producing good trees for every agent, which is needed so that bad trees have a chance of being produced. Finally, the third assumption implies that the cost of producing good trees grows faster than the cost of producing bad trees. That is, high (low) cost agents have a comparative advantage in producing bad (good) trees. Thus, one can interpret $q_B(\xi)$ as the *efficiency type* of the agent, and the difference $q_G(\xi) - q_B(\xi)$ as the *effort cost* required to obtain a good tree. Thus, for less efficient agents, the cost of increasing the quality of the tree produced is higher. Below, I discuss the robustness of my results to these assumptions.

Trees deliver fruits in final consumption good in period 2. A unit of good tree pays Z with certainty at maturity. On the other hand, only a fraction α of bad trees deliver fruit in period 2, so that one unit of bad tree in period 0 pays αZ in period 2. The fraction of bad trees that deliver fruit is known one period in advance. Thus, in period 1 the fraction α is common knowledge. However, in period 0 agents believe that α is a random variable distributed according to the cumulative distribution function F in the interval $[\underline{\alpha}, \bar{\alpha}] \subseteq [0, 1]$. One can interpret α as an aggregate shock to the productivity of bad trees, so that higher α implies higher quality of bad trees, or $1 - \alpha$ as a default rate of bad trees in period 2. Initially I assume that F is continuous and non-degenerate. I analyze what happens if this assumption is violated later in this section.

Finally, I assume that the investment opportunities are private information of the agents. Moreover, only the owner of a tree can determine its quality. These elements will be important when I describe the financial markets below.

Denote by H_t^G and H_t^B the total amount of good and bad trees in the economy in period t , respectively. Let λ_t^E denote the fraction of good trees in the economy in t , that is $\lambda_t^E \equiv \frac{H_t^G}{H_t^G + H_t^B}$.

Financial Markets. Due to the idiosyncratic liquidity shocks in period 1, there are gains from trade among agents. I assume that financial markets are incomplete. In particular, I limit the financial markets to trade of existing trees. This market is meant to be a metaphor of collateralized debt markets, like "repos" or short-term commercial papers.¹¹

I follow Kurlat (2013) and Bigio (2015) and assume that there is one market in which trees are traded, that buyers cannot distinguish the quality of a specific unit of tree but can predict what fraction of each type there is in this market, and that the market is anonymous, non-exclusive and

¹¹Bigio (2015) presents an equivalence result between a market for trading assets and a repo contract when there is no cost of defaulting besides delivering the collateral to the creditor. This is a standard assumption in papers of collateralized debt. See for example Geanakoplos (2010) and Simsek (2013).

competitive. These assumptions imply that the market features a pooling price, P_1^M .¹² Buyers get a diversified pool of trees from the market, where λ_1^M is the fraction of good trees in the pool. Note that since agents don't hold any trees initially, there is no trade in period 0.

In order to make the distinction between good and bad trees stark, I make the following assumptions.

Assumption 2. *The expected payoff of each type of tree satisfies*

1. $Z > 1 = q_G(0)$,
2. $E[\mu_1 \alpha Z] < 1 = q_B(0)$,
3. $E[\mu_1 Z] < q_B(1) < q_G(1)$.

The first assumption states that at least some agents will always find it profitable to produce good trees, even if there were no market for trees in period 1. The second assumption states that if the quality of trees was observable in the market, the net present value of bad trees would be lower than the production cost of the most efficient agent. This implies that in an economy with perfect information bad trees would never be produced. The third assumption implies that the agents with the highest costs do not produce trees.

Aggregate State and Timing. In period 1, the exogenous state of the economy is given by the distribution of liquidity shocks in the population and the realized quality of bad trees, α . The endogenous state is given by the cross-section distribution of trees and shocks across agents. Hence, the aggregate state of the economy in period 1 is $X_1 \equiv \{\alpha, \Gamma_1\} \in \mathbb{X}_1$, where $\Gamma_1(h_G, h_B, \mu_1)$ is the cumulative distribution of agents over holdings of each type of tree and liquidity shocks. In period 2, the state of the economy is given by the quality of bad trees in the current period, and the cross-section distribution of trees across agents, $X_2 \equiv \{\alpha, \Gamma_2\} \in \mathbb{X}_2$, where $\Gamma_2(h_G, h_B)$ is the cumulative distribution of agents over holdings of each type of tree.

To summarize, the timing of the economy is as follows. Agents start period 0 with an endowment of final consumption good W_0 . At the beginning of the period they are assigned a type, indexed by ζ , which determines their cost of producing trees of different qualities. Given the production costs they face, agents decide whether to produce trees, and in case they do, of what quality, or consume.

In period 1, agents receive an endowment of final consumption good W_1 . The aggregate shock α is realized, and agents receive an idiosyncratic liquidity shock. Since some agents may hold trees that they produced in period 0, the secondary market in period 1 may be active. Agents choose among two possible uses of the consumption goods they hold, which I call *liquid wealth*: consume or buy trees in the secondary market.

Finally, in period 2 all good trees pay Z , a fraction α of bad trees pays Z , and agents consume. Figure 1 summarizes the timing.

¹²There is a literature that assumes exclusive markets and assets of different qualities can trade at different prices. See for example Chari, Shourideh and Zetlin-Jones (2014) and Guerrieri and Shimer (2014a).

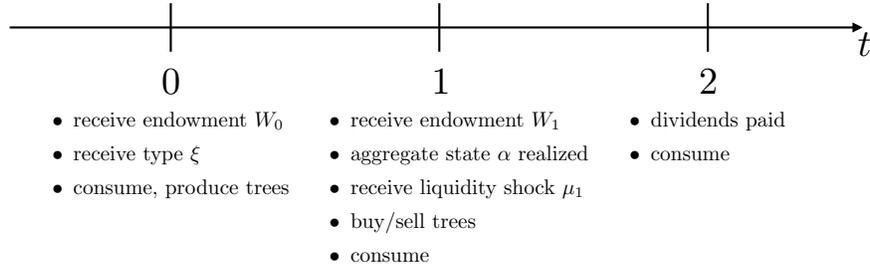


FIGURE 1: Timing

I find the equilibrium of this economy in steps. First, I solve the agents' problem. I show that the policy functions are linear in both the quantity of good and bad trees. This implies an aggregation result by which equilibrium prices and aggregate quantities are independent of the portfolio distribution of the agents in period 1. Second, I study the market for trees in period 1 and define a partial equilibrium for this market, which is an intermediate step for solving the full equilibrium of the economy. I show that finding an equilibrium of the economy simplifies to solving a fixed point problem in the fraction of good trees in the economy in period 1, λ_1^E . Finally, I study the equilibrium properties of the model and some comparative statics.

2.2 Agents' Problem

The problem the agents face in period 2 is very simple. They just collect the dividends from the trees they own and consume. Their value function is given by

$$V_2(h_G, h_B; X_2) = Zh_G + \alpha Zh_B, \quad (\text{P2})$$

where h_G and h_B are their holdings of good and bad trees, respectively.

Let's turn to period 1. Denote purchases of trees in the secondary market by m . If an agent buys m units of trees, a fraction λ_1^M of them is good, while a fraction $1 - \lambda_1^M$ is bad. Let s_B denote sales of bad trees and s_G sales of good trees. The agents' problem in state X_1 is given by:

$$V_1(h_G, h_B; \mu_1, X_1) = \max_{\substack{d, m, s_G, s_B, \\ h'_G, h'_B}} \mu_1 d + V_2(h'_G, h'_B; X_2), \quad (\text{P1})$$

subject to

$$d + P_1^M(X_1)(m - s_G - s_B) \leq W_1, \quad (1)$$

$$h'_G = h_G + \lambda_1^M(X_1)m - s_G, \quad (2)$$

$$h'_B = h_B + (1 - \lambda_1^M(X_1))m - s_B, \quad (3)$$

$$d \geq 0, \quad m \geq 0, \quad s_G \in [0, h_G], \quad s_B \in [0, h_B].$$

Constraint (1) is the agent's budget constraint, that states that consumption plus net purchases of trees cannot be larger than the endowment W_1 . Constraints (2) and (3) are the laws of motion

of good and bad trees respectively, which are given by the agents' initial holdings of trees plus a fraction of the purchases they make (where the fraction is given by the market composition of each type) minus the sales they make.

Given the linearity of the budget constraint and the utility function, both in current consumption, d , and the holdings of each type of trees for period 2, h'_G and h'_B , the agents' decisions are characterized by two thresholds on μ_1 : μ_1^B , that determines when to consume or buy trees, and μ_1^S , that determines when to sell good trees.

Lemma 1 (Agents' Choice in Period 1). *Consider an agent with liquidity shock μ_1 . There exists thresholds μ_1^B and μ_1^S that may depend on the state of the economy, X_1 , such that*

- if $\mu_1 \leq \mu_1^B$, then the agent buys trees ($m > 0$), keeps all his good trees ($s_G = 0$), and if $\mu_1 < \mu_1^B$ his consumption in period 1 is zero ($d = 0$);
- if $\mu_1 > \mu_1^B$ and $\mu_1 \leq \mu_1^S$, then the agent's consumption in period 1 is positive ($d > 0$), and he does not buy trees nor sells good trees ($m = s_G = 0$);
- if $\mu_1 > \mu_1^S$, then the agent's consumption is positive ($d > 0$), his purchases of trees are zero ($m = 0$), and he sells all his good trees in order to consume the proceeds ($s_G = h_G$).

All agents always sell their holding of bad trees, i.e. $s_B = h_B$. If π is sufficiently big, then $\mu_1^B = 1$.

The result in Lemma 1 is fairly straightforward. First, all agents sell their holdings of bad trees because there is an arbitrage opportunity. By selling one unit of bad tree they get P_1^M units of final good, which they can use to buy trees in the secondary market to obtain λ_1^M units of good trees and $1 - \lambda_1^M$ units of bad trees. Since $\lambda_1^M \in [0, 1]$, this strategy is always weakly optimal.¹³ Second, the return from buying trees in the market is given by $\mu_1^B \equiv \frac{\lambda_1^M Z + (1 - \lambda_1^M) \alpha Z}{P_1^M}$, which is the same for all agents. Because the utility from consuming in period 1 and the return from the market are both linear, agents just compare μ_1 and μ_1^B to decide whether to use their liquid wealth to consume or to buy trees. If $\mu_1 > \mu_1^B$ agents strictly prefer to consume, while they prefer to buy trees if $\mu_1 < \mu_1^B$. If π is sufficiently big, there are enough agents with $\mu_1 = 1$ so that they have enough wealth to purchase all the trees in the market, pushing the market price up until the return is equal to 1. In what follows, I will proceed under the assumption that $\mu_1^B = 1$. Note that in this case, μ_1 is also the marginal utility of *liquid wealth*, that is, the marginal utility of holding an extra unit of final consumption good, in contrast to just holding wealth in illiquid form, like trees.

The decision to sell good trees involves similar calculations. The market price of trees is always below the fundamental value of good trees, Z . This implies that the market price is lower than the payoff the agent would obtain if he kept the good tree until maturity. Hence, the only reason the agent would sell his good trees is if the utility derived from consuming in period 1 instead of period 2 compensates for the loss. This happens if $\mu_1 \geq \mu_1^S$, where $\mu_1^S \equiv \frac{Z}{P_1^M} \geq \mu_1^B$. Figure 2 summarizes these choices.

¹³Note that the arbitrage opportunity is independent of the price level. It does not rely on the market price being higher than the bad trees fundamental value αZ , but on the fact that the market composition cannot be worse than getting only bad trees.

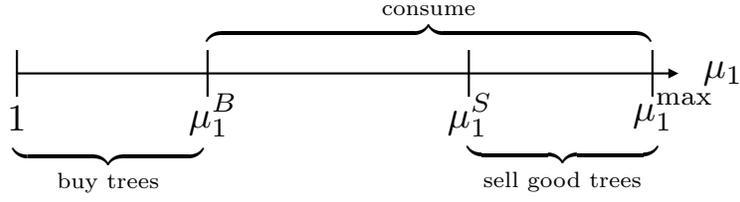


FIGURE 2: Agents' Choice in period 1

An important result that will greatly simplify the analysis that follows is the linearity of the agents' value function with respect to their holdings of each type of tree.

Lemma 2. *The value function in period 1, $V_1(h_G, h_B; \mu_1, X_1)$, is linear in each type of tree:*

$$V_1(h_G, h_B; \mu_1, X_1) = \mu_1 W_1 + \tilde{\gamma}_1^G(\mu_1, X_1) h_G + \tilde{\gamma}_1^B(\mu_1, X_1) h_B,$$

where

$$\tilde{\gamma}_1^G(\mu_1, X_1) = \max\{\mu_1 P_1^M(X_1), Z\}, \quad (4)$$

$$\tilde{\gamma}_1^B(\mu_1, X_1) = \mu_1 P_1^M(X_1). \quad (5)$$

This result follows directly from the linearity of the objective function and the budget constraint, and it is already using that $\mu_1^B(X_1) = 1$. For the agents, the marginal utility of an extra unit of consumption good is given by μ_1 . If $\mu_1 > 1$, this utility comes from consuming in period 1. If $\mu_1 = 1$, the agent is indifferent between consuming and buying trees in order to consume in period 2, which report a utility of 1. Since agents always sell their holdings of bad trees, their liquid wealth is no less than $W_1 + P_1^M(X_1) h_B$. As described above, agents might not be willing to sell their good trees unless their preference for consumption in period 1 is high enough. By selling a unit of good tree and consuming the proceeds, the agent gets $\mu_1 P_1^M(X_1)$ in period 1. On the other hand, by keeping the tree until maturity, the agents gets Z in period 2. Since there is no extra discounting between periods 1 and 2, the value of an extra unit of good tree is given by $\max\{\mu_1 P_1^M(X_1), Z\}$. Note that the coefficient on bad trees does not directly depend on its payoff in period 2. This is because no agent that *starts* the period owning bad trees, holds them until maturity.

As a consequence of the linearity of the value function, prices and aggregate quantities do not depend on the distribution of portfolios in the population. Therefore, the relevant state in periods 1 and 2 is $X = \{\lambda_1^E, H_1, \alpha\} \in \mathbb{X}$.

Finally, the problem of an agent in period 0 is given by

$$V_0(\xi) = \max_{\substack{d, i_G, i_B, \\ h'_G, h'_B}} d + E_0[V_1(h'_G, h'_B; \mu_1, X)], \quad (P0)$$

subject to

$$d + q_G(\xi) i_G + q_B(\xi) i_B \leq W_0, \quad (6)$$

$$h'_G = i_G, \quad h'_B = i_B. \quad (7)$$

$$d \geq 0, \quad i_G \geq 0, \quad i_B \geq 0.$$

Constraint (6) is the agent's budget constraint, that states that consumption plus expenditures in the production of trees cannot be larger than the endowment W_0 , and constraint (7) are the laws of motion of good and bad trees respectively, which are simply given by the investment agents make.

In order to decide whether to invest or not, agents compare their cost of production and their shadow valuation of holding trees in period 1, with the utility they get from consumption, which is equal to 1. Next, I define the shadow value of trees in this economy.

Definition 1 (Shadow Value of Trees). *The shadow value of trees are given by*

$$\begin{aligned} \gamma_0^G &\equiv E_0 \left[\tilde{\gamma}_1^G(\mu_1, X) \right] = E_0 \left[\max \left\{ \mu_1 P_1^M(X), Z \right\} \right], \\ \gamma_0^B &\equiv E_0 \left[\tilde{\gamma}_1^B(\mu_1, X) \right] = E_0 \left[\mu_1 P_1^M(X) \right]. \end{aligned}$$

The shadow value of trees is just the expected value of the marginal utility of each type of tree in period 1, given by (4) and (5). They can be decomposed in three different elements: a fundamental value, a liquidity premium, and an adverse selection tax/subsidy. That is:

$$\gamma_0^G = E \left[\underbrace{Z}_{\text{fund. value}} + \underbrace{(\mu_1 - 1)Z}_{\text{liq. premium}} - \underbrace{\min\{\mu_1(Z - P_1^M(X)), (\mu_1 - 1)Z\}}_{\text{adv. sel. tax}} \right], \quad (8)$$

$$\gamma_0^B = E \left[\underbrace{\alpha Z}_{\text{fund. value}} + \underbrace{(\mu_1 - 1)\alpha Z}_{\text{liq. premium}} + \underbrace{\mu_1(P_1^M(X) - \alpha Z)}_{\text{adv. sel. subs.}} \right]. \quad (9)$$

First, the fundamental value is given by the dividends each type of tree pays in period 2, given by Z for good trees, and αZ for bad trees. Second, trees in this economy derive value from the fact that they can be traded in period 1, transforming some payoff in period 2 into resources in period 1, when they are more valuable. The liquidity premium is a consequence of the *liquidity services* tradable assets provide in economies with incomplete markets, as emphasized by Holmström and Tirole (2001). Finally, the asymmetric information problem in the market for trees introduces a wedge in the market price that is negative for good trees and positive for bad trees. Let's focus on the shadow value of good trees first, given by (8). As I show below, the market price of trees is always between the fundamental value of good and bad trees, that is, $P_1^M(X) \in [\alpha Z, Z]$. Therefore, the adverse selection tax is always weakly positive. This tax is charged only if the tree is sold. Hence, the agents have a choice: sell the tree and pay the tax, generating a utility loss of $\mu_1(Z - P_1^M(X))$, or keep the tree and give up the liquidity services associated to it, generating a utility loss of $(\mu_1 - 1)Z$. The agent optimally chooses the option that generates the smallest loss. On the other hand, the pooling price implies an implicit subsidy for bad trees, as the last term in (9) shows. It is the size of this cross-subsidization between good and bad trees that shapes the incentives to produce different qualities. Moreover, note that all agents have the same ex-ante valuation for an extra unit of tree (good or bad) in the following period. This result relies mainly on the linearity of the agents' problem and greatly simplifies the analysis.¹⁴

¹⁴It also depends on the fact that liquidity shocks in period 1 are independent of the types in period 0. However,

A consequence of these expressions is that the shadow values have heterogeneous elasticities to market prices. Let $\gamma_0^i(P_1^M)$ be the shadow value of type $i \in \{G, B\}$ as a function of future prices $\{P_1^M(X)\}_{X \in \mathbb{X}}$, and let $D_\kappa \gamma_0^i(P_1^M)$ be the associated directional derivative with respect to future prices in the direction $\kappa(X)$.

Proposition 1 (Sensitivity of Shadow Values to Prices). *The shadow price of bad trees is more sensitive to future prices than the shadow value of good trees, that is*

$$\frac{D_\kappa \gamma_0^B(P_1^M + \kappa)}{\gamma_0^B(P_1^M)} > \frac{D_\kappa \gamma_0^G(P_1^M + \kappa)}{\gamma_0^G(P_1^M)} > 0,$$

for $\kappa(X) > 0 \forall X \in A$ with $\nu(A) > 0$ for some $A \subseteq \mathbb{X}$, where ν is the measure associated to \mathbb{X} .

This is the key result of the model. It says that the private valuation of bad trees is more sensitive to changes in expected market prices than that of good trees. Or put differently, the private valuation of good trees is more insulated to shocks from the market than that of bad trees. As explained above, and explicit in equation (8), good trees have the option value of being kept until maturity if market conditions are not sufficiently good, or if liquidity needs are low, while this strategy is always dominated for bad trees. Bad trees are produced only to be sold in the future, that is, for *speculative motives*. Since the fundamental value of bad trees is lower than its cost, it is never profitable to produce bad trees in order to keep them until maturity. The only reason to produce bad trees is the expectation of high prices in the secondary market, that can produce high returns when bad trees are sold as good ones. On the other hand, good trees have a high fundamental value. Since their market price is always below the discounted value of its dividends, agents only sell their good trees if their liquidity shock is high enough, that is, if the benefits of current consumption are sufficiently attractive so as to compensate for the loss from selling good trees below their private valuation. Thus, there are states of nature in which agents strictly prefer not to sell their good trees, isolating its value from price changes. This channel is at the core of the positive and normative analysis that follows. Moreover, it is important to note that this result is independent of Assumption 1. It only relies on the the cross-subsidization between good and bad trees due to the pooling price, independently of their costs.

Now I'm ready to characterize the agents' choice in period 0. As in period 1, the linearity of the agents' problem implies that their decisions are characterized by cutoffs. Given the shadow valuation of trees, γ_0^G and γ_0^B , agents decide whether to produce trees or not by comparing the return per unit invested of each option (good or bad) and the utility of consumption (which is 1). Since agents with low ξ have a comparative advantage in producing good trees, there always exists a threshold ξ_G such that agents with $\xi \leq \xi_G$ produce good trees. Agents with $\xi > \xi_G$ have a comparative advantage in producing bad trees. However, the cost of production might not be low enough to compensate for the opportunity cost of consuming immediately. If $\frac{\gamma_0^B}{q_B(\xi_G)} \leq 1$, then the shadow value of bad trees is too low compared to the cost of production. In this case, the marginal investor equalizes the return from producing good trees with the utility of consuming immediately,

allowing for correlation would not complicate the analysis, since at the individual level the shadow values would still be independent of the individual portfolio, which is the key property for tractability.

that is, $\frac{\gamma_0^G}{q_G(\xi_G)} = 1$. Agents with $\xi \in (\xi_G, 1]$ consume all their endowment.

On the other hand, if $\frac{\gamma_0^B}{q_B(\xi_G)} > 1$, then there are agents with $\xi \in (\xi_G, \xi_G + \varepsilon)$, for some $\varepsilon > 0$, that face a cost of producing good trees that is too high, but have a positive return if they produce bad trees. Hence, there exists $\xi_B > \xi_G$ such that if $\xi \in (\xi_G, \xi_B]$ the agent produces bad trees. The marginal investors of each type are determined as follows. The marginal investor of good trees is indifferent between producing good trees and bad trees, so ξ_G satisfies $\frac{\gamma_0^G}{q_G(\xi_G)} = \frac{\gamma_0^B}{q_B(\xi_G)}$. The marginal investor of bad trees is indifferent between producing bad trees and consuming in period 0, thus ξ_B satisfies $\frac{\gamma_0^B}{q_B(\xi_B)} = 1$. Finally, all agents for which $\frac{\gamma_0^B}{q_B(\xi)} < 1$ do not produce trees but consume. In order to simplify notation, I set $\xi_B = \xi_G$ whenever $\frac{\gamma_0^B}{q_B(\xi_G)} < 1$ (that is, there is no production of bad trees). The next lemma summarizes this result.

Lemma 3. *There exists $\xi_G \in (0, 1)$ such that $i_G(\xi) = \frac{W_0}{q_G(\xi)}$ if and only if $\xi \leq \xi_G$. If $\frac{\gamma_0^B}{q_B(\xi_G)} \leq 1$, then ξ_G satisfies $\frac{\gamma_0^G}{q_G(\xi_G)} = 1$, and $i_B(\xi) = 0$ for all ξ . On the other hand, if $\frac{\gamma_0^B}{q_B(\xi_G)} > 1$, then ξ_G is such that $\frac{\gamma_0^G}{q_G(\xi_G)} = \frac{\gamma_0^B}{q_B(\xi_G)}$. In this case, there exists $\xi_B \in (\xi_G, 1]$ such that $i_B(\xi) = \frac{W_0}{q_B(\xi)}$ if and only if $\xi \in (\xi_G, \xi_B]$, where ξ_B satisfies $\frac{\gamma_0^B}{q_B(\xi_B)} = 1$.*

Define aggregate investment in good and bad trees as $I_0^G = \int_0^1 i_G(\xi) d\xi$ and $I_0^B = \int_0^1 i_B(\xi) d\xi$, respectively. Then

$$I_0^G = \int_0^{\xi_G} \frac{W_0}{q_G(\xi)} d\xi,$$

$$I_0^B = \int_{\xi_G}^{\xi_B} \frac{W_0}{q_B(\xi)} d\xi.$$

In Proposition 1 I showed that the shadow value of bad trees is more sensitive to changes in the market conditions than the shadow value of good trees. Now, I extend the result to the behavior of aggregate investment.

As future prices increase, both the shadow value of good and bad trees increase. However, the shadow value of bad trees increases proportionally more. If $I_0^B > 0$, then ξ_G is defined such that $\frac{\gamma_0^G}{q_G(\xi_G)} = \frac{\gamma_0^B}{q_B(\xi_G)}$, or $\frac{\gamma_0^G}{\gamma_0^B} = \frac{q_G(\xi_G)}{q_B(\xi_G)}$. When expected prices increase, the left hand side of the expression *decreases*, since the shadow value of bad trees increases by more than the shadow value of good trees by Proposition 1, hence ξ_G *decreases* and the production of bad trees partially crowds out the production of good trees. The intuition is simple. Before the change in prices, the marginal agent was indifferent between producing good and bad trees. Now that prices increased, the production of bad trees is more profitable, hence the production of bad trees partially crowds out the production of good trees. Moreover, ξ_B , the type of the marginal investor of bad trees, increases, reinforcing the increase in the production of bad trees. Thus, an increase in expected prices reduces the production of good trees while increases the production of bad trees.

On the other hand, if $\frac{\gamma_0^B}{q_B(\xi_G)} < 1$, so there is no production of bad trees, then ξ_G is defined so that $\gamma_0^G = q_G(\xi_G)$. Therefore, a small increase in expected prices *increases* ξ_G . Therefore, when $I_0^B = 0$, an increase in expected prices increases the production of good trees. The next proposition summarizes this result.

Proposition 2. Let $I_0^G(P_1^M)$ and $I_0^B(P_1^M)$ be the aggregate investment functions of good and bad capital, respectively, as a function of future prices $\{P_1^M(X)\}_{X \in \mathbb{X}}$. If $I_0^B(P_1^M) = 0$, then $D_\kappa I_0^G(P_1^M + \kappa) > 0$. If $I_0^B(P_1^M) > 0$, then

$$D_\kappa I_0^B(P_1^M + \kappa) > 0 > D_\kappa I_0^G(P_1^M + \kappa),$$

for $\kappa(X) > 0 \forall X \in A$ with $\nu(A) > 0$ for some $A \subseteq \mathbb{X}$, where ν is the measure associated to \mathbb{X} .

While the result on the sensitivity of shadow values in Proposition 1 does not depend on Assumption 1, the result in Proposition 2 does. For the result in shadow valuations to translate into a result on quantities produced, some structure is necessary on the mass of agents that change their behavior after expected prices change. In particular, for Proposition 2 to hold, it is necessary that when the shadow value of bad trees moves more than that of good trees, a bigger mass of agents decide to produce bad trees than good trees. The perfect correlation of the cost functions and the comparative advantage assumptions are sufficient conditions for that. Moreover, the result that the production of good trees decreases because of the crowding-out effect is a partial equilibrium one. In general equilibrium, shocks that increase market prices can generate an increase in the production of both types of trees. I will analyze general equilibrium effects later in this section.

Proposition 2 implies that the production of lemons is more elastic to *future* prices than the production of non-lemons. It is an extension of the result in Akerlof (1970), who shows that the decision to *sell* non-lemons is more sensitive to prices than the decision to sell lemons. In my model, this result still holds in the secondary market for trees. But the lower exposure of the private valuation of good trees to market shocks reverts the result when considering production.

An immediate corollary of Proposition 2 is that the fraction of good trees in the economy in period 1, λ_1^E , decreases when agents expect higher market prices in the future. Moreover, the total amount of trees in the economy, $H_1 = H_1^G + H_1^B$, increases.

Corollary 2.1. Let $\lambda_1^E(P_1^M)$ be the fraction of good trees in the economy in period 1, and $H_1(P_1^M)$ the total amount of trees in period 1, as a function of future prices $\{P_1^M(X)\}_{X \in \mathbb{X}}$. Then,

$$D_\kappa \lambda_1^E(P_1^M + \kappa) \leq 0, \quad \text{with strict inequality if } I_0^B > 0,$$

and

$$D_\kappa H_1(P_1^M + \kappa) > 0,$$

for $\kappa(X) > 0 \forall X \in A$ with $\nu(A) > 0$ for some $A \subseteq \mathbb{X}$, where ν is the measure associated to \mathbb{X} .

Next, I turn to the equilibrium in the secondary market for trees.

2.3 Market for Trees

The economy features a unique market in which all trees for sale are traded, as in Kurlat (2013) and Bigio (2015). By assuming that π is big, the market for trees becomes a market with a demand and supply of quality, rather than quantity, in which agents with $\mu_1 = 1$ are willing and able to buy all the trees in the market as long as the price is fair. The inverse demand of tree quality is then given

by

$$P_1^M = \lambda_1^M Z + (1 - \lambda_1^M) \alpha Z,$$

and hence the demand is

$$\lambda_1^M = \frac{P_1^M - \alpha Z}{(1 - \alpha)Z}. \quad (10)$$

On the other hand, Lemma 1 states that there exists μ_1^S such that agents with $\mu_1 = \mu_1^S$ are indifferent between selling their good trees and keeping them. All agents with $\mu_1 > \mu_1^S$ sell their holdings of good trees (recall that all agents sell their bad trees). Therefore, the supply of trees is given by

$$S = \int_{\mu_1^S}^{\bar{\mu}} H_1^G dG(\mu_1) + H_1^B = \left[1 - G\left(\mu_1^S\right)\right] H_1^G + H_1^B.$$

Using that $\mu_1^S = \frac{Z}{P_1^M}$, the implied fraction of good trees supplied is given by

$$\begin{aligned} \lambda_1^M &= \frac{\left[1 - G\left(\frac{Z}{P_1^M}\right)\right] H_1^G}{S}, \\ &= \frac{\left[1 - G\left(\frac{Z}{P_1^M}\right)\right] \lambda_1^E}{\left[1 - G\left(\frac{Z}{P_1^M}\right)\right] \lambda_1^E + (1 - \lambda_1^E)}. \end{aligned} \quad (11)$$

In order to organize the analysis of the equilibrium of the economy, it is useful to define the partial equilibrium of this market for each state, taking λ_1^E as given.

Definition 2 (Partial Equilibrium in the Market for Trees). *A partial equilibrium in the market for trees in state X is a price P_1^M and a fraction of good trees in the market λ_1^M such that, given λ_1^E and α , the demand for tree quality (10) equals the supply of tree quality (11).*

There are two well-known characteristics of the set of partial equilibria in markets that suffer from adverse selection. The first one is what I call a *market collapse*, also known as *market unraveling*. If at every price greater than αZ the fraction of good trees supplied by sellers is too low compared to the *break-even* condition of buyers given by (10), then the only possible partial equilibrium has $P_1^M = \alpha Z$ and $\lambda_1^M = 0$. Because bad trees are inefficient (Assumption 2), if agents expected the price to be αZ in all states of the economy, no one would have incentives to produce bad trees. Since this paper studies how the incentives to produce different qualities varies with the underlying conditions of the economy, I will restrict the analysis to parameter values and functional forms such that for any realization of the exogenous state $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, there exists a threshold $\bar{\lambda}_1^E(\alpha) \in [0, 1)$ such that if the fraction of good trees in the economy is greater than $\bar{\lambda}_1^E(\alpha)$, then (10) and (11) intersect at an interior point with $\lambda_1^M > 0$. A necessary condition for this is that G_μ is convex at least over some interval of its support $[1, \mu_1^{\max}]$. In order to simplify exposition, I make the following assumption.

Assumption 3. *The distribution function G_μ is (weakly) convex in all its support $[1, \mu_1^{\max}]$.*

The second characteristic of markets that suffer from adverse selection is the multiplicity of partial equilibria. Consider figure 3. The panel (a) shows a market in which the quality of bad trees

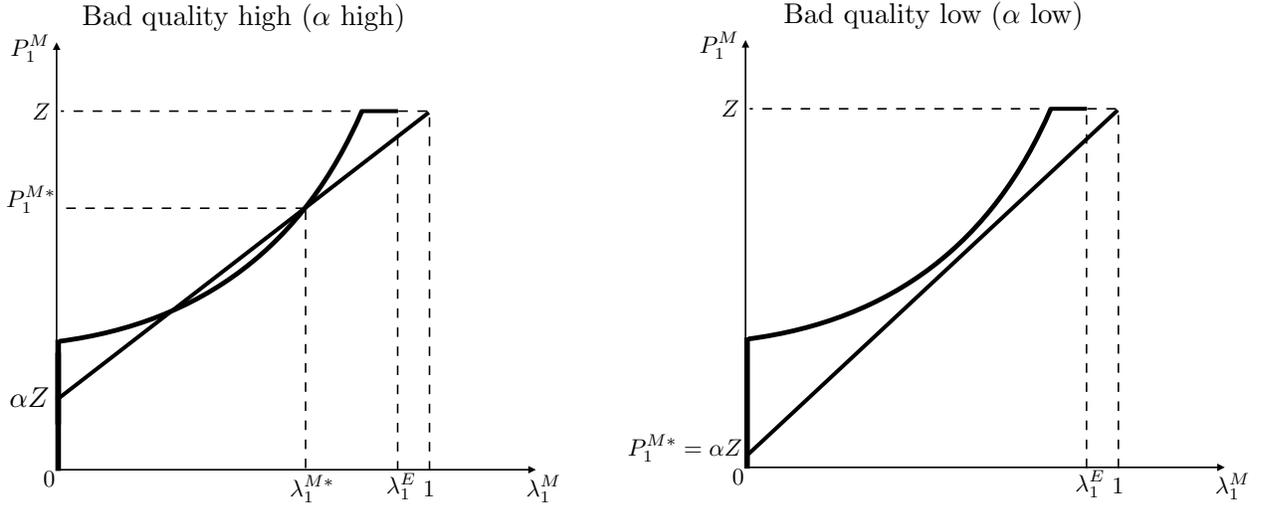


FIGURE 3: Market Equilibrium in period 1. (a) Multiple Equilibria: Maximal Volume of Trade Selected. (b) Unique Equilibrium: Market Collapse.

is high and there are multiple partial equilibria. The literature has adopted the convention of selecting the partial equilibrium that features the highest volume of trade (see Kurlat (2013), Bigio (2015), Chari, Shourideh and Zetlin-Jones (2014)). Later in this section I discuss the microfoundations of this selection criterion and how it affects the equilibrium of the economy. For now, I make the same selection.

As the quality decreases, the demand function (10) moves down. When α is low enough, the economy transitions to the market depicted in figure 3(b). In this case, the highest volume of trade equilibrium disappears, generating a market collapse. This has two implications. First, there exists a threshold $\alpha^*(\lambda_1^E)$ such that if $\alpha < \alpha^*$, then the market collapses and only bad trees are traded. On the other hand, if $\alpha \geq \alpha^*$, then both good and bad trees are traded in the market. Second, as λ_1^E increases, the threshold α^* decreases, meaning that the set of states such that there is a market collapse shrinks. This leads to the following definition of *market fragility*.

Definition 3. Define *market fragility* as

$$MF_1(\lambda_1^E) \equiv Prob(\alpha \leq \alpha^*(\lambda_1^E)).$$

Market fragility is the probability of a market collapse, that is, the probability that the economy features a market in which only bad trees are traded. It is straightforward to see that market fragility, $MF_1(\lambda_1^E)$, is decreasing in λ_1^E .

Even though market fragility is not a direct measure of welfare, it is a property that is tightly connected to the efficiency of the economy. The collapse of a market is the extreme case in which the flow of resources is severely impaired.

2.4 Equilibrium

Let's define an equilibrium for this economy.

Definition 4 (Equilibrium). *An equilibrium in this economy consists of prices $\{P_1^M(X)\}$; fraction of good trees in the market $\{\lambda_1^M(X)\}$; decision rules $\{d_0(\xi), d_1(h_G, h_B; \mu_1, X), d_2(h_G, h_B; X)\}$, $\{i_G(\xi), i_B(\xi)\}$, $\{h'_G(h_G, h_B; \mu_1, X), h'_B(h_G, h_B; \mu_1, X)\}$, $\{m(h_G, h_B; \mu_1, X), s_G(h_G, h_B; \mu_1, X), s_B(h_G, h_B; \mu_1, X)\}$; a fraction of good trees in the economy, λ_1^E , and a total amount of trees H_1 , such that*

1. $\{d_0(\xi), d_1(h_G, h_B; \mu_1, X), d_2(h_G, h_B; X)\}$, $\{i_G(\xi), i_B(\xi)\}$, $\{h'_G(h_G, h_B; \mu_1, X), h'_B(h_G, h_B; \mu_1, X)\}$, $\{m(h_G, h_B; \mu_1, X), s_G(h_G, h_B; \mu_1, X), s_B(h_G, h_B; \mu_1, X)\}$ solve the agents' problems (P0), (P1) and (P2), taking $\{P_1^M(X)\}$, $\{\lambda_1^M(X)\}$, λ_1^E and H_1 as given;
2. $\{P_1^M(X)\}$ and $\{\lambda_1^M(X)\}$ are the maximum volume of trade partial equilibrium state by state;
3. λ_1^E and H_1 are consistent with individual decisions.

Because of the linearity of the agents' problem in period 1, prices are independent of the total amount of trees, H_1 , while aggregate variables are linear in H_1 . Hence, in order to complete the characterization of the equilibrium, I just need to determine the fraction of good trees in period 1, λ_1^E , which is given by

$$\lambda_1^E = \frac{I_0^G}{I_0^G + I_0^B}.$$

Note that the decision to produce trees in period 0, and of what quality, depends on market prices in period 1. But prices in period 1 depend on the fraction of good trees in the economy, which in turn are determined by aggregate investment in period 0. It is useful to define the following mapping

$$T(\lambda_1^E) = \frac{I_0^G(\lambda_1^E)}{I_0^G(\lambda_1^E) + I_0^B(\lambda_1^E)}. \quad (12)$$

An equilibrium of this economy requires that $T(\lambda_1^E) = \lambda_1^E$. The mapping T is decreasing in λ_1^E , since higher λ_1^E implies higher expected prices, and the result follows from Proposition 2. When the distribution of α , F , is continuous, then $I_0^G(\lambda_1^E)$ and $I_0^B(\lambda_1^E)$ are continuous, and hence T is continuous. Therefore, the equilibrium of the economy exists and is unique. The following proposition summarizes these results.

Proposition 3. *An equilibrium of the economy always exists and is unique.*

Next, I study some properties and comparative statics of the economy. Propositions 4 and 5 formalize the idea that positive shocks to fundamentals distort the quality production decisions, since they increase the production of bad trees relative to that of good trees so that the average tree quality in the economy decreases. Next, I show that a reduction of transaction costs has a similar effect, and I lay out a plausible story for the development of the US financial sector in the last 30 years that could have led to the financial crisis of 2008. Finally, I show that the endogenous production of asset quality can interact with markets that suffer from adverse selection in such a way that the amplification of risk in the economy can be very large, to the extreme that endogenous risk remains positive and bounded away from zero even as exogenous risk vanishes away.

The Quality of Bad Trees

Consider the effect of an anticipated (from period 0 point of view) increase in the expected quality of bad trees (or an expected reduction of *default rates*).¹⁵ In particular, suppose that the distribution of α is indexed by a parameter $\theta : F(\alpha|\theta)$, where a higher θ means a better distribution in the sense of first-order stochastic dominance. It can be shown that an increase in θ is equivalent to an increase in prices for all states under the initial distribution. From Proposition 2, we know that the partial equilibrium effect is an increase in the investment of bad trees, a reduction in the investment of good trees, and a reduction in the fraction of good trees in the economy, λ_1^E . This reduction in λ_1^E feeds back to the prices, through a general equilibrium effect. This partially offsets the increase in production of bad trees and the reduction in production of good trees. However, the overall effect is an increase in the investment in bad trees, a reduction in the fraction of good trees in the economy, an ambiguous effect on the investment in good trees, but an increase in the total production of trees, $H_1 = I_0^G + I_0^B$.

Since λ_1^E decreases, the market price for each realization of α decreases, so the threshold α^* increases. This endogenous adjustment of the economy is a force towards more fragility. However, the direct effect of the shock is an improvement in the distribution of shocks, which is a force towards less fragility. In general, the result is ambiguous and depends on the nature of the shock and the elasticities of production of trees. Recall that market fragility is the probability that the quality of bad trees, α , is below the threshold, α^* , that is $MF = F(\alpha^*(\lambda_1^E)|\theta)$. Differentiating this expression with respect to θ we get

$$\frac{dMF}{d\theta} = \underbrace{\frac{\partial F(\alpha^*|\theta)}{\partial \theta}}_{\leq 0} + f(\delta^*; \theta) \underbrace{\frac{\partial \alpha^*(\lambda_1^E)}{\partial \lambda_1^E}}_{< 0} \underbrace{\frac{\partial \lambda_1^E}{\partial \theta}}_{< 0}.$$

For example, suppose the change in F is concentrated in very high values of α , so that $\frac{\partial F(\alpha^*|\theta)}{\partial \theta} = 0$. Then, the effect of the endogenous adjustment mechanism of the economy dominates, and market fragility *increases*. On the other hand, consider what would happen if the fraction of good trees in the economy was exogenously given, as in Eisfeldt (2004) and Kurlat (2013). In that case, $\frac{\partial \lambda_1^E}{\partial \theta} = 0$, so that market fragility would *decrease* after the shock. The next proposition summarizes these results.

Proposition 4 (Increase in Bad Trees' Expected Quality). *Consider an anticipated increase in θ , so that $F(\alpha|\theta)$ increases in FOSD sense. Then,*

1. *total investment in trees, $I_0^G + I_0^B$, increases;*
2. *the fraction of good trees in the economy, λ_1^E , decreases;*
3. *market prices in period 1, P_1^M , decrease in every state;*
4. *the threshold α^* increases;*
5. *the effect on market fragility is ambiguous.*

¹⁵Or equivalently, consider two economies with different distributions of bad tree quality.

This is an important result since it states that a “positive” shock to the economy can endogenously increase the fragility of its financial markets, in the sense that the probability of a market collapse is higher. Thus, it formalizes the idea that positive shocks can set the stage for a financial crisis. Next, I show that changes to the agents’ liquidity needs have similar effects.

Liquidity Shocks

An increase in the distribution of liquidity shocks increases the value of trees coming from their medium of exchange role. This is a positive shock in the sense that it improves the functioning of the market for trees.¹⁶ Since liquidity shocks and market prices enter symmetrically in the expressions for the shadow value of trees, an increase in liquidity shocks triggers a qualitatively similar response from period 0.

Proposition 5 (Increase in Liquidity Shocks). *Consider an anticipated change in the distribution of μ_1 from $G(\mu_1)$ to $\tilde{G}(\mu_1)$ such that $\tilde{G} > G$ in FOSD sense. Then,*

1. *total investment in trees, $I_0^G + I_0^B$, increases;*
2. *the fraction of good trees in the economy, λ_1^E , decreases;*
3. *the effect in market prices in period 1, P_1^M , is ambiguous;*
4. *the effect on the threshold α^* is ambiguous;*
5. *the effect on market fragility is ambiguous.*

The incentives to produce lemons increase with the value of liquidity services. However, the effect on market fragility is, again, ambiguous. On the one hand, as G increases, more agents sell their good trees so market fragility decreases. On the other hand, the endogenous response of the economy reduces the average quality of trees, increasing fragility. The overall effect depends on the interaction between these two forces.

Note that if the change in expectations does not reflect a change in the actual distributions (in the sense that it is just unfounded *optimism*) then fragility always increases for both types of shocks. Moreover, even though the effect of shocks to the economy’s fundamentals on market fragility is ambiguous on impact, in the infinite horizon extension I show that if the shock is transitory, then market fragility increases as the shock dies out. This is another way in which good times sow the seeds of the next crisis.

Finally, in the next section I extend the model of this section and microfound these shocks so that changes in the distribution of G arise from shocks to the “real economy”, or shocks to the supply of government bonds. This introduces a new set of comparative statics and sources of risk build-up in the economy.

¹⁶In this model liquidity shocks are “good” shocks in the sense that they increase the agents’ valuation for consumption. Similarly one could assume that the shocks are “bad” and they reduce the utility of consumption in period 2. In both cases, an increase in the distribution of liquidity shocks are good news for the functioning of the market.

Transaction Costs

Financial innovation can reduce the cost of trading financial assets. Many scholars argue that in the last 30 years the financial sector underwent a process that facilitated the transformation of illiquid assets (e.g. mortgages) into liquid ones (e.g. MBS, ABS, CDOs).¹⁷ Securitization and repo contracts seem to have been some of the stars of this process. Here, I show that a reduction in transactions costs naturally leads to a deterioration of the quality of assets in the economy.

Consider a variant of the economy described before in which sellers receive $P_1^S = P_1^M - c$ per tree sold, where P_1^M is the price paid by buyers, and c is a pecuniary cost that summarizes all the costs the seller has to incur in order to be able to transfer property of the tree to another agent. The main characteristics of the equilibrium with trading costs follow from the previous discussion, in particular existence and uniqueness. An important difference is that the market for trees can be inactive for some values of c , or have only good trees being traded. Obviously, if $c = 0$ the equilibrium is exactly the one described above.

Suppose $c \geq Z$. Since prices cannot be higher than Z , agents get no net resources from the sale of trees. Therefore, there will be no active market for trees in this economy, and producers of trees keep them until maturity. Since $E[\mu_1 \alpha Z] < 1$ by assumption, no agent produces bad trees, and the economy has $\lambda_1^E = 1$. Since the maximum utility agents can get from consumption is μ_1^{\max} , this result holds for all $c \in (c_1, \infty)$, where $c_1 \equiv \frac{\mu_1^{\max} - 1}{\mu_1^{\max}} Z$.

For a cost c slightly lower than c_1 , one of two things can happen, depending on parameter values. If μ_1^{\max} is relatively high, then the cost c can be high and still incentivize some agents with high μ_1 to sell their good trees. But if c is high, then the price the sellers receive is low, so the returns from selling trees are not sufficiently high to incentivize speculative production of bad trees. In that case, there exists a $c_2 < c_1$ such that if $c \in (c_2, c_1)$ there is an active market of trees in period 1, $\lambda_1^E = 1$, and $P_1^M = Z$ in all states of the economy. Also note that I_0^G increases as c decreases in this region. The reason is that the liquidity premium increases as the cost of trading trees decreases, and while the production of bad trees is inefficient, the incentives of producing good trees increases. On the other hand, if $c < c_2$, the transaction cost is sufficiently low to attract the production of bad trees, so $\lambda_1^E \in (0, 1)$.

If μ_1^{\max} is relatively low, then the cost c has to be low in order to incentivize agents with good trees to sell in the market. In this case, the price sellers get from selling trees, $P_1^S = Z - c$ is relatively high when there are no bad trees. Thus, if c is low enough, some agents will have incentives to produce bad trees. Therefore, when μ_1^{\max} is low, if $c < c_1$ there is an active market in period 1 and $\lambda_1^E \in (0, 1)$. For notational convenience I set $c_2 = c_1$ when this happens.

Finally, the fraction of good trees in the economy decreases as c decreases in the region $c \in [0, c_2)$. The next proposition summarizes these results.

Proposition 6. *Suppose sellers receive $P_1^S = P_1^M - c$ per tree sold, where c is a transaction cost. There exists c_1 and c_2 with $c_1 \geq c_2$ such that*

- if $c > c_1$, there is no market for trees and $I_B = 0$,

¹⁷See for instance Adrian and Shin (2010).

- if $c \in (c_2, c_1)$, there is an active market for trees in period 1, $I_B = 0$, and $\frac{\partial I_G}{\partial c} < 0$,
- if $c < c_2$, there is an active market for trees in period 1, $I_B > 0$, and $\frac{\partial \lambda_1^E}{\partial c} > 0$.

This result introduces a plausible story for the development of the US financial sector in the last 30 years. When the main financial innovations were introduced, the cost of trading certain assets (e.g., ABS, MBS, CDOs) decreased. However, if the reduction in costs was gradual, then the economy could have spent some time in the range at which there was an active market but no production of low quality assets, since the market return did not make their production profitable. Hence, the economy completely benefited from further innovation and cost reductions, increasing the high-quality asset production and volume traded, and improving the allocation of resources. However, at some point the transaction costs could have decreased so much that some agents found it profitable to produce low quality assets to take advantage of the market. Further reductions of the transaction tax further reduced the average quality of the assets, which exposed the economy to financial risk, as experienced in 2008.

Financial Risk

The previous exercises were meant to convey the idea that "positive" shocks give bad incentives in terms of asset quality production, since they improve the functioning of markets and increase prices, which in turn reduces the incentives to produce high quality assets. Here I make a digression in order to show that the interaction between the production of asset quality and the presence of markets that suffer from adverse selection can generate a large amplification of exogenous shocks, to the point that endogenous risk can remain positive and bounded away from zero even as exogenous risk vanishes away.

Consider an economy in which the distribution of bad tree quality is given by

$$\alpha = \tilde{\alpha} + u, \quad u \sim \mathcal{U}[-\epsilon, \epsilon], \quad (13)$$

for some $\epsilon > 0$, and where \mathcal{U} denotes the uniform distribution. Let $P_1^M(\alpha|\epsilon)$ denote the equilibrium price in period 1 when the exogenous state is α and the bounds of the uniform distribution is given by ϵ . I want to determine what happens with the variance of the price as the exogenous risk vanishes away, that is, as $\epsilon \rightarrow 0$.

In order to understand how the economy behaves as exogenous risk vanishes away it is useful to note that prices perform two roles in this economy. First, they clear markets, which in this case means that the quality supplied has to be consistent with the quality demanded. Second, prices send signals to the agents and shape investment decisions in period 0. Note that this dual role of prices is not special to this economy but it appears every time agents have investment opportunities and there is a market for that investment (think of physical capital in a standard neoclassical model, in which the rental rate clears the market for available capital but also gave incentives to produce capital in the past). What is special about markets that suffer from adverse selection is that prices can be discontinuous in state variables. In particular, the market price in a given state α is discontinuous in the fraction of good trees in the economy, λ_1^E . This discontinuity will be key to

understand the role of risk in the economy.

As $\epsilon \rightarrow 0$, the fundamentals of the economy in every state get very similar to each other. If prices were continuous, the prices in different states would also get closer to each other. At what level should they be? If prices were low in every state, such that markets collapse for every realization of α , then no agent would produce bad trees, contradicting that the prices in period 1 are low. On the other hand, if all prices are high, then it might be that too many bad trees are produced so that it is inconsistent with prices being high. Hence, in order for prices to give the right incentives to invest, they should be of a "middle" range. However, those prices can be inconsistent with market clearing. Does this mean that there is no equilibrium for some pair $\tilde{\alpha}$ and ϵ , with ϵ small but positive? We already know the answer is no, because Proposition 3 guarantees existence for any continuous distribution function of the aggregate shock α . Hence, what this is saying is that the equilibrium cannot feature prices that are *continuous* in the aggregate state. Hence, even though the difference between the lowest state $\tilde{\alpha} - \epsilon$ and the highest state $\tilde{\alpha} + \epsilon$ can be made arbitrarily small, the economy might need discontinuous prices to give the right incentives to the agents in period 0. The risk introduced by market fragility allows the economy to obtain a "middle range" price on average, when that price is not consistent with market clearing in any state in period 1. The next proposition summarizes this result.

Proposition 7. *Consider an economy in which α is distributed according to (13). There exists an open set $B \subset [0, 1]$ such that if $\tilde{\alpha} \in B$ then*

$$\lim_{\epsilon \rightarrow 0} \text{Var}[P_1^M(\alpha|\epsilon)] = \sigma^2(\tilde{\alpha}),$$

for some $\sigma^2(\tilde{\alpha}) > 0$.

Finally, the result in Proposition 7 is related to what happens to the economy if the distribution of α , F , has atoms. As noted above, the proof of existence of equilibrium uses the fact that F is continuous so that the mapping T is continuous, which guarantees that a fixed point exists. I now show that the limit $\sigma^2(\tilde{\alpha})$ is the variance of the price in an economy with no exogenous aggregate risk, that is, F is degenerate at $\alpha = \tilde{\alpha}$, and an equilibrium definition that allows for sunspots.

In order to explain the role of sunspot in the perfect foresight economy, it is useful to take a step back and study the theoretical justifications for the selection of the maximal volume of trade partial equilibrium I made before. The choice of the maximal volume of trade equilibrium can be justified as being the generic outcome of a game in which buyers can make different offers but choose not to in equilibrium (see, for instance, Mas-Colell, Whinston and Green (1995) and Attar, Mariotti and Salanié (2011)). Consider the cases depicted in figure 4. Figure 4(a) shows the case in which the game-theoretic approach selects the highest volume of trade equilibrium. The intuition is fairly simple: if the equilibrium featured prices P_1^* or P_2^* , some buyer could offer a price slightly higher than P_2^* , and attract a relatively large number of sellers of good trees, and make a profit. P_3^* is the only price at which there is no profitable deviation. On the other hand, figure 4(b) shows a case in which both P_1^* and P_2^* are consistent with equilibrium. Suppose the equilibrium has P_1^* . There is no deviation for buyers that can get them positive profits. The same happens with P_2^* . Hence, both prices are consistent with agents' optimization. This case is not relevant when the distribution of exogenous aggregate risk F is continuous, since given λ_1^E there is only one state α in which the

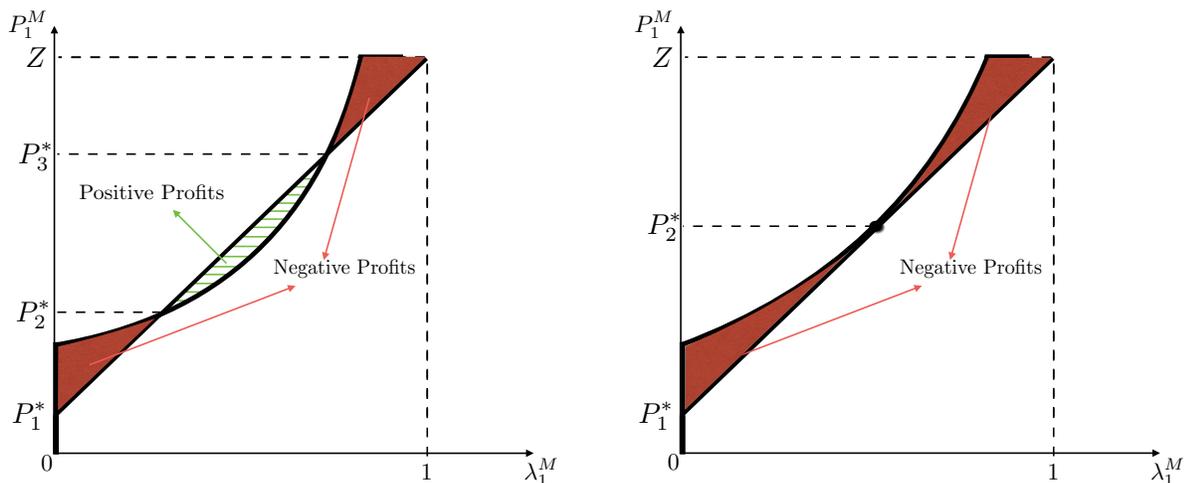


FIGURE 4: Market Equilibrium in period 1. (a) Unique Equilibrium. (b) Multiple Equilibria.

multiplicity can arise. Since that state has probability zero from the point of view of period 0, selecting the maximal volume of trade had no impact on agents choices in period 0. However, this logic doesn't hold when F has atoms.

Consider the case in which F is degenerate in some $\tilde{\alpha}$, so the economy does not face any exogenous aggregate risk (agents still face idiosyncratic liquidity shocks). As before, an equilibrium of the economy requires that $T(\lambda_1^E) = \lambda_1^E$, with the mapping T defined in (12). However, the mapping T can be discontinuous in λ_1^E . Let $\lambda_1^{E*} \equiv \sup\{\lambda_1^E \in [0, 1] : P_1^M(\lambda_1^E; \tilde{\alpha}) = \tilde{\alpha}Z\}$, that is, the threshold fraction of good trees in the economy such that if $\lambda_1^E < \lambda_1^{E*}$ the market in period 1 collapses. Note that λ_1^{E*} corresponds to figure 4(b), so that both prices can be part of an equilibrium. The key to finding an equilibrium in this economy is to determine what happens when $\lambda_1^E = \lambda_1^{E*}$. Since bad trees are inefficient, I already know that if the low price equilibrium is selected, $T(\lambda_1^{E*}) = 1$. If the high price is selected, then existence depends on whether $T(\lambda_1^{E*})$ is greater or smaller than λ_1^{E*} . If $T(\lambda_1^{E*}) \geq \lambda_1^{E*}$, the discontinuity in T does not prevent a fixed point from existing, so the equilibrium of the economy has the same properties as the economies with continuous F . This case is depicted in figure 5(a).

On the other hand, if $T(\lambda_1^{E*}) < \lambda_1^{E*}$, then a fixed point does not exist. In order to obtain existence of equilibrium in this case as well, I need to modify the definition of equilibrium. Motivated by the fact that the economy in the limit to perfect foresight featured positive endogenous risk, I define a Sunspot Equilibrium (SE) in which there is a random variable that selects a partial equilibrium in period 1. Note that when the fixed point of T exists (that is, cases like figure 5(a)), then the SE coincides with the previous equilibrium definition. But when the mapping T does not have a fixed point, the sunspot convexifies the mapping T so that it crosses the 45° line, as shown in figure 5(b). Moreover, the SE is unique.

When the sunspot is not trivial, the economy faces strictly positive endogenous aggregate risk even though the exogenous aggregate risk is zero. The reason for this result is the tension between the discontinuity of prices with respect to λ_1^E and the endogenous production decisions/*portfolio choices* of the agents, as in the limit above. When prices cannot align agents' incentives, risk helps,

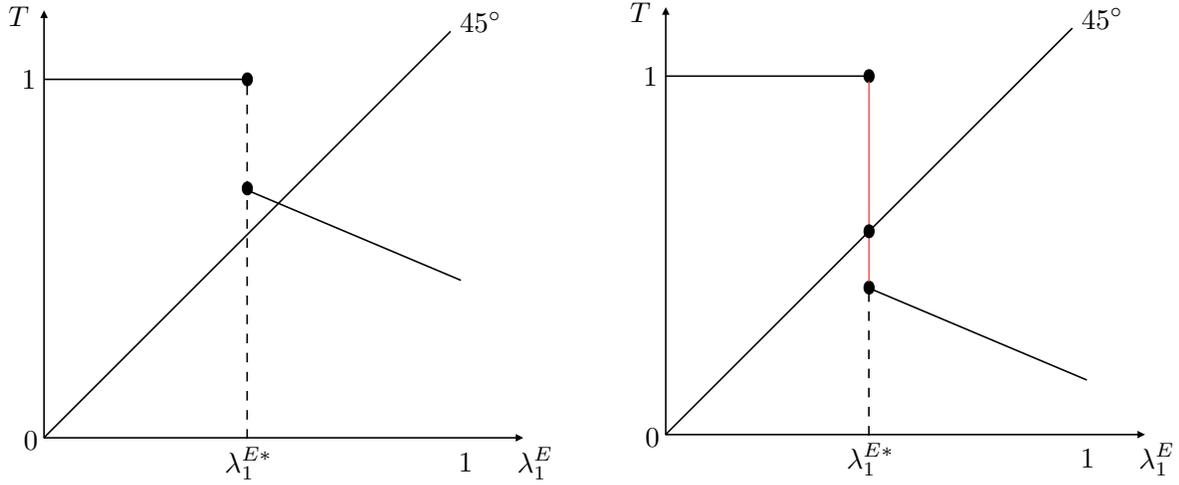


FIGURE 5: Equilibrium in period 0 when F is degenerate. (a) No Aggregate Risk. (b) Positive Aggregate Risk (Sunspot Equilibrium).

and that is what the sunspot is doing. In this sense, I view the financial markets as not just amplifying exogenous risk but as *creating* endogenous risk. Moreover, it turns out that

$$\mathbb{V}ar[P_1^M(\tilde{\alpha})] = \sigma^2(\tilde{\alpha}).$$

That is, the limit of the variance of an economy with exogenous risk vanishing away coincides with the variance introduced by the sunspot in a perfect foresight equilibrium.

The next proposition summarizes these results.

Proposition 8 (Fundamental Endogenous Financial Risk). *A Sunspot Equilibrium (SE) always exists and is unique. It coincides with the maximal volume of trade equilibrium whenever the latter exists. If it doesn't, the SE features strictly positive randomization. Moreover, the SE is the limit of the maximal volume of trade equilibrium with uniform exogenous aggregate risk and vanishing volatility, in the sense that*

$$\lim_{\epsilon \rightarrow 0} \mathbb{V}ar[P_1^M(\alpha|\epsilon)] = \sigma^2(\tilde{\alpha}) = \mathbb{V}ar[P_1^M(\tilde{\alpha})].$$

3 EXTENDED MODEL AND POSITIVE IMPLICATIONS

The analysis in the previous section shows that it is the dual role that trees play that exposes the economy to financial risk. On the one hand, they are a form of real investment, in the sense of being a technology that transforms units of goods in one period into units of goods in others. On the other hand, they facilitate transactions in period 1, so that resources can flow among agents even if the tree did not produce any dividend. In reality, the government is an important provider of instruments that perform the second role, through government bonds. There are both theoretical (see for example Woodford (1990), Holmström and Tirole (1998)) and empirical work (see for example Krishnamurthy and Vissing-Jorgensen (2012), Greenwood, Hanson and Stein (2015), Sunderam (2015), Krishnamurthy and Vissing-Jorgensen (2015)) that study the interaction between private and public liquidity. On the theoretical side they show that government bonds can be welfare enhancing

when the economy cannot produce enough financial instruments to optimally transfer resources among agents (for example, because markets are incomplete or there is limited pledgeability of future income). On the empirical side, they show that the production of private liquid instruments increases when the supply of government bonds decreases, which seems to be driven by changes in the liquidity premium.

In this section, I extend the basic model and incorporate decreasing returns to liquidity in order to obtain a more stable demand for liquid assets and be able to study the interaction between private and public liquidity in a meaningful way. To do so, I change the source of the liquidity risk that agents face. In particular, I now assume that instead of receiving a shock to preferences, agents are endowed with a technology that transforms final consumption good into physical capital (denoted by k), and the marginal rate of transformation is random and idiosyncratic. The agents' preferences are now given by

$$d_0 + E[d_1 + d_2].$$

Moreover, agents operate a linear technology that transforms final consumption good into capital at a rate A , where A is distributed independently across agents according to the cumulative distribution function G in $[0, A^{\max}]$.

In period 2, agents then rent the capital they own to a representative firm that operates the following technology

$$Y = Z^Y f(K),$$

where $f'(K) > 0$, $f''(K) < 0$, and it satisfies the Inada conditions, K is the amount of capital operated by the firm, and Z^Y is the TFP level. I assume that the market for renting capital is competitive, hence the rental rate is $r(K) = Z^Y f'(K)$. Moreover, I assume that the profits of the firm, $\Pi = f(K) - r(K)K$, are transferred to the agents uniformly in period 2. The state of the economy in period 1 is given by $X_1 = \{\lambda_1^E, H_1, \alpha\} \in \mathbb{X}_1$, and the state of the economy in period 2 is given by $X_2 = \{\lambda_1^E, H_1, K, \alpha\} \in \mathbb{X}_2$.

This extension allows me to study the interaction between private and public liquidity in a model that is only a small departure from the one in the previous section. The specific modeling choices rely on two main reasons. First, they incorporate decreasing returns to liquidity in a way that keeps the linearity of the agents' problem, so that cross-section distributions of agents' portfolios are not necessary to determine aggregate allocations.¹⁸ Second, they incorporate a different sector of the economy in a parsimonious way, and formally establishes the connection between financial markets and the "real economy". Better functioning markets imply a better allocation of resources and hence a higher efficiency of investment, but also higher productivity of the real economy implies a higher demand for liquidity and hence affects the quality production decisions of the agents.

Because of the intertemporal linkages between period 1 and 2 that capital introduces, the model requires a modification of the definition of equilibrium. It turns out that all it is needed is to allow

¹⁸A different approach would have been to incorporate decreasing marginal utility of consumption at the individual level in the basic model. However, this would have implied that the agents' problem is not linear, hence losing some tractability of the problem.

for a richer set of markets. Instead of forcing that all transactions take place in the same market, I allow for the existence of many markets that operate simultaneously.¹⁹ Each market ω is defined by a positive price $P_1^M(\omega) \in \mathbb{R}_+$. Without loss of generality, I assume that if $\omega' > \omega$ then $P_1^M(\omega') > P_1^M(\omega)$. The set of all markets is denoted by Ω . As in the previous section, only sellers know the quality of the asset they hold. Buyers do not observe the quality of a tree being offered, and they can only form some expectation about the quality distribution in each market. Moreover, markets need not clear. A fraction of the trees offered in a specific market may remain unsold.

Importantly, I keep the assumption that markets are non-exclusive. Sellers can offer the same unit of tree for sale in any subset of markets simultaneously. They are only restricted not to sell more trees than they own. From the seller's point of view, markets are characterized both by their prices, $P_1(\omega)$, as well as an amount of rationing, $\eta(\omega)$. The amount of rationing specifies the fraction of supplied trees a seller will be able to sell in market ω . I assume that trees are perfectly divisible, so $\eta(\omega)$ is the fraction of trees the seller actually sells rather than being the probability of selling an indivisible unit. The amount of rationing η is an equilibrium object that results from the equilibrium supply and demand decisions of the agents in each market and state of the economy. Finally, let Ω^B be the set of markets with positive supply.

Next, I state the agents' problem for this economy. I show that the main features of the equilibrium are isomorphic to the basic economy of the previous section, so the main insights still hold. The main difference is that the marginal utility of liquid wealth is now decreasing in total liquidity, creating a two-way feedback effect between the financial markets and the real economy. In the next section, I use this result to study the effect of the supply of public liquidity on the incentives to produce tree quality.

3.1 Agents' Problem

The problem agents' face in period 2 now is

$$V_2(h_G, h_B, k; X_2) = Zh_G + \alpha Zh_B + r(X_2)k + \Pi(X_2). \quad (\text{P2}')$$

The only difference between (P2') and (P2) is that in (P2'), besides the dividend from the trees, agents receive the rental rate $r(X_2)$ for their holdings of capital k , and the profits of the representative firm, $\Pi(X_2)$.

The problem that agents face in period 1 is slightly more complicated. The program they solve has to accommodate for the new investment opportunities and the availability of many markets. Therefore, the agents solve the following program:

$$V(h_G, h_B; A, X_1) = \max_{\substack{d, i_K, m, s_G, \\ s_B, h'_G, h'_B, k'}} d + V_2(h'_G, h'_B, k'; X_2), \quad (\text{P1}')$$

¹⁹See, for example, Guerrieri and Shimer (2014a), Guerrieri and Shimer (2014b), and Kurlat (2016) for models with adverse selection and many markets.

subject to

$$d + i_K + \sum_{\omega \in \Omega^B} P_1^M(\omega) m(\omega) \leq W_1 + \sum_{\omega \in \Omega} P_1^M(\omega) (s_G(\omega) + s_B(\omega)) \eta(\omega; X_1), \quad (14)$$

$$h'_G = h_G + \sum_{\omega \in \Omega^B} \lambda_1^M(\omega; X_1) m(\omega) - \sum_{\omega \in \Omega} s_G(\omega) \eta(\omega; X_1), \quad (15)$$

$$h'_B = h_B + \sum_{\omega \in \Omega^B} (1 - \lambda_1^M(\omega; X_1)) m(\omega) - \sum_{\omega \in \Omega} s_B(\omega) \eta(\omega; X_1), \quad (16)$$

$$k' = A i_K, \quad (17)$$

$$\sum_{\omega \in \Omega} s_G(\omega) \eta(\omega; X_1) \leq h_G, \quad \text{and} \quad \sum_{\omega \in \Omega} s_B(\omega) \eta(\omega; X_1) \leq h_B, \quad (18)$$

$$d \geq 0, \quad i_K \geq 0,$$

$$m(\omega) \geq 0, \quad s_G(\omega) \in [0, h_G], \quad s_B(\omega) \in [0, h_B], \quad \forall \omega \in \Omega.$$

Constraint (14) is the agent's budget constraint, that states that consumption plus investment in capital and purchases in all markets cannot be larger than the endowment W_1 plus the sale of trees in different markets. Constraints (15) and (16) are the laws of motion of good and bad trees respectively, while constraint (17) is the law of motion of capital. Finally, constraint (18) establishes that agents cannot sell more trees than they hold. Note that the measure used is $\eta(m; X_1)$, implying that the restriction is over the actual sales, not on the number of trees the agents send to the market. This is the non-exclusivity assumption.

Following Kurlat (2016), I focus on solutions to this problem that are robust to small perturbations of η , in order to rule out self-fulfilling equilibria in which sellers do not supply in certain markets because there are no buyers, and buyers do not demand in some markets because there are no sellers, even though a small amount of trade would trigger a response from them. See Appendix B for the details.

It is useful to define $\tilde{\omega}(X)$ as the market with the lowest price such that if an agent sends his trees to all markets with prices at least as high, they would be able to sell all their holdings in equilibrium. Formally,

$$\tilde{\omega}(X) \equiv \max \left\{ \omega' \in \Omega : \sum_{\omega \geq \omega'} \eta(\omega; X) \geq 1 \right\}. \quad (19)$$

The interpretation of $\tilde{\omega}(X)$ is that it is the market with the lowest price that can have active trading, given the rationing in the other markets.

The solution to (P1') is the analogous of Lemma 1 in the previous section.

Lemma 4 (Agents' Choice). *Consider an agent with investment opportunity A . There exists thresholds A_1^B and $A_1^S(\omega)$ (with $A_1^S(\omega)$ decreasing in ω) that may depend on the state of the economy, X_1 , such that*

- if $A \leq A_1^B$, then the agent does not produce capital ($i_K = 0$), consumes or buys trees in some markets ($d \geq 0, m(\omega) \geq 0$) and does not sell his good trees ($s_G(\omega) = 0$ for all $\omega \in \Omega$);
- if $A > A_1^B$, then the agent produces capital ($i_K > 0$), does not consume ($d = 0$), does not buy trees

($m(\omega) = 0$ for all $\omega \in \Omega$), and might sell his good trees in some markets ($s_G(\omega) = h_G$ for all $\omega \in \Omega$ such that $A > A_1^S(\omega)$ and $s_G(\omega) = 0$ for all $\omega \in \Omega$ such that $A < A_1^S(\omega)$).

All agents always sell their holding of bad trees in the markets with the highest prices, i.e. $s_B(\omega) = h_B$ for all $\omega > \tilde{\omega}$ and $s_B(\omega) = 0$ for all $\omega < \tilde{\omega}$.

The decisions of the agents are very similar to the ones in the basic model. First, they compare their productivity A with the return from buying trees in the market and the utility from consumption (which is equal to 1). Since all agents face the same alternatives to investing, there is a threshold A_1^B common to all agents such that only those with productivity higher than A_1^B produce capital. Those with productivity below A_1^B use their liquid wealth to buy trees in the market and consume, whichever provides the highest utility. Moreover, while all agents sell all of their bad trees, only agents with high enough productivity sell their good trees. Since there are many markets, they choose in which market to sell given their productivity. The thresholds $\{A_1^S(\omega)\}_{\omega \in \Omega}$ determine exactly that.

In the previous section, the marginal utility of liquid wealth coincided with the liquidity shocks μ_1 . Now, it is given by the Lagrange multiplier associated to the agents' budget constraint

$$\mu_1(A, X_1) = \max \left\{ 1, Ar(X_2), \left\{ \frac{\lambda_1^M(\omega; X_1)Z + (1 - \lambda_1^M(\omega; X_1))\alpha Z}{P_1(\omega)} \right\}_{\omega \in \Omega^B} \right\}.$$

It represents the utility derived from the use of resources that provides the maximum return in the margin. Agents have three possible uses. They can consume and obtain 1 unit of utility; they can produce capital at a rate A and obtain a payoff $Ar(X_2)$ in period 2 per unit invested; or they can buy trees in some markets and obtain a return of $\frac{\lambda_1^M(\omega; X_1)Z + (1 - \lambda_1^M(\omega; X_1))\alpha Z}{P_1(\omega)}$.

Even though the model is richer, the mapping between the liquidity shocks of the previous section and the liquidity services of this section is very direct. Let $\mu_1^B(X_1)$ be the return from buying trees in the secondary market.²⁰ Agents with low A have a marginal utility of liquid wealth of $\mu_1(A, X_1) = \max\{1, \mu_1^B(X_1)\}$. Following the same logic as in the previous section, if W_1 is high enough, then there are enough agents willing to buy trees rather than invest in capital, so that $\mu_1^B(X_1) = 1$. Thus, liquidity services simplify to

$$\mu_1(A, X_1) = \max\{1, Ar(X_1)\}. \quad (20)$$

Therefore, there is a mass of agents with low enough A such that $\mu_1(A, X) = 1$, analogous to the mass π of agents with $\mu_1 = 1$ in the previous section. Note that this case implies that aggregate consumption is positive in period 1, hence restricting parameter values so that there is positive aggregate consumption every period leads to the same result. Moreover, as A increases, $\mu_1(A, X_1)$ increases, and the cross-section distribution of $\mu_1(A, X_1)$ is ultimately governed by the distribution of A , and the value of $r(X_2)$.

²⁰That is

$$\mu_1^B(X_1) \equiv \max_{\omega \in \Omega^B} \frac{\lambda_1^M(\omega; X_1)Z + (1 - \lambda_1^M(\omega; X_1))\alpha Z}{P_1(\omega)}.$$

In order to maintain the assumption that bad trees are an inefficient investment, I assume that the amount of investment in period 1 that would prevail in an economy with no markets for trees would be such that the liquidity services are not too large. Let $\tilde{\mu}_1(A, X_1)$ denote the liquidity services that would prevail in such economy.

Assumption 4. *The payoff of bad trees is such that*

$$E[\tilde{\mu}_1(A, X_1)\alpha Z] < 1.$$

This assumption holds if W_1 is large enough.

Finally, the program the agents solve in period 0 does not change except that V_1 is now given by (P1'). Next, I briefly describe the determination of partial equilibria in the markets for trees, define the equilibrium of the full economy and characterize it.

3.2 Equilibrium

Most of the analysis in the previous section follows through after these modifications. The main difference is that more than one market may be active in equilibrium. It turns out that under my assumptions, at most two markets can be active: a high price market in which good and bad trees are traded, and a low price market in which only bad trees are traded. The low price market can have positive volume of trade only if there is rationing in the high price market (market collapse is an extreme case in which there is 100% rationing in the high price market). Moreover, the structure of active markets follows very closely the discussion on the multiplicity of partial equilibria of the previous section when the economy was forced to have only one active market, as depicted in figure 4.

Consider once again the demand and supply of trees studied in the previous section, modified to the specifics of the economy in this section. The demand in each market is given by

$$\lambda_1^M(P_1) = \frac{P_1 - \alpha Z}{(1 - \alpha)Z}, \quad (21)$$

while the supply is given by

$$\lambda_1^M(P_1) = \frac{\left[1 - G\left(\frac{Z}{r(K)P_1}\right)\right] \lambda_1^E}{\left[1 - G\left(\frac{Z}{r(K)P_1}\right)\right] \lambda_1^E + (1 - \lambda_1^E)}. \quad (22)$$

Given λ_1^E and K , the partial equilibrium of the markets for trees can take 3 different forms:

1. if one of the intersections between (21) and (22) happens at a point in which $\lambda_1^M > 0$ and the game-theoretic foundation developed in the previous section selects the maximal volume of trade partial equilibrium, then the economy with many potential markets has only one active market in equilibrium which corresponds to the maximal volume of trade equilibrium and there is no rationing in the market;
2. if there is only one intersection between (21) and (22) and happens at $\lambda_1^M = 0$, then there is

also only one active market in equilibrium, which corresponds to a market collapse;

3. if one of the intersections between (21) and (22) happens at a point in which $\lambda_1^M > 0$ and the game-theoretic foundation developed in the previous section *does not* select the maximal volume of trade partial equilibrium, then there can be *two* active markets in equilibrium, which correspond to the two intersections of (21) and (22), as in figure 4(b). Sellers of bad trees send their trees to both markets. They sell all they can in the high price market and then sell the rest in the low price market. Sellers of good trees only send their trees to the high price market. If there is rationing, they keep the units they were not able to sell.

See Appendix B for details.

Therefore, the equilibrium is pooling when there is only one active market, and semi-separating when there are more than one active markets. In both cases, there is some degree of cross-subsidization among types of trees. Since the low price market is active only when there is rationing in the high price market, and all agents try to sell their trees in the high price market before trying to sell in the low price market, I denote by $\eta(X_1)$ the rationing in the high price market and $1 - \eta(X_1)$ the fraction of trees sold in the low price market. Note that $\eta(X_1)$ indexes all the possibilities described above. Moreover, I denote by $P_1^M(\omega_H; X_1)$ and $P_1^M(\omega_L; X_1)$ the high price and the low price, respectively.

Let's define an equilibrium for this economy.

Definition 5 (Equilibrium). *An equilibrium in this economy consists of prices $\{P_1^M(\omega_H; X_1), P_1^M(\omega_L; X_1), r(X_2)\}$; fraction of good trees in the market $\omega_H \{\lambda_1^M(\omega_H; X_1)\}$; a rationing function $\eta(X_1)$; decision rules $\{d_0(\xi), d_1(h_G, h_B; A, X_1), d_2(h_G, h_B, k; X_2)\}$, $\{i_G(\xi), i_B(\xi), i_K(h_G, h_B; A, X_1)\}$, $\{h'_G(h_G, h_B; A, X_1), h'_B(h_G, h_B; A, X_1)\}$, $\{m(h_G, h_B; \omega_H, A, X_1), m(h_G, h_B; \omega_L, A, X_1), s_G(h_G, h_B; A, X_1), s_B(h_G, h_B; A, X_1)\}$; a fraction of good trees in the economy, λ_1^E , a total amount of trees H_1 , and aggregate capital $\{K(X_1)\}$, such that*

1. $\{d_0(\xi), d_1(h_G, h_B; A, X_1), d_2(h_G, h_B, k; X_2)\}$, $\{i_G(\xi), i_B(\xi), i_K(h_G, h_B; A, X_1)\}$, $\{h'_G(h_G, h_B; A, X_1), h'_B(h_G, h_B; A, X_1)\}$, $\{m(h_G, h_B; \omega_H, A, X_1), m(h_G, h_B; \omega_L, A, X_1), s_G(h_G, h_B; A, X_1), s_B(h_G, h_B; A, X_1)\}$ solve the agents' problems (P0), (P1') and (P2'), taking $\{P_1^M(\omega_H; X_1), P_1^M(\omega_L; X_1)\}$, $\{\lambda_1^M(\omega_H; X_1)\}$, $\eta(X_1)$, λ_1^E , H_1 , and $\{K(X_1)\}$ as given;
2. $\{P_1^M(\omega_H; X_1), P_1^M(\omega_L; X_1)\}$, $\{\lambda_1^M(\omega_H; X_1)\}$ and $\eta(X_1)$ are the partial equilibrium of the markets for trees state by state;
3. the rental rate $r(X_2)$ equals the marginal product of capital, $r(X_2) = f'(K(X_1))$;
4. λ_1^E , H_1 and $\{K(X_1)\}$ are consistent with individual decisions.

It is important to note that the change in the definition of equilibrium does not imply a fundamental change in the functioning of the economy. In particular, if I used this definition of equilibrium in the previous section, all the results when the distribution F is continuous would hold. This is reassuring in the sense that the main forces of the economy do not change by allowing for a richer set of markets.

Finding an equilibrium involves similar steps than in the previous section. In particular, shadow prices are defined following the same logic. There are two differences. First, the investment in physical capital in period 1 connects the outcomes of period 1 and period 2, so finding an equilibrium

of the economy starting in period 1 is a little more involved than before. Second, the economy does not scale linearly in H_1 , so the fixed point I will need to solve is two dimensional in λ_1^E and H_1 .

I solve for the equilibrium by backward induction. First, I find an equilibrium of the economy starting in period 1. Then, I move to period 0 and solve for the equilibrium of the full economy.

Define aggregate investment in physical capital as

$$I_1^K(X_1) = \int_0^{A^{\max}} Ai_K(h_G, h_B; A, X_1) d\Gamma_1(h_G, h_B, A),$$

where $\Gamma_1(h_G, h_B, A)$ is the cross section distribution of portfolio holdings and investment opportunities. Then

$$I_1^K(X_1) = \int_{A_1^B(X_1)}^{A^{\max}} A[W_1 + [\eta(X_1)P_1^M(\omega_H; X_1) + (1 - \eta(X_1))P_1^M(\omega_L; X_1)]H_1^B]dG(A) + \int_{A_1^S(X_1)}^{A^{\max}} A\eta(X_1)P_1^M(\omega_H; X_1)H_1^G dG(A). \quad (23)$$

Since the return on capital that the agents get depends on the aggregate capital of the economy, $A_1^B(X_1)$ and $A_1^S(X_1)$ depend on K , which in turn affect the market prices and rationing functions. Hence, it is useful to define the mapping $T_K(K; X_1) = I_1^K(K; X_1)$. An equilibrium of the economy in period 1 requires that $T_K(K; X_1) = K$. If I didn't allow for multiple markets and rationing, the mapping T_K could be discontinuous in K . Hence, the extension in the market for trees guarantees that there is a fixed point for any value of X_1 .

Let's turn to period 0. In the previous section, finding an equilibrium involved finding a fixed point of a mapping that depended on λ_1^E , but not on H_1 . The reason for this was that the economy starting in period 1 was linear in H_1 since there were constant returns to liquidity. Now, because f has decreasing returns in capital (i.e., r is decreasing in K), this is not true anymore. Therefore, I define a vector mapping $T(\lambda_1^E, H_1)$ given by

$$T(\lambda_1^E, H_1) = \left[\begin{array}{c} \frac{I_0^G(\lambda_1^E, H_1)}{I_0^G(\lambda_1^E, H_1) + I_0^B(\lambda_1^E, H_1)} \\ I_0^G(\lambda_1^E, H_1) + I_0^B(\lambda_1^E, H_1) \end{array} \right] \quad (24)$$

An equilibrium requires that

$$T(\lambda_1^E, H_1) = \left[\begin{array}{c} \lambda_1^E \\ H_1 \end{array} \right]$$

The next proposition establishes existence of the equilibrium of the full economy.

Proposition 9. *An equilibrium of the economy always exists.*

While the equilibrium may not be unique, I will focus on the equilibrium with the highest fraction of good trees. This equilibrium is stable.

Next, I use the model to characterize the interaction between the financial markets and the real economy. First, better functioning markets increase the flow of resources to those with the best investment opportunities, hence aggregate capital in the economy increases. Second, higher

productivity in the real economy, both through higher TFP of the representative firm, Z^Y , and investment opportunities, A , increase market prices and hence *worsens* the tree quality production in period 0.

Interaction Between Financial Markets and Real Investment

Real investment and the financial markets relate to each other through two channels. First, if there is more liquidity in the market then agents with good investment opportunities can invest more and aggregate capital in the economy goes up. Second, if TFP of the firm in period 2 goes up, investment opportunities are more profitable so more agents sell their good trees, which improves liquidity of the market. For future reference, define investment efficiency as

$$\frac{K(X_1)}{\int_0^{A^{\max}} i_K(h_G, h_B; A, X_1) d\Gamma_1(h_G, h_B, A)}$$

where the denominator is the total amount of resources used in the production of K .

Consider first how the functioning of the secondary markets affects the real economy.

Lemma 5 (Contagion). *Aggregate capital and investment efficiency are increasing in α and in H_1^G .*

Even though the two sectors of the economy are not directly related, the efficiency of the economy's investment depends on how well the secondary markets function. If the liquidity in the market is high, agents with good investment opportunities will be able to invest their endowment and obtain funds from the market. This implies that the total amount of capital they produce is relatively high, which crowds out low productivity agents. Hence, the efficiency of investment increases. Interestingly, if the crowding-out effect is strong enough, the aggregate consumption in period 1 may also increase. This is because the low productivity agents that switch from investing to not investing will now consume and buy trees. Hence, if the flow of new consumers is larger than the increased expenditures due to the price increase, total consumption in the economy goes up.

Next, I study the interaction between the real sector and the incentives to produce asset qualities. In particular, I consider the effects of an increase in the TFP level of the representative firm, Z^Y , and an increase in agents' investment opportunities, from A to ϕA , for some $\phi > 1$.

Lemma 6 (Shocks to the Real Economy). *An increase in the TFP level, Z^Y , or in the investment opportunities, from A to ϕA , increases $\mu_1(A, X_1)$ for every state (A, X_1) . As a consequence, the production of trees, H_1 , increases, and the fraction of good trees in the economy, λ_1^E , decreases.*

This lemma is an extension of Proposition 5 in the previous section. A higher demand for intermediation driven by a stronger real sector increases the liquidity premium and the incentives to produce low quality assets. Conditional on λ_1^E and H_1 , P_1^M and λ_1^M are increasing in Z^Y and ϕ for every realization of α . Hence, λ_1^E decreases in equilibrium. This result can be quantitatively important to understand the build-up to the crisis. Bigio (2015) finds that, in the years previous to the crisis, the measured TFP for the US economy was above trend, and the crisis is triggered by a substantial drop in TFP *followed* by an increase in the adverse selection problem in financial

markets. An interpretation of the data through the lens of my model is that the abnormally high TFP worsened the asset quality distribution in the years previous to the crisis, which was latent while TFP remained high but generated a collapse in financial markets when TFP declined.

4 NORMATIVE IMPLICATIONS

In this section, I use the model to analyze how the economy interacts with government intervention. First, I study how the economy reacts to changes in the public supply of liquidity, with particular interest in how it affects the incentives to produce tree quality. Then, I analyze the role that transaction taxes and purchase programs play in shaping incentives and improve liquidity.

4.1 Government Bonds

Government bonds contribute to the total amount of liquidity in the economy. Intuitively, a higher volume of bonds allows for a greater volume of transactions, which increases investment. However, because of the decreasing returns to capital, this in turn reduces the marginal return to liquidity, reducing the liquidity premium on *all* tradable assets. Therefore, the incentives to produce trees decreases. Because bad trees are more sensitive to changes in the value of liquidity services, an increase in the supply of government bonds reduces the shadow value of bad trees disproportionately more than that of good trees, so that the fraction of good trees in the economy increases. However, government bonds can also have negative effects. For a given fraction of good trees in the economy, a larger supply of government bonds *increases* the adverse selection problem in the market. The reason is that government bonds crowd out private markets. Since bad trees are always sold, it is some of the good trees that leave the market, increasing the adverse selection wedge. Which effect dominates depends on the relative strength of each channel.

Consider the following timing. As before, agents start with an endowment W_0 of final goods. Agents receive a type ξ and decide whether to produce trees or not. But now, agents have a different alternative to consumption. They can buy government bonds at price P_0^{GB} . Government bonds pay one unit of final good in period 2. I still want to focus on economies that have positive consumption in all periods and states, so I assume that the supply of government bonds, B_0 , is not too large compared to W_0 and W_1 . In that case,

$$P_0^{GB} = \gamma_0^{GB} = E[\mu_1(A, X_1)].$$

That is, the price is equal to the liquidity services the bonds provide in period 1. Note that I am already imposing that the market price in period 1 is equal to one. The reason for this is that as long as aggregate consumption is positive, the return of bonds between periods 1 and 2 has to be equal to the intertemporal marginal rate of substitution of the buyers (who are the non-investors), which is equal to one.

For simplicity, I assume that the government rebates the proceeds of selling bonds in period 0 to the agents lump-sum, and then taxes agents lump-sum in period 2. In order to keep the mechanics of the model as close as possible to the previous sections, I assume that the government's transfers

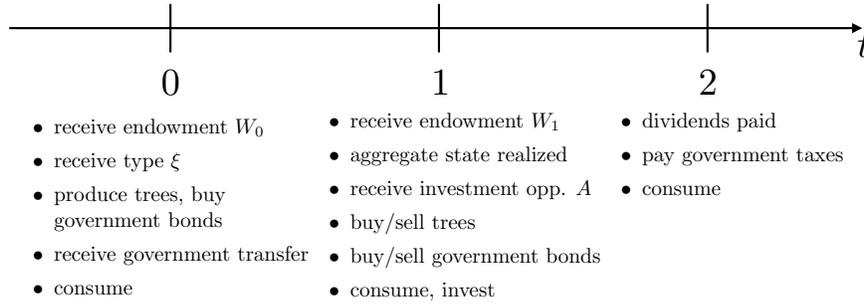


FIGURE 6: Timing with government bonds.

in period 0 occur after investment takes place, so that they cannot be used for investment. I make this assumption to isolate the market incompleteness in period 0 from the market incompleteness in period 1. Allowing the alternative would not change the main message, but incorporate a distributive role of government bonds that is unlikely to be relevant in reality.²¹ Figure 6 summarizes the new timing.

The quantity of government bonds affects the liquidity services and hence the *risk free* interest rate of the economy, which is given by

$$i_0 \equiv \frac{1}{E[\mu_1(A, X_1)]} - 1.$$

The first result shows that incomplete reallocation pushes interest rates down.

Lemma 7 (Laissez-faire Interest Rates). *Consider an equilibrium with positive consumption in every period and state. The interest rate in the laissez-faire equilibrium is lower than in first best.*

In first best, $\mu(A, X_1) = 1$ in all states if aggregate consumption is positive, since there is no limitation to the reallocation of resources among agents. As long as there is incomplete reallocation, $\mu_1(A, X_1) > 1$ for some A , hence the interest rate is lower.

Government bonds affect the economy through the quantity of liquid instruments, which in turn affects the liquidity premium of assets. This has three effects: the direct effect is to increase the flow of resources in the economy, since more government bonds implies more instruments to trade for goods, that is, more liquidity; second, it reduces the incentives to sell good trees, reducing both the quantity of assets traded as well as their price in the secondary market, which increases the adverse selection wedge for a given λ_1^E ; last, in period 0, anticipating the effect government bonds have in period 1, it reduces the incentives to produce bad trees, hence increasing the equilibrium fraction of good trees in the economy. Next, I formally study these effects.

Consider the economy in period 1. Suppose that agents hold a total of $B_0 > 0$ of government bonds, distributed uniformly among all agents.²² These bonds pay one unit of consumption good in period 2. How does the equilibrium in period 1 change with an increase in B_0 ? Assuming that B_0 is not too large, the price of government bonds between period 1 and 2 is equal to one. Keeping

²¹I could alternatively assume there is a different set of agents with linear preferences and no liquidity needs that receive the transfers. The result would be the same. This is the assumption in Holmström and Tirole (1998).

²²Because of the linearity of the value function and the iid assumption on investment opportunities, this is without loss of generality.

everything else fixed, an increase in B_0 increases investment:

$$I_1^K(X_1) = \int_{A_1^B(X_1)}^{A^{\max}} A[W_1 + [\eta(X_1)P_1^M(\omega_H; X_1) + (1 - \eta(X_1))P_1^M(\omega_L; X_1)]H_1^B + B_0]dG(A) + \int_{A_1^S(X_1)}^{A^{\max}} A\eta(X_1)P_1^M(\omega_H; X_1)H_1^G dG(A).$$

However, as K increases, $r(K)$ decreases. This has two separate effects. On the one hand, $A_1^B(X_1)$ increases. As the return on capital decreases, investment becomes less attractive, so the agents that have the marginal productivity, $A_1^B(X_1)$, decide not to invest under the new conditions. That is, the presence of government bonds improves the flow of resources so that high productivity agents are able to invest more, while low productivity agents choose not to invest. Hence, a higher supply of government bonds increases investment efficiency.

On the other hand, $A_1^S(X_1)$ also increases. Since the return of investing in capital decreases, less agents are willing to sell their good trees to produce capital. Note that this can have perverse effects in the secondary market for trees. While the demand for trees is not affected, the supply of trees decreases. Hence, the market price or the rationing η decrease.

It is useful to define the total amount of liquidity in the economy. Total liquidity is the value of all the assets available for trade. In this economy it is given by

$$TL(X_1) \equiv \underbrace{B_0}_{\text{public liquidity}} + \underbrace{\eta(X_1)P_1^M(\omega_H; X_1) \left[[1 - G(A_1^S(X_1))]H_1^G + H_1^B \right] + (1 - \eta(X_1))P_1^M(\omega_L; X_1)H_1^B}_{\text{private liquidity}}.$$

Hence, a higher volume of government bonds in period 1 increases the investment in the economy and its efficiency, but it partially crowds out the market for trees, increasing the adverse selection wedge for a fixed λ_1^E . The next proposition summarizes these results.

Proposition 10. *Consider an economy in period 1 with some fraction of good trees, λ_1^E , and total amount of trees, H_1 . Suppose the total amount of government bonds in the hands of agents increases. Then*

1. *the total amount of liquidity in the economy increases;*
2. *aggregate capital and investment efficiency increase;*
3. *the volume traded in the market for trees decreases;*
4. *liquidity services, $\mu_1(A, X_1)$, decreases for every state (A, X_1) .*

Now, let's switch to period 0. The government sells government bonds to agents. Agents anticipate that more public liquidity in period 1 reduces the liquidity premium and hence the shadow value of trees in period 0. This has a bigger impact on the shadow value of bad trees, so production of bad trees, I_0^B , decreases, and the fraction of trees in the economy, λ_1^E , increases. Moreover, because the liquidity premium decreases, the risk-free interest rate of the economy increases. The next proposition summarizes the results.

Proposition 11. *An increase in the supply of government bonds reduces the production of bad trees, I_0^B , and increases the fraction of good trees in the economy, λ_1^E . The equilibrium interest rate in period 0 increases.*

The overall effect of B_0 on market fragility is ambiguous. On the one hand, the incentives to sell good trees decreases, but on the other hand the fraction of good trees in the economy increases. If the asset quality composition of the economy is sufficiently inelastic (exogenously given quality distribution is an extreme case), then a higher supply of government bonds increases fragility. However, below I describe an example that shows the forces at play, and why a reduction in market fragility is a plausible outcome.

But first consider an extension of the model in which an external agent demands domestic government bonds. Even though the model is of a closed economy, one could easily extend it to incorporate international financial transactions. Suppose there is a foreign agent that buys government bonds. This reduces the local supply of government bonds (while increasing current consumption). The effect over the production of trees is analogous to a reduction in the supply of government bonds.

Corollary 11.1. *If a foreign agent buys government bonds, λ_1^E falls. The risk free interest rate also falls.*

This result also connects to stories of safe asset shortages due to the world's savings glut in the early 2000s, which put excessive pressure on the US financial sector to produce safe assets.²³

Next I consider a particular technology for the representative firm that shows that government bonds can reduce financial fragility.

Example: Government Bonds Reduce Financial Fragility

Suppose the production function in period 2 is given by

$$f(K) = Z^Y \max\{K, K^*\}, \quad (25)$$

where K^* is a technological parameter. This production function has the special feature of being linear in the region $[0, K^*)$, and the marginal product of capital drops to zero when $K \geq K^*$. In order for the concave part of the production function to affect the economy, I assume that K^* is such that if α is high, $K(\alpha) = K^*$. That is, I assume that when the liquidity in the market is high, the economy achieves the first best quantity of capital (note that this does not imply that the allocation is first best for two reasons: first, bad trees were produced, which is socially inefficient; second, the composition of the investment in capital is also inefficient, since some investment is undertaken by low productivity agents). If α is low, then $K(\alpha) < K^*$. Moreover, K^* is high enough so that the threshold α^* is such that $K(\alpha^*) < K^*$.

Consider an increase in B_0 . In low α states, K increases but there is no impact on $r(K)$, hence the market is not affected. In high α states, government bonds partially crowd out the private market, in this extreme case by increasing rationing. Hence, the direct effect of the increase in the supply of public liquidity is a reduction in the shadow value of trees, with the shadow value of

²³See, for instance, Caballero (2006).

bad trees decreasing more than that of good trees. Therefore, the production of bad trees decreases and the fraction of good trees in the economy, λ_1^E , increases. Since the state α^* features $K < K^*$, the overall effect of an increase in government bonds is a drop in the probability of a market collapse, that is, market fragility decreases when the supply of public liquidity increases. The reason why market fragility unambiguously decreases here is that, with this production function, government bonds *crowd out* private liquidity in high liquidity states, but *complement* private liquidity in low liquidity states. While this sounds like a reasonable result, it does not immediately hold for more general production functions. In those cases, the overall effect also depends on the elasticity of production of tree quality. This will be particularly interesting in the infinite horizon version of the model, where the elasticity of the fraction of good trees in the economy does not only depend on the elasticity of production but also on the stock and composition of trees in the economy from previous periods' production.

Optimal Policy

It is well known that allocations in economies with markets that suffer from adverse selection are usually interim-constrained Pareto Optimal, since it is not possible to improve efficiency in the economy without lowering the well-being of those benefiting from the asymmetric information.²⁴ However, here I am interested in aggregate allocations rather than distributional concerns. Therefore, in order to study optimal policy, I assume that the planner maximizes a utilitarian welfare function that puts equal weight to all agents. Hence, the planner maximizes:

$$W = D_0 + E_0[D_1(X_1) + D_2(X_2)], \quad (26)$$

subject to the equilibrium conditions

$$\begin{aligned} \frac{\gamma_0^G}{q_G(\xi_G)} &= \frac{\gamma_0^B}{q_B(\xi_G)}, \\ \frac{\gamma_0^B}{q_B(\xi_B)} &= \frac{\gamma_0^{GB}}{P_0^{GB}} = 1, \end{aligned}$$

where D_0 , D_1 and D_2 are the aggregate consumption functions. This program is isomorphic to one in which the planner maximizes the expected utility of the representative agent *before* its type ξ is realized in period 0.

From the previous analysis one could conclude that the optimal policy should involve issuing enough government bonds so as to completely crowd out the private market. That is, the government could use its taxing power to become the monopolist producer of liquid instruments in the economy. This is an appealing solution since it separates the liquidity value of assets from their dividend value, so that assets are produced only for fundamental reasons. This logic resembles the Friedman Rule for monetary policy, that is, the government should completely satiate the liquidity needs of the agents.

However, there are at least two problems with this solution. First, the amount of bonds needed

²⁴See Bigelow (1990).

can be very large, so that the fiscal cost of the intervention can be unbounded. In order for the liquidity premium to be equal to zero, the agents with $A = A^{\max}$ need to hold enough liquidity to invest the optimal amount in period 1. Since investment opportunities are random, all agents that have a chance of getting the best investment opportunity need to be holding enough government bonds in advance. Moreover, the smaller the set of agents that can get the best shock, the larger the reallocation that is needed in period 1, so the larger the supply of bonds needed. In the limit in which the measure of agents with $A = A^{\max}$ is zero, the amount of bonds the government has to issue in period 1 is infinite.

Second, even if the fiscal cost was zero, the dynamics of the economy might cause the bonds to end up in the wrong hands. Suppose the government is willing and able to issue all the bonds needed to completely satiate agents' liquidity needs in period 1. The problem is that some agents will prefer to invest (for fundamental reasons) instead of buying government bonds. And it is exactly because of this that *securitization* has a valuable social role. It allows investors to mitigate the trade-off they face between undertaking investment opportunities and keeping enough liquidity available to satisfy future needs. Hence, even if it wanted, it is unlikely the planner can satisfy the full demand for liquidity with government bonds.

Given this discussion, I will continue my analysis under the assumption that if the government issues bonds in period 0, in period 2 it has to pay a cost q_{GB} per unit of bond issued (the shadow cost of taxation). This is a similar strategy to the one adopted by Holmström and Tirole (1998) and Tirole (2012). The benefit of this assumption, instead of using the model to determine the costs of taxation, is that the model was not built to take a stand on the cheapest way of collecting revenue. However, the exercise is still insightful to understand how optimal policy should look like. With this positive cost, the government will choose to complement the market rather than fully substitute it.

In an interior solution, it must be that²⁵

$$\gamma_0^{GB} + E \left[\frac{\partial \tilde{\gamma}_1^G(X_1)}{\partial P_1^M(\omega_H; X_1)} \frac{\partial P_1^M(\omega_H; X_1)}{\partial B_0} + \frac{\partial \tilde{\gamma}_1^G(X_1)}{\partial \eta(X_1)} \frac{\partial \eta(X_1)}{\partial B_0} \right] H_G + E \left[\frac{\partial \tilde{\gamma}_1^B(X_1)}{\partial P_1^M(\omega_H; X_1)} \frac{\partial P_1^M(\omega_H; X_1)}{\partial B_0} + \frac{\partial \tilde{\gamma}_1^B(X_1)}{\partial \eta(X_1)} \frac{\partial \eta(X_1)}{\partial B_0} \right] H_B = 1 + q_{GB}. \quad (27)$$

The LHS is the sum of the liquidity value of an extra unit of government bond, γ_0^{GB} , and the change in the value of private liquidity for a fixed level of K . The first term is analogous to the force that justifies government intervention in Woodford (1990) and Holmström and Tirole (1998). This effect would still be there even if there was perfect information in private markets.

But government bonds also affect the functioning of private markets. The change in the value of private liquidity depends on how prices and rationing react to government bonds. For a fixed fraction of good trees in the economy, λ_1^E , both prices and rationing decrease because of the crowding-out effect of government bonds. Moreover, the drop in the liquidity premium disproportionately reduces the incentives to produce bad trees, so the average asset quality in the economy increases. Hence, while previous work suggested that optimal policy should equalize the liquidity premium

²⁵See the details of the derivation in Appendix C.

to the shadow cost of taxation, when private markets are fragile it should also take into account potentially negative crowding-out effects.

Still, the government should try to smooth the changes of the liquidity premium to shocks. To see this, note that, in an optimum, the second order condition (SOC) has to be negative. But the effect of B_0 over the variables in the SOC works indirectly through the liquidity premium. The quantity of bonds affects the amount of investment and hence the amount of capital for period 2, which determines the rate of return of capital, $r(K)$, and hence the liquidity services $\mu_1(A, X_1)$. And it is the change in $\mu_1(A, X_1)$ that affects γ_0^B , $A_1^S(X_1)$, H_1^G , H_1^B , and $P_1^M(\omega_h; X_1)$. This is important because then I can sign the effect of any shock that affects the FOC only through the liquidity premium by determining if its effect has the same or opposite sign to the SOC. For example, an increase in Z^Y has the opposite effect than government bonds, hence the FOC increases with Z^Y , and optimal B_0 increases with Z^Y .

The next proposition summarizes this result.

Proposition 12. *Optimal policy takes the form of increasing the supply government bonds when the liquidity premium is high, and lowering it when the liquidity premium is low.*

4.2 Transaction Tax

An alternative policy tool the government could use are transaction taxes and subsidies (or purchase programs). In fact, the government used purchase programs to improve liquidity in financial markets after the crisis hit. Tirole (2012) and Philippon and Skreta (2012) study how to optimally intervene in markets that collapse due to adverse selection from an ex-post point of view. Here, I analyze the problem from an ex-ante perspective. Since subsidies and purchase programs are equivalent in this setting, I assume that the government uses subsidies for notational convenience, even though purchase programs are better from a practical point of view.²⁶

Suppose the price the sellers receive is $P_1^S(X) = P_1^M(X) + c(X)$, where $P_1^M(X)$ is the price paid by the buyers and $c(X)$ is the government's subsidy (or tax if negative).²⁷ By manipulating the price, the government is effectively doing two things. First, given H_1 and λ_1^E , it is deciding how much liquidity there is in the market. Second, it shapes the incentives to invest in period 0. While the government wants the highest possible liquidity in the markets and the highest possible quality production in period 0, transaction taxes and subsidies trade-off one for the other. So the question is what is the optimal way to balance these forces. In particular, what states should be taxed and what states should be subsidized? Below I show that the answer depends on whether the marginal value of liquidity in low liquidity states is high enough compared to the marginal value in high liquidity states. In the likely case that the value of liquidity in low liquidity states is sufficiently higher than in high liquidity states, then optimal policy requires that taxes are pro-cyclical (and potentially subsidize low liquidity states), in a *leaning against the liquidity* type of policy.

²⁶With subsidies to transactions, agents could just buy and sell the same asset from one another repeatedly only to receive the subsidy. By buying the asset, the government avoids this type of behavior. However, the two policies differ with respect to the timing of payments, even though they have the same net present value.

²⁷I use this notation instead of the more standard ad-valorem subsidy/tax for analytical convenience. It is always possible to define the implicit ad-valorem subsidy/tax as $\tau(X) \equiv \frac{P_1^M(X)+c(X)}{P_1^M(X)} - 1$.

For simplicity, suppose that the quality of bad trees can only take two values: α_H and α_L , with $\alpha_H > \alpha_L$. The probability that $\alpha = \alpha_H$ is denoted by ζ_H . Moreover, assume that the production function is given by

$$f(K) = \begin{cases} Z^Y K & \text{if } K \leq K^* \\ Z^Y (K^* + \delta K) & \text{if } K > K^* \end{cases} \quad (28)$$

with $\delta \in [0, 1]$. Note that (25) is a particular case of (28) with $\delta = 0$. For this exercise it is more convenient to work with this form. I choose K^* such that if the state is α_H , in the laissez-faire equilibrium $K > K^*$. On the other hand, if the state is α_L , then $K < K^*$. This implies that $r(K(\alpha_H)) = Z^Y \delta < Z^Y = r(K(\alpha_L))$.

Maximizing (26) by choosing the values for $\{c(\alpha)\}$ gives the following FOC

$$\begin{aligned} \frac{\partial \mathbb{W}}{\partial c(\alpha')} &= E \left[\left[\int_{A_1^B(\alpha)}^{A_1^{\max}} Ar(K(\alpha)) dG(A) + G(A_1^B(\alpha)) \right] \left[\frac{\partial P_1^M(\alpha)}{\partial c(\alpha')} + \frac{\partial c(\alpha)}{\partial c(\alpha')} \right] \right] H_1^B + \\ &E \left[\int_{A_1^S(\alpha)}^{A_1^{\max}} Ar(K(\alpha)) dG(A) \left[\frac{\partial P_1^M(\alpha)}{\partial c(\alpha')} + \frac{\partial c(\alpha)}{\partial c(\alpha')} \right] \right] H_1^G = E \left[\frac{\partial T(\alpha)}{\partial c(\alpha')} \right], \end{aligned}$$

where $T(\alpha) = c(\alpha) [[1 - G(A_1^S(\alpha))] H_1^G + H_1^B]$, is the fiscal cost (revenues if negative) of the policy $\{c(\alpha)\}$.

So, should taxes be pro-cyclical or counter-cyclical? The answer depends on the value of δ and ζ_H . If $\delta = 1$, it could be optimal to subsidize the high α state and tax the low α state. The reason is that market liquidity is convex in the selling price. When the production function is linear, $A_1^B(\alpha_H) = A_1^B(\alpha_L)$, since $r(K(\alpha_H)) = r(K(\alpha_L))$, but $A_1^S(\alpha_H) < A_1^S(\alpha_L)$, since $P_1^M(\alpha_H) > P_1^M(\alpha_L)$. Therefore, the direct benefits from the subsidy $E \left[\left[\int_{A_1^B(\alpha)}^{A_1^{\max}} Ar(K(\alpha)) dG(A) + G(A_1^B(\alpha)) \right] \frac{\partial c(\alpha)}{\partial c(\alpha')} \right] H_1^B + E \left[\int_{A_1^S(\alpha)}^{A_1^{\max}} Ar(K(\alpha)) dG(A) \frac{\partial c(\alpha)}{\partial c(\alpha')} \right] H_1^G$ are higher for the high liquidity state. If ζ_H is not too high, $\frac{\partial P_1^M(\alpha)}{\partial c(\alpha_L)} < \frac{\partial P_1^M(\alpha)}{\partial c(\alpha_H)} < 0$, since a one unit increase in the price of the low α state induces a higher production of lemons than a unit increase in the price of the high α state. Therefore, the optimal policy would require to increase liquidity in high liquidity states and lower it in low liquidity states.

This result is counter-intuitive and an artifact of the fact that agents are risk neutral, so that the elasticity of substitution across states of nature is infinite. Moreover, it goes in the opposite direction than the result in Tirole (2012), who finds that subsidies should be higher for low liquidity states. Even though Tirole (2012) also has agents with linear preferences, the production function has an extreme form of concavity at the individual level. I can achieve a similar result by choosing a δ that is low enough. In that case, extra liquidity in the high liquidity state is less valuable than in the low liquidity state because in the former it gives a return of $AZ^Y \delta$ while in the latter the return is AZ^Y . It is straightforward to see that as δ goes to zero, the benefits from extra liquidity in the high liquidity state vanish away. In that case, the optimal policy prescribes a pro-cyclical transaction tax (whether it implies subsidizing the low state depends on parameter values). The next proposition summarizes these results.

Proposition 13. *There exists $\delta^* \in (0, 1]$ such that if $\delta < \delta^*$, the optimal transaction tax is pro-cyclical.*

5 INFINITE HORIZON

In the previous sections I presented a three-period model which allowed me to study the interaction between the incentives to produce assets of different qualities and changes in the economy's fundamentals and government policy. This section builds a tractable extension to an infinite horizon model in order to get some insights about the dynamic behavior of these mechanisms.

5.1 The Environment

The model is analogous to the three-period version with two main differences. First, agents operate the technology to produce trees and physical capital every period. Second, there are markets for trees every period.

There is a continuum of infinitely lived agents. There are three types of goods: a final consumption good, Lucas (1978) trees (which can be good or bad), and physical capital. Agents maximize utility:

$$U_t = E_t \left[\sum_{s=t}^{\infty} \beta^s d_s \right],$$

where d_s is consumption in $s = \{t, t+1, t+2, \dots\}$, β is the agents' discount factor, and the expectation is taken with respect to their idiosyncratic investment opportunities and an aggregate state of the economy, both described below. For convenience, I assume that agents receive an endowment W of final goods every period. This will give me a flexible way of guaranteeing that there is positive aggregate consumption in all states around the stochastic steady state of the economy (described below), so that pricing is risk neutral like in the previous sections.

Agents have access to two technologies every period: one that produces trees and one that produces physical capital. The technologies and payoff of trees and capital are a natural extension of the ones described in the three-period models, with some simplifying assumptions.

I assume that trees are long lived and depreciate at a rate δ_H . While good trees pay a dividend Z every period, bad trees pay αZ , where $\alpha \sim F(\alpha)$ with support in $[0, 1]$, for some non-degenerate continuous cumulative distribution function F . For simplicity, I assume that α is iid over time. That is, trees of different qualities die at the same rate, but while good trees always pay Z , bad trees pay a fraction of that. Moreover, I assume that all agents face the same cost of producing trees. I normalize the cost of producing bad trees to one, so that $q_B = 1$. On the other hand, the cost of producing good trees has two components: producing one unit of good tree costs $q_G \phi(I_G)$, where I_G is the aggregate production of good trees. The term q_G could be interpreted as the unit cost of production, with $q_G > q_B$. The term $\phi(I_G)$ is an investment adjustment cost, with $\phi(0) = 1$, $\phi' > 0$, and $\phi'' > 0$. This avoids that the stock of good trees grows without bound in this linear environment.²⁸

I keep the production technology of physical capital from the previous section. That is, agents receive a productivity A drawn from a convex and continuous cumulative distribution function G

²⁸There are different assumptions that would bound the amount of trees in equilibrium, and convex investment adjustment costs is a very tractable one. Note that, by making the adjustment cost depend on aggregate investment, the problem of the agents remains linear, so there is no need to keep track of cross-section distribution of agents' portfolio holdings to determine aggregate allocations.

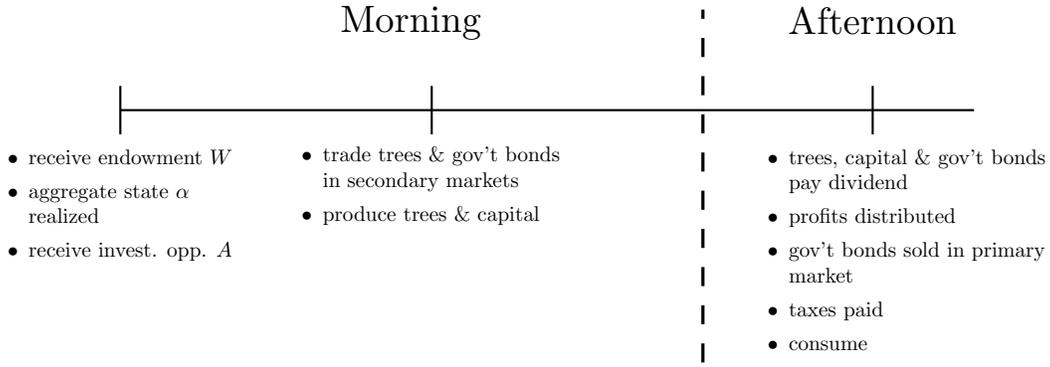


FIGURE 7: Within Period Timing

with support $[0, A^{\max}]$. In order to simplify the dynamic interactions of the economy, I assume that physical capital fully depreciates after use. There is a representative firm that operates a concave production function $f(K)$ and it rents capital from the agents in competitive markets, so that the rental rate of capital is given by $r(K) = f'(K)$. Firm's profits are distributed uniformly across all agents.

The structure of the market for capital is analogous to the extended three-period model from the previous section. Every period there could be up to two active markets. In one market only bad trees are traded, and there is no rationing. In the other market, good and bad trees are traded at a pooling price and there can be rationing in equilibrium.

Finally, the government supplies an amount B of one-period bonds every period, that pay 1 unit of final good at maturity. The per-period budget constraint of the government is given by

$$B - P_{GB}B = T,$$

where P_{GB} is the price at which the government sells the bonds in the primary market, and T is a lump-sum tax to the agents.

The timing within a period is as follows. For tractability, I assume that each period is divided into two sub-periods, which I denote by "morning" and "afternoon". In the morning, agents receive the endowment W , and both the aggregate state, α , and the idiosyncratic investment opportunity, A , are realized. Moreover, the secondary markets for trees and government bonds open and production of trees and physical capital takes place. Note that this implies that only the own endowment W and the proceeds from trading assets can be used in production. In the afternoon, trees pay their dividend, production in the representative firm takes place, the rental rate of capital is paid and profits distributed, the government pays the outstanding bonds and sells new bonds in the primary market, and agents pay taxes and consume. This timing makes each period in the infinite horizon model as close as possible to the timing in the three-period economy from the previous sections and greatly simplifies the dynamics of the economy, as described below. Figure 7 depicts the timing within a period.

I will look for a recursive competitive equilibrium of the economy with $X \equiv \{\lambda_E, H, K, B; \alpha\}$ as a state variable, where λ_E is the fraction of good trees, H is the total amount of trees, K is the

amount of physical capital, B is the supply of government bonds, and α is the exogenous quality of bad trees.

5.2 Agents' Problem and Equilibrium

Agents start the period with a portfolio of trees (good and bad), capital, and government bonds. An agent's investment opportunity is given only by A , since all agents face the same cost of producing trees. Every period there can be two markets for trees active. The first features a price $P_M(\omega_H, X)$ and both good and bad trees are traded, and $\lambda_M(X)$ denotes the fraction of good trees in the market. However, only a fraction $\eta(X)$ of the trees supplied are actually sold. In the second market, the price is $P_M(\omega_L, X)$ and only bad trees are sold. Since a fraction $\eta(X)$ of the bad trees are sold in the high-price market, only the remainder $1 - \eta(X)$ is sold in this low-price market. Moreover, buyers can decide in which market to trade. Let $m(\omega_H)$ denote the purchases in the high price market, and $m(\omega_L)$ denote the purchases in the low price market (where they know they are getting bad trees with probability one). Finally, let $Q(X)$ denote the price of the government bonds traded in the secondary markets and \tilde{b} the holdings of government bonds at the end of the morning.

Thus, an agent's budget constraint in the morning is given by

$$\begin{aligned} \phi(I_G)q_G i_G + i_B + i_K + P_M(\omega_H, X)m(\omega_H) + P_M(\omega_L, X)m(\omega_L) + Q(X)\tilde{b} \leq W + \\ P_M(\omega_H, X)\eta(X)(s_G + s_B) + P_M(\omega_L, X)(1 - \eta(X))s_B + Q(X)b, \end{aligned} \quad (29)$$

which states that expenditures in investment and purchases of trees and government bonds cannot exceed the sum of endowment, trees sold and government bonds sold.

I assume that the final good cannot be stored between periods, but it can be stored between morning and afternoon. Let Δ denote the surplus in the morning (note that $\Delta \geq 0$). Then, the agent's budget constraint in the afternoon is given by

$$\begin{aligned} d + P_{GB}(X)b' \leq \Delta + [h_G + \lambda_M(X)m(\omega_H) - \eta(X)s_G]Z + [h_B + (1 - \lambda_M(X))m(\omega_H) + m(\omega_L) - s_B]\alpha Z + \\ r(X)k + \tilde{b} + \Pi(X) - T(X), \end{aligned} \quad (30)$$

which states that expenditures in consumption and purchases of government bonds in the primary market cannot exceed the sum of the surplus from the morning, the dividends received (from trees, capital, firms and government bonds), and government transfers.

Finally, agents face the following laws of motion of their portfolio holdings

$$h'_G = (1 - \delta_H)[h_G + \lambda_M(X)m(\omega_H) - \eta(X)s_G] + i_G, \quad (31)$$

$$h'_B = (1 - \delta_H)[h_B + (1 - \lambda_M(X))m(\omega_H) + m(\omega_L) - s_B] + i_B, \quad (32)$$

$$k' = Ai_K. \quad (33)$$

Therefore, the problem of an agent with investment opportunity A is given by

$$V(h_G, h_B, k, b; A, X) = \max_{\substack{d, i_G, i_B, i_K, m, s_G, s_B \\ h'_G, h'_B, k', b', \Delta}} d + \beta E[V(h'_G, h'_B, k', b'; A', X')|X], \quad (\text{P})$$

subject to (29), (30), (31), (32) and (33), and

$$d \geq 0, \quad i_G \geq 0, \quad i_B \geq 0, \quad i_K \geq 0, \quad m \geq 0, \quad b' \geq 0,$$

$$s_G \in [0, h_G], \quad s_B \in [0, h_B].$$

As in the three period models, the value function V is linear in *each* element of the agents' portfolio:

$$V(h_G, h_B, k, b; A, X) = \tilde{\gamma}_G(A, X)h_G + \tilde{\gamma}_B(A, X)h_B + \tilde{\gamma}_K(A, X)k + \tilde{\gamma}_{GB}(A, X)b,$$

where

$$\tilde{\gamma}_G(A, X) = \max\{\mu(A, X)\eta(X)P_M(\omega_H, X) + (1 - \eta(X))[Z + (1 - \delta_H)\gamma_G(X)], Z + (1 - \delta_H)\gamma_G(X)\},$$

$$\tilde{\gamma}_B(A, X) = \mu(A, X)[\eta(X)P_M(\omega_H, X) + (1 - \eta(X))P_M(\omega_L, X)],$$

$$\tilde{\gamma}_K(A, X) = r(X),$$

$$\tilde{\gamma}_{GB}(A, X) = \mu(A, X),$$

and liquidity services are given by

$$\mu(A, X) = \max\left\{1, \frac{\gamma_G(X)}{\phi(I_G)q_G}, \gamma_B(X), A\gamma_K(X), \mu_B(X), \frac{1}{Q(X)}, \frac{\gamma_{GB}(X)}{P_{GB}(X)}\right\}, \quad (34)$$

where $\mu_B(X) \equiv \max\left\{\frac{\lambda_M(X)[Z + (1 - \delta_H)\gamma_G(X)] + (1 - \lambda_M(X))[\alpha Z + (1 - \delta_H)\gamma_B(X)]}{P_M(\omega_H, X)}, \frac{\gamma_B(X)}{P_1^M(\omega_L, X)}\right\}$ is the return from buying trees in the secondary market.²⁹ Finally, the shadow prices are given by

$$\gamma_j(X) = \beta E[\tilde{\gamma}_j(A', X')|X], \quad j \in \{G, B, K, GB\}.$$

The agents' choices follow the same logic than in the simple three-period model. Consider the problem of an agent with investment opportunity given by A . There exists $A_B(X)$ such that the agent chooses to produce physical capital if and only if $A \geq A_B(X)$. Moreover, there exists $A_S(X) \geq A_B(X)$ such that if $A \geq A_S(X)$, the agent sells his good trees in order to invest in capital. Agents with $A < A_B(X)$ use their liquid wealth to consume, produce trees or buy trees and government bonds in the market. Since at least one market for trees is always active in equilibrium, optimality

²⁹There is a slight abuse of notation since there are states in which only one market is active. In that case $\mu_B(X)$ is the return on the active market.

requires that

$$\mu_B(X) \geq 1, \quad (35)$$

$$\frac{\gamma_G(X)}{\phi(I_G)q_G} \leq \mu_B(X), \quad (36)$$

$$\gamma_B(X) \leq \mu_B(X), \quad (37)$$

$$\frac{1}{Q(X)} \leq \mu_B(X), \quad (38)$$

$$\frac{\gamma_{GB}(X)}{P_{GB}(X)} \leq \mu_B(X). \quad (39)$$

Therefore, $A_B(X) \equiv \frac{\mu_B(X)}{\gamma_K(X)}$ and $A_S(X) \equiv \frac{\gamma_G(X)}{P_M(\omega_H, X)\gamma_K(X)}$.

Consider now the market for trees. Let $M(\omega_H, X)$ and $M(\omega_L, X)$ be the total demand in markets ω_H and ω_L respectively. They must satisfy

$$P_M(\omega_H, X)M(\omega_H, X) + P_M(\omega_L, X)M(\omega_L, X) \leq \int_0^{A_B(X)} WdG(A). \quad (40)$$

Moreover, (35) imposes the following restrictions on prices

$$P_M(\omega_H, X) \leq \lambda_M(X)[Z + (1 - \delta_H)\gamma_G(X)] + (1 - \lambda_M(X))[\alpha Z + (1 - \delta_H)\gamma_B(X)], \quad (41)$$

$$P_M(\omega_L, X) \leq \alpha Z + (1 - \delta_H)\gamma_B(X), \quad (42)$$

where

$$\lambda_M(X) = \frac{[1 - G(A_S(X))]\lambda_E}{[1 - G(A_S(X))]\lambda_E + (1 - \lambda_E)}. \quad (43)$$

On the other hand, let $S(\omega_H, X)$ and $S(\omega_L, X)$ be the supply of trees in market ω_H and ω_L respectively. Then

$$S(\omega_H, X) = \int_{A_S(X)}^{A^{\max}} H_G dG(A) + H_B, \quad (44)$$

$$S(\omega_L, X) = H_B. \quad (45)$$

A partial equilibrium in the markets for trees requires that $M(\omega_H, X) = \eta(X)S(\omega_H, X)$ and $M(\omega_L, X) = (1 - \eta(X))S(\omega_L, X)$, and that prices are the highest consistent with (41) and (42) (maximal volume of trade equilibrium). If (40) is satisfied with strict inequality, then (41) and (42) hold with strict equality. In the previous sections I simplified the problem by assuming that the analogous to (40) was always satisfied with strict inequality. Here, I will assume that the same holds around the stochastic steady state I define below.

Finally, the laws of motion of λ_E , H and K are given by

$$\lambda'_E(X) = \lambda_E \theta(X) + \frac{I_G(X)}{I_G(X) + I_B(X)}(1 - \theta(X)), \quad (46)$$

where $\theta(X) \equiv \frac{(1-\delta_H)H}{(1-\delta_H)H+I_G(X)+I_B(X)}$,

$$H'(X) = (1 - \delta_H)H + I_G(X) + I_B(X), \quad (47)$$

and

$$K'(X) = \int_{\frac{1}{\gamma_K(X)}}^{A^{\max}} A[W + [\eta(X)P_M(\omega_H, X) + (1 - \eta(X))P_M(\omega_L, X)]H_B + Q(X)B]dG(A) + \int_{\frac{\gamma_G(X)}{\gamma_K(X)P_M(\omega_H, X)}}^{A^{\max}} A\eta(X)P_M(\omega_H, X)H_GdG(A). \quad (48)$$

where $\lambda'_E(X)$, $H'(X)$ and $K'(X)$ are the fraction of good trees, total amount of trees, and aggregate physical capital, respectively, one period ahead. Note that (48) is the analogous to its three-period counterpart (23).

I define an equilibrium for this economy.

Definition 6 (Equilibrium). *An equilibrium consists of prices $\{P_M(\omega_H, X), P(\omega_L, X), Q(X), P_{GB}(X), r(X)\}$; market fraction of good trees $\lambda_M(X)$ and rationing $\eta(X)$ of market ω_H ; a value function $V(h_G, h_B, k, b; A, X)$, shadow values $\{\gamma_G(X), \gamma_B(X), \gamma_K(X), \gamma_{GB}(X)\}$ and decision rules $\{d, i_G, i_B, i_K, m(\omega_H), m(\omega_L), s_G, s_B, h'_G, h'_B, k', \tilde{b}, b', \Delta\}$ that depend on $(h_G, h_B, k, b; A, X)$, such that*

1. $\{d, i_G, i_B, i_K, m(\omega_H), m(\omega_L), s_G, s_B, h'_G, h'_B, k', \tilde{b}, b', \Delta\}$, and $V(h_G, h_B, k, b; A, X)$ solve program (P) taking $P_M(\omega_H, X)$, $P_M(\omega_L, X)$, $Q(X)$, $P_{GB}(X)$, $r(X)$, $\lambda_M(X)$, and $\eta(X)$ as given;
2. the markets for trees clear: $M(\omega_H, X) = S(\omega_H, X)$ and $M(\omega_L, X) = S(\omega_L, X)$;
3. $P_M(\omega_H, X)$, $P_M(\omega_L, X)$ and $\lambda_M(X)$ satisfy (40), (41), (42) and (43);
4. the primary and secondary markets for government bonds clear;
5. the rental rate $r(X)$ equals the marginal product of capital, $r(X) = f'(K)$;
6. the laws of motion of λ_E , H and K , given by (46), (47) and (48), are consistent with individual decisions and rationing $\eta(X)$.

5.3 Stochastic Steady State

I study the economy around a stochastic steady state. This is a natural starting point, and as I will show, provides a tractable laboratory to study the dynamics of the economy. I do this in stages. First, I analyze the characteristics of the steady state. I show that there exists an equilibrium in which λ_E and H are constant over time, and K fluctuates with the aggregate shock α . Then, I characterize the dynamic properties of the economy around this equilibrium. Finally, I perform some comparative statics exercises.

I guess and verify that the equilibrium with λ_E and H constant over time exists. The laws of

motion of λ_E and H (equations (46) and (47)) imply that

$$\Delta\lambda_E = 0 \Leftrightarrow \lambda_E = \frac{I_G}{I_G + I_B}, \quad (49)$$

$$\Delta H = 0 \Leftrightarrow H = \frac{I_G + I_B}{\delta_H}. \quad (50)$$

Thus, (λ_E, H) are constant over time if and only if I_G and I_B are constant over time, independently of K and α . Since $\lambda_E \in (0, 1)$, both good and bad trees have to be produced in equilibrium. Moreover, I assume that W is large enough so that $\mu_B = 1$ in all states in the steady state (below I put a lower bound on W so that this is satisfied), (41) and (42) are satisfied with equality, and $Q(X) = 1$ and $P_{GB}(X) = \gamma_{GB}(X)$. Therefore, agents that produce trees have to be indifferent between this and consuming, thus

$$\frac{\gamma_G(X)}{\phi(I_G)q_G} = \gamma_B(X) = 1,$$

which implies that γ_G and γ_B have to be constant over time. Two things remain to be shown. First, that constant γ_G and γ_B are consistent with the definitions of the shadow values. Second, that there exists λ_E and H consistent with this equilibrium. The shadow values of trees are given by

$$\gamma_G(X) = \beta E[\max\{\mu(A', X')\eta(X')P_M(\omega_H, X') + (1 - \eta(X'))[Z + (1 - \delta_H)\gamma_G(X')], \\ Z + (1 - \delta_H)\gamma_G(X')\} | X], \quad (51)$$

$$\gamma_B(X) = \beta E[\mu(A', X')[\eta(X')P_M(\omega_H, X') + (1 - \eta(X'))P_M(\omega_L, X')]| X], \quad (52)$$

where $\mu(A, X) = \max\{1, A\gamma_K(X)\}$. Then, (51) and (52) are constant over time conditional on (λ_E, H) being constant, if and only if they are independent of α and K . First, since α is iid, the shadow values do not directly depend on α . Second, the shadow values do not depend explicitly on K and $K'(X)$. Hence, it is sufficient to show that $K'(X)$ does not depend on K . But it is immediate from (48) that $K'(X)$ does not depend on K , since prices P_M , fraction λ_M , and rationing η only depend on shadow values and current realization of α , proving that the shadow values of trees are constant over time if (λ_E, H) are constant over time. This result relies on three assumptions: first, aggregate shocks are iid; second, capital fully depreciate after use; third, trees and capital pay their dividend after trade and investment (in trees and capital) takes place. Below I discuss how changing these assumptions would change the results.

Finally, I need to show that there exists a pair (λ_E, H) consistent with the equilibrium. Recall that a constant path of λ_E and H solves

$$\lambda_E = \frac{I_G}{I_G + I_B}, \quad (53)$$

and

$$H = \frac{I_G + I_B}{\delta_H}, \quad (54)$$

where I_G and I_B have to be consistent with individual optimality conditions. These two equations determine two curves in the space (λ_E, H) . A steady state is characterized by an intersection of

these curves. To see that at least one intersection exists consider the following. On the one hand, (53) is strictly greater than zero when $H = 0$ because $I_G > 0$, and has a limit $\tilde{\lambda}_E < 1$ when $H \rightarrow \frac{W}{\delta_H}$, since $I_B > 0$. On the other hand, (54) is greater than zero when $\lambda_E = 0$ since $I_G > 0$, and less than $\frac{W}{\delta_H}$ when $\lambda_E = \tilde{\lambda}_E$, since at least some endowment is used to produce physical capital. Since they are both continuous functions, an intersection exists. However, there could be multiple intersections. Although this could potentially be an interesting phenomenon to study, it is beyond the scope of this paper. Therefore, I select the one that features the highest λ_E .

Finally, a sufficient restriction on W is that

$$G(A_M(X))W > P_M(\omega_H, X)\eta(X)[[1 - G(A_S(X))]H_G + H_B] + P_M(\omega_L, X)(1 - \eta(X))H_B + [1 - G(A_M(X))]B,$$

for all states α . Since the left hand side is increasing in W (because $A_M(X)$ is increasing in W) and unbounded, while the right hand side is decreasing in W and bounded, there exists an open set in \mathbb{R}_+ such that the condition is satisfied.

In the stochastic steady state the realization of α only affects the liquidity in the market and therefore the production of capital. Higher α implies higher volume traded and therefore more reallocation towards the agents with the highest productivities. Thus, this infinite horizon extension keeps the main insights from the previous sections while maintaining tractability.

Finally, it is easy to see that in the intersection with the highest level of λ_E , (53) crosses (54) from below in the space (λ_E, H) . This property is key to show that the steady state is stable and the economy converges monotonically to the steady state from any initial conditions (λ_E, H) that are sufficiently close to it. I show this in Appendix D.

Next, I study the dynamic response of the economy to a transitory shock.

5.4 Transitory Shock

When studying the effect of shocks on market fragility, a general result stated that the overall effect was ambiguous. While the fundamental shock reduces the fragility of the system, the endogenous response generates an opposite effect. The overall result depends on functional forms and parameter values. A natural question is what happens if the economy goes back to its initial value or trend, in a impulse response type of exercise.

Here I study a transitory increase in the distribution of the quality of bad trees, α . Suppose the economy is in its stochastic steady state in period T and the distribution of α in $T + 1$ increases from F to \tilde{F} such that $\tilde{F} > F$ in first order stochastic dominance sense, and then it goes back to F in $T + 2$. On impact, this generates an increase in the shadow value of trees, with the shadow value of bad trees increasing proportionally more than that of good trees. Therefore, production of bad trees increases and the fraction of good trees in the economy decreases. This is the same effect I found in the three-period model. In $T + 1$, the agents anticipate that fundamentals go back to their initial level, so their incentives to produce bad trees decreases. Moreover, the incentives to produce good trees is also lower than in the steady state. To see this note that

$$\lambda'_E = \lambda_E \theta(\lambda_E, H) + \frac{I_G}{I_G + I_B} (1 - \theta(\lambda_E, H)).$$

Therefore, if λ_E is lower, $\frac{I_G}{I_G+I_B}$ has to be higher. But for each level of investment, the shadow value of good trees is lower than in the stochastic steady state, since it results in a lower fraction of good trees and hence in lower expected prices. Therefore, market fragility in $T + 2$ is higher than in the stochastic steady state. The next proposition states the main result of this section.

Proposition 14. *Consider an economy that starts in its stochastic steady state. Suppose in period T the economy is hit by a shock that increases the distribution of the quality of bad trees α in FOSD sense for one period. Then, the fraction of good trees in $T + 1$ decreases and market fragility in period $T + 2$ is higher than in the stochastic steady state.*

This result is important because it shows that a transitory shock sows the seeds of the a crisis by generating perverse incentives in the boom that exposes the economy to a bust when conditions go back to "normal". The result can also be extended to the other sources of risk studied before, like a transitory increase in the TFP of the representative firm or a transitory reduction in the supply of government bonds. Note that the intuition is a natural extension from the insight gained in the three period models: a positive shock worsens the composition of assets in the economy, and since assets are long lived, when the shock vanishes away the fraction of good assets in the economy is smaller than before the shock, while the exogenous state goes back to its initial level, so fragility increases.

Moreover, this setting allows me to study the effects of the timing of unexpected government intervention. The government could issue bonds as soon as the shock hits or wait until the economy goes back to its trend. If the issuance occurs when the shock hits, higher public liquidity reduces the liquidity premium, which reduces the incentives to produce bad trees. Therefore, a small increase in government bonds reduces market fragility with respect to a situation of no government intervention. On the other hand, if the intervention occurs when the shock dies out, the increase in public liquidity crowds out the production of *good* trees, increasing market fragility even more.

Proposition 15. *A small increase in government bonds in T reduces market fragility. A small increase in government bonds in $T + 1$ increases market fragility.*

5.5 Discussion

In order to keep the tractability of the infinite horizon model I made several assumptions about the fundamentals and the timing of the economy. Without them, the steady state I analyzed would not exist, and both λ_E and H would fluctuate with the realization of the aggregate shock α . Here I briefly discuss what would change in the more general setting.

If shocks were not iid and capital did not fully depreciate after use, shocks in one period would carry information about the economy in future periods. I conjecture that under different assumptions, as long as the aggregate shock has positive auto-correlation, the main results should not change. That is, a positive transitory shock would disproportionately increase the production of bad trees and, thus, reduce the fraction of good trees in the economy. As the shock vanishes away, the economy faces the same conditions than in my simplified economy: the same fundamentals than before the shock (for a *given* path of realizations of the aggregate state) but a worse asset quality composition in the economy. Therefore, market fragility would increase.

Moreover, I assumed that trees and capital pay after the market for trees close and investment is undertaken. Each assumption performs a different role. Trees need to pay after the market closes so that with iid aggregate shocks there is some risk in the market. On the other hand, if the dividends from capital were used for investment, it would introduce a term that connects past shocks with current control variables, so that a steady state with constant λ_E and H would not exist. However, the main forces of the economy do not change, so the results are likely to survive. But more research is needed to fully understand the implications of the infinite horizon economy.

6 CONCLUSION

I have developed a model in which ex-ante production of assets interacts with ex-post adverse selection in financial markets. The production of low-quality assets is more sensitive to changes in markets conditions and the value of liquidity services than that of high-quality assets. Therefore, shocks that improve market functioning, such as reductions in the "default rate" of low-quality assets, increases in the productivity of the real economy, or reductions in transaction costs, deteriorate the asset quality composition of the economy and can even increase the probability of a financial crisis, defined as an event in which the financial markets collapse. Moreover, the supply of public liquidity also affects the private incentives to produce asset quality. I show that an increase in government bonds increases the total liquidity available in the economy and reduces the incentives to produce low-quality assets, but it can also exacerbate the adverse selection problem in private markets. If the production of trees is sufficiently elastic, then a reduction in government bonds can increase market fragility.

All these comparative statics point to plausible sources of risk build-up in the US before the Great Recession: perceived low risk on subprime mortgages that ended when house prices started to decrease; strong growth rates at the wake of the "dot-com" crisis; financial innovation that reduced the costs of trading illiquid assets; safe asset shortage due to fiscal surpluses in the late 90s as well as foreign demand in the early 2000s (the "savings glut").

Moreover, I study optimal policy in this setting. I find that the government should take into account the crowding-out effect on private markets when choosing the supply of bonds. Still, supply should increase when the liquidity premium increases (and viceversa). Moreover, I show that if the liquidity in low-liquidity states is sufficiently more valuable than in high-liquidity states, transaction taxes (or subsidies) that "lean against liquidity" are optimal.

Finally, I extend the insights from the basic models to an infinite horizon setting. I show that financial fragility is a natural outcome after transitory shocks, and that government intervention through the issuance of bonds should take place when the shock hits rather than when it dies out, since in the latter case it can exacerbate the negative effects of the lower asset quality distribution. In this analysis, I had to make strong assumptions in order to keep the model tractable. A natural next step would be to study whether these results survive in more realistic models (which would probably need to be solved numerically).

REFERENCES

- Adrian, Tobias, and Hyun Song Shin.** 2009. "Money, Liquidity, and Monetary Policy." *American Economic Review: Papers & Proceedings*, 99(2): 600–05.
- Adrian, Tobias, and Hyun Song Shin.** 2010. "The Changing Nature of Financial Intermediation and the Financial Crisis of 2007–2009." *Annual Review of Economics*, 2(1): 603–18.
- Adrian, Tobias, and Hyun Song Shin.** 2010. "Financial Intermediaries and Monetary Economics." In *Handbook of Monetary Economics*. Vol. 3, , ed. Benjamin M. Friedman and Michael Woodford, 601–50. Amsterdam: Elsevier.
- Akerlof, George A.** 1970. "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism." *The Quarterly Journal of Economics*, 84(3): 488–500.
- Angeletos, George-Marios, Fabrice Collard, and Harris Dellas.** 2016. "Public Debt as Private Liquidity: Optimal Policy."
- Ashcraft, Adam B, and Til Schuermann.** 2008. "Understanding the Securitization of Subprime Mortgage Credit." *Foundations and Trends® in Finance*, 2(3): 191–309.
- Attar, Andrea, Thomas Mariotti, and François Salanié.** 2011. "Nonexclusive competition in the market for lemons." *Econometrica*, 79(6): 1869–1918.
- Bank for International Settlements.** 2001. "Collateral in Wholesale Financial Markets: Recent Trends, Risk Management and Market Dynamics." *Committee on the Global Financial System*.
- Bigelow, John P.** 1990. "Efficiency and Adverse Selection." *Journal of Economic Theory*, 52(2): 380–405.
- Bigio, Saki.** 2015. "Endogenous Liquidity and The Business Cycle." *American Economic Review*, 105(6): 1883–1927.
- Bolton, Patrick, Tano Santos, and Jose A Scheinkman.** 2011. "Outside and Inside Liquidity." *Quarterly Journal of Economics*, 126(1): 259–321.
- Brunnermeier, Markus K.** 2009. "Deciphering the Liquidity and Credit Crunch 2007–2008." *The Journal of Economic Perspectives*, 23(1): 77–100.
- Caballero, Ricardo J.** 2006. "On the Macroeconomics of Asset Shortages." *The Role of Money: Money and Monetary Policy in the Twenty-First Century, The Fourth European Central Banking Conference 9-10 November 2006, Andreas Beyer and Lucrezia Reichlin, editors.*, 272–83.
- Caballero, Ricardo J.** 2010. "The "Other" Imbalance and the Financial Crisis." *NBER Working Paper No. 15636*.
- Caballero, Ricardo J, and Arvind Krishnamurthy.** 2009. "Global Imbalances and Financial Fragility." *American Economic Review: Papers & Proceedings*, 99(2): 585–88.
- Caballero, Ricardo J, and Emmanuel Farhi.** 2014. "The Safety Trap." *NBER Working Paper No. 19927*.
- Calvo, Guillermo.** 2013. "Puzzling over the Anatomy of Crises: Liquidity and the Veil of Finance." *Working Paper*.
- Chari, V V, Ali Shourideh, and Ariel Zetlin-Jones.** 2014. "Reputation and Persistence of Adverse Selection in Secondary Loan Markets." *American Economic Review*, 104(12).
- Chemla, Gilles, and Christopher A Hennessy.** 2014. "Skin in the game and moral hazard." *The Journal of Finance*, 69(4): 1597–1641.
- Dang, Tri Vi, Gary Gorton, and Bengt Holmstrom.** 2009. "Opacity and the Optimality of Debt for Liquidity Provision." *Manuscript Yale University*.
- Demiroglu, Cem, and Christopher James.** 2012. "How Important is Having Skin in the Game? Originator-Sponsor Affiliation and Losses on Mortgage-Backed Securities." *Review of Financial Studies*, 25(11): 3217–58.
- Diamond, Douglas W, and Raghuram G Rajan.** 2012. "Illiquid Banks, Financial Stability, and Interest Rate Policy." *Journal of Political Economy*, 120(3): 552–591.
- Downing, Chris, Dwight Jaffee, and Nancy Wallace.** 2009. "Is the Market for Mortgage-Backed Securities a Market for Lemons?" *Review of financial Studies*, 22(7): 2457–2494.
- Eisfeldt, Andrea L.** 2004. "Endogenous Liquidity in Asset Markets." *The Journal of Finance*, 59(1): 1–30.

- Farhi, Emmanuel, and Jean Tirole.** 2012. "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts." *American Economic Review*, 102(1): 60–93.
- Fukui, Masao.** 2016. "Asset Quality Cycles." *Working Paper*.
- Geanakoplos, John.** 2010. "The leverage cycle." In *NBER Macroeconomics Annual 2009, Volume 24*. 1–65. University of Chicago Press.
- Gennaioli, Nicola, Andrei Shleifer, and Robert W Vishny.** 2013. "A Model of Shadow Banking." *The Journal of Finance*, 68(4): 1331–63.
- Gertler, Mark, and Nobuhiro Kiyotaki.** 2010. "Financial Intermediation and Credit Policy in Business Cycle Analysis." In *Handbook of Monetary Economics*. Vol. 3, , ed. Benjamin M. Friedman and Michael Woodford, 547–599. Amsterdam: Elsevier.
- Gertler, Mark, and Peter Karadi.** 2011. "A Model of Unconventional Monetary Policy." *Journal of monetary Economics*, 58(1): 17–34.
- Gorton, Gary.** 2010. *Slapped by the Invisible Hand: The panic of 2007*. Oxford University Press.
- Gorton, Gary.** 2016. "The History and Economics of Safe Assets." *NBER Working Paper No. 22210*.
- Gorton, Gary, and Andrew Metrick.** 2012. "Securitization." *NBER Working Paper No. 18611*.
- Gorton, Gary, and George Pennacchi.** 1990. "Financial Intermediaries and Liquidity Creation." *The Journal of Finance*, 45(1): 49–71.
- Gorton, Gary, and Guillermo Ordonez.** 2013. "The Supply and Demand for Safe Assets." *NBER Working Paper No. 18732*.
- Gorton, Gary, and Guillermo Ordoñez.** 2014. "Collateral Crises." *American Economic Review*, 104(2): 343–378.
- Gorton, Gary, and Guillermo Ordoñez.** 2016. "Good Booms, Bad Booms." *NBER Working Paper No. 22008*.
- Gorton, Gary, Stefan Lewellen, and Andrew Metrick.** 2012. "The Safe-Asset Share." *American Economic Review: Papers & Proceedings*, 102(3): 101–106.
- Greenwood, Robin, Samuel G Hanson, and Jeremy C Stein.** 2015. "A Comparative-Advantage Approach to Government Debt Maturity." *The Journal of Finance*, 70(4): 1683–1722.
- Guerrieri, Veronica, and Robert Shimer.** 2014a. "Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality." *American Economic Review*, 104(7): 1875–1908.
- Guerrieri, Veronica, and Robert Shimer.** 2014b. "Markets with Multidimensional Private Information." *NBER Working Paper No. 20623*.
- He, Zhiguo, and Arvind Krishnamurthy.** 2012. "A Model of Capital and Crises." *The Review of Economic Studies*, 79(3): 753–77.
- Hill, Andy.** 2014. "Collateral is the New Cash: The Systemic Risks of Inhibiting Collateral Fluidity." *Journal of Securities Operations & Custody*, 7(1): 6–20.
- Holmström, Bengt, and Jean Tirole.** 1998. "Private and Public Supply of Liquidity." *Journal of Political Economy*, 106(1): 1–40.
- Holmström, Bengt, and Jean Tirole.** 2001. "LAPM: A Liquidity-Based Asset Pricing Model." *The Journal of Finance*, 56(5): 1837–67.
- Jermann, Urban, and Vincenzo Quadrini.** 2012. "Macroeconomic Effects of Financial Shocks." *American Economic Review*, 102(1): 238–71.
- Keys, Benjamin, Tanmoy Mukherjee, Amit Seru, and Vikrant Vig.** 2010. "Did Securitization Lead to Lax Screening? Evidence from Subprime Loans." *The Quarterly Journal of Economics*, 125(1): 307–62.
- Kiyotaki, Nobuhiro, and John Moore.** 2012. "Liquidity, Business Cycles, and Monetary Policy." *NBER Working Paper No. 17934*, 1–59.
- Krainer, John, and Elizabeth Laderman.** 2014. "Mortgage Loan Securitization and Relative Loan Performance." *Journal of Financial Services Research*, 45(1): 39–66.
- Krishnamurthy, Arvind, and Annette Vissing-Jorgensen.** 2012. "The Aggregate Demand for Treasury Debt." *Journal of Political Economy*, 120(2): 233–67.
- Krishnamurthy, Arvind, and Annette Vissing-Jorgensen.** 2015. "The Impact of Treasury Supply

- on Financial Sector Lending and Stability." *Journal of Financial Economics*, 118(3): 571–600.
- Krishnamurthy, Arvind, Stefan Nagel, and Dmitry Orlov.** 2014. "Sizing Up Repo." *The Journal of Finance*, 69(6): 2381–2417.
- Kurlat, Pablo.** 2013. "Lemons Markets and the Transmission of Aggregate Shocks." *American Economic Review*, 103(4): 1463–89.
- Kurlat, Pablo.** 2016. "Asset Markets with Heterogeneous Information." *Econometrica*, 84(1): 33–85.
- Lorenzoni, Guido.** 2008. "Inefficient Credit Booms." *The Review of Economic Studies*, 75(3): 809–33.
- Lucas Jr, Robert E.** 1978. "Asset prices in an exchange economy." *Econometrica: Journal of the Econometric Society*, 1429–1445.
- Mas-Colell, Andreu, Michael Dennis Whinston, and Jerry R Green.** 1995. *Microeconomic Theory*. Oxford university press New York.
- Mian, Atif, and Amir Sufi.** 2009a. "The Consequences of Mortgage Credit Expansion: Evidence from the US Mortgage Default Crisis." *The Quarterly Journal of Economics*, 1449–1496.
- Mian, Atif, and Amir Sufi.** 2009b. "The Consequences of Mortgage Credit Expansion: Evidence from the US Mortgage Default Crisis." *The Quarterly Journal of Economics*, 124(4): 1449–1496.
- Mian, Atif R, and Amir Sufi.** 2015. "Fraudulent Income Overstatement on Mortgage Applications During the Credit Expansion of 2002 to 2005." *NBER Working Paper No. 20947*.
- Moreira, Alan, and Alexi Savov.** 2016. "The Macroeconomics of Shadow Banking." *The Journal of Finance*, forthcoming.
- Neuhann, Daniel.** 2016. "Macroeconomic Effects of Secondary Market Trading." *Working Paper*.
- Philippon, Thomas, and Vasiliki Skreta.** 2012. "Optimal Interventions in Markets with Adverse Selection." *The American Economic Review*, 102(1): 1–28.
- Piskorski, Tomasz, Amit Seru, and James Witkin.** 2015. "Asset Quality Misrepresentation by Financial Intermediaries: Evidence from the RMBS Market." *The Journal of Finance*, 70(6): 2635–78.
- Pozsar, Zoltan.** 2014. "Shadow Banking: the Money View." *Working Paper*.
- Shin, Hyun Song.** 2010. "Financial Intermediation and the Post-Crisis Financial System." Citeseer.
- Simsek, Alp.** 2013. "Belief Disagreements and Collateral Constraints." *Econometrica*, 81(1): 1–53.
- Stein, Jeremy C.** 2012. "Monetary Policy as Financial Stability Regulation." *The Quarterly Journal of Economics*, 127(1): 57–95.
- Sunderam, Adi.** 2015. "Money Creation and the Shadow Banking System." *Review of Financial Studies*, 28(4): 939–77.
- Tirole, Jean.** 2012. "Overcoming Adverse Selection: How Public Intervention can Restore Market Functioning." *American Economic Review*, 102(1): 29–59.
- Vanasco, Victoria.** 2016. "The Downside of Asset Screening for Market Liquidity." *The Journal of Finance*, forthcoming.
- Woodford, Michael.** 1990. "Public Debt as Private Liquidity." *American Economic Review: Papers & Proceedings*, 80(2): 382–88.
- Zryumov, Pavel.** 2015. "Dynamic Adverse Selection: Time-Varying Market Conditions and Endogenous Entry." *Working Paper*.

A BASIC MODEL

Proof of Lemma 1. Let $\kappa_1(h_G, h_B; \mu_1, X_1)$ be the Lagrange multiplier associated to the budget constraint of program (P1). The FOC with respect to d is

$$\mu_1 - \kappa_1(h_G, h_B; \mu_1, X_1) \leq 0.$$

Moreover, the FOC with respect to m is

$$-\kappa_1(h_G, h_B; \mu_1, X_1)P_1^M(X_1) + \lambda_1^M(X_1)Z + (1 - \lambda_1^M(X_1))\alpha Z \leq 0.$$

Therefore,

$$\kappa_1(h_G, h_B; \mu_1, X_1) = \max \left\{ \mu_1, \frac{\lambda_1^M(X_1)Z + (1 - \lambda_1^M(X_1))\alpha Z}{P_1^M(X_1)} \right\}.$$

Define $\mu_1^B(X_1) \equiv \frac{\lambda_1^M(X_1)Z + (1 - \lambda_1^M(X_1))\alpha Z}{P_1^M(X_1)}$. This is the return from the market, which is the same for all agents. Therefore, if $\mu_1 < \mu_1^B(X_1)$, then $m > 0$ and $d = 0$. If $\mu_1 = \mu_1^B(X_1)$, the agent is indifferent between consuming and buying trees in the market. On the other hand, if $\mu_1 > \mu_1^B(X_1)$, then $m = 0$ and $d > 0$.

Moreover, the FOC with respect to s_G is

$$\kappa_1(h_G, h_B; \mu_1, X_1)P_1^M(X_1) - Z.$$

Therefore, if $\mu_1 \leq \mu_1^B(X_1)$, $\kappa_1(h_G, h_B; \mu_1, X_1) = \mu_1^B(X_1)$, and hence $\kappa_1(h_G, h_B; \mu_1, X_1)P_1^M(X_1) - Z < 0$ as long as $\lambda_1^M(X_1) < 1$, and $s_G = 0$. Let $\mu_1^S(X_1) \equiv \frac{Z}{P_1^M(X_1)}$. If $\mu_1 > \mu_1^S(X_1)$, then $s_G = h_G$, and zero otherwise. It is straightforward to see that $s_B = h_B$ for all μ_1 .

Finally, note that the demand for trees is

$$M(X_1) = \int_1^{\mu_1^B(X_1)} \frac{W_1}{P_1^M(X_1)} dG(\mu_1),$$

while the supply is

$$S(X_1) = \int_{\mu_1^S(X_1)}^{\mu_1^{\max}} H_G dG(\mu_1) + H_B.$$

As π increases, M increases for all prices, while S decreases. If π is high enough, agents with μ_1 have enough wealth to buy all the trees in the market and also consume. Therefore, they have to be indifferent between the return from the market, $\mu_1^B(X_1)$ and the utility from consumption, 1. Hence, $\mu_1^B(X_1) = 1$ and $P_1^M(X_1) = \lambda_1^M(X_1)Z + (1 - \lambda_1^M(X_1))\alpha Z$. ■

Proof of Lemma 2. Since agents always sell their bad trees, and agents with $\mu_1 > 1$ consume, while agents with $\mu_1 = 1$ consume and buy trees in the market (with a return of one), the utility from a unit of good tree is given by $\mu_1 P_1^M(X_1)$. Similarly, the utility from the endowment W_1 is given by μ_1 . On the other hand, only agents with $\mu_1 > \mu_1^S(X_1)$ sell their good trees, in which case they get a utility of $\mu_1 P_1^M(X_1)$. If they don't sell, they get a utility of Z in period 2. Note that

$\mu_1 > \mu_1^S(X_1) \Leftrightarrow \mu_1 P_1^M(X_1) > Z$. Hence, the utility from holding one unit of good tree un period 1 is given by $\max\{\mu_1 P_1^M(X_1), Z\}$. Therefore, the value function in period 1 is

$$V(h_G, h_B; \mu_1, X_1) = \mu_1 W_1 + \max\{\mu_1 P_1^M(X_1), Z\} h_G + \mu_1 P_1^M(X_1) h_B.$$

■

Proof of Proposition 1. First, let's calculate $D_\kappa \gamma_0^B(P_1^M + \kappa)$:

$$D_\kappa \gamma_0^B(P_1^M + \kappa) = \lim_{\varepsilon \rightarrow 0} \frac{\gamma_0^B(P_1^M + \varepsilon \kappa) - \gamma_0^B(P_1^M)}{\varepsilon} = E_0[\mu_1 \kappa(X)] > 0.$$

On the other hand, $D_\kappa \gamma_0^G(P_1^M + \kappa)$ is given by:

$$D_\kappa \gamma_0^G(P_1^M + \kappa) = E_{0,\alpha} \left[E_{0,\mu_1} \left[\mu_1 \kappa(X) \mid \mu_1 \geq \frac{Z}{P_1^M(X)} \right] \left[1 - G \left(\frac{Z}{P_1^M(X)} \right) \right] \right] > 0.$$

Hence, $D_\kappa \gamma_0^B(P_1^M + \kappa) > D_\kappa \gamma_0^G(P_1^M + \kappa) > 0$ as long as $P_1^M(X) < Z$ in some states with positive measure. Moreover, since $\gamma_0^B < \gamma_0^G$, then

$$\frac{D_\kappa \gamma_0^B(P_1^M + \kappa)}{\gamma_0^B(P_1^M)} > \frac{D_\kappa \gamma_0^G(P_1^M + \kappa)}{\gamma_0^G(P_1^M)} > 0.$$

■

Proof of Lemma 3. Let $\kappa_0(\xi)$ be the Lagrange multiplier associated to the budget constraint of program (P0). The FOC with respect to d is

$$1 - \kappa_0(\xi) \leq 0.$$

Moreover, the FOCs with respect to i_G and i_B are

$$\kappa_0(\xi) q_G(\xi) + \gamma_0^G \leq 0,$$

$$\kappa_0(\xi) q_B(\xi) + \gamma_0^B \leq 0.$$

Therefore

$$\kappa_0(\xi) = \max \left\{ 1, \frac{\gamma_0^G}{q_G(\xi)}, \frac{\gamma_0^B}{q_B(\xi)} \right\}.$$

First, note that by Assumption 2, $Z > 1$ and so $\gamma_0^G \geq Z > 1 = q_G(0)$. Second, since $\gamma_0^B > \gamma_0^G$, then $\frac{\gamma_0^G}{q_G(0)} > \frac{\gamma_0^B}{q_B(0)}$. By continuity of q_G and q_B , there exists $\xi_G(0, 1)$ such that $\max \left\{ 1, \frac{\gamma_0^G}{q_G(\xi)}, \frac{\gamma_0^B}{q_B(\xi)} \right\} = \frac{\gamma_0^G}{q_G(\xi)}$, and hence $i_G(\xi) = \frac{W_0}{q_G(\xi)}$ if and only if $\xi \leq \xi_G$. Agents with $\xi > \xi_G$ will choose to consume or produce bad trees. If $\frac{\gamma_0^B}{q_B(\xi_G)} \leq 1$ then $\max \left\{ 1, \frac{\gamma_0^G}{q_G(\xi)}, \frac{\gamma_0^B}{q_B(\xi)} \right\} = 1$ for all $\xi > \xi_G$, and ξ_G is defined such that the marginal investors is indifferent between producing trees and consuming, $\frac{\gamma_0^G}{q_G(\xi_G)} = 1$. If $\frac{\gamma_0^B}{q_B(\xi_G)} > 1$, then there exists $\xi_B \in (\xi_G, 1)$ such that $\max \left\{ 1, \frac{\gamma_0^G}{q_G(\xi)}, \frac{\gamma_0^B}{q_B(\xi)} \right\} = \frac{\gamma_0^B}{q_B(\xi)}$, and $i_B(\xi) = \frac{W_0}{q_B(\xi)}$ if and only

if $\xi \in (\xi_G, \xi_B]$. In this case, ξ_G is defined so that the marginal investor of good trees is indifferent between producing good and bad trees, $\frac{\gamma_0^G}{q_G(\xi_G)} = \frac{\gamma_0^B}{q_B(\xi_G)}$, and the marginal investor of bad trees is indifferent between producing bad trees and consuming, $\frac{\gamma_0^B}{q_B(\xi_B)} = 1$. ■

Proof of Proposition 2. From Proposition 1 we know that shadow values increase, but γ_0^B increases by more than γ_0^G when prices increase. If $I_0^B = 0$, then optimality implies that $\frac{\gamma_0^G}{q_G(\xi_G)} = 1$. Since γ_0^G increases, ξ_G has to increase, so I_G increases.

On the other hand, if $I_0^B > 0$, the optimality conditions are $\frac{\gamma_0^G}{q_G(\xi_G)} = \frac{\gamma_0^B}{q_B(\xi_G)}$ and $\frac{\gamma_0^B}{q_B(\xi_B)} = 1$. Hence, as prices increase, ξ_G decreases and ξ_B increases, giving the desired result. ■

Proof of Corollary 2.1. That λ_1^E decreases with prices is immediate from Proposition 2. To see that H_1 increases note two things. First, the mass of agents that invests increases since it is given by ξ_B . Second, since $q_B(\xi) < q_G(\xi) \forall \xi \in [0, 1]$, when ξ_G decreases to $\xi_G - \Delta$ for some $\delta > 0$, investment of the agents in $\xi \in (\xi_G, \xi_G - \Delta]$ goes up, since $\frac{W_0}{q_G(\xi)} < \frac{W_0}{q_B(\xi)}$. ■

Proof of Proposition 3. Since the cdf of α , F is continuous, the shadow values of good and bad trees are continuous in λ_1^E even if market prices are discontinuous in the state of the economy. Because I_0^G and I_0^B are continuous functions of the shadow values, the mapping T is continuous in λ_1^E . Moreover, since prices are increasing in λ_1^E , Proposition 2 implies that I_0^G is decreasing in λ_1^E while I_0^B is increasing, so the mapping T is decreasing in λ_1^E . Therefore, a fixed point of T exists and is unique. ■

Proof of Proposition 4. Note that the change in the distribution of F has no effect on the equilibrium in period 1 as long as λ_1^E doesn't change. So the key is to see how λ_1^E changes, which reduces to determining how the mapping T defined in (12) changes.

First note that since P_1^M is increasing in α for any value of λ_1^E and H_1 , the increase in F to \tilde{F} is mathematically equivalent to an increase in prices in each state α by $\phi(\alpha) \geq 0$. To see this note that

$$\text{Prob}_{\tilde{F}}(P_1^M(X) \leq P) \leq \text{Prob}_F(P_1^M(X) \leq P) \Rightarrow \text{Prob}_{\tilde{F}}(P_1^M(X) \leq P) = \text{Prob}_F(P_1^M(X) + \phi(X) \leq P).$$

By Proposition 2, an increase in prices reduces I_0^G and increases I_0^B as functions of λ_1^E , so that the mapping T decreases for all λ_1^E . Hence, the fixed point $\lambda_1^{E*} = T(\lambda_1^{E*})$ decreases. Note that the sign of the change in I_0^G is ambiguous since the partial equilibrium effect of prices reduces it but the endogenous change in λ_1^E increases it.

Because ξ_B increases and ξ_G decreases, total investment increases (recall that those who switch from producing good trees to bad trees face a lower cost, so they produce more trees). Moreover, because λ_1^E decreases, equilibrium prices decrease in all states, so the threshold α^* increases.

Finally, market fragility is ambiguous since the change in F reduces it but the endogenous change in λ_1^E increases it. The overall effect depends on parameters and functional forms. ■

Proof of Proposition 5. The change in the distribution G is equivalent to an increase in μ_1 to

$\mu_1 + \phi(\mu_1)$ with $\phi(\mu_1) \geq 0$. Keeping the prices fixed, the change in the shadow values are given by

$$\begin{aligned}\Delta\gamma_0^G &= E[\max\{(\mu_1 + \phi(\mu_1))P_1^M, Z\}] - E[\max\{\mu_1 P_1^M, Z\}], \\ &= E_{0,\alpha} \left[E_{0,\mu_1} \left[\phi(\mu_1)P_1^M | \mu_1 \geq \frac{Z}{P_1^M} \right] \left[1 - G\left(\frac{Z}{P_1^M}\right) \right] + \right. \\ &\quad \left. E_{0,\mu_1} \left[(\mu_1 + \phi(\mu_1))P_1^M - Z | \mu_1 < \frac{Z}{P_1^M} \ \& \ \mu_1 + \phi(\mu_1) \geq \frac{Z}{P_1^M} \right] \left[G\left(\frac{Z}{P_1^M}\right) - \tilde{G}\left(\frac{Z}{P_1^M}\right) \right] \right], \\ \Delta\gamma_0^B &= E[(\mu_1 + \phi(\mu_1))P_1^M] - E[\mu_1 P_1^M] = E[\phi(\mu_1)P_1^M].\end{aligned}$$

Since $(\mu_1 + \phi(\mu_1))P_1^M - Z < \phi(\mu_1)P_1^M$ when $\mu_1 < \frac{Z}{P_1^M}$, then $\Delta\gamma_0^B > \Delta\gamma_0^G$ and hence $\frac{\Delta\gamma_0^B}{\gamma_0^B} > \frac{\Delta\gamma_0^G}{\gamma_0^G}$. Therefore, I_0^G decreases and I_0^B increases as functions of λ_1^E , so the mapping T decreases for all λ_1^E . Hence, $\lambda_1^{E*} = T(\lambda_1^{E*})$ decreases.

Note that now the effect on P_1^M is ambiguous since more agents want to sell good trees, but there is a smaller fraction of good trees in the economy. Hence the effect on α^* and MF are ambiguous.

■

Proof of Proposition 6. Let $c_1 \equiv \frac{\mu_1^{\max}-1}{\mu_1^{\max}}Z$. Therefore, if $c > c_1$, the price sellers get is $P_1^S < Z - \frac{\mu_1^{\max}-1}{\mu_1^{\max}}Z = \frac{Z}{\mu_1^{\max}}$. Hence, no agent sells their good trees.

Consider now $c \leq c_1$. If $c = c_1$, and only good trees were produced, the shadow value of bad trees would be

$$\gamma_0^B = E \left[\mu_1 \frac{Z}{\mu_1^{\max}} \right] = E[\mu_1] \frac{Z}{\mu_1^{\max}}.$$

Fixing $E[\mu_1]$, note that if $\mu_1^{\max} \geq ZE[\mu_1]$, then $\gamma_0^B \leq 1$ even when $\lambda_1^E = 1$, so no agent will produce bad trees when the cost is in the neighborhood of c_1 . On the other hand, note that as $\mu_1^{\max} \rightarrow E[\mu_1]$, then $\gamma_0^B \rightarrow Z > 1$ when $\lambda_1^E = 1$. Hence, there exists $\bar{\mu}_1^{\max}$ such that if $\mu_1^{\max} \geq \bar{\mu}_1^{\max}$, there exists $c_2 \leq c_1$ such that if $c \in (c_2, c_1)$, only good trees are produced and there is some trade in the secondary market. On the other hand, if $\mu_1^{\max} < \bar{\mu}_1^{\max}$, $\gamma_0^B > 1$ and there is some production of bad trees.

Moreover, since an increase in c is equivalent to a reduction in prices, it is straightforward to see that if $c \in (c_2, c_1)$, then $\frac{\partial \gamma_0^B}{\partial c} < 0$. If $c < c_2$, note that a reduction of c moves the mapping T down, so $\frac{\partial \lambda_1^E}{\partial c} < 0$.

■

Sketch of Proof of Proposition 7. Consider an economy with F degenerated at $\tilde{\alpha}$. I choose $\tilde{\alpha}$ such that $T(\lambda_1^{E*}) < \lambda_1^{E*}$, where λ_1^{E*} is the fraction of good trees such that if $\lambda_1^E < \lambda_1^{E*}$ the market collapses. Denote the associated price at λ_1^{E*} as P_1^{M*} .

Now consider another economy in which α is distributed according to

$$\alpha = \tilde{\alpha} + u, \quad u \sim \mathcal{U}[-\epsilon, \epsilon],$$

where \mathcal{U} denotes the uniform distribution. The objective is to show that as $\epsilon \rightarrow 0$, $\text{Var}(P_1^M(\alpha|\epsilon)) \rightarrow 0$.

Suppose $\text{Var}(P_1^M(\alpha|\epsilon)) \rightarrow 0$ as $\epsilon \rightarrow 0$. This means that $P_1^M(\alpha|\epsilon) \rightarrow \tilde{P}_1^M$, for some \tilde{P}_1^M . Note that $P_1^M(\alpha|\epsilon)$ needs to be higher than αZ for all α as $\epsilon \rightarrow 0$, since otherwise agents would not produce bad trees, which cannot be an equilibrium. Moreover, the mapping $T(\lambda_1^E|\epsilon)$ is continuous in ϵ and converges pointwise to $T(\lambda_1^E)$ (the mapping of an economy with F degenerate at $\tilde{\alpha}$), except at λ_1^{E*} . Therefore, $\tilde{P}_1^M = P_1^{M*}$. But then $T(\lambda_1^{E*}(\epsilon)|\epsilon) < \lambda_1^{E*}(\epsilon)$, so this cannot be an equilibrium. Therefore, $P_1^M(\alpha|\epsilon) \not\rightarrow \tilde{P}_1^M$ and $\text{Var}(P_1^M(\alpha|\epsilon)) \not\rightarrow 0$. In particular, there exists $\alpha^*(\epsilon)$ such that $P_1^M(\alpha|\epsilon) = \alpha Z$ for all $\alpha < \alpha^*(\epsilon)$, and $P_1^M(\alpha|\epsilon) > \alpha Z$ for all $\alpha \geq \alpha^*(\epsilon)$.

Moreover, since λ_1^{E*} and market prices in period 1 are continuous in $\tilde{\alpha}$, there exists an open set $B \subset [0, 1]$ such that if $\tilde{\alpha} \in B$, then $\text{Var}(P_1^M(\alpha|\epsilon)) \rightarrow 0$. ■

Sketch of Proof of Proposition 8. I need to show that $MF(\epsilon) \rightarrow \zeta$ as $\epsilon \rightarrow 0$, where ζ is the sunspot. Since $\lambda_1^E(\epsilon) \rightarrow \lambda_1^{E*}$, it holds that $E[P_1^M(\lambda_1^E(\epsilon), \alpha)] \rightarrow E[P_1^M(\lambda_1^{E*}, \tilde{\alpha})]$. Note that

$$E[P_1^M(\lambda_1^E(\epsilon), \alpha)] = \int_{\alpha^*(\epsilon)}^{\hat{\alpha}+\epsilon} \frac{P_1^M(\lambda_1^E(\epsilon), \alpha)}{2\epsilon} d\alpha + \int_{\hat{\alpha}-\epsilon}^{\alpha^*(\epsilon)} \frac{\alpha Z}{2\epsilon} d\alpha.$$

Hence,

$$E[P_1^M(\lambda_1^E(\epsilon), \alpha)] \geq \text{Prob}(\alpha \geq \alpha^*(\epsilon))P_1^M(\lambda_1^E(\epsilon), \alpha^*(\epsilon)) + \text{Prob}(\alpha < \alpha^*(\epsilon))(\hat{\alpha} - \epsilon)Z,$$

and

$$E[P_1^M(\lambda_1^E(\epsilon), \alpha)] \leq \text{Prob}(\alpha \geq \alpha^*(\epsilon))P_1^M(\lambda_1^E(\epsilon), \hat{\alpha} + \epsilon) + \text{Prob}(\alpha \geq \alpha^*(\epsilon))\alpha^*(\epsilon)Z.$$

As ϵ goes to zero we get

$$\lim_{\epsilon \rightarrow 0} E[P_1^M(\lambda_1^E(\epsilon), \alpha)] = E[P_1^M(\lambda_1^{E*}, \alpha)] = (1 - \tilde{\zeta})P_1^M(\lambda_1^{E*}, \tilde{\alpha}) + \tilde{\zeta}\tilde{\alpha}Z.$$

Hence, $\zeta = \tilde{\zeta}$. ■

B EXTENDED MODEL AND POSITIVE IMPLICATIONS

Proof of Lemma 4.

First I formally define the robustness of the solution to (P1') to small perturbations of η , which is a modification of the analysis in Kurlat (2016) adjusted to the present setting.

Definition 7. A solution to (P1') is robust if there exists a sequence of strictly positive real numbers $\{z_n\}_{n=1}^{\infty}$ and a sequence of consumption, investment, buying and selling decisions

$\{d^n, i_k^n, m^n, s_G^n, s_b^n, h_G^n, h_B^n, k'^n\}$ such that, defining

$$\eta^n(\omega; X_1) = \eta(\omega; X_1) + z_n, \quad \forall \omega \in \Omega$$

1. $\{d^n, i_k^n, m^n, s_G^n, s_b^n, h_G^n, h_B^n, k'^n\}$ solve the program

$$V(h_G, h_B; A, X_1) = \max_{\substack{d, i_k, m, s_G, \\ s_B, h'_G, h'_B, k'}} d + V_2(h'_G, h'_B, k'; X_2), \quad (\text{P1'.A})$$

subject to

$$d + i_K + \sum_{\omega \in \Omega} P_1^M(\omega) m(\omega) \leq W_1 + \sum_{\omega \in \Omega} P_1^M(\omega) (s_G(\omega) + s_B(\omega)) \eta^n(\omega; X_1),$$

$$h'_G = h_G + \sum_{\omega \in \Omega} \lambda_1^M(\omega; X_1) m(\omega) - \sum_{\omega \in \Omega} s_G(\omega) \eta^n(\omega; X_1),$$

$$h'_B = h_B + \sum_{\omega \in \Omega} (1 - \lambda_1^M(\omega; X_1)) m(\omega) - \sum_{\omega \in \Omega} s_B(\omega) \eta^n(\omega; X_1),$$

$$k' = A i_K,$$

$$\sum_{\omega \in \Omega} s_G(\omega) \eta^n(\omega; X_1) \leq h_G, \quad \text{and} \quad \sum_{\omega \in \Omega} s_B(\omega) \eta^n(\omega; X_1) \leq h_B,$$

$$d \geq 0, \quad i_K \geq 0,$$

$$m(\omega) \geq 0, \quad s_G(\omega) \in [0, h_G], \quad s_B(\omega) \in [0, h_B], \quad \forall \omega \in \Omega.$$

2. $z_n \rightarrow 0$

3. $\{d^n, i_K^n, m^n, s_G^n, s_B^n, h'_G, h'_B, k'^n\} \rightarrow \{d, i_K, m, s_G, s_B, h'_G, h'_B, k'\}$.

Let $\mu_1(A, X_1)$ be the Lagrange multiplier associated to the budget constraint in program (P1'). The FOC with respect to d is

$$1 - \mu_1(A, X_1) \leq 0.$$

Moreover, the FOC with respect to $m(\omega)$ is

$$-\mu_1(A, X_1) P_1^M(\omega) + \lambda_1^M(\omega; X_1) Z + (1 - \lambda_1^M(\omega; X_1)) \alpha Z \leq 0,$$

while the FOC with respect to i_K and k' combined is

$$-\mu_1(A, X_1) + Ar(X_2) \leq 0.$$

Therefore

$$\mu_1(A, X_1) = \max \left\{ 1, Ar(X_2), \left\{ \frac{\lambda_1^M(\omega; X_1) Z + (1 - \lambda_1^M(\omega; X_1)) \alpha Z}{P_1^M(\omega)} \right\}_{\omega \in \Omega^B} \right\}.$$

Now, define $\mu_1^B(X_1) \equiv \max_{\omega \in \Omega} \frac{\lambda_1^M(\omega; X_1) Z + (1 - \lambda_1^M(\omega; X_1)) \alpha Z}{P_1^M(\omega)}$. Moreover, define $A_1^B(X_1) \equiv \frac{\max\{1, \mu_1^B(X_1)\}}{r(K(X_1))}$. If $A \leq A_1^B(X_1)$, the return from investing in capital, $Ar(K(X_1))$ is too low compared to the return of the best alternative between consuming and buying trees in some market. Hence, agents with $A \leq A_1^B(X_1)$ do not produce capital, and consume or buy trees. Moreover, since $P_1^M < Z$ in all markets in equilibrium, and ruling out arbitrage opportunities that are not consistent with equilibrium, agents do not sell good trees to consume or to buy trees in the markets, so that $s_G(\omega) = 0$ if $A \leq A_1^B(\omega)$.

On the other hand, if $A > A_1^B(X_1)$, agents produce capital and do not consume or buy trees.

Now let's switch to selling decisions. The decision to sell bad trees is straightforward: agents offer bad trees in all markets with the highest prices until they all the holdings are sold. On the other hand, the decision to sell is more involved and uses the definition of robust solution.

First, let's show that optimality requires that if $\omega' > \omega$, then if $s_G(\omega) > 0$, $s_G(\omega') = h_G$. Suppose this didn't hold. Then, the agent can increase its utility by reducing $s_G(\omega)$ by ϵ and increasing $s_G(\omega')$ by $\epsilon \frac{\eta(\omega; X_1)}{\eta(\omega'; X_1)}$, for some $\epsilon > 0$. This policy is feasible and non-trivial unless $\eta(\omega; X_1) = 0$ or $\eta(\omega'; X_1) = 0$. So consider a sequence $\eta^n(\omega; X_1) > 0$ and $\eta^n(\omega'; X_1) > 0$. The solution to (P1'.A) must satisfy that if $s_G^n(\omega) > 0$ then $s_G^n(\omega') = h_G$ from the previous argument. But then $s_G^n(\omega') \rightarrow h_G$. Hence, agents sell there good trees only in markets that feature a high enough price.

Second, the FOC with respect to $s_G(\omega)$ is

$$\mu_1(A, X_1)P_1^M(\omega)\eta(\omega; X_1) - Z\eta(\omega; X_1).$$

Define $\tilde{A}(\omega; X_1) \equiv \frac{Z}{r(K(X_1))P_1^M(\omega)}$. It is straightforward to see that $\tilde{A}_1^S(\omega; X_1)$ is decreasing in ω . Then, an agent with productivity A sells in market ω if and only if $A > \tilde{A}_1^S(\omega; X_1)$ and $\omega \leq \tilde{\omega}$, where $\tilde{\omega}$ is defined in (19). Therefore

$$A_1^S(\omega) \equiv \begin{cases} \tilde{A}_1^S(\omega; X_1) & \text{if } \omega \geq \tilde{\omega} \\ A^{\max} & \text{if } \omega < \tilde{\omega}. \end{cases}$$

■

Partial Equilibrium

If W_1 is sufficiently high, then buyers have enough wealth to drive down the return of the markets in which they participate to 1, so that they both buy trees and consume. In that case, active markets satisfy

$$P_1^M(\omega) = \lambda_1^M(\omega; X_1)Z + (1 - \lambda_1^M(\omega; X_1))\alpha Z, \quad (55)$$

with

$$\begin{aligned} \lambda_1^M(\omega; X_1) &= \frac{[1 - G(A_1^S(\omega; X_1))]\lambda_E}{[1 - G(A_1^S(\omega; X_1))]\lambda_E + (1 - \lambda_E)}, \\ &= \frac{\left[1 - G\left(\frac{Z}{r(K(X_1))P_1^M(\omega)}\right)\right]\lambda_E}{\left[1 - G\left(\frac{Z}{r(K(X_1))P_1^M(\omega)}\right)\right]\lambda_E + (1 - \lambda_E)}. \end{aligned} \quad (56)$$

The intersection of these two curves defines the set of partial equilibria analogous to the previous section. I will show that a subset of these partial equilibria will determine active markets. First, recall that for each state X_1 , the set of intersections between (55) and (56) could be one of three cases:

1. the two curves intersect only once, which can be at a price for which good and bad trees are traded, or a price at which only bad trees are traded, as in figure 3(b);

2. the two curves intersect three times, as depicted in figure 3(a);
3. the two curves intersect twice, as in figure 4(b).

First I show that active markets have to feature prices that belong to the set of partial equilibria. Then I establish what subset of these prices are actually active markets in equilibrium.

Suppose there was an active market in which $P_1^M(\omega) > \lambda_1^M(\omega; X_1)Z + (1 - \lambda_1^M(\omega; X_1))\alpha Z$. Then, buyers would get a higher utility from consuming than from buying in this market. Hence, $m(\omega) = 0$, which contradicts that the market was active. On the other hand, suppose that there are some markets in which $P_1^M(\omega) < \lambda_1^M(\omega; X_1)Z + (1 - \lambda_1^M(\omega; X_1))\alpha Z$. Define $\bar{\omega} \equiv \max_{\omega \in \Omega^B} \frac{\lambda_1^M(\omega; X_1)Z + (1 - \lambda_1^M(\omega; X_1))\alpha Z}{P_1^M(\omega)}$. Then, buyers would want to spend all their liquid wealth in buying trees from market $\bar{\omega}$, but because the endowment W_1 is big, the demand for trees is greater than the supply, which is inconsistent with equilibrium. Hence, only markets that satisfy $P_1^M(\omega) = \lambda_1^M(\omega; X_1)Z + (1 - \lambda_1^M(\omega; X_1))\alpha Z$ can have active trading.

This immediately implies that in case 1., there is only one active market in equilibrium. Can there be rationing? The answer is no. Suppose there were rationing. Then, there would be some agents with high investment opportunity that were not able to sell all they wanted. Then, from Lemma (4) we know that this agent will offer its trees in a market with a slightly lower price. Since the rationing in the markets is uniform, the fraction of good trees offered at that price is still given by

$\lambda_1^M(\omega; X_1) = \frac{\left[1 - G\left(\frac{Z}{r(K(X_1))P_1^M(\omega)}\right)\right]^{\lambda_E}}{\left[1 - G\left(\frac{Z}{r(K(X_1))P_1^M(\omega)}\right)\right]^{\lambda_E + (1 - \lambda_E)}}$. But then, in that market $P_1^M(\omega') < \frac{\left[1 - G\left(\frac{Z}{r(K(X_1))P_1^M(\omega')}\right)\right]^{\lambda_E}}{\left[1 - G\left(\frac{Z}{r(K(X_1))P_1^M(\omega')}\right)\right]^{\lambda_E + (1 - \lambda_E)}}$, which we already know cannot be active in equilibrium. Hence, there cannot be rationing in this case.

In case 2., I now show that only the market with the highest price can be active. Suppose another market is active. Then, in a robust solution, seller are offering trees in markets with higher prices also, even though they are inactive. There are markets with price just below the highest partial equilibrium price such that $P_1^M(\omega) < \lambda_1^M(\omega; X_1)Z + (1 - \lambda_1^M(\omega; X_1))\alpha Z$. But then buyers would prefer to trade in this market instead, a contradiction. Hence, only the highest price market is active. Moreover, for the same reasons as in case 1., there is no rationing in equilibrium.

Finally, in case 3. both markets can be active in equilibrium, since there is no possible deviation of buyers and sellers that can rule out any of them. Moreover, the lower price market is active only if there is rationing in the higher price market. In fact, any rationing level in the high price market is consistent with partial equilibrium. However, no rationing can occur in the low price market, since some rationed agents, those with high A , would be willing to sell at a slightly lower price, a contradiction.

Sketch of Proof of Proposition 9.

The proof has two parts. First, I show that there is an equilibrium of the economy starting in period 1 for any value of X_1 . Then, I move to period 0 and show that an equilibrium of the full economy exists.

Given X_1 , the mapping $T_K(L; X_1)$ has the following properties:

1. if K is "low", then the return on capital is "high" so many agents sell their trees in order to

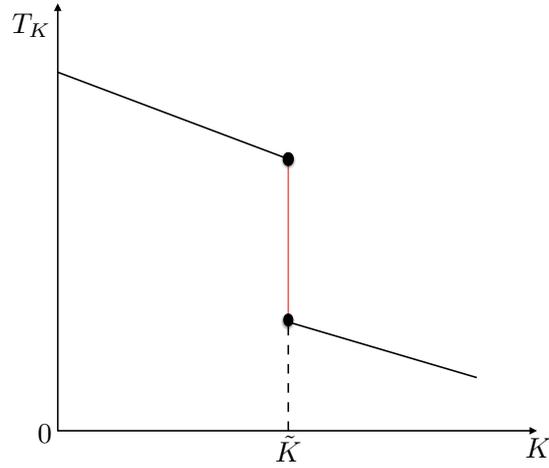


FIGURE 8: Mapping $T_K(K; X_1)$

invest. Then there is a unique active market that features a “high” price, so T_K is high;

2. if K is “high”, then the return on capital is “low” so few agents are willing to sell their trees in order to invest, hence only the market that trades bad trees is active and the price is “low”, so T_K is low;

3. there exists a unique \tilde{K} such the economy is in case 3., and two markets can be active.

Figure 8 shows the mapping $T_K(K; X_1)$. For $K < \tilde{K}$, T_K is high. For $K > \tilde{K}$, T_K is low. Note that T_K is always decreasing since the higher K is the lower the return on investment and hence total investment. Finally, if $K = \tilde{K}$, T_K can take a continuum of values indexed by η . This shows that extending the definition of equilibrium is necessary to guarantee that the mapping T_K is continuous in K and an equilibrium of the economy starting in period 1 always exists.

So the equilibrium can take one of three forms. If the state α is high, then the market for trees is liquid, so that the price is high and the equilibrium level of capital is high. This is the case depicted in figure 9(a). If the state α is low, then the market for trees collapses, only bad trees are traded at a low price, and the equilibrium capital is low. Figure 9(b) shows this case. Finally, if the state α is “middle-range”, then the economy features two markets and the high price market is rationed. How is the amount of rationing determined? So that total investment is equal to \tilde{K} . Figure 10 shows this case.

So I established that an equilibrium of the economy in period 1 always exists. Let $K(\lambda_1^E, H_1; \alpha)$ denote the equilibrium capital as a function of the fraction of good trees in the economy, the total amount of trees, and the aggregate state α . Note that K is increasing in H_1 . Now I switch to the determination of equilibrium period 0.

Unlike in the basic model where the economy was linear in H_1 , finding an equilibrium of this economy requires finding a fixed point of the two-dimensional mapping

$$T(\lambda_1^E, H_1) = \left[\begin{array}{c} \frac{I_0^G(\lambda_1^E, H_1)}{I_0^G(\lambda_1^E, H_1) + I_0^B(\lambda_1^E, H_1)} \\ I_0^G(\lambda_1^E, H_1) + I_0^B(\lambda_1^E, H_1) \end{array} \right]$$

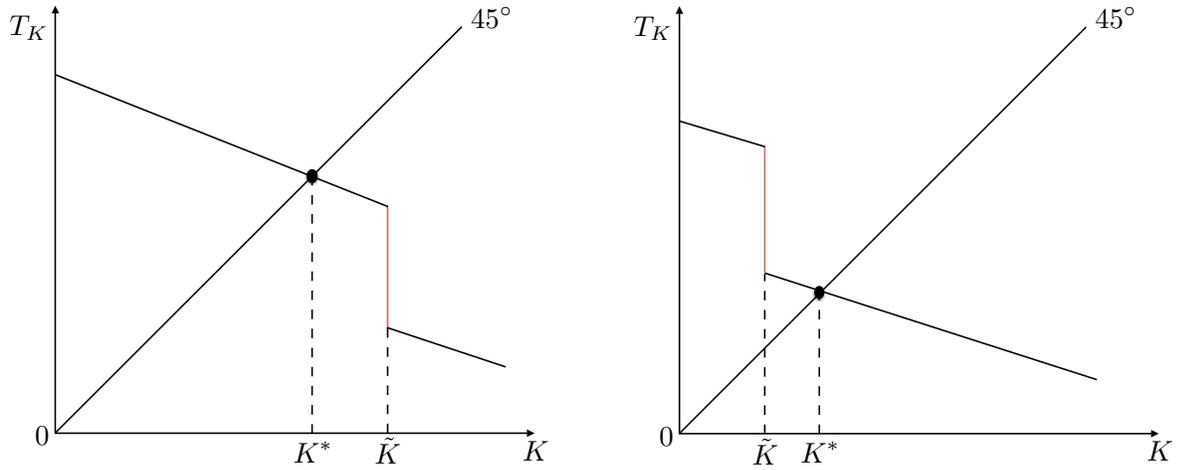


FIGURE 9: Equilibrium in period 1. (a) High state α : high price. (b) Low state α : low price.

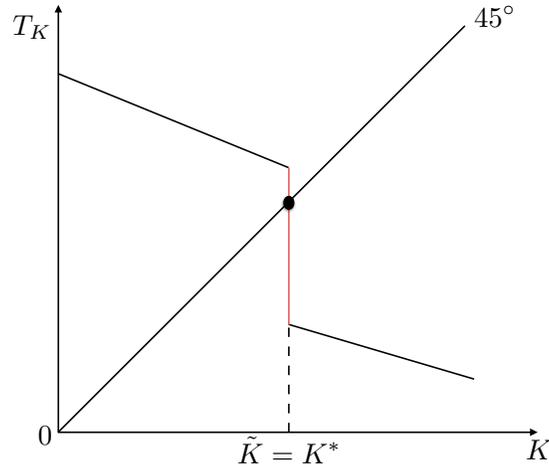


FIGURE 10: Mapping $T_K(K; X_1)$

Consider first the mapping $T_H(H_1; \lambda_1^E) \equiv I_0^G(\lambda_1^E H_1) + I_0^B(\lambda_1^E, H_1)$, which takes λ_1^E as given. Since higher H_1 increases K , which reduces the liquidity services of trees, T_H is decreasing in H_1 , so equilibrium implies that $H_1(\lambda_1^E)$ is a continuous function of λ_1^E . Hence, finding an equilibrium of the economy reduces to finding a fixed point of the mapping $T_\lambda(\lambda_1^E) = \frac{I_0^G(\lambda_1^E, H_1(\lambda_1^E))}{I_0^G(\lambda_1^E, H_1(\lambda_1^E)) + I_0^B(\lambda_1^E, H_1(\lambda_1^E))}$. Since T_λ is continuous in λ_1^E and belongs to the compact space $[0, 1]$, a fixed point exists. However, there can be multiple fixed points. Though this could be a potentially interesting phenomena (generating a channel for self-fulfilling equilibria), it is beyond the scope of the paper. I will select the equilibrium that features the highest fraction of good trees. Note that this equilibrium is stable, since T_λ crosses the 45° from above.

■

Proof of Lemma 5. To prove this result it is enough to show that T_K increases for all K .

First note that for a fixed K (and hence a fixed return on capital), the equilibrium price in market ω_H is increasing in α and H_G . Consider two economies in period 1 with the same λ_1^E and H_1 , but one has quality of bad trees α and the other α' , with $\alpha' > \alpha$. For a fixed K , the demand of trees is higher in the economy with α' , while the supplies are the same. Moreover, since the supply of trees

is decreasing in K (since $r(K)$ is decreasing in K), $\tilde{K}(\alpha') > \tilde{K}(\alpha)$. Therefore, the equilibrium level of capital is higher in the economy with state α' , strictly so if the market ω_H is active.

Similarly, consider two economies with the same H_B and α , but one has H_G of good trees and the other H'_G , with $H'_G > H_G$. For a fixed K , market liquidity increases for two reasons. First, there are more trees in the economy, so for the same price, volume traded is higher. Second, for a fixed H_B , the higher the fraction of good trees in the economy, and hence the higher the price in the market ω_H . Hence, the mapping T_K increases with H_G , so equilibrium capital increases.

■

Proof of Lemma 6.

First I need to show that increases in Z^Y and A increase $\mu_1(A, X_1)$. Then I show that this increase in $\mu_1(A, X_1)$ generates a reduction in λ_1^E and an increase in H_1 .

For a fixed level of capital, an increase in Z^Y increases investment, both because more agents find it optimal to invest and because more agents sell their good trees to invest. Therefore, for each state X_1 , T_K increases and hence the equilibrium level of capital increases. However, what matters for period 0 is what happens with the return on capital, since $\mu_1(A, X_1) = \max\{1, Ar(X_2)\}$. Since $r(K(X_1)) = Z^Y f'(K(X_1))$, r increases because of Z^Y but decreases because of $f'(K(X_1))$. Suppose $r(K(X_1))$ decreases as a result, then less agents invest and less agents sell good trees to invest. But then total investment decreases, which contradicts that $K(X_1)$ increases.

Note that an increase of A to ϕA for some $\phi > 1$ has similar implications and enters the expression for $\mu_1(A, X_1)$ in an analogous ways as Z^Y , so $\mu_1(A, X_1)$ also increases when A increases to ϕA . Moreover, $P_1^M(X_1)$ increases because more agents sell their good trees (for fixed λ_1^E and H_1).

Now let's return to period 0. Recall that shadow values are given by

$$\begin{aligned}\gamma_0^G &= E[\max\{\mu_1(A, X_1)P_1^M(\omega_H; X_1)\eta(X_1) + (1 - \eta(X_1))Z, Z\}], \\ \gamma_0^B &= E[\mu_1(A, X_1)[\eta(X_1)P_1^M(\omega_H; X_1) + (1 - \eta(X_1))P_1^M(\omega_L; X_1)]].\end{aligned}$$

For fixed λ_1^E and H_1 , γ_0^G and γ_0^B increase when μ_1 increases, but γ_0^B increases by more. Therefore, $H(\lambda_1^E)$ defined as the fixed point of $T_H(H_1; \lambda_1^E) \equiv I_0^G(\lambda_1^E, H_1) + I_0^B(\lambda_1^E, H_1)$ taking λ_1^E as given, increases.

Finally, I need to show that $T_\lambda(\lambda_1^E) \equiv \frac{I_0^G(\lambda_1^E, H(\lambda_1^E))}{I_0^G(\lambda_1^E, H(\lambda_1^E)) + I_0^B(\lambda_1^E, H(\lambda_1^E))}$ decreases as a function of λ_1^E . From previous analysis we know that if μ_1 increases, then $T_\lambda(\lambda_1^E)$ decreases. Hence, $T_\lambda(\lambda_1^E)$ can increase only if the increase in $H(\lambda_1^E)$ provides so much liquidity in period 1 that μ_1 decreases. But a decrease in μ_1 contradicts that H increases in the first place, hence T_λ decreases.

Finally, since I am selecting the equilibrium with the highest λ_1^E , and this equilibrium happens in the intersection of T_λ with the 45° line from above, a decrease in T_λ reduces the equilibrium λ_1^E .

■

C NORMATIVE IMPLICATIONS

Proof of Lemma 7. In first best, only agents with $A = A^{\max}$ invest and they do so until $A^{\max}r(K(X_1)) = 1$. Therefore, $\mu_1(A, X_1) = 1$ for all A, X_1 , and $i_0 = 0$.

In laissez-faire, $\mu_1(A^{\max}, X_1) > 1$ for all X_1 , hence $E[\mu_1(A, X_1)] > 1$ and $i_0 > 0$. ■

Proof of Proposition 10.

It is straightforward to see that for state X_1 , the mapping $T_K(K; X_1)$ increases with B_0 . Therefore, $K(X_1)$ increases with B_0 (this effect is strict except for states in which $K = \tilde{K}$). This immediately implies that $TL(X_1)$ increases. However, because the return on capital decreases, the supply of good trees decreases, so that $P_1^M(\omega_H; X_1)$ decreases, strictly so except of states in which $K = \tilde{K}$, where $\eta(X_1)$ decreases. Therefore, $\mu_1(A, X_1)$ decreases for every state (A, X_1) , strictly so except for states with $K = \tilde{K}$.

■

Proof of Proposition 11.

It follows directly from Lemma 6 since from the point of view of period 0 it only matters that $\mu_1(A, X_1)$ decreased as a function of λ_1^E and H_1 .

■

Proof of Corollary 11.1.

Let $\tilde{B}_0 < B_0$ be the amount of government bonds bought by the foreign agent. The government collects revenues from selling to this agent of $P_0^{GB}\tilde{B}_0$, which are distributed after investment takes place. Hence, consumption in period 0 goes up and aggregate variables in period 1 are equivalent to those in an economy in which the government issues $B'_0 \equiv B_0 - \tilde{B}_0$ bonds.

■

Optimal Policy: Government Bonds

The planner solves

$$W = D_0 + E_0[D_1(X_1) + D_2(X_2)],$$

subject to

$$\begin{aligned} D_0 &= (1 - \xi_B)W_0, \\ D_1(X_1) &= G(A_1^B(X_1))W_1 - P_1^M(\omega_H; X_1)\eta(X_1) \left[[1 - G(A_1^S(X_1))]H_G + [1 - G(A_1^B(X_1))]H_B \right] - \\ &\quad P_1^M(\omega_L; X_1)(1 - \eta(X_1))[1 - G(A_1^B(X_1))]H_B, \\ D_2(X_2) &= [H_G + \alpha H_B]Z + f(K) - q_{GB}B_0, \end{aligned}$$

$$K(X_1) = \int_{A_1^B(X_1)}^{A^{\max}} A[W_1 + [P_1^M(\omega_H; X_1)\eta(X_1) + P - 1^M(\omega_L; X_1)(1 - \eta(X_1))]]H_B + B_0]dG(A) + \int_{A_1^S(X_1)}^{A^{\max}} AP_1^M(\omega_H; X_1)\eta(X_1)H_G dG(A),$$

$$H_G = \int_0^{\xi_G} \frac{W_0}{q_G(\xi)} d\xi,$$

$$H_B = \int_{\xi_G}^{\xi_B} \frac{W_0}{q_B(\xi)} d\xi,$$

and the equilibrium conditions

$$\frac{\gamma_0^G}{q_G(\xi_G)} = \frac{\gamma_0^B}{q_B(\xi_B)},$$

$$\frac{\gamma_0^B}{q_B(\xi_B)} = \frac{\gamma_0^{GB}}{P_0^{GB}} = 1.$$

The first order condition is

$$\frac{\partial D_0}{\partial B_0} + E \left[\frac{\partial D_1(X_1)}{\partial B_0} + \frac{\partial D_2(X_2)}{\partial B_0} \right] = 0,$$

where

$$\frac{\partial D_0}{\partial B_0} = -\frac{\partial \xi_B}{\partial B_0} W_0,$$

$$\begin{aligned} \frac{\partial D_1(X_1)}{\partial B_0} &= g(A_1^B(X_1)) \frac{\partial A_1^B(X_1)}{\partial B_0} W_1 - \left[\frac{\partial P_1^M(\omega_H; X_1)}{\partial B_0} \eta(X_1) + P_1^M(\omega_H; X_1) \frac{\partial \eta(X_1)}{\partial B_0} \right] \\ &[[1 - G(A_1^S(X_1))]H_G + [1 - G(A_1^B(X_1))]H_B] - P_1^M(\omega_H; X_1)\eta(X_1) \left[-g(A_1^S(X_1)) \frac{\partial A_1^S(X_1)}{\partial B_0} H_G + \right. \\ &\quad \left. [1 - G(A_1^S(X_1))] \frac{\partial H_G}{\partial B_0} - g(A_1^B(X_1)) \frac{\partial A_1^B(X_1)}{\partial B_0} H_B + [1 - G(A_1^B(X_1))] \frac{\partial H_B}{\partial B_0} \right] + \\ &P_1^M(\omega_L; X_1) \frac{\partial \eta(X_1)}{\partial B_0} [1 - G(A_1^B(X_1))]H_B + P_1^M(\omega_L; X_1)(1 - \eta(X_1))g(A_1^B(X_1)) \frac{\partial A_1^B(X_1)}{\partial B_0} H_B - \\ &P_1^M(\omega_L; X_1)(1 - \eta(X_1))[1 - G(A_1^B(X_1))] \frac{\partial H_B}{\partial B_0} + g(A_1^B(X_1)) \frac{\partial A_1^B(X_1)}{\partial B_0} B_0 - [1 - G(A_1^B(X_1))], \\ \frac{\partial D_2(X_2)}{\partial B_0} &= \left[\frac{\partial H_G}{\partial B_0} + (1 - \delta) \frac{\partial H_B}{\partial B_0} \right] Z + r(K) \frac{\partial K}{\partial B_0} - q_{GB}, \end{aligned}$$

and

$$\begin{aligned}
\frac{\partial K(X_1)}{\partial B_0} &= -\frac{\partial A_1^B(X_1)}{\partial B_0} A_1^B(X_1) [W_1 + [P_1^M(\omega_H; X_1)\eta(X_1) + P_1^M(\omega_L; X_1)(1 - \eta(X_1))]H_B + B_0] \\
&\quad g(A_1^B(X_1)) + \int_{A_1^B(X_1)}^{A^{\max}} A \left[\left[\frac{\partial P_1^M(X_1)}{\partial B_0} \eta(X_1) + [P_1^M(\omega_H; X_1) - P_1^M(\omega_L; X_1)] \frac{\partial \eta(X_1)}{\partial B_0} \right] H_B + \right. \\
&\quad \left. [P_1^M(\omega_H; X_1)\eta(X_1) + P_1^M(\omega_L; X_1)(1 - \eta(X_1))] \frac{\partial H_B}{\partial B_0} + 1 \right] G(dA) - \\
&\quad \frac{\partial A_1^S(X_1)}{\partial B_0} A_1^S(X_1) P_1^M(X_1) \eta(X_1) H_G g(A_1^S(X_1)) + \\
&\quad \int_{A_1^S(X_1)}^{A^{\max}} A \left[\left[\frac{\partial P_1^M(\omega_H; X_1)}{\partial B_0} \eta(X_1) + P_1^M(\omega_H; X_1) \frac{\partial \eta(X_1)}{\partial B_0} \right] H_G + P_1^M(\omega_H; X_1) \eta(X_1) \frac{\partial H_G}{\partial B_0} \right] G(dA)
\end{aligned}$$

After some algebra, it simplifies to

$$\begin{aligned}
\frac{\partial \mathbb{W}}{\partial B_0} &= E \left[\int_{A_1^B(X_1)}^{A^{\max}} Ar(K(X_1)) dG(A) + G(A_1^B(X_1)) \right] + \\
&\quad E \left[\left[\int_{A_1^B(X_1)}^{A^{\max}} Ar(K(X_1)) dG(A) + G(A_1^B(X_1)) \right] \right. \\
&\quad \left. \left[\frac{\partial P_1^M(X_1)}{\partial B_0} \eta(X_1) + [P_1^M(\omega_H; X_1) - P_1^M(\omega_L; X_1)] \frac{\partial \eta(X_1)}{\partial B_0} \right] \right] H_B + \\
&\quad E \left[\int_{A_1^S(X_1)}^{A^{\max}} Ar(K(X_1)) dG(A) \left[\frac{\partial P_1^M(\omega_H; X_1)}{\partial B_0} \eta(X_1) + P_1^M(\omega_H; X_1) \frac{\partial \eta(X_1)}{\partial B_0} \right] - \right. \\
&\quad \left. \frac{\partial \eta(X_1)}{\partial B_0} [1 - G(A_1^S(X_1))] Z \right] H_G = 1 + q_{GB},
\end{aligned}$$

or

$$\begin{aligned}
\frac{\partial \mathbb{W}}{\partial B_0} &= \gamma_0^{GB} + E \left[\frac{\partial \tilde{\gamma}_1^G(X_1)}{\partial P_1^M(\omega_H; X_1)} \frac{\partial P_1^M(\omega_H; X_1)}{\partial B_0} + \frac{\partial \tilde{\gamma}_1^G(X_1)}{\partial \eta(X_1)} \frac{\partial \eta(X_1)}{\partial B_0} \right] H_G + \\
&\quad E \left[\frac{\partial \tilde{\gamma}_1^B(X_1)}{\partial P_1^M(\omega_H; X_1)} \frac{\partial P_1^M(\omega_H; X_1)}{\partial B_0} + \frac{\partial \tilde{\gamma}_1^B(X_1)}{\partial \eta(X_1)} \frac{\partial \eta(X_1)}{\partial B_0} \right] H_B = 1 + q_{GB}.
\end{aligned}$$

D INFINITE HORIZON

It is possible to guess and verify that the value function of an agent with portfolio given by $\{h_G, h_B, k, b\}$ and investment opportunity A , when the aggregate state is $X = \{\lambda_E, H, K, B; \alpha\}$, is given by

$$V(h_G, h_B, k, b; A, X) = \tilde{\gamma}_G(A, X)h_G + \tilde{\gamma}_B(A, X)h_B + \tilde{\gamma}_K(A, X)k + \tilde{\gamma}_{GB}b,$$

where

$$\begin{aligned}
\tilde{\gamma}_G(A, X) &= \max \{ \mu(A, X)\eta(X)P_M(\omega_H; X) + (1 - \eta(X))[Z + (1 - \delta_H)\gamma_G(X)], Z + (1 - \delta_H)\gamma_G(X) \}, \\
\tilde{\gamma}_B(A, X) &= \mu(A, X)[\eta(X)P_M(\omega_H; X) + (1 - \eta(X))P_M(\omega_L; X)], \\
\tilde{\gamma}_K(A, X) &= r(X), \\
\tilde{\gamma}_{GB}(A, X) &= \mu(A, X), \\
\gamma_j(X) &= \beta E[\tilde{\gamma}_j(A', X')|X] \quad \text{for } j \in \{H, B, K, GB\}, \tag{57}
\end{aligned}$$

and

$$\mu(A, X) = \max \left\{ 1, \frac{\gamma_G(X)}{\phi(I_G)q_G}, \gamma_B(X), A\gamma_K(X), \left\{ \frac{\lambda_M(\omega; X)\gamma_G(X) + (1 - \lambda_M(\omega; X))\gamma_B(X)}{P_M(\omega)} \right\}_{\omega \in \Omega^B} \right\}.$$

Assuming that the endowment W is big, then the markets feature risk neutral pricing. Thus, $P_1^M(\omega_H; X)$ is determined by the intersection between

$$P_M = \lambda_M[Z + (1 - \delta_H)\gamma_G(X)] + (1 - \lambda_M)[\alpha Z + (1 - \delta_H)\gamma_B(X)], \tag{58}$$

and

$$\lambda_M = \frac{\left[1 - G\left(\frac{\gamma_G(X)}{\gamma_K(X)P_M}\right) \right] \lambda_E}{\left[1 - G\left(\frac{\gamma_G(X)}{\gamma_K(X)P_M}\right) \right] \lambda_E + (1 - \lambda_E)}, \tag{59}$$

and $P_M(\omega_L; X)$ is just the value of bad trees

$$P_M(\omega_L; X) = \alpha Z + (1 - \delta_H)\gamma_B(X). \tag{60}$$

Since all agents have the same cost of producing trees, it means that buyers, consumers and producers of trees derive the same utility. This implies that it must hold

$$\frac{\gamma_G(X)}{\phi(I_G)q_G} = 1, \tag{61}$$

$$\gamma_B(X) \leq 1. \tag{62}$$

Moreover, capital is given by

$$\begin{aligned}
K(X) &= \int_{A_B(X)}^{A^{\max}} A[W + [\eta(X)P_M(\omega_H; X) + (1 - \eta(X))P_M(\omega_L; X)]H_B + P_{GB}(X)B]dG(A) + \\
&\quad \int_{A_S(X)}^{A^{\max}} A\eta(X)P_M(\omega_H; X)H_G dG(A), \tag{63}
\end{aligned}$$

where $A_B(X) \equiv \frac{1}{\gamma_K(X)}$ and $A_S(X) \equiv \frac{\gamma_G(X)}{\gamma_K(X)P_M(\omega_H; X)}$. Note that this expression is analogous to (23).

Finally, the laws of motion of the fraction of good trees and the total amount of trees is given by

$$\lambda'_E(X) = \lambda_E \theta(X) + \frac{I_G(X)}{I_G(X) + I_B(X)} (1 - \theta(X)), \quad (64)$$

$$H'(X) = (1 - \delta_H)H + I_G(X) + I_B(X), \quad (65)$$

where $\theta(X) \equiv \frac{(1 - \delta_H)H}{(1 - \delta_H)H + I_G(X) + I_B(X)}$.

To summarize, an equilibrium for this economy is characterized by:

1. Laws of motion of λ_E , H and K , which are given by (64), (65) and (63) respectively,
2. Shadow values of H_G , H_B , and K , given by (57),
3. Prices $P_M(\omega_H; X)$ and $P_M(\omega_L; X)$, and fraction of good trees $\lambda_M(X)$, that satisfy (58), (59) and (60), and rationing $\eta(X)$,
4. Equilibrium conditions (61) and (62).

To find an equilibrium of this economy I will follow similar steps than in the three period model. But first note that $X = (\lambda_E, H, K, B; \alpha)$ belongs to a compact set as long as B is bounded, because λ_E and α are bounded between 0 and 1, H is bounded between 0 and $\frac{W}{\delta_H}$, and K is bounded between 0 and $A^{\max}W$. Second, for a given non-exploding sequence of prices $P_M(\omega_H)$ and $P_M(\omega_L)$, rationing η , and rental rate r , γ_B and γ_K are well defined and unique. Moreover, the mapping defining γ_G is well defined and satisfies Blackwell's sufficient conditions hence γ_G exists and is unique.

Now, I proceed as in the previous section. First I characterize the partial equilibrium in the markets for trees, selecting the maximal volume of trade equilibrium whenever possible. Fixing γ_G , γ_B and γ_K , the partial equilibrium can take one of three cases:

1. only the market ω_H is active, hence $\eta = 1$,
2. only the market ω_L is active, hence $\eta = 0$,
3. both ω_H and ω_L are active, and $\eta \in [0, 1]$.

Note that market prices are decreasing in α and increasing in λ_E . Importantly, they are independent of K .

First, I will find K' for fixed λ'_E and H' . First, note that K' does not directly depend on K . This is important for tractability, since past shocks do not affect future capital. Second, $\tilde{\gamma}_G$ and $\tilde{\gamma}_B$ are independent of K , so γ_G and γ_B are independent of K' . Define the mapping T_K as

$$T_K(K'; X) = \int_{A_B(K'; X)}^{A^{\max}} A[W + [\eta(K'; X)P_M(\omega_H, K'; X) + (1 - \eta(K'; X))P_M(\omega_L, K'; X)]H_B + P_{GB}(K'; X)B]dG(A) + \int_{A_S(K'; X)}^{A^{\max}} A\eta(K'; X)P_M(\omega_H, K'; X)H_G dG(A).$$

It is straightforward to see that the mapping T_K is continuous in K' for fixed λ'_E and H' (the rationing η smooths out the discontinuity in the market). Moreover, because γ_K is decreasing in K' , which

makes volume traded (prices and rationing) decreasing in K' , the mapping T_K is decreasing in K' . Therefore, there exists a unique fixed point $K' = T_K(K')$.

Second, I find H' for fixed λ'_E . Note that if H increases, K' increases, and hence μ and market volume (prices and rationing) decreases. Therefore, γ_G and γ_B are decreasing in H' . Define the mapping T_H as

$$T_H(H'; X) = (1 - \delta_H)H + I_G(H'; X) + I_B(H; X).$$

It is straightforward to see that T_H is continuous in H' . Since in equilibrium $\gamma_G(X) = \phi(I_G)q_G$ and $\gamma_B(X) \leq 1$, I_G and I_B decrease with H' . Therefore, a fixed point $H' = T_H(H')$ for a given λ'_E exists and is unique.

Finally, define the mapping T_λ as

$$T_\lambda(\lambda'_E; X) = \lambda_E \theta(\lambda'_E; X) + \frac{I_G(\lambda'_E; X)}{I_G(\lambda'_E; X) + I_B(\lambda'_E; X)} (1 - \theta(\lambda'_E; X)).$$

This mapping is continuous in a compact space, so a fixed point $\lambda'_E = T_\lambda(\lambda'_E; X)$ exists. Let $L \equiv \{\tilde{\lambda}'_E \in [0, 1] : \tilde{\lambda}'_E = T_\lambda(\tilde{\lambda}'_E; X)\}$. Since $T_\lambda(1; X) < 1$, the slope of T_λ at $\lambda'_E = \max L$ is less than one, that is, T_λ crosses the 45° line from above. If there are many intersections, I select the one with the highest λ'_E .

Stochastic Steady State

I look for a stochastic steady state of the economy. I guess and verify that there exists an equilibrium of this economy in which λ_E and H are constant over time.

The laws of motion of λ_E and H imply that

$$\Delta \lambda_E = 0 \Leftrightarrow \lambda_E = \frac{I_G}{I_G + I_B}, \quad (66)$$

$$\Delta H = 0 \Leftrightarrow H = \frac{I_G + I_B}{\delta_H}. \quad (67)$$

Hence, if λ_E and H are constant over time, I_G and I_B are constant over time. Moreover, the fact that capital fully depreciates, that α is iid, and the timing assumption on the payout of dividends, imply that $K(X)$ is connected to past periods only through λ_E and H . Therefore, if λ_E and H are constant over time, the distribution of K in the following period is constant over time. This means that $\gamma_K(X)$ is constant over time. But then, there is a solution to the recursive equations determining $\gamma_G(X)$ and $\gamma_B(X)$ that is constant over time.

So it only remains to be shown that there exists λ_E and H such that this equilibrium exists. First note that as H increases, $K(X)$ increases so that $\mu(A, X)$ decreases. Since γ_B is more sensitive to changes in the liquidity premium than γ_G , $\frac{I_G}{I_G + I_B}$ increases. Hence,

$$\lim_{H \rightarrow 0} \lambda_E = \lim_{H \rightarrow 0} \frac{I_G}{I_G + I_B} > 0.$$

Second, as $H \rightarrow \infty$, the liquidity premium goes to zero so $\mu(A, X) \rightarrow 1$. But as long as there is some

trade in the market for trees, $\lim_{H \rightarrow \infty} I_B > 0$, hence

$$\lim_{H \rightarrow \infty} \lambda_E = \tilde{\lambda}_E < 1.$$

On the other hand, as $\lambda_E \rightarrow 0$, production of good trees remains positive, hence

$$\lim_{\lambda_E \rightarrow 0} H = \lim_{\lambda_E \rightarrow 0} \frac{I_G + I_B}{\delta_H} > 0.$$

And as $\lambda_E \rightarrow \tilde{\lambda}_E$ the production of trees remains finite, so

$$\lim_{\lambda_E \rightarrow \tilde{\lambda}_E} H < \infty.$$

Hence, (66) and (67) intersect at least once. Analyzing the possibility of multiple steady states is beyond the scope of this paper, so I choose the equilibrium that features the maximum fraction of good trees. It is possible to see that in that equilibrium (67) crosses (66) from above.

Moreover, this steady state is stable. Denote by (λ_E^*, H^*) the steady state levels of the fraction of good trees in the economy and the total amount of trees. If $\lambda_E < \lambda_E^*$, then γ_G and γ_B are lower (for all H'), but $\frac{\gamma_G}{\gamma_B}$ is higher, hence, $\frac{I_G}{I_G + I_B} > \lambda_E$. The opposite is true if $\lambda_E > \lambda_E^*$. Similarly, if $H < H^*$, the liquidity premium is high, so γ_G and γ_B are high, so $I_G + I_B > \delta_H H$. The opposite holds if $H > H^*$.