Corporate Finance, Collateralized Borrowing, and Monetary Policy

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Abstract

We construct a general equilibrium model where entrepreneurs facing uncertainty in the input cost and return of projects may finance investment internally and with bank credit. The key transmission channels of monetary policy are that money used as down payment and government bonds as collateral in external financing can reduce lenders' risk exposure and save the bankruptcy cost. Lower nominal policy rate and open market sales can compress risk spreads and increase investment. The central bank's private asset purchases improve the availability of credit, and the innovation is to enhance the efficacy of the risk-reducing role of bonds, as if the government had supplied more safe assets to expand the aggregate stock of collateral. Risk retention requirements associated with the asset purchase program are essential for welfare. As the input cost uncertainty and risk of investment return intensify, the central bank should lower the optimal risk retention rate to encourage lending.

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1 Introduction

It is usually difficult to finance risky projects due to several obstacles—low successful rates, volatile returns, uncertain input cost, as pointed out by Hall and Lerner (2010). The inherent riskiness of investment projects causes banks to be especially cautious in lending, and to require collateral and higher compensation than otherwise. Evidence shows that collateralized borrowing is widespread—77% of firms' borrowing from financial institutions requires collateral.[1] As the heightened uncertainty caused by the pandemic has seriously disrupted economic activity, defaults and risk spreads surged.[2] To counter the serious risks to the economy, central banks around the world made tremendous efforts to provide the market with sufficient liquidity and credit. The purpose of this paper is to build a general equilibrium model with uncertainty in the input cost and returns of investment, to study transmission channels through which conventional and unconventional monetary policy affect liquidity, risk spreads, business failures, and output.

Our model features corporate financing choices under various types of uncertainty. Entrepreneurs may not find suitable suppliers offering the input for investment projects, and access to bank credit is not ensured.[3] This, together with uncertainty in the input cost of investment, creates entrepreneurs' needs for cash holdings (liquid retained earnings) and bank credit. Moreover, the investment return is uncertain and is private information to the entrepreneur, and the lender needs to incur a monitoring cost to observe the return when borrowers declare default (interpreted as the bankruptcy cost). The basic assets in the economy are money and government bonds. The difficulty in external financing implies a dual role for money: An unbanked entrepreneur can finance investment internally, and this is the insurance role; a banked entrepreneur may use money as down payment, which lowers the default probability and saves the expected monitoring cost, and helps the entrepreneur to obtain credit in a more favorable term, and this is the risk-reducing role. Risk-free and fully recognizable government bonds served as collateral have a similar risk-reducing

[1]See Fan, Nguyen, and Qian (2020). They use the World Bank Enterprise Surveys (WBES) to investigate firms' collateralized borrowing in 131 countries between 2005–2017. They find that the average loan-to-value ratio is about 60%, but when bonds, equities, and other financial assets are pledged as collateral, the loan-to-value is 83%.

[2]According to S&P report, "2020 Annual Global Corporate Default And Rating Transition Study" (April 7, 2021), 226 rated issuers defaulted in 2020, resulting in a rise of global corporate defaults by 91% from 2019. High-yield spreads in the U.S. climbed to almost 10% in March 2020 at the onset of the pandemic.

[3]The WBES in 2019 indicates that, 33.7% of firms are currently with a bank loan/LOC, 11.1% of loan applications are rejected.

role as down payment. While money is universally acceptable in this economy, the use of bonds relies on bank's asset transformation. Due to limited access to banking, money and bonds are not perfect substitutes, which has crucial implications for open market operations.[4]

Specifically, we introduce corporate finance into a monetary model, as Rocheteau, Wright, and Zhang (2018), with an intermediary structure featuring costly state verification similar to Williamson (1986, 2012). Each entrepreneur is endowed an indivisible risky investment project, and after the idiosyncratic shock to the input cost is realized, entrepreneurs are heterogeneous in the aspect whether the input cost is fully covered by their internal funds. Those with the input cost larger than the internal funds can resort to external financing if they have access to bank credit. Consequently, the loan amounts, interest payments, and default probabilities are different across borrowers. The marginal borrowers, who have the largest input cost among those receiving bank credit, bear the highest default risk. Entrepreneurs finance investment with internal funds and borrowing collateralized by bonds, before they resort to loans backed by the risky investment return which involves monitoring costs. This result is consistent with the pecking-order theory of capital structure in Myers (1984), but in our model because money and bonds have the identical risk-reducing role in external finance, entrepreneurs are indifferent between internal funds as down payment and borrowing collateralized by bonds.

We derive an investment threshold such that if an entrepreneur's randomly drawn input cost is above this threshold, the investment project will not be implemented either because banks may find it too risky to lend, or the entrepreneur finds it unprofitable to invest even if credit is available. The investment threshold is mainly determined by entrepreneurs' holdings of internal funds and bonds, and the severity of credit market frictions captured by the risk of project returns and the monitoring cost. A rise in the investment threshold may imply that more investment projects obtain external finance. More investment projects receiving bank loans, however, does not necessarily result in higher net output (final output subtracting the bankruptcy cost) because marginal borrowers have the highest tendency to default.

Monetary policy works mainly by affecting entrepreneurs' holdings of money and bonds, and

[4] Some studies consider exogenous restrictions to prevent bonds from being used as a means of payment in certain trades, for examples, Shi (2008) and Lagos (2010).

the investment threshold. Lower nominal policy rate, by increasing entrepreneurs cash holdings as down payment, reduces the lending rate and risk spreads. This policy results in higher aggregate investment and net output. Open market sales, by increasing bonds as collateral to reduce the lender's risk exposure, can compress risk spreads and boost the aggregate economy. Different from the conventional wisdom is that though an open market sale withdraws the most liquid asset, it reduces the average lending rate. The reason is that, while both assets have the same risk-reducing function in external finance, money enjoys a larger liquidity premium due to its insurance role in financing investment for unbanked entrepreneurs, and claims a higher price than bonds. Consequently, open market sales is expansionary. The result highlights the importance of bonds used as collateral in corporate financing and in evaluating the effects of monetary policy. Empirical evidence suggests that collateralized loans are prevalent in business; moreover, bonds are used prevalently in the repurchase agreement (repo) market, which is a main source of funds for financial institutions and business.

During the Great Recession, several central banks, including the Federal Reserve and European Central Bank, purchased a large amount of private sector assets such as asset-backed securities and corporate bonds. To mitigate the negative impacts caused by the coronavirus outbreak, the Federal Reserve launched several new credit facilities. In the Main Street Lending Program, for example, the central bank would buy 95 percent of loans to qualified firms, while the issuing bank keeps 5 percent stake.[5] One of the goals of purchasing private assets, in response to an adverse economic event, is to restore the credit market activity. Caballero, Farhi, and Gourinchas (2017) argue that if a shortage of safe assets is the main reason behind the economic downturn, then to mitigate the problem a central bank engaged in quantitative easing should purchase riskier assets. To address this issue, the type of unconventional monetary policy we consider is private asset purchases.

The private assets purchased by the central bank in the model are bank loans, which can be interpreted as securities backed by collateralized corporate loans. When conducting the policy, the central bank requires that the issuers of private assets retain a certain fraction of the assets, to encourage liquidity provision while containing risk-taking.[6] We show that, the asset purchase

[5]Similarly, the Dodd-Frank Act requires that securitization sponsors retain not less than a 5% share of the aggregate credit risk of the assets they securitize.

[6]Some studies resort to moral hazard as the concern for asset retention; for example, Li, Rocheteau, and Weill

program increases private banks' incentives to lend by sharing a fraction of the lending risk; that is, it enlarges the fraction of a loan collateralized by bonds for a given borrowing amount. The innovation of the program is to enhance the efficacy of the risk-reducing role of government bonds, and it functions as if the government has supplied more safe assets to expand the aggregate stock of collateral. As such, the purchase program reverses the pecking order of corporate finance—entrepreneurs exhaust borrowing collateralized by government bonds before they use internal funds.

A novel result is that risk retention requirements are essential for welfare. The optimal risk retention rate trades off the benefit of more projects implemented and the cost of possible higher bankruptcy incidences. As uncertainty of the input cost and the investment return increases, the central bank should lower the risk retention rate to encourage lending. The policy implications we draw from our analysis is this. The private asset purchase program helps to compress risk spreads, increase aggregate lending, and reduce business failures. And as uncertainty and difficulties facing business firms intensify during the pandemic, the central bank should bear a larger fraction of the lending risk by imposing a lower risk retention rate.

Related literature

Our model has features related to two strands of the literature. The first includes studies on financial frictions using costly state verification proposed by Townsend (1979); see, e.g., Williamson (1987), Bernanke and Gertler (1989), and Christiano, Motto, and Rostagno (2014), which are contributions to the role of agency problem in business cycles.[7] The second strand includes explicit models of money, liquidity, and exchange, based on Lagos and Wright (2005) and Rocheteau and Wright (2005) (see references in Lagos, Rocheteau, and Wright 2017). Regarding the second strand, the closely related papers include Rocheteau, Wright, and Zhang (2018) who impose exogenous pledgeability constraints on firms, Imhof, Monnet, and Zhang (2018) who introduce limited commitment by banks and risky loans due to moral hazard, and Bethune et al. (2020) who consider lending relationships to study optimal monetary policy in the aftermath of a crisis. The distinction is that we consider firms' internal and external finances under uncertainty in both the input cost

(2012) show that the incentive to produce fraudulent assets rationalize retention mechanisms in markets for asset-backed securities.

[7]Some research resorts to exogenous pledgeability constraints to study credit frictions; e.g., Kiyotaki and Moore (1997) and Gertler and Kiradi (2010).

and return of investment, and we focus on the implications of conventional and unconventional monetary policy.

Our study on financing choices under uncertainty is also motivated by the empirical findings in several studies that firms hold a larger position of safe assets such as cash and government bonds when facing risky investment decisions; moreover, the inherent riskiness of projects is likely to cause banks to require more collateral in lending to high-tech firms. See, for instance, Bates, Kahle, and Stulz (2009), Hall (2002), and Brown, Fazzari, and Petersen (2009).

Our approach to risk-free government bonds as collateral to back loans when only a part of firms' value (or, the expected return of investment) is pledgeable is related to Holmstrom and Tirole' (1998) corporate finance model, where moral hazard generates endogenous pledgeability constraints to firms, and there is a role for government bonds in providing liquidity. Our paper is related to research that studies the role (and shortage) of safe assets as collateral and its impact on aggregate liquidity, including, for example, Caballero, Farhi, and Gourinchas (2017), Gorton (2017), and Williamson (2016, 2018).

The result that open market sales boost the economy is similar to some studies that consider the consumption channel wherein assets are used as means of payment, or collateral for consumer credit (e.g., Andolfatto and Williamson 2015, and Dong and Xiao 2019), while we focus on the investment channel where assets are used in corporate financing. In Williamson (2012), open market sales enable banks to hold more bonds to back deposits which households use to purchase consumption goods; while open market sales in our paper promote privately produced liquidity by reducing the lending risk.[8]

The next section presents the model. Section 3 derives properties of the debt contract. Section 4 incorporates the solution to the debt contract problem to study monetary equilibrium with bank credit. Section 5 studies the conventional money policy, and section 6 studies private asset purchases by the central bank. Section 7 concludes. All proofs and omitted derivations of equations are contained in the Appendix.

[8]Studies that use search monetary models to focus on the role of assets as collateral include the following. For example, Ferrais and Watanabe (2008) consider a productive input as collateral to study the effect on capital accumulation. Li and Li (2013) examine how the endogenously derived loan-to-value ratios vary with the efficiency of debt enforcement. Geromichalos et al. (2016) study how the degree of liquidity in the secondary market affects the price of the asset used as collateral.

2 The model

The model is based on Rocheteau, Wright, and Zhang (2018), with the intermediary structure similar to Williamson (1986, 2012). Time is indexed by $t = 0, 1, 2, ...$, and there are two stages within each period. In stage 1, there are a competitive market for capital and a credit market; in stage 2, agents settle debt and trade the general good and assets in a frictionless centralized market. Capital and the general good are perfectly divisible, fully perished within a period.

There are three types of agents—entrepreneurs, suppliers, and banks. Entrepreneurs are potential investors and need capital as the input for implementing investment projects. Suppliers can produce capital at unit cost in stage 1; all agents can produce the general good at unit cost in stage 2. Each type of entrepreneurs and suppliers has a unit measure, and the measure of banks is irrelevant as long as it is small relative to the mass of entrepreneurs who need external finance so that a bank can hold a diversified portfolio. Agents of each type have the same linear utility function in stage 2,

$$u(C, H) = C - H, \tag{1}$$

where C and H are units of the general good consumed and produced, respectively. All agents discount across periods according to $\beta = \frac{1}{1+r}$, where $r > 0$ is the rate of time preference.

At the beginning of stage 1 in each period, an entrepreneur is endowed with an indivisible investment project, which requires random units of capital, $\tilde{k}$, to implement. The input, $\tilde{k}$, is distributed according to the distribution function, $F(k)$, with the continuously differentiable density function $f(\cdot)$, which is strictly positive on $[\underline{k}, \overline{k}] \subset (0, \infty)$. An entrepreneur enters the market for capital with probability $\alpha \in (0, 1]$, which captures the frictions in searching for the right supplier to provide with the input necessary for an investment project. An entrepreneur's investment project becomes obsolete at the end of stage 2.

A project, which requires k units of capital, produces random units of the general good, $\tilde{\omega}$, in stage 2 if k units of capital is used as input, and produces zero units otherwise. The project return is drawn from the distribution function, $G(\omega)$. The associated density function, $g(\cdot)$, is continuously differentiable and positive on $[\underline{\omega}, \overline{\omega}] \subset [0, \infty)$. The random return, $\tilde{\omega}$, and random capital input, $\tilde{k}$, are independent, and both are independently and identically distributed. The realization of $\tilde{\omega}$, denoted by ω, is costlessly observable only to the entrepreneur who implements the project.

An entrepreneur enters the credit market with probability $\phi \in (0, 1)$, where he can borrow from a bank to finance the purchase of capital. The limited access to external finance illustrates that the availability of credit can be affected by financial development or regulations.

The model setup described above implies that entrepreneurs face four types of idiosyncratic uncertainty: the amount of capital required for implementing a project, the possibility to meet with suppliers offering capital for investment, external financing opportunities, and the project return.

Banks transform illiquid corporate loans into liquid liabilities. The lending activity is interfered by the ex post asymmetric information between the lender and the borrower: an entrepreneur's project return is private information, and the lender needs to incur the monitoring cost, γ, to observe the project return. Following Williamson (1986), the debt contract considered here has the following features: the interest payment is non-contingent, and if an entrepreneur does not make repayment, the lender monitors, and seize the defaulter's property. When extending loans, a bank issues redeemable claims (called banknotes), which entrepreneurs use as a means of payment to purchase capital from suppliers. Assume that banknotes are perfectly recognizable, and we consider the case where the issuers of banknotes commit to redeem the notes with the general good at par value in stage 2.

Suppliers produce capital and trade with entrepreneurs in a competitive market. We assume lack of commitment, enforcement, and record keeping between entrepreneurs and suppliers, and thus trade credit is infeasible. Suppliers accept fiat money and banknotes as a means of payment.

A government provides two types of perfectly recognizable assets: infinitely lived fiat money and one-period real bonds. Let $M_{s,t}$ denote the nominal money supply per entrepreneur in period t. The fiat money supply grows at a fixed gross rate, σ, according to $M_{s,t} = \sigma M_{s,t-1}$ via lump-sum transfers if $\sigma > 1$, or taxes if $\sigma < 1$, in stage 2. Assume that $\sigma \geq \beta$. Let q_t denote the value of money in terms of the general good, and $m_{s,t} = q_t M_{s,t}$ the real balances in period t. The other asset is one-period bonds with supply, $z_{s,t}$, per entrepreneur. Each unit of bonds pays the bearer one unit of the general good in stage 2. The government fully commits to redeem bonds at par, so bonds are considered as risk-free. The price of newly-issued bonds in terms of the general good is ψ_t. Assume that bonds are book-keeping entries and cannot be used as a medium of exchange; however, entrepreneur can collateralize government bonds when borrowing from banks. We assume that banks incur no cost to seize government bonds if borrowers default.[9]

3 The debt contract and bank credit

We study stationary equilibria where the real balances are constant: $q_t M_{s,t} = q_{t-1} M_{s,t-1} = m_s$, implying that $\sigma = \frac{q_{t-1}}{q_t}$; i.e., the money growth rate equals the inflation rate. Let m and z denote an entrepreneur's holdings of real balances and government bonds, respectively.

Because producing capital requires unit cost and the market is competitive, the equilibrium price of capital is 1. For a project which requires k units of capital to implement, its input cost is k. Uncertainty of the amount of capital required as the input for investment is thus considered as the input cost uncertainty. To ensure that potentially profitable projects exist, we consider $F(\mu_\omega) > 0$; that is, there are projects with input cost lower than mean return.

Consider first the case in which an entrepreneur's randomly drawn input cost, k, can be covered by her internal funds (real balances), m, and bank credit is not needed. The entrepreneur pays k with fiat money for capital in stage 1, and obtains the return, ω, in stage 2. Moreover, she receives lump-sum transfer, T, produces H, consumes C units of the general good, and adjusts the holding of real balances, m', and bonds, z', for next period.

Let $V(m, z)$ and $W(m - k, z)$ denote an entrepreneur's expected life-time utility from entering stages 1 and 2, respectively. The entrepreneur solves the following problem in stage 2:

$$\begin{aligned} \max_{\{C,H,m',z'\}} \quad & W(m-k,z) = C - H + \beta\,\mathbb{E}\,V(m',z') \\ s.t. \quad & C = H + m - k + z + T + \omega - \sigma m' - \psi z'. \end{aligned}$$

where expectations for $\mathbb{E}\,V(m', z')$ are with respect to, e.g., the input cost and the project return. Substituting C from the budget constraint into the objective, we derive the following first-order

[9] See Chu and Li (2020) for a model in which the payoff of assets served as collateral are risky, and lenders monitor once defaults occur.

conditions:

$$-\sigma + \beta\, \mathbb{E}\, V_m(m', z') \quad \geq \quad 0, \text{ `` = '' if } \quad m' > 0, \tag{2}$$

$$-\psi + \beta\, \mathbb{E}\, V_z(m', z') \quad \geq \quad 0, \text{ `` = '' if } \quad z' > 0, \tag{3}$$

where $\mathbb{E}\, V_m(m', z')$ and $\mathbb{E}\, V_z(m', z')$ are marginal expected values of taking an additional unit of real balances and bonds, respectively, to the next period. Conditions (2) and (3) determine the optimal portfolio, (m', z'), independent of the initial holdings of m and z.

3.1 The debt contract

We now consider the case wherein the randomly drawn input cost is larger than the entrepreneur's real balances, so external financing is necessary for implementing the project. We derive first an entrepreneur's default rule and solve for the contract problem. Then we show that, by using more internal funds as down payment and more bonds as collateral to reduce the bank's exposure to default risk, entrepreneurs can lower the interest payment and increase their expected payoff.

An entrepreneur's default rule. Consider an entrepreneur holding (m, z), endowed with a project with the input cost $k > m$, has access to banking. The debt contract, (d, a, x), specifies the down payment $d \leq m$, units of bonds as collateral, $a \leq z$, and a non-contingent interest payment, x, for borrowing the loan, $\ell = k - d$, to maximize the entrepreneur's expected payoff.

After the project return, ω, is realized in stage 2, the entrepreneur chooses whether to repay the debt or default. Let $W^j(m-d, z, a, x)$, $j = R, D$, be the expected value to an entrepreneur who repays or defaults, respectively, and $\mathbb{E}\, V^j(m'^j, z'^j)$, the associated expected value from entering stage 1 of next period with m'^j real balances and z'^j bonds. If an entrepreneur repays x, his continuation value is

$$\max_{\{m'^R, z'^R\}} \quad W^R(m - d, z, a, x) = C - H + \beta\, \mathbb{E}\, V^R(m'^R, z'^R)$$
$$s.t. \quad C = H + m - d + z + (\omega - x) + T - \sigma m'^R - \psi z'^R.$$

If an entrepreneur does not repay x, her return and collateral, $\omega + a$, is seized by the bank, and the continuation value becomes

$$\max_{\{m'^D, z'^D\}} \quad W^D(m-d,z,a,x) = C - H + \beta\, \mathbb{E}\, V^D(m'^D, z'^D)$$
$$s.t. \quad C = H + m - d + z - a + T - \sigma m'^D - \psi z'^D.$$

The optimal portfolio choices satisfy the following first-order conditions:

$$-\sigma + \beta\, \mathbb{E}\, V_m^j(m'^j, z'^j) \geq 0, \text{ `` = '' if } \quad m'^j > 0, \tag{4}$$
$$-\psi + \beta\, \mathbb{E}\, V_z^j(m'^j, z'^j) \geq 0, \text{ `` = '' if } \quad z'^j > 0. \tag{5}$$

Because confiscation of project returns and collateral is the only punishment on defaulters, and entrepreneurs are *ex ante* identical before the input cost is realized, we have $\mathbb{E}\, V^D(\cdot) = \mathbb{E}\, V^R(\cdot)$. Therefore, from (4) and (5), entrepreneurs bring identical portfolios to the next period whether or not they have defaulted. This, together with (2) and (3), implies that entrepreneurs choose identical portfolios whether or not they have used external finance; that is, $m'^D = m'^R = m'$, and $z'^D = z'^R = z'$.

Given the optimal asset holdings, (m', z'), we rewrite $W^j(m-d,z,a,x)$ as

$$W^R(m-d,z,a,x) = m - d + z + T + (\omega - x) - \sigma m' - \psi z' + \beta\, \mathbb{E}\, V(m', z'),$$
$$W^D(m-d,z,a,x) = m - d + z - a + T - \sigma m' - \psi z' + \beta\, \mathbb{E}\, V(m', z').$$

An entrepreneur chooses to default if $W^D(m-d,z,a,x) > W^R(m-d,z,a,x)$, which implies $\omega + a < x$; that is, default occurs if

$$\omega < \underbrace{x - a.}_{:=y} \tag{6}$$

where we denote the threshold, $x - a$, as y, and refer it as the 'effective repayment'. Thus, the default probability is

$$\Pr(\omega < x - a) = \Pr(\omega < y) = G(y).$$

With the default rule described by (6), the expected utility of an entrepreneur who has borrowed to implement a project is

$$W(m-d,z,a,x) \equiv \mathbf{E} \max(W^D, W^R) = \int_{x-a}^{\overline{\omega}} W^R(m-d,z,a,x) dG(x) + \int_{\underline{\omega}}^{x-a} W^D(m-d,z,a,x) dG(x).$$

With the linearity of $W(\cdot)$, we have

$$\begin{aligned} W(m-d,z,a,x) &= -d + \int_{x-a}^{\overline{\omega}} (\omega - x)\, dG(\omega) + \int_{\underline{\omega}}^{x-a} (\omega - \omega - a)\, dG(\omega) + W(m,z,0,0) \\ &= -d - a + \int_{x-a}^{\overline{\omega}} \omega\, dG(\omega) - (x-a)[1 - G(x-a)] + W(m,z,0,0), \end{aligned} \tag{7}$$

where $W(m,z,0,0)$ is the expected value of forgoing the project.

The contract problem. An entrepreneur's net expected payoff from borrowing to implement the project k with the contract, (d,a,x), is $\pi_e = W(m-d,z,a,x) - W(m,z,0,0)$. Substituting the effective repayment, $y = x - a$, into (7), we rewrite the entrepreneur's net expected payoff as

$$\pi_e(y) = -d - a + \int_{y}^{\overline{\omega}} \omega\, dG(\omega) - y[1 - G(y)]. \tag{8}$$

The contract, (d,a,x), yields the following expected payoff to the bank from extending a loan, $\ell = k - d$:

$$\pi_b = \int_{x-a}^{\overline{\omega}} x\, dG(\omega) + \int_{\underline{\omega}}^{x-a} (\omega + a - \gamma) dG(\omega) - \ell. \tag{9}$$

The right side of (9) shows that, if $\omega \geq x - a$, the bank receives the constant repayment x; if $\omega < x - a$, the borrower defaults, and the bank pays cost, γ, to observe the project return, and seize the project return and collateral, $\omega + a$. The last term, ℓ, implies that the bank needs to redeem banknotes at par in sage 2. The expected payoff must be non-negative for the bank to accept the contract.

With the notation, $y = x - a$, we rewrite (9) as

$$\pi_b(y) = y[1 - G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y) - \underbrace{(k - d - a)}_{:=\ell_y} \tag{10}$$

As shown in (10), the bank's expected payoff from the original contract, (d,a,x), can be expressed as that from a contract with down payment, $d + a$, loan amount, $\ell_y = k - d - a$, which is solely backed by risky project return, and the effective repayment, y. The down payment d, and collateral a, play the same role in reducing the bank's risk exposure.[10]

[10] Suppose that there is no transaction cost of selling bonds to banks. Then, collateralizing risk-free bonds functions like selling bonds to the bank for banknotes which can be used to pay for capital.

With $\pi_e(y)$ and $\pi_b(y)$ expressed in (8) and (10), we depict the original contract problem as follows:[11]

$$\{d^*, a^*, y^*\} = \arg\max_{\{d,a,y\}} -d - a + \int_y^{\overline{\omega}} \omega \, dG(\omega) - y[1 - G(y)]$$
$$\text{s.t.} \quad \pi_b(y) \geq 0; \; \pi_e(y) \geq 0; \; m \geq d \geq 0; \; z \geq a \geq 0. \tag{11}$$

The original contract characterized by (d, a, x) is equivalent to the contract expressed in (11), that the entrepreneur uses down payment, $d + a$, borrows $\ell_y = k - d - a$, and promises to repay y if $\omega \geq y$; otherwise, he defaults, and the bank seizes ω.[12] We will use the contract, (d, a, y), for the following analysis.

Under the intermediary structure with costly state verification, there is an asymmetry in the payoff functions of borrowers and lenders—here the entrepreneur's profit is decreasing in the effective repayment, y, while the bank's expected payoff is not monotone increasing in y. This creates endogenously credit constraints, which we elaborate below.

First note that $\pi_e'(y) = -[1 - G(y)] < 0$ for all $y \in [0, \overline{\omega})$. Thus, the bank's participation constraint in (11) must be binding, $\pi_b(y) = 0$. Substituting $\pi_b(y) = 0$ from (10) into the objective in (11), we have

$$\pi_e(y; k) = \mu_\omega - \gamma G(y) - k, \tag{12}$$

where μ_ω is the mean return of projects.

Next we derive the maximum risky loan amount, $\overline{\ell}_{yb}$, that banks are willing to issue. Assume that $\pi_b(y)$ in (10) strictly concave in y over $[0, \overline{\omega}]$. Thus, there exists a $y_b \in [0, \overline{\omega}]$, such that $\pi_b'(y_b) = 0$.[13] From (10), let $B(y)$ denote the bank's expected payoff backed solely by the risky project return, where

$$B(y) = y[1 - G(y)] + \int_{\underline{\omega}}^{y} \omega \, dG(\omega) - \gamma G(y). \tag{13}$$

[11] The original contract is that, given the entrepreneur's initial real balances, m, the input cost, k, and the solution of down payment, d^*, the loan amount received by the entrepreneur is $\ell = k - d^* \geq 0$, and the noncontingent repayment is x^*.

[12] Entrepreneurs whose borrowing with $\ell_y = 0$ will not incur monitoring cost. In this case, $k = d + a$, the input cost is fully covered by internal funds and bonds, and the interest payment is $x = a$.

[13] The first-order condition, $\pi_b'(y) = 1 - G(y) - \gamma g(y)$, implies $\pi_b'(\overline{\omega}) = -\gamma g(\overline{\omega}) < 0$, and if γ is small enough (e.g. $\pi'(0) = 1 - \gamma g(0) > 0$), then it ensures that there exists y_b maximizing the bank's expected payoff because of the concavity assumption, $\pi_b''(y) = -g(y) - \gamma g'(y) < 0$. See Williamson (1987) for a similar assumption.

Thus, repayment y_b yields the maximum expected returns, $B(y_b)$. Observe from (13) that y_b depends on the distribution of project return, $G(y)$; that is, y_b is identical across debt contacts regardless of the borrower's input cost. The bank's binding participation constraint implies that there is a maximum risky loan amount, $\bar{\ell}_{yb}$, such that $B(y_b) - \bar{\ell}_{yb} = 0$. This implies that if an entrepreneur wishes to take a loan of which the risky loan amount $\ell_y > \bar{\ell}_{yb}$, she will find no repayment that satisfies the bank's participation condition, and the entrepreneur is considered as credit constrained.

The following proposition establishes the existence and properties of the effective repayment.

Proposition 1. *Given the strict concavity assumption on $\pi_b(y)$ and $\ell_y \in [0, \bar{\ell}_{yb}]$, there exists an effective repayment, y^*, such that $y^* = \min\{y|\pi_b(y) = 0\}$ solves the contract problem, (11). Furthermore, the effective repayment is increasing in the risky loan amount, i.e. $\frac{\partial y^*}{\partial \ell_y} > 0$, for all $\ell_y \in [0, \bar{\ell}_{yb}]$.*

From Proposition 1, an entrepreneur, by using more internal funds and more collateral to lower the risky loan amount, will reduce the repayment and increase his expected payoff, while satisfying the bank's participation condition. A lower repayment reduces the default probability and saves the bankruptcy cost, $\gamma G(y^*)$, which increases the entrepreneur's profit.

Proposition 2 shows that an entrepreneur will use all real balances and collateralize all bonds before she resorts to the risky loan.

Proposition 2. *Given the input cost, $k > m$, an entrepreneur is indifferent between using internal funds as down payment and collateralized borrowing with bonds. Suppose an entrepreneur chooses the down payment, $d^* = m$. The amount of bonds collateralized is $a^* = \min\{z, k - m\}$.*

The intuition underlying Proposition 2 is this. One unit of real balances or bonds can reduce one unit of risky loan, while each asset is valued at one unit of the general good in stage 2. That is, both money and bonds have the same benefits in reducing risky loan amount and identical opportunity costs. Proposition 2 shows the pecking-order of financing investment: Entrepreneurs exhaust retained earning and debt secured by safe assets, before they resort to borrowing backed by risky project return which involves a monitoring cost.

So far we have established the properties of the debt contract. A bank in our model lends to a large number of entrepreneurs, and commits to redeem banknotes at par. In equilibrium the bank is perfectly diversified by making loans to a positive mass of entrepreneurs, and receives a certain return for lending, as shown in Williamson (1986). Specifically, a bank holds a positive mass of loans for a given input cost k, which ensures that the bank receives certain return ex post to redeem banknotes, $\ell = k - d$, at par. In equilibrium,

$$\int_{x-a}^{\overline{\omega}} x\, dG(\omega) + \int_{\underline{\omega}}^{x-a} (\omega + a - \gamma) dG(\omega) = (k - d).$$

Using $y = x - a$ to express the above condition, we have that, for a given k,

$$\pi_b(y) = y[1 - G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y) - (k - d - a) = 0.$$

By lending to a large number of entrepreneurs, a bank receives a certain return and ensures the redemption value of banknotes.[14]

Bank credit. By the properties of the debt contract described in Propositions 1 and 2, whether a project with the input cost $k > m$ will be implemented is determined by two factors: banks are accessible and willing to extend credit; given that the external finance is feasible, the entrepreneur's profit is positive. First, given $\overline{\ell}_{yb}$, an entrepreneur holding portfolio (m, z) and a project with the input cost $k > m + z + \overline{\ell}_{yb}$, is not able to obtain bank credit; denoting this threshold as $k_b = m + z + \overline{\ell}_{yb}$. Second, even if the contract is acceptable to a bank, an entrepreneur's expected payoff must be positive. From (12), $\pi_e(y)$ is decreasing in k—given (m, z), a higher input cost implies a higher risky loan and repayment, which lowers the entrepreneur's profit. We define the threshold, k_e, as such that $k_e \leq k_b$ (so that bank credit is feasible) and satisfies $\pi_e(y; k_e) = 0$.

Given the definition of k_e and the fact that $\pi_e(y)$ is decreasing in k, if k_e does not exist, a project with $k \leq k_b$ yields $\pi_e(y; k \leq k_b) \geq 0$. This implies that entrepreneurs with projects $k > k_b$ may find it profitable to invest and are willing to offer a higher repayment, but banks do not want to lend because the implied risky loan amount is higher than $\overline{\ell}_{yb}$. The entrepreneurs are thus credit

[14] In Williamson (1986), individual lenders collectively act as a perfectly diversified financial intermediary to economize the monitoring cost. As the financial intermediary lends to a large number of borrowers, it can ensure a certain return to each individual lender, so that they need not monitor the intermediary. A similar argument applies here that if a bank does not lend to a large number of entrepreneurs, it cannot ensure the redemption value of banknotes. Then, entrepreneurs who borrow from the bank may ask for a lower interest payment to compensate for suppliers who accept the banknotes with possibly uncertain value.

constrained, and we call this situation as credit rationing. If k_e exists, then all profitable projects can be financed with credit when bank are accessible, a situation with no credit rationing. We summarize the discussion as follows.

Definition 1. *An entrepreneur with (m, z) is not credit rationed if there exists a $k_e \leq k_b$ satisfying $\pi_e(y; k_e) = 0$; and the investment threshold is $k^T = k_e$. Otherwise, the entrepreneur is credit rationed, and $k^T = k_b = m + z + \bar{\ell}_{yb}$.*

4 Monetary equilibrium with bank credit

In this section we incorporate the solution to the debt contract problem into a general equilibrium framework, and determine entrepreneurs' optimal portfolios and financing choices. We define some aggregate variables that are key to describing the performance of aggregate economy.

4.1 Optimal portfolios and financing choices

From Proposition 2 and Definition 1, the expected value of an entrepreneur holding (m, z) is

$$\mathbb{E}\,V(m,z) = \alpha\Big[\underbrace{(1-\phi)\int_{\underline{k}}^{m}(\mu_\omega - k)dF(k)}_{\text{(a)}} + \underbrace{\phi\int_{\underline{k}}^{k^T}(\mu_\omega - \gamma G(y^*) - k)dF(k)}_{\text{(b)}}\Big] + W(m,z,0,0) \quad (14)$$

where y^* is the solution to the contract of an entrepreneur with the input cost k. With probability α, an entrepreneur enters the capital market. Term (a) in (14) shows that, with probability, $1-\phi$, the entrepreneur has no access to bank credit, and she can implement a project with the input cost lower than the internal funds; and term (b) shows that, if the entrepreneur is banked, she can finance a project with an input cost up to k^T, depending on whether or not she is credit rationed, which yields the expected profit, $\mu_\omega - \gamma G(y^*) - k$.

From (14), the expected marginal values of real balances and bonds are

$$\mathbb{E}\,V_m(m,z) = \alpha\left\{(1-\phi)\underbrace{f(m)(\mu_\omega - m)}_{\text{(c)}} + \phi\left\{\begin{array}{l}\overbrace{f(k^T)\frac{\partial k^T}{\partial m}\Big[\mu_\omega - \gamma G(y^\dagger) - k^T\Big]}^{\text{(d)}} \\ \underbrace{-\int_m^{k^T}\gamma g(y^*)\frac{\partial y^*}{\partial m}dF(k)\Big\}}_{\text{(e)}}\end{array}\right\}\right\} + 1, \quad (15)$$

$$\mathbb{E}\,V_z(m,z) = \alpha\phi\Big\{\underbrace{f(k^T)\frac{\partial k^T}{\partial z}\Big[\mu_\omega - \gamma G(y^\dagger) - k^T\Big]}_{\text{(f)}} - \underbrace{\int_m^{k^T}\gamma g(y^*)\frac{\partial y^*}{\partial z}dF(k)}_{\text{(g)}}\Big\} + 1, \quad (16)$$

where

$$\frac{\partial k^T}{\partial m} = \frac{\partial k^T}{\partial z} = \begin{cases}\frac{\gamma g(y^\dagger)}{1-G(y^\dagger)} < 1, & \text{when } k^T = k_e, \\ 1, & \text{when } k^T = k_b,\end{cases} \quad (17)$$

and $y^\dagger$ is the effective repayment for the marginal borrower with $k = k^T$. Term (c) in (15) depicts the benefit of an additional unit of real balances in expanding the chance to implement a project when the entrepreneur has to rely solely on internal finance. If the entrepreneur has access to bank credit, holding more real balances or government bonds expands the investment threshold, k^T, as depicted in (d) and (f), and reduces the risky loan for a given k, which, in turn, decreases the repayment, y^*. A lower repayment consequently results in a lower default probability and saving of the expected enforcement cost, as depicted in (e) and (g).

Note that an extra unit of real balances and bonds have identical effects in raising the investment threshold; that is, $\frac{\partial k^T}{\partial m} = \frac{\partial k^T}{\partial z}$, as shown in (17). Moreover, they have identical benefits in reducing the risky loan and, consequently, leading to lower repayment and saving of the expected enforcement cost ($\frac{\partial y^*}{\partial m} = \frac{\partial y^*}{\partial z} < 0$).[15] These effects serve as an important transmission mechanism of monetary policy.

Using (4) and (5) lagged one period to eliminate $\mathbb{E}\,V_m(m,z)$ and $\mathbb{E}\,V_z(m,z)$ from (15) and (16), respectively, we have an entrepreneur's optimal asset holdings satisfying

$$\frac{\sigma}{\beta} = \underbrace{\alpha\Big\{(1-\phi)f(m)(\mu_\omega - m) + \phi\Big[f(k^T)(\mu_\omega - k^T) - \gamma\int_m^{k^T} G(y^*)f'(k)dk\Big]\Big\}}_{\text{liquidity premium of money}} + 1, \quad (18)$$

[15]The effective repayment, y^*, is the solution to $\pi_b(y^*) = y^*[1-G(y^*)] + \int_{\underline{\omega}}^{y^*}\omega dG(\omega) - \gamma G(y^*) - (k-m-z) = 0$. Thus, $\frac{\partial y^*}{\partial m} = \frac{\partial y^*}{\partial z} = \frac{-1}{1-G(y^*)-\gamma g(y^*)} < 0$ for all $y^* < y_b$, given the concavity assumption of $\pi_b(y)$.

$$\frac{\psi}{\beta} = \underbrace{\alpha\phi\Big\{f(k^T)(\mu_\omega - k^T) - \gamma\int_m^{k^T} G(y^*)f'(k)dk\Big\}}_{\text{liquidity premium of bond}} + 1. \tag{19}$$

Observing from equations (18) and (19) that, if $\phi = 1$, money and bonds are perfect substitutes. The (im)perfect substitution between money and bonds is captured in two aspects: liquidity and ability in facilitating external finance. For liquidity, fiat money is universally accepted by suppliers as a means of payment, while bonds have to rely on banks to be transformed into banknotes; that is, the access to banking affects the liquidity role of bonds. As for external financing ability, an extra unit of real balances and bonds have identical effects in raising the investment threshold, and in decreasing the risky loan amount. As a result, when the access to banks is not always ensured, $\phi < 1$, money enjoys a higher liquidity premium than bonds.

Definition 2. *Given $\{G(\cdot), F(\cdot), \gamma, \sigma, m_s, z_s\}$, a monetary equilibrium with bank credit is $\{m, \psi\}$ satisfying (18) and (19). The market clearing conditions for money and bonds, $m = m_s$ and $z = z_s$, hold. The effective repayment, $y^*(k)$, solves the contract problem (11) for $k \leq k^T$, where $k^T = k_e$, and the equilibrium features no credit rationing; otherwise, $k^T = k_b$, and there is credit rationing.*

By Definition 2, we illustrate the existence of monetary equilibrium with bank credit as follows.[16]

Proposition 3. *Consider that the input cost and project return follow uniform distributions: $k \sim \mathcal{U}[0, 2\mu_k]$, $\omega \sim \mathcal{U}[0, 2\mu_\omega]$ where $\mu_k = \frac{\epsilon_k}{2}$ and $\mu_\omega = \frac{\epsilon_\omega}{2}$, and $\gamma < \frac{\epsilon_\omega}{2}$. Suppose $\sigma \geq \beta$. In a monetary equilibrium,*

$$\begin{aligned}
\psi &= \beta\Big[1 + \frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T)\Big], \\
y^* &= (\overline{\omega} - \gamma)^2 - \sqrt{(\overline{\omega} - \gamma)^2 - 2\epsilon_\omega \ell_y},
\end{aligned}$$

where $\ell_y = k - m - z$. There are two cases:

[16] In equilibrium in which assets bear strictly positive liquidity premia, $\sigma > \beta$ and $\psi > \beta$, suppliers do not have strictly positive gains from holding money and bonds as a store of value, and thus, they do not have demand for assets. If $\sigma = \beta$, money does not bear liquidity premium, and it does not matter whether suppliers hold money. A similar argument holds for bonds when $\psi = \beta$.

(1) when there is credit rationing,

$$\begin{aligned} k^T &= m + z + \bar{\ell}_{yb}, \\ m &= \mu_\omega - \phi(z + \bar{\ell}_{yb}) - \frac{(\sigma - \beta)\epsilon_k}{\alpha\beta}, \\ \bar{\ell}_{yb} &= \mu_\omega - \gamma + \frac{\gamma^2}{2\epsilon_\omega}; \end{aligned}$$

(2) when there is no credit rationing,

$$k^T = \begin{cases} \mu_\omega - \gamma\Big(1 - \sqrt{\frac{2(m+z)}{\epsilon_\omega}}\Big), & \textit{if } \sigma > \beta, \\ \mu_\omega, & \textit{if } \sigma = \beta, \end{cases}$$

$$m \textit{ solves} \begin{cases} (1-\phi)(\mu_\omega - m) + \phi\gamma\Big(1 - \sqrt{\frac{2(m+z)}{\epsilon_\omega}}\Big) - \frac{(\sigma-\beta)\epsilon_k}{\alpha\beta} = 0, & \textit{if } \sigma > \beta, \\ (1-\phi)(\mu_\omega - m) = 0, & \textit{if } \sigma = \beta. \end{cases} \tag{20}$$

In this economy, money and credit can coexist: an entrepreneur may have insufficient internal funds to cover the input cost so external financing is necessary, while monitoring is costly so internal funds are valuable. The inflation rate, σ, is the cost of internal financing, and the monitoring cost, γ, which affects repayment through banks' participation condition, is the main cost of external financing. The existence of equilibrium depends on how entrepreneurs trade off the two costs to make financing choices.

From Proposition 3 we have the following observations. When the monitoring cost is so high that $\bar{\ell}_{yb} < 0$, bank credit is not feasible.[17] Bank credit is feasible if the monitoring cost is sufficiently small. When the monitoring cost approaches 0, banks act like a super-efficient supervision third party. They take the risk of projects, charge the risk-adjusted loan rate, and $\bar{\ell}_{yb} = \mu_\omega$. Under this situation, all banked entrepreneurs can finance profitable investment. Nonetheless, entrepreneurs still hold some internal funds because access to bank credit is not guaranteed ($\phi < 1$).

Because inflation makes holding money costly, entrepreneurs may not choose to hold sufficiently enough internal funds to avoid being credit constrained. This implies that whether or not credit rationing occurs is endogenously determined in equilibrium. When inflation is low enough, entrepreneurs hold sufficient internal funds and there is no credit rationing. according to (20), as

[17] This implies that banks' expected payoff backed solely by the risky project returns, $B(y)$, defined in (13), remains negative for all $y \in [0, \overline{\omega}]$.

$\sigma = \beta$, holding money is costless, and entrepreneurs choose $m = \mu_\omega$, and implement the project whenever $k \le \mu_\omega$. External financing is not needed, and bonds are priced at the fundamental value: $\psi = \beta$. Friedman's rule implies efficiency. When inflation rises above β, bonds starts to play a role, and they enjoy a positive liquidity premium.

4.2 Aggregate variables

After the idiosyncratic input cost shock is realized, and financing choices are made, entrepreneurs are heterogenous in terms of whether to invest in a project, the amount of loan taken, and the repayment. We thus need to define aggregate variables that are key to studying the macroeconomic implications of policy.[18]

Firms entry and business failures. Entrepreneur are regarded as entering the business as they decide to implement a project. The firm entry rate is defined as $N = \alpha[(1-\phi)F(m) + \phi F(k^T)]$, which is determined by the probability of entering the capital market, α, the measure of unbanked entrepreneurs whose input costs are less than internal funds, m, and the measure of banked entrepreneurs whose input cost is less than the investment threshold, k^T. The intermediary structure implies that an entrepreneur who has collateralized all bond holdings and defaults will lose internal funds, collateral, and project returns, and is considered as a business failure. The aggregate default probability is defined as $D = \alpha\phi \int_{m+z}^{k^T} G(y^*(k))dF(k)$. The business failure rate is defined as the measure of defaulting firms divided by the measure of firms: D/N.

Aggregate lending and interest rates. The aggregate lending is defined as total bank loans, $TL = \alpha\phi \int_m^{k^T} (k-m)dF(k)$. As for the lending rate, we first define the average interest payment as $X = \alpha\phi \int_m^{k^T} x^* \, dF(k)$, where x^* is the repayment of the original contract. The average real lending rate, R, is defined as average interest payment divided by loans; i.e., $R = \frac{X}{TL}$. The real yield of risk-free government bond is $r_b = \frac{1}{\psi} - 1$. The risk spread is captured by the difference between the lending rate and the risk-free rate adjusted with the same time span as loans, $\frac{\beta}{\psi} - 1$; i.e., the risk spread is $S = R - \frac{\beta}{\psi}$.[19]

[18]The aggregate variables defined below are the expected value per entrepreneur.

[19]In this economy loans are taken in stage 1 and repaid in stage 2. Bond are sold at the price ψ in stage 2, and are redeemed in one unit of goods at stage 2 of next period. To define a sensible risk spread, we use the discounted

Aggregate investment and net output. The aggregate investment is defined as total capital used in implementing projects:

$$K = \alpha\Big[(1-\phi)\int_{\underline{k}}^{m} k\,dF(k) + \phi\int_{\underline{k}}^{k^T} k\,dF(k)\Big],$$

which also equals the suppliers' proceeds. Because external financing involves the monitoring cost, we define net output, Y, as aggregate output subtracting bankruptcy costs, $Y = \mu_\omega N - \gamma D$, where D is the average default rate, and γD is total resources used in monitoring the project return of defaulting firms. An entrepreneur's expected profit is

$$P = \alpha\Big\{(1-\phi)\int_{\underline{k}}^{m} (\mu_\omega - k)dF(k) + \phi\int_{\underline{k}}^{k^T} [\mu_\omega - \gamma G(y^*) - k]dF(k)\Big\}. \tag{21}$$

The net output turns out equal to the amount of goods consumed by entrepreneurs and suppliers; i.e., $Y = P + K$.

5 Conventional monetary policy and financial development

In this section, we study macroeconomic implications of conventional monetary policy and financial development such as reducing the monitoring cost or enhancing access to bank credit.

5.1 Conventional monetary policy

The monetary authority commits a constant money growth rate and the fiscal sector issues real bonds. The consolidated government budget constraint is

$$(\sigma-1)m_s + (\psi-1)z_s = T + G, \tag{22}$$

where G is government expenditure, T is lump-sum transfers, or taxes if $T < 0$, $(\sigma-1)m_s$ is seigniorage from printing new money, and $(\psi-1)z_s$ is net revenue from issuing debt. Assume that government employs an accommodating fiscal policy; that is, fiscal policy is passive, with T adjusting to satisfy (22).

The central bank can adjust the money growth rate (inflation), σ, or conduct open market operations as its conventional monetary policy tools. Let i denote the return on an illiquid nominal

price of bonds ψ/β, instead of ψ, to calculate the risk-free rate.

bonds, defined by the Fisher equation, $i = \sigma(1+r) - 1$. One can interpret i as the central bank policy rate. The type of open market operations (OMOs) we consider is that the monetary-fiscal authority changes the supply of government bonds by withdrawing or injecting money, and T is adjusted so that the government continues to issue the new level of bonds forever, and money still grows at the rate σ.[20] Taking the policy rules as given, entrepreneurs determine holdings of real balances and bonds according to (18) and (19).

We assume specific distributions for the input cost and project return in Proposition 4 to illustrate monetary policy implications in a credit rationing equilibrium, and then we show policy effects for a non-credit rationing equilibrium from numerical examples.

Proposition 4. *Consider the specific distributions,* $k \sim \mathcal{U}[0, 2\mu_k]$, $\omega \sim \mathcal{U}[0, 2\mu_\omega]$, *where* $\mu_k = \frac{\epsilon_k}{2}$ *and* $\mu_\omega = \frac{\epsilon_\omega}{2}$. *In a credit rationing equilibrium,*
(1) the effects of adjusting the nominal policy rate, i, *are:* $\frac{\partial m}{\partial i} < 0$, $\frac{\partial k^T}{\partial i} < 0$, $\frac{\partial N}{\partial i} < 0$, $\frac{\partial r_b}{\partial i} < 0$, $\frac{\partial S}{\partial i} > 0$, $\frac{\partial K}{\partial i} < 0$, $\frac{\partial Y}{\partial i} < 0$, $\frac{\partial P}{\partial i} < 0$, $\frac{\partial TL}{\partial i} = \frac{\partial R}{\partial i} = \frac{\partial D}{\partial i} = 0$;
(2) the effects of open market operations are: $\frac{\partial m}{\partial z_s} < 0$, $\frac{\partial k^T}{\partial z_s} > 0$, $\frac{\partial TL}{\partial z_s} > 0$, $\frac{\partial R}{\partial z_s} < 0$, $\frac{\partial r_b}{\partial z_s} > 0$, $\frac{\partial S}{\partial z_s} < 0$, $\frac{\partial K}{\partial z_s} > 0$, $\frac{\partial P}{\partial z_s} < 0$, $\frac{\partial N}{\partial z_s} = \frac{\partial D}{\partial z_s} = \frac{\partial Y}{\partial z_s} = 0$.

The key transmission channel of monetary policy is two fold. First, higher nominal policy rate, i, raises the cost of holding real balances and reduces internal funds, which lowers unbanked entrepreneurs' liquidity to purchase capital, and banked entrepreneurs' down payment. Second, the policy affects the investment threshold, which is $k^T = m + z + \bar{\ell}_{yb}$ in a credit rationing equilibrium. The maximum risky loan that banks are willing to lend, $\bar{\ell}_{yb}$, is affected only by the enforcement cost and the distribution of project returns (recall from Proposition 3, $\bar{\ell}_{yb} = \mu_\omega - \gamma + \frac{\gamma^2}{2\epsilon_\omega}$); moreover, bond holdings, z, is given by the fiscal authority's supply of bonds. Thus, k^T falls because m is reduced by higher i, as was shown in (17) that $\frac{\partial k^T}{\partial m} = 1$ when there exists credit rationing. Lower internal funds to finance investment for unbanked entrepreneurs, together with lower k^T for banked entrepreneurs, reduce firms entry, aggregate investment, and net output. As money holdings fall, the demand for bonds to facilitate external finance increases, which drives down the bond yield, r_b,

[20]See, e.g., Rocheteau, Wright, and Xiao (2018). Note that in a stationary monetary equilibrium, a one-time change in the aggregate nominal money supply, M_s, is neutral, because the value of money, q, changes to make the aggregate real balances, $m_s = qM_s$, constant over time. Thus, an OMO that swaps money for bonds has the same real impact as changing only the bond supply, z_s.

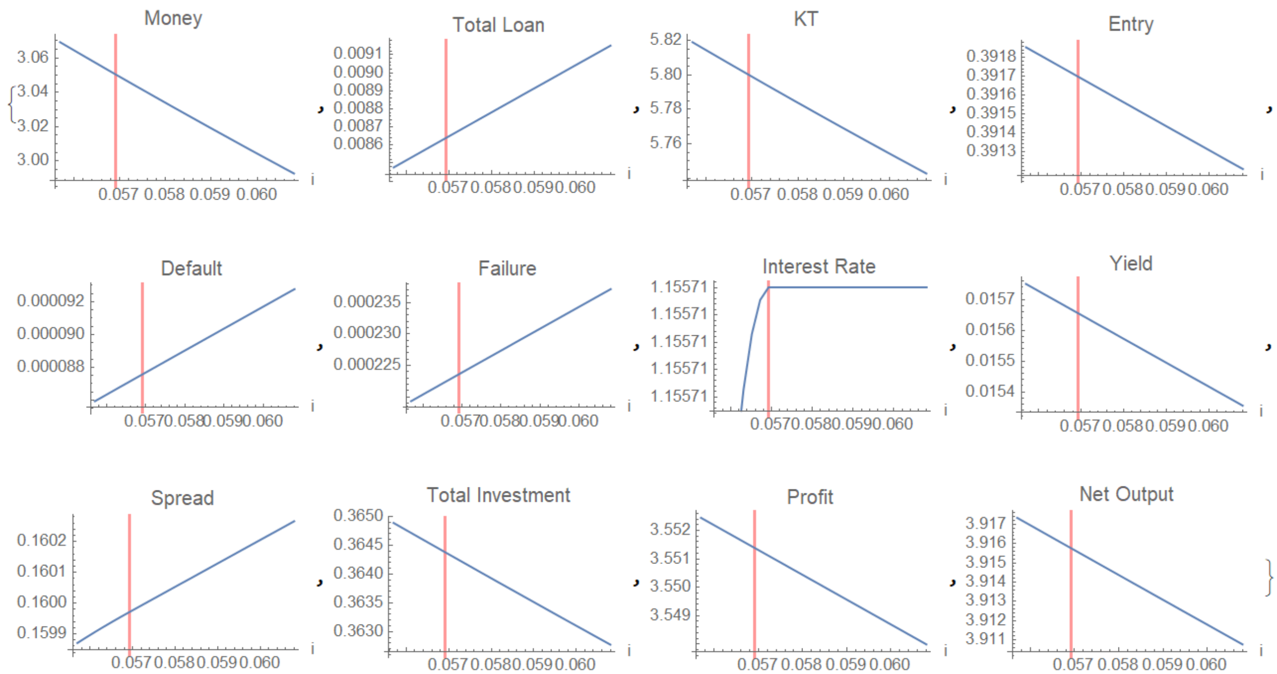

Figure 1: Effects of changes in the policy rate

and raises the risk spread.

Given the assumed uniform distributions for k and ω, Proposition 4 shows $\frac{\partial TL}{\partial i} = \frac{\partial R}{\partial i} = \frac{\partial D}{\partial i} = 0$; however, numerical simulations with uniform distributions show that in a non-credit rationing equilibrium, higher i raises aggregate lending, the lending rate, and defaults. To gain further insights on policy implications, we consider that k follows an exponential distribution while maintain the uniform distribution for ω in Figures 1 and 2.[21] We have the following observations from Figure 1. First, non-credit rationing equilibria exist when policy rate (inflation) is low, while when i is above a threshold (right of red line), equilibria exhibit credit rationing. Second, as the central bank raises the policy rate, aggregate lending and defaults rise, and net output falls, whether or not there is credit rationing.

When the central bank conducts open market sales, entrepreneurs have more bonds pledged as

[21] The followings are parameter settings for numerical examples for Figures 1 and 2:. $\omega \sim U[0, 20]$, $k \sim f(k) = \lambda e^{-\lambda k}$ where $\lambda = 1$, $\{\alpha, \beta, \gamma, \phi, \sigma, z_s\} = \{0.4, 1/1.2, 14, 0.6, 1.035, 1.85\}$.
We also consider lognormal distributions for project returns, ω, in other numerical examples. Most main results regarding monetary policy implications described in the main text hold.

collateral. As more bonds substitute for money as down payment, entrepreneurs tend to reduce money holdings, consistent with the evidence that money demand decreases with pledgeability. Notice that, with a fixed $\bar{\ell}_{yb}$, a larger z raises k^T even though money holdings decrease. The reason is that money enjoys higher liquidity premium, and thus claims a higher value, than bonds. When the central bank sells one unit of bonds, it withdraws less than one unit of real balances from the market, and consequently, k^T increases. We thus observe higher aggregate lending and investment. Open market sales drive up the bond yield, reduce the lending rate, and compress the risk spread.

Proposition 4 shows $\frac{\partial D}{\partial z} = \frac{\partial Y}{\partial z} = 0$ under assumed uniform distributions. However, in a non-credit rationing equilibrium, the investment threshold is $k^T = k_e$, satisfying $\pi_e(y; k_e) = 0$, and entrepreneurs are not constrained by $\bar{\ell}_{yb}$. Numerical simulations with uniform distributions show that more bonds decrease risky loan to such an extent that defaults and aggregate bankruptcy cost fall, and net output rises. From Figure 2 we observe that when bonds supply is scarce (left of red line), entrepreneurs are credit rationed, whereas as bonds are plentiful, the aggregate stock of collateral is so sufficient that there is no credit rationing. Moreover, firms entry and net output rise when more bonds are injected into the economy, whether or not there is credit rationing.

Open market sales boost the economy—higher investment threshold accompanied by lower real balances implies that more banknotes are issued to facilitate the purchase of capital. This result is similar to Williamson's (2012) finding, where banks hold government bonds as assets, and more bonds enables banks to issue more deposits to households as a means of payment to purchase consumption goods. In our model, open market sales promote privately produced liquidity by reducing the lending risk.

Notice that the effectiveness of OMOs in our model relies on the premise that money and bonds are imperfect substitutes. If entrepreneurs have full access to bank credit ($\phi = 1$), bonds become perfect substitutes for money, and this neutralizes OMOs as in Wallace (1981). The monetary policy effects in our model are in line with the conventional wisdom that ample liquidity can stimulate the economy. What is different from the conventional wisdom is that, while an open market sale withdraws the most liquid assets in the economy, it lowers the lending rate and risk spread, and raises net output. Our result suggests that the role of bonds in collateralized borrowing should be incorporated when evaluating the effects of OMOs.

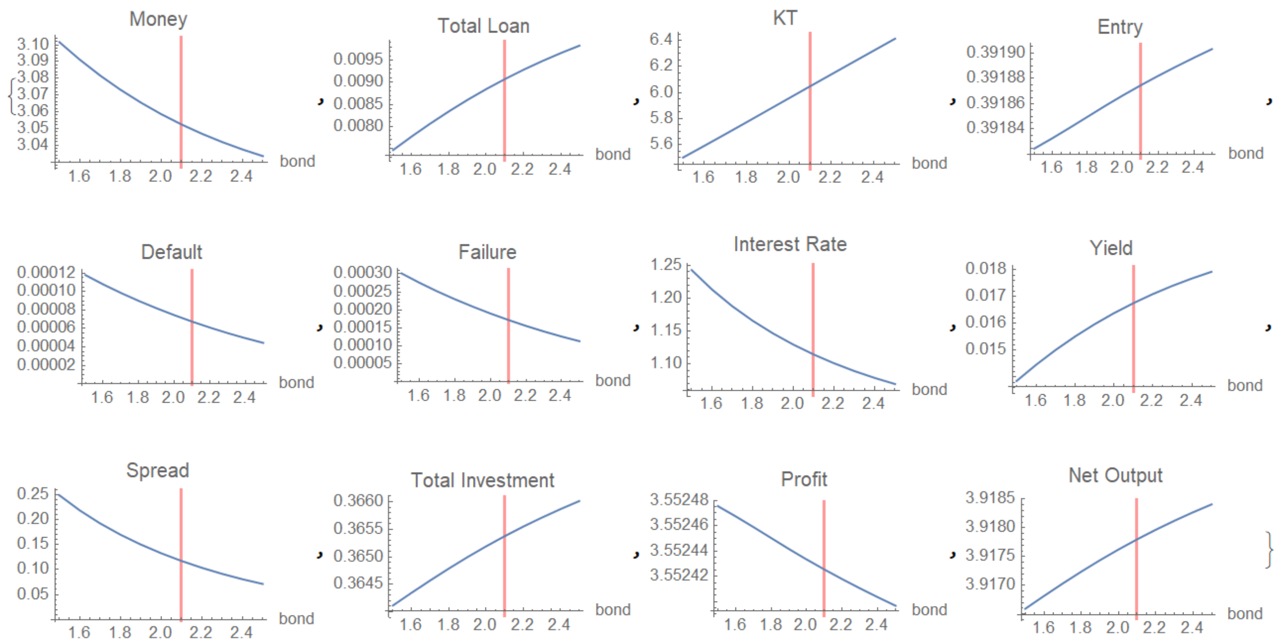

Figure 2: Effects of changes in the bond supply

5.2 Financial development

Technological improvements and financial development can lower the monitoring cost or enhance the access to banking. Financial regulations may restrict or enhance the availability of bank credit. Proposition 5 summarizes the macroeconomic impacts of changes in the monitoring cost and access to bank credit.

Proposition 5. *Consider the specific distributions, $k \sim \mathcal{U}[0, 2\mu_k]$, $\omega \sim \mathcal{U}[0, 2\mu_\omega]$ where $\mu_k = \frac{\epsilon_k}{2}$ and $\mu_\omega = \frac{\epsilon_\omega}{2}$. In a credit rationing equilibrium,*
(1) the effects of changes in the monitoring cost are: $\frac{\partial m}{\partial \gamma} > 0$, $\frac{\partial k^T}{\partial \gamma} < 0$, $\frac{\partial D}{\partial \gamma} < 0$, $\frac{\partial TL}{\partial \gamma} < 0$, $\frac{\partial r_b}{\partial \gamma} < 0$, $\frac{\partial K}{\partial \gamma} < 0$, $\frac{\partial N}{\partial \gamma} = 0$, and when γ is big, $\frac{\partial R}{\partial \gamma} < 0$, $\frac{\partial S}{\partial \gamma} < 0$, $\frac{\partial Y}{\partial \gamma} > 0$, $\frac{\partial P}{\partial \gamma} > 0$;
(2) the effects of changes in the access to banking are: $\frac{\partial m}{\partial \phi} < 0$, $\frac{\partial k^T}{\partial \phi} < 0$, $\frac{\partial D}{\partial \phi} > 0$, $\frac{\partial TL}{\partial \phi} > 0$, $\frac{\partial r_b}{\partial \phi} < 0$, $\frac{\partial S}{\partial \phi} > 0$, $\frac{\partial Y}{\partial \phi} < 0$, $\frac{\partial N}{\partial \phi} = \frac{\partial R}{\partial \phi} = 0$, and when ϕ is small, $\frac{\partial K}{\partial \phi} > 0$, $\frac{\partial P}{\partial \phi} < 0$.

A rise in the monitoring cost impacts the economy mainly through its effects on asset holdings and the cost of risk in lending. Demands for money and bonds rise because both assets can reduce the probability to incur the monitoring cost, and the bond yield falls. Higher monitoring costs reduce $\bar{\ell}_{yb}$, and aggregate lending falls. The investment threshold, $k^T = m + z + \bar{\ell}_{yb}$, is lowered since the rise in m cannot compensate the decrease in $\bar{\ell}_{yb}$, and aggregate investment falls as well. Given bonds holding, z, a lower k^T implies a smaller risky loan amount, which leads to lower repayment. When the monitoring cost is big ($\gamma \geq \frac{\epsilon_\omega}{4}$), as γ rises further, the effect of reduction in the interest payment dominates the effect of fewer loans, and thus, the average lending rate and risk spread fall. As such, a rise in γ can reduce defaults and result in lower aggregate bankruptcy cost, and higher net output.[22]

As the access to banking rises, the usefulness of bonds increases. Entrepreneurs have a higher demand for bonds that drives lower the bond yield, and they reduce real balances, which results in lower k^T. When more loans are granted due to higher access to bank credit, defaults and aggregate bankruptcy cost are more likely to rise, and net output falls.[23] Unlike Rocheteau, Wright, and Zhang (2018), improving the access to bank loans is not always beneficial in the economy considered here, and the key lies in the bankruptcy cost associated with external finance.

5.3 Risk shocks

We capture risk shocks to entrepreneurs' productivity by the volatility of input costs and project returns. From simulations we have the following observations.[24] As the dispersion of firms' input cost, ϵ_k, increases, the marginal benefit of money decreases because it is less likely to help entrepreneurs cover the input cost. Consequently, real balances, m, and investment threshold, k^T fall, and we observe a macroeconomic downturn—characterized by drops in firm creation, investment, net output, and firms' profits, while risk spread and business failures rise. We also notice that results depend on the distribution of the input cost.

[22]In numerical simulations wherein k follows an exponential distribution, firms entry may drop so much that net output falls with an increase in the monitoring cost.

[23]In Proposition 5, the result, $\frac{\partial N}{\partial \phi} = \frac{\partial R}{\partial \phi} = 0$, is due to the uniform distribution of the input cost. Take $\frac{\partial N}{\partial \phi} = 0$ as an example. The loss in the liquidity due to a lower real balance and a lower investment threshold is offset by a higher probability to borrow from a bank, and thus N do not change. When k follows an exponential distribution, we find that as ϕ increases, there is higher firms entry, which can dominate the effect of higher aggregate bankruptcy cost, so that net output increases.

[24]We first replace the original uniform distribution assumption with log-normal distribution in project returns while the input cost distribution remains the same as the previous one where k ~U[0,20]. Then, we simulate the risk surges by fixing the mean return of project at 10 and increases the volatility; or increases the volatility of input cost distribution by increasing the upper bound. Our numerical examples focus on the credit rationing equilibrium. The followings are the benchmark parameter setting and numerical result.$\omega \sim$ Log N (5,2), k ~U[0,20], $\{\alpha, \beta, \gamma, \phi, \sigma, z_s\} = \{1, 0.9, 8, 0.7, 1.03, 8\}$

The risk of project return affects the economic activity through the lending channel. Increasing dispersion of project return is similar to a rise in the monitoring cost, both of which reduce banks' willingness to lend. Entrepreneurs thus tend to hold more internal funds, which leads to higher down payment and lower defaults. Though the investment threshold and aggregate investment decrease, net output and firms' profits may increase. This seemly counter-intuitive results lie in the fact that firms adjust their financing choices so that they rely less on external finances and therefore, incur less bankruptcy cost.

Finally, an increase in μ_ω implies higher productivity for a given input cost, which leads to a macroeconomic boom—increases in aggregate investment, firms creation, and net output, and fewer business failures. Higher opportunities to purchase capital, α, have similar effects as a rise in μ_ω on the aggregate economic performance.

6 Private asset purchases by the central bank

In this section, we study how the central bank's private asset purchases work to improve liquidity and investment, and why this type of unconventional policy is beneficial to the aggregate economy. The central bank's purchase of private assets is captured in our model as buying loans from the issuing banks, and one can interpret bank loans as securities backed by collateralized corporate loans or mortgage. To encourage liquidity provision for risky investment while containing risk-taking, the central bank requires that the issuer of private assets retain a certain fraction of the assets. Specifically, we consider the private asset purchase program that requires the issuing bank retain a fraction, $\theta \in (0, 1)$, of asset on the book. The central bank pays reserves to purchase $1 - \theta$ fraction of assets, and receives $1 - \theta$ fraction of expected returns which is backed solely by risky investment returns.[25] We call θ the risk retention rate. Assume that the central bank has the same monitoring technology as private banks.

6.1 Effects of the private asset purchase policy

This section shows that the policy, by strengthening the role of safe assets as collateral, effectively reduces banks' risk exposure, increases entrepreneurs' expected profits for a given input cost

[25] Reserves are account balances, and each unit of real reserves is valued at one unit of the general good in stage 2.

realization, and raises the investment threshold.

We first show how the issuing bank's payoff is changed by policy. The bank's expected payoff from a contract with down payment d and collateral a, as shown in (10), is re-expressed as follows:

$$\pi_b = \Big\{ y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y) \Big\} - (k-d-a). \tag{23}$$

Because the safe asset as collateral works like down payment, the actual funding cost for the lending bank is $k-d-a$, while $\ell = k-d$ is the loan balance on the bank's balance sheet. The issuing bank receives reserves from the central bank for a fraction of the loan on the banking book, $(1-\theta)(k-d)$, retains θ fraction of the expected returns backed solely by risky investment returns. The bank's expected payoff under the policy thus becomes

$$\begin{aligned} \pi_b^p(y) &= (1-\theta)(k-d) + \theta\Big[y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y) \Big] - (k-d-a) \\ &= \theta \Big\{ y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y) - \underbrace{(k-d-\frac{a}{\theta})}_{\ell_y^p} \Big\} \end{aligned} \tag{24}$$

Condition (24) shows that, from the bank's view-point, the 'effective' down payment is $d + \frac{a}{\theta}$, and the risky loan amount is $k - d - \frac{a}{\theta}$.

Next, we show how the policy affects the contract problem and entrepreneurs' expected profits. Proposition 2 shows that an entrepreneur is indifferent between using money as down payment and collateralizing borrowing, because both assets have the same benefits in reducing risky loan amount and have identical opportunity costs. The private asset purchase program makes bonds more effective than money in reducing risk—one unit of bond can reduce $\frac{1}{\theta} > 1$ units of risky loan. Let k^p denote the investment threshold under policy. The contract problem becomes:

$$\begin{aligned} \{d^*,\ a^*,\ y^*\} &= \arg\max_{\{d,a,y\}} -d - a + \int_{y}^{\overline{\omega}} \omega\, dG(\omega) - y[1-G(y)] \\ \text{s.t.} \quad \pi_b^p(y) &= \theta\{ y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y) - (k-d-\frac{a}{\theta}) \} \geq 0, \\ & 0 \leq d \leq m,\ 0 \leq a \leq z, \end{aligned} \tag{25}$$

with the solution satisfies $\pi_b^p(y) = 0$, and

$$a^* = \begin{cases} \theta k, & \text{if } \underline{k} \le k \le \frac{z}{\theta} \\ z, & \text{if } \frac{z}{\theta} < k \le k^p \end{cases}, \qquad d^* = \begin{cases} 0, & \text{if } \underline{k} \le k \le \frac{z}{\theta} \\ k - \frac{z}{\theta}, & \text{if } \frac{z}{\theta} < k \le m + \frac{z}{\theta} \\ m, & \text{if } m + \frac{z}{\theta} < k \le k^p. \end{cases} \tag{26}$$

From (26), if the entrepreneur's input cost is such that $\underline{k} \le k \le \frac{z}{\theta}$, bonds are collateralized, and no down payment is needed. Money will be used if $k > \frac{z}{\theta}$, under which all bond holdings are collateralized.

Proposition 6. *The private asset purchases program enhances the efficacy of bonds as collateral, and changes the pecking-order of financing investment. Under the policy, entrepreneurs collateralize bonds before using internal funds.*

Given the solution to the contract problem, the expected profit of entrepreneurs with access to bank credit is

$$\pi_e = \begin{cases} \mu_\omega - \theta k & \text{if } \underline{k} \le k \le \frac{z}{\theta} \\ \mu_\omega - k + \frac{1-\theta}{\theta} z, & \text{if } \frac{z}{\theta} < k < m + \frac{z}{\theta} \\ \mu_\omega - \gamma G(y^*) - k + \frac{1-\theta}{\theta} z, & \text{if } m + \frac{z}{\theta} \le k \le k^p \end{cases}, \tag{27}$$

The expected profit of unbanked entrepreneurs, who use internal funds only to purchase capital, is

$$\pi_e = \mu_\omega - k, \qquad \text{if } \underline{k} \le k \le m, \tag{28}$$

Lemma 1 shows that the policy effectively raises the investment threshold.

Lemma 1. *Given an entrepreneur's portfolio, (m, z), the private asset purchase policy raises the investment threshold by $\frac{1-\theta}{\theta} z$, no matter whether there is credit rationing.*

The private asset purchase policy enhances bank's incentives to lend by sharing a fraction of the lending risk. The policy effectively enlarges the collateralizable bonds from a^* to $\frac{a^*}{\theta}$, reducing the risky loan amount and repayment for a given input cost realization; that is, the policy raises the 'risk-less' part of the loan for a given loan amount (and a given input cost). According to Lemma 1, banks now extend loans to entrepreneurs with input cost up to $k^p = k^T + \frac{1-\theta}{\theta} z$. Entrepreneurs with the input cost higher than the initial investment threshold, k^T, who could not borrow or find it profitable to invest without policy can receive bank credit under policy. The asset purchase program improves the availability of credit.

By (27) and (28), an entrepreneur's expected profit becomes

$$\begin{aligned} \pi_e^p &= \alpha(1-\phi)\int_{\underline{k}}^{m}(\mu_\omega - k)dF(k) \\ &+\alpha\phi\left\{\begin{array}{c} \int_{\underline{k}}^{\frac{z}{\theta}}(\mu_\omega-\theta k)dF(k)+\int_{\frac{z}{\theta}}^{m+\frac{z}{\theta}}[\mu_\omega - z-(k-\frac{z}{\theta})]dF(k) \\ +\int_{m+\frac{z}{\theta}}^{k^p}[\mu_\omega-\gamma G(y^*)-k+\frac{1-\theta}{\theta}z]dF(k) \end{array}\right\} \end{aligned} \tag{29}$$

Notice that (29) incorporates the entrepreneur's decision exhaust collateralized borrowing before using internal funds. An entrepreneurs's lifetime expected value under policy becomes

$$\mathbb{E}\,V^p(m,z) = \pi_e^p + W(m,z,0,0). \tag{30}$$

Given (30), an entrepreneur's optimal portfolio choices are characterized by the following conditions (see Appendix for derivation):

$$\frac{\sigma}{\beta} = \alpha\Big\{(1-\phi)(\mu_\omega - m)f(m) + \phi\Big[(\mu_\omega - k^p + \frac{1-\theta}{\theta}z)f(k^p) - \gamma\int_{m+\frac{z}{\theta}}^{k^p} G(y^*)f'(k)dk\Big]\Big\}+1 \tag{31}$$

$$\frac{\psi}{\beta} = \frac{\alpha\phi}{\theta}\Big[(\mu_\omega - k^p + \frac{1-\theta}{\theta}z)f(k^p) - \gamma\int_{m+\frac{z}{\theta}}^{k^p} G(y^*)f'(k)dk + (1-\theta)[F(k^p)-F(\frac{z}{\theta})]\Big]+1 \tag{32}$$

Conditions (31) and (32) show that the liquidity premia of money and bonds are changed by the risk retention rate, θ.[26]

6.2 The optimal risk retention rate

We now incorporate the solution to the contract problem, (d^*, a^*, y^*), and the entrepreneur's optimal portfolio choices, (m, z), into the aggregate economy, and derive the optimal risk retention rate associated with the private asset purchase program. Our result suggests that risk retention requirements are essential.

When conducting the policy, the central bank knows the decision rules of entrepreneurs and banks. To choose the optimal risk retention rate, θ^*, the central bank anticipates the equilibrium

[26] In equilibrium a bank holds a positive mass of loans for a given input cost k, receives a certain return for lending under policy, and ensures the redemption value of banknotes at par.

m and ψ given by (31) and (32). Entrepreneurs and banks take θ^* as given, and (31) and (32) ensure that the equilibrium m and ψ are as anticipated.

The central bank, by giving reserves, $(1-\theta)(k-d^*)$, to the lending bank, is entitled to $1-\theta$ of the expected return of the loan backed solely by the project return. Given the investment threshold, k^p, the central bank's expected payoff is:

$$\pi_c = \alpha\phi \int_{\underline{k}}^{k^p} \Big\{(1-\theta)\Big[y[1-G(y)] + \int_{\underline{\omega}}^{y} \omega\, dG(\omega) - \gamma G(y)\Big] - (1-\theta)(k-d^*)\Big\} dF(k).$$

Using the bank's participation condition from (25) with (d^*, a^*, y^*), we obtain

$$\pi_c = -\alpha\phi\Big[\int_{\underline{k}}^{k^p} \frac{(1-\theta)a^*}{\theta} dF(k)\Big] < 0. \tag{33}$$

The central bank runs a deficit from purchasing private assets, and it needs transfers from the fiscal authority. Condition (33) suggests that the cost of the policy is equal to the benefit of increased collateralizable bonds, and is the subsidy to entrepreneurs who receive bank credit.

From the solution to the contract, (26), an entrepreneur's collateralized bonds is $a^* = \theta k$ if $\underline{k} \le k \le \frac{z}{\theta}$, and $a^* = z$ if $\frac{z}{\theta} < k \le k^p$. Substituting a^* into (33), we obtain

$$\pi_c = -\alpha\phi\Big\{\int_{\underline{k}}^{\frac{z}{\theta}} (1-\theta)k dF(k) + \frac{(1-\theta)z}{\theta}\Big[F(k^p) - F(\frac{z}{\theta})\Big]\Big\}. \tag{34}$$

A lower θ leads to a larger increase in the investment threshold, which can potentially increase aggregate output. The 'subsidy' is not without a cost, however, as we have seen from (33) that the program runs a deficit, and more reserves are needed as the investment threshold is raised higher. The right side of (34) equals the expected total subsidy to entrepreneurs through the central bank's purchase of private assets. For borrowers with $k < \frac{z}{\theta}$, the central bank subsidize $(1-\theta)k$; for those with $\frac{z}{\theta} < k < k^p$, the central bank subsidizes $\frac{(1-\theta)z}{\theta}$.

When choosing the risk retention rate, the central bank's objective is to maximize the society's welfare. At the beginning of a period before all shocks are realized, the expected steady state lifetime utility of the representative entrepreneur, Π_e, is $(1-\beta)\Pi_e = \pi_e^p + C^* - H^*$,where π_e^p is the consumption of expected profits of investment. Suppliers' net expected utility from producing capital and consuming the payment is zero. Therefore, we use $\Omega = \pi_e^p + \pi_c$ as our welfare criterion, which is the expected utility of entrepreneurs subtracting the cost of policy (financed by lump-sum taxes on entrepreneurs), as our welfare criterion, where π_e^p is from (29) and π_c is from (34). Using (27) and (28), we have[27]

$$\Omega = \alpha\Big\{(1-\phi)\int_{\underline{k}}^{m}(\mu_\omega - k)dF(k) + \phi\Big[\int_{\underline{k}}^{k^p}(\mu_\omega - k)dF(k) - \int_{m+\frac{z}{\theta}}^{k^p}\gamma G(y^*)dF(k)\Big]\Big\}. \qquad (35)$$

To find the optimal risk retention rate, θ^*, the central bank maximizes Ω in (35), subject to $\pi_b^p(y) = 0$, and government budget constraint, $(\sigma - 1)m - (1-\psi)z - G + \pi_c = T$. Proposition 7 summarizes the main result.

Proposition 7. *Consider that the input cost and project return follow uniform distributions:* $\omega \sim U[\mu_\omega - \frac{\epsilon_\omega}{2}, \mu_\omega + \frac{\epsilon_\omega}{2}]$ *and* $k \sim U[\mu_k - \frac{\epsilon_k}{2}, \mu_k + \frac{\epsilon_k}{2}]$. *In a monetary equilibrium, there exists an optimal risk retention rate,* $\theta^* \in (0, 1)$, *such that*
(1) the investment threshold is raised and equals the mean return of projects; i.e., $k^p = \mu_\omega$;
(2) the optimal risk retention rate should be lower, when the input cost and project return become more risky, or it is harder to meet with the supplier; i.e., $\frac{\partial\theta^*}{\partial\epsilon_\omega} < 0$, $\frac{\partial\theta^*}{\partial\epsilon_k} < 0$, *and* $\frac{\partial\theta^*}{\partial\alpha} > 0$.

By assuming specific distributions in Proposition 7, we gain clear policy implications and intuition. The optimal risk retention rate, θ^*, is chosen to raise the investment threshold to such a level that all socially efficient projects ($k \le \mu_\omega$) would be implemented. By sharing a fraction of the lending risk, the central bank's purchase of private assets can enhance welfare. As a negative shock increases the risk of investment, uncertainty of the proceeds facing business firms, and difficulty of meeting the right supplier, the central bank should lower the risk retention rate to encourage lending.

To further investigate the transmission mechanism of the private asset purchase program, we illustrate how it affects three terms in the welfare measure, (35). As the policy improves liquidity of bonds as collateral, entrepreneurs may lower the demand for money. The first term in (35), $\int_{\underline{k}}^{m}(\mu_\omega - k)dF(k)$, which is the surplus generated by unbanked entrepreneurs, may decrease due to lower real balances.[28] The second term is the surplus of entrepreneurs who obtain bank credit,

[27]The welfare measure, Ω, is bounded because, when the investment threshold is raised by policy, so is the marginal borrower's input cost, which may result in the implementation of ex-ante inefficient investment with $k > \mu_\omega$.

[28]From the proof of Proposition 7, when k is uniformly distributed, money demand is not affected by the policy. In numerical examples wherein k follows an exponential distribution and ω uniform distribution, aggregate bankruptcy cost and risk spreads are reduced by the policy.

$\int_{\underline{k}}^{k^p}(\mu_\omega - k)dF(k)$, which increases as the policy raises k^p, and it starts to decrease once k^p is above μ_ω. The last term is aggregate bankruptcy cost, $\int_{m+\frac{z}{\theta}}^{k^p}\gamma G(y^*)dF(k)$. The policy, by raising the investment threshold, results in more projects to be implemented; however, these additional projects that would not have been implemented without policy are those with relatively high input costs. The policy also increases collateralizable bonds. The range of the input cost with strictly positive risky loan amount (and may be subject to default) is changed from $k \sim (m+z, k^T)$ to $k \sim (m+\frac{z}{\theta}, k^p)$, while the corresponding expected bankruptcy cost, $\gamma G(y^*)$ of a given risky loan amount remains the same. Whether and how much the bankruptcy cost would occur thus depends on the distribution of k within the above two regions. If, for instance, $F(m+\frac{z}{\theta} < k < k^p)$ becomes very small and so default is not very likely to occur, then the policy may benefit the society by saving aggregate bankruptcy cost. Finally, we observe that unbanked entrepreneurs are hurt and banked entrepreneurs are subsidized by the private asset purchase program. Hence, though the policy raises welfare, it has a distributional effect.

Corollary 1 summarizes the effects of the policy on aggregate economy.

Corollary 1. *Given the assumptions and results of Proposition 7, the private asset purchase program raises firms entry, aggregate lending, investment, and net output, and reduces the average lending rate and business failures.*

7 Conclusion

This paper considers corporate financing decisions to implement risky investment projects to study the macroeconomic implications of conventional monetary policy and private asset purchase program. We have established that the key transmission channels of monetary policy are that money used as down payment and government bonds as collateral in external financing can reduce lenders' risk exposure and save the bankruptcy cost. Lower nominal policy rate and open market sales can compress risk spreads and increase investment. The central bank's private asset purchases improve the availability of credit, and risk retention requirements are essential.

We observe that negative impacts caused by the covid-19 pandemic on the economy include raising uncertainty in investment and proceeds of business, and disrupting the supply chain of firms. The policy implications we draw from our analysis is this. The private asset purchase program with the optimal risk retention rate helps to compress the risk spread, improve credit availability, and lower business failures. As the uncertainty and difficulties for business firms are intensified by the pandemic, the central bank should bear a larger fraction of the lending risk by requiring a lower risk retention rate.

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Appendix

Proof of Proposition 1. Given (d^*, a^*), input cost k and corresponding effective loan amount $\ell_y \in [0, \overline{\ell}_{yb}]$ where $\overline{\ell}_{yb} > 0$, the contract problem is rewritten as

$$y^* = \arg\max_y -d^* - a^* + \int_y^{\overline{\omega}} \omega \, dG(\omega) - y[1 - G(y)]$$
$$s.t. \quad \pi_b = y[1 - G(y)] + \int_{\underline{\omega}}^{y} \omega \, dG(\omega) - \gamma G(y) - \ell_y = 0.$$

First, we know $\overline{\ell}_{yb} > 0$ implies $\pi_b(y_b) \geq 0$. If $y = 0$, then we have $\pi_b(0) = -\ell_y < 0$. Second, if we have γ small enough such that $\pi_b'(0) > 0$, then by the assumption by the assumption of strictly concavity of $\pi_b(y)$, there exists a $y^* \in [0, y_b] \subset [0, \overline{\omega}]$ solves this contract problem. Furthermore, we know $\pi_b'(y) > 0$ for all $y \in [0, y_b)$. Therefore, it must be $\frac{\partial y^*}{\partial \ell_y} > 0$ to keep bank's participation condition satisfied.

Proof of Proposition 2. According to (12), $\pi_e(y; k) = \mu_\omega - \gamma G(y) - k$, the entrepreneur's expected payoff is decreasing in the repayment y^*. From Proposition 1, $\frac{\partial y^*}{\partial \ell_y} > 0$, and $\ell_y = k - d - a$, an entrepreneur with a given input cost, k, has an incentive to use more money and more bonds to reduce the risky loan amount, ℓ_y. Since $\frac{\partial \ell_y}{\partial d} = \frac{\partial \ell_y}{\partial a}$, the effects of real balances and bonds in reducing the risky loan amount are identical. Moreover, $\frac{\partial y^*}{\partial \ell_y} \frac{\partial \ell_y}{\partial d} = \frac{\partial y^*}{\partial \ell_y} \frac{\partial \ell_y}{\partial a}$, which implies that the two options reduce the effective repayment y^* by the same amount. Thus, she is indifferent between using money as down payment and bonds as collateral. Without loss of generality, we consider that entrepreneurs use real balances before bonds in the contract problem. Thus, $d^* = \min\{m, k\}$ and $a^* = \min\{z, k - m\}$.

Derivation of (18) and (19). Using (4) and (5) lagged one period to eliminate $\mathbb{E}\,V_m(m,z)$ and $\mathbb{E}\,V_z(m,z)$ from (15) and (16), respectively, we derive (18) and (19) as follows.

$$
\begin{aligned}
\frac{\sigma}{\beta} &= \alpha\left\{\begin{array}{l}(1-\phi)(\mu_\omega - m)f(m)+\\ \phi\left[\left[\mu_\omega - \gamma G(y^\dagger) - k^T\right] f(k^T)\dfrac{\partial k^T}{\partial m} - \displaystyle\int_{m+z}^{k^T}\gamma g(y^*)\dfrac{\partial y^*}{\partial m} f(k)dk\right]\end{array}\right\} + 1 \\
&= \alpha\left\{\begin{array}{l}(1-\phi)(\mu_\omega - m)f(m)+\\ \phi\left[\left[\mu_\omega - \gamma G(y^\dagger) - k^T\right] f(k^T)\dfrac{\partial k^T}{\partial m} + \underbrace{\displaystyle\int_{m+z}^{k^T}\gamma g(y^*)\dfrac{\partial y^*}{\partial k} f(k)dk}_{\text{Integration by part}}\right]\end{array}\right\} + 1 \\
&= \alpha\left\{\begin{array}{l}(1-\phi)(\mu_\omega - m)f(m)+\\ \phi\left[\left[\mu_\omega - \gamma G(y^\dagger) - k^T\right] f(k^T)\dfrac{\partial k^T}{\partial m} + \left[\gamma G(y^*)f(k)\right]_{k=m+z}^{k=k^T} - \displaystyle\int_{m+z}^{k^T}\gamma G(y^*)f'(k)dk\right]\end{array}\right\} + 1 \\
&= \alpha\left\{\begin{array}{l}(1-\phi)(\mu_\omega - m)f(m)+\\ \phi\left[\left[\mu_\omega - \gamma G(y^\dagger) - k^T\right] f(k^T)\dfrac{\partial k^T}{\partial m} + \left[\gamma G(y^\dagger)f(k^T) - 0\right] - \displaystyle\int_{m+z}^{k^T}\gamma G(y^*)f'(k)dk\right]\end{array}\right\} + 1 \\
&= \alpha\left\{(1-\phi)(\mu_\omega - m)f(m) + \phi\left[(\mu_\omega - k^T)f(k^T) - \int_{m+z}^{k^T}\gamma G(y^*)f'(k)dk\right]\right\} + 1,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\psi}{\beta} &= \alpha\phi\left[\left[\mu_\omega - \gamma G(y^\dagger) - k^T\right] f(k^T)\frac{\partial k^T}{\partial m} - \int_{m+z}^{k^T}\gamma g(y^*)\frac{\partial y^*}{\partial m} f(k)dk\right] + 1 \\
&= \alpha\phi\left[\left[\mu_\omega - \gamma G(y^\dagger) - k^T\right] f(k^T)\frac{\partial k^T}{\partial m} + \underbrace{\int_{m+z}^{k^T}\gamma g(y^*)\frac{\partial y^*}{\partial k} f(k)dk}_{\text{Integration by part}}\right] + 1 \\
&= \alpha\phi\left[\left[\mu_\omega - \gamma G(y^\dagger) - k^T\right] f(k^T)\frac{\partial k^T}{\partial m} + \left[\gamma G(y^*)f(k)\right]_{k=m+z}^{k=k^T} - \int_{m+z}^{k^T}\gamma G(y^*)f'(k)dk\right] + 1 \\
&= \alpha\phi\left[\left[\mu_\omega - \gamma G(y^\dagger) - k^T\right] f(k^T)\frac{\partial k^T}{\partial m} + \left[\gamma G(y^\dagger)f(k^T) - 0\right] - \int_{m+z}^{k^T}\gamma G(y^*)f'(k)dk\right] + 1 \\
&= \alpha\phi\left[(\mu_\omega - k^T)f(k^T) - \int_{m+z}^{k^T}\gamma G(y^*)f'(k)dk\right] + 1,
\end{aligned}
$$

where we have used $\frac{\partial k^T}{\partial m} = \frac{\partial k^T}{\partial z} = 1$ if there is credit rationing, and $\mu_\omega - \gamma G(y^\dagger) - k^T = 0$ if there is no credit rationing.

Proof of Proposition 3 If $k \sim \mathcal{U}[0, 2\mu_k]$, $\omega \sim \mathcal{U}[0, 2\mu_\omega]$ where $\mu_k = \frac{\epsilon_k}{2}$ and $\mu_\omega = \frac{\epsilon_\omega}{2}$, and $\gamma < \frac{\epsilon_\omega}{2}$, then we can first calculate the efficient repayment y^* by equation (10). Thus, when $\ell_y = k - m - z$, we get

$$y^* = (\overline{\omega} - \gamma)^2 - \sqrt{(\overline{\omega} - \gamma)^2 - 2\epsilon_\omega \ell_y}$$

which is defined when $0 \leq \ell_y \leq \frac{(\overline{\omega}-\gamma)^2}{2\epsilon_\omega} = \mu_\omega - \gamma + \frac{\gamma^2}{2\epsilon_\omega} \equiv \overline{\ell}_{yb}$.

According to (18) and (19), and the uniform distribution assumption, we have the following conditions in a monetary equilibrium.

$$\begin{aligned}
\frac{\sigma}{\beta} &= \frac{\alpha}{\epsilon_k}[(1-\phi)(\mu_\omega - m) + \phi(\mu_\omega - k^T)] + 1, \\
\frac{\psi}{\beta} &= \frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T) + 1
\end{aligned}$$

From the second equation, we conclude that

$$\psi^* = \beta\left[1 + \frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T)\right],$$

while the value of k^T depends on whether or not there is a credit rationing. When there is a credit rationing, $k^T = k_b$, and thereby

$$\begin{aligned}
k^T &= m + z + \overline{\ell}_{yb}, \\
m^* &= \mu_\omega - \phi(z + \overline{\ell}_{yb}) - \frac{(\sigma - \beta)\epsilon_k}{\alpha\beta}.
\end{aligned}$$

When there is no credit rationing, we find $k^T = k_e$ by solving

$$\pi_e = \mu_\omega - \gamma G(y^*) - k^T = 0$$

given $\{m,\ z\}$. This implies

$$k^T = \mu_\omega - \gamma\left(1 - \sqrt{\frac{m+z}{\mu_\omega}}\right)$$

when $\sigma > \beta$. The real balances solve

$$m^* = (1-\phi)(\mu_\omega - m) + \phi\gamma\left(1 - \sqrt{\frac{m+z}{\mu_\omega}}\right) - \frac{\sigma - \beta}{\alpha\beta}\epsilon_k = 0$$

based on (18). Notice that holding real balances is costless when $\sigma = \beta$. Because access to banks is not ensured, entrepreneurs would hold sufficient real balances to finance all potential profitable projects. That is, $m^* = \mu_\omega = k^T$ when $\sigma = \beta$.

Proof of Proposition 4 and 5 With the uniform distribution assumptions and Proposition 3, we have following conditions in credit rationing equilibrium.

$$\begin{aligned}
m^* &= \mu_\omega - \phi(z + \bar{\ell}_{yb}) - \frac{(\sigma - \beta)\epsilon_k}{\alpha\beta}, \\
\psi^* &= \beta\left[1 + \frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T)\right], \\
k^T &= m + z + \bar{\ell}_{yb}, \\
\bar{\ell}_{yb} &= \mu_\omega - \gamma + \frac{\gamma^2}{2\epsilon_\omega}, \\
y^* &= (\varpi - \gamma)^2 - \sqrt{(\varpi - \gamma)^2 - 2\epsilon_\omega \ell_y}.
\end{aligned}$$

Then, according to the definitions in Section 4, the aggregate variables are as follows.

$$\begin{aligned}
N &= \frac{\alpha}{\epsilon_k}\left(\mu_\omega - \frac{\sigma - \beta}{\alpha\beta}\epsilon_k\right), \\
D &= \frac{\alpha\phi\gamma}{\epsilon_k}\frac{\epsilon_\omega - \gamma}{3}\bar{\ell}_{yb}, \\
TL &= \frac{\alpha\phi}{\epsilon_k}\frac{(z + \bar{\ell}_{yb})^2}{2}, \\
X &= \frac{\alpha\phi}{\epsilon_k}\left[\frac{z^2}{2} + z\bar{\ell}_{yb} + \frac{\epsilon_\omega - \gamma}{3}\bar{\ell}_{yb}\right], \\
R &= \frac{X}{TL}, \\
r_b &= \frac{1}{\beta\left[1 + \frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T)\right]} - 1, \\
S &= R - \left[1 + \frac{\alpha\phi}{\epsilon_k}(\mu_\omega - k^T)\right], \\
K &= \frac{\alpha}{2\epsilon_k}\left[(1 - \phi)m^2 + \phi(m + z + \bar{\ell}_{yb})^2\right], \\
Y &= \mu_\omega N - \gamma D, \\
P &= Y - K.
\end{aligned}$$

Taking derivative with respect to each parameter change, we get Proposition 4 and 5.

Proof of Proposition 6 For the completeness, we start from entrepreneurs' default decision under the policy. Given down payment, d^*, and collateralized bonds, a^*, the entrepreneur follows the same default rule in (6). As a result, we have the same expected profit for entrepreneur as (8). The expected payoff for bank is changed because of the policy:

$$\pi_b^p(y) = \theta \left[y[1 - G(y)] + \int_{\underline{\omega}}^{y} \omega \, dG(\omega) - \gamma G(y) - \underbrace{(k - d - \frac{a}{\theta})}_{\ell_y^p} \right]$$

as shown in (24). Therefore, the contract problem under policy can be described as

$$\{d^*,\ a^*,\ y^*\} = \arg \max_{\{d,a,y\}} -d - a + \int_{y}^{\overline{\omega}} \omega \, dG(\omega) - y[1 - G(y)]$$

$$\text{s.t.} \quad \pi_b^p(y) = \theta \left[y[1 - G(y)] + \int_{\underline{\omega}}^{y} \omega \, dG(\omega) - \gamma G(y) - \underbrace{(k - d - \frac{a}{\theta})}_{\ell_y^p} \right] = 0,$$

$$0 \le d \le m,\ 0 \le a \le z.$$

Recall in Proposition 2, the entrepreneur is indifferent between using down payment or collateralizing bonds when she borrows. The intuition underlying Proposition 2 is that one unit of real balances or bonds can reduce one unit of risky loan, while each asset is valued at one unit of the general good in stage 2. That is, both money and bonds have the same benefits in reducing risky loan amount, $\ell_y = k - d - a$, and identical opportunity costs. However, under policy the risky loan becomes $\ell_y^p = k - d - \frac{a}{\theta}$, and one unit of bonds collateralized can reduce $\frac{a}{\theta}$ unit of risky loans which is larger than using one unit of down payment. Hence, banked entrepreneurs collateralize bonds before they use down payment, which is shown in (26) and is different from Proposition 2. For example, when $k \in (\underline{k}, \frac{z}{\theta})$, the entrepreneur with access to bank will propose a contract with $\{d^*,\ a^*,\ y^*\} = \{0,\ \theta k,\ 0\}$. This does not mean the original repayment on the contract is 0. Recall that the effective repayment is defined as $y^* = x^* - a^*$, which implies the original repayment in the contract is $x^* = y^* + a^* = \theta k$. This also implies that bank experiences loss by extending loan. The bank's participation condition still holds because of the subsidy from central bank by purchasing loans. Finally, by binding bank's participation condition and the solution of the contract problem (26), we obtain (27) and (29).

Proof of Lemma 1. Let $B(y)$ denote the bank's expected payoff backed solely by the risky project return:

$$B(y) = y[1 - G(y)] + \int_{\underline{\omega}}^{y} \omega \, dG(\omega) - \gamma G(y).$$

The bank's expected payoff under policy, (24), can be rewritten as

$$\pi_b^p(y) = \theta \left[B(y) - \ell_y^p \right], \tag{36}$$

where $\ell_y^p = k - d - \frac{a}{\theta}$.

First, consider the case with credit rationing. Let $B(y_b)$ denote the bank's maximum expected payoff of loan without policy when the repayment is y_b, and $\overline{\ell}_{yb}$ is the associated maximum risky loan amount such that $B(y_b) - \overline{\ell}_{yb} = 0$. From Definition 1, the investment threshold is $k_b = m + z + \overline{\ell}_{yb}$ without policy. Under policy, let $B(y_b^p)$ denote the bank's maximum expected payoff when the repayment is y_b^p, and the associated maximum risky loan amount is $\overline{\ell}_{yb}^p$. Therefore, we have $\theta \left[B(y_b^p) - \overline{\ell}_{yb}^p \right] = 0$ and the investment threshold under policy is $k_b^p = m + \frac{z}{\theta} + B(y_b^p)$. Note that the repayment associated with the maximum risky loan amount depends only on the monitoring cost and the distribution of project return. Hence, $y_b^p = y_b$, and $B(y_b) = B(y_b^p)$. Thereby, we conclude that $k_b^p - k_b = \frac{1-\theta}{\theta} z$.

Next, consider the case without credit rationing. Without policy, no-credit rationing implies that there exist (y_1, k_e) that solves

$$\begin{cases} \mu_\omega - \gamma G(y_1) - k_e = 0 \\ B(y_1) - (k_e - m - z) = 0, \end{cases} \tag{$*$}$$

while with policy, (y_2, k_e^p) solves

$$\begin{cases} \mu_\omega - \gamma G(y_2) - k_e^p + \frac{1-\theta}{\theta} z = 0 \\ B(y_2) - (k_e^p - m - \frac{z}{\theta}) = 0. \end{cases} \tag{$*'$}$$

Given that (y_1, k_e) solves the system, $(*)$, we let $(y_2, k_e^p) = (y_1, k_e + \frac{1-\theta}{\theta} z)$ and verify this is the solution for the system, $(*')$. Therefore, $k_e^p - k_e = \frac{1-\theta}{\theta} z$.

Derivation of (31) and (32). From (30), we have

$$\begin{aligned}
\frac{\sigma}{\beta} &= \alpha \left\{ \begin{array}{l} (1-\phi)(\mu_\omega - m) f(m) + \\ \phi \left[\begin{array}{l} (\mu_\omega - m - z) f(m + \frac{z}{\theta}) + \\ (\mu_\omega - \gamma G(y^\dagger) - k^p + \frac{1-\theta}{\theta} z) f(k^p) \frac{\partial k^p}{\partial m} - (\mu_\omega - m - z) f(m + \frac{z}{\theta}) - \\ \int_{m+\frac{z}{\theta}}^{k^p} \gamma g(y^*) \frac{\partial y^*}{\partial m} f(k) dk \end{array} \right] \end{array} \right\} + 1 \\
&= \alpha \left\{ (1-\phi)(\mu_\omega - m) f(m) + \phi \left[\begin{array}{l} (\mu_\omega - \gamma G(y^\dagger) - k^p + \frac{1-\theta}{\theta} z) f(k^p) \frac{\partial k^p}{\partial m} + \\ \underbrace{\int_{m+\frac{z}{\theta}}^{k^p} \gamma g(y^*) \frac{\partial y^*}{\partial k} f(k) dk}_{\text{Integration by part}} \end{array} \right] \right\} + 1 \\
&= \alpha \left\{ (1-\phi)(\mu_\omega - m) f(m) + \phi \left[\begin{array}{l} (\mu_\omega - \gamma G(y^\dagger) - k^p + \frac{1-\theta}{\theta} z) f(k^p) \frac{\partial k^p}{\partial m} + \\ \Big[\gamma G(y^*) f(k) \Big]_{k=m+\frac{z}{\theta}}^{k=k^p} - \int_{m+\frac{z}{\theta}}^{k^p} \gamma G(y^*) f'(k) dk \end{array} \right] \right\} + 1 \\
&= \alpha \left\{ (1-\phi)(\mu_\omega - m) f(m) + \phi \left[\begin{array}{l} (\mu_\omega - \gamma G(y^\dagger) - k^p + \frac{1-\theta}{\theta} z) f(k^p) \frac{\partial k^p}{\partial m} + \\ \Big[\gamma G(y^\dagger) f(k^T) - 0 \Big] - \int_{m+\frac{z}{\theta}}^{k^p} \gamma G(y^*) f'(k) dk \end{array} \right] \right\} + 1 \\
&= \alpha \left\{ (1-\phi)(\mu_\omega - m) f(m) + \phi \left[(\mu_\omega - k^p + \frac{1-\theta}{\theta} z) f(k^p) - \int_{m+\frac{z}{\theta}}^{k^p} \gamma G(y^*) f'(k) dk \right] \right\} + 1,
\end{aligned}$$

where we have used $\frac{\partial k^p}{\partial m} = 1$ if there is credit rationing, and $\mu_\omega - \gamma G(y^\dagger) - k^p + \frac{1-\theta}{\theta} z = 0$ if there is no credit rationing, to obtain the last line.

Let $\hat{z} = \frac{z}{\theta}$, then

$$\begin{aligned}
\frac{\partial \mathbb{E} V^p(m,z)}{\partial \hat{z}} &= \alpha\phi \begin{bmatrix} (\mu_\omega - z) f(\hat{z}) + \\ (\mu_\omega - m - z) f(m+\hat{z}) - (\mu_\omega - z) f(\hat{z}) + \int_{\hat{z}}^{m+\hat{z}} (1-\theta) dF(k) + \\ (\mu_\omega - \gamma G(y^\dagger) - k^p + \frac{1-\theta}{\theta} z) f(k^p) \frac{\partial k^p}{\partial \hat{z}} - (\mu_\omega - m - z) f(m+\hat{z}) - \\ \int_{m+\hat{z}}^{k^p} \gamma g(y^*) \frac{\partial y^*}{\partial \hat{z}} f(k) dk + \int_{m+\hat{z}}^{k^p} (1-\theta) dF(k) \end{bmatrix} + \theta \\
&= \alpha\phi \begin{bmatrix} (\mu_\omega - \gamma G(y^\dagger) - k^p + \frac{1-\theta}{\theta} z) f(k^p) \frac{\partial k^p}{\partial \hat{z}} + \\ \underbrace{\int_{m+\hat{z}}^{k^p} \gamma g(y^*) \frac{\partial y^*}{\partial k} f(k) dk}_{\text{Integration by part}} + (1-\theta)[F(k^p) - F(\hat{z})] \end{bmatrix} + \theta \\
&= \alpha\phi \begin{bmatrix} (\mu_\omega - \gamma G(y^\dagger) - k^p + \frac{1-\theta}{\theta} z) f(k^p) \frac{\partial k^p}{\partial \hat{z}} + \\ \Big[\gamma G(y^*) f(k)\Big]_{k=m+\hat{z}}^{k=k^p} - \int_{m+\hat{z}}^{k^p} \gamma G(y^*) f'(k) dk + (1-\theta)[F(k^p) - F(\hat{z})] \end{bmatrix} + \theta \\
&= \alpha\phi \begin{bmatrix} (\mu_\omega - \gamma G(y^\dagger) - k^p + \frac{1-\theta}{\theta} z) f(k^p) \frac{\partial k^p}{\partial \hat{z}} + \\ \Big[\gamma G(y^\dagger) f(k^p) - 0\Big] - \int_{m+\hat{z}}^{k^p} \gamma G(y^*) f'(k) dk + (1-\theta)[F(k^p) - F(\hat{z})] \end{bmatrix} + \theta \\
&= \alpha\phi \begin{bmatrix} (\mu_\omega - k^p + \frac{1-\theta}{\theta} z) f(k^p) - \int_{m+\hat{z}}^{k^p} \gamma G(y^*) f'(k) dk + \\ (1-\theta)[F(k^p) - F(\hat{z})] \end{bmatrix} + \theta,
\end{aligned}$$

where we have used $\frac{\partial k^p}{\partial \hat{z}} = 1$ if there is credit rationing, and $\mu_\omega - \gamma G(y^\dagger) - k^p + \frac{1-\theta}{\theta} z = 0$ if there is no credit rationing, to obtain the last line. Then,

$$\begin{aligned}
\frac{\psi}{\beta} &= \frac{\partial \mathbb{E} V^p}{\partial z} = \frac{\partial \mathbb{E} V^p}{\partial \hat{z}} \frac{\partial \hat{z}}{\partial z} = \frac{1}{\theta} \frac{\partial \mathbb{E} V^p}{\partial \hat{z}} \\
&= \frac{\alpha\phi}{\theta} \begin{bmatrix} (\mu_\omega - k^p + \frac{1-\theta}{\theta} z) f(k^p) - \int_{m+\frac{z}{\theta}}^{k^p} \gamma G(y^*) f'(k) dk + \\ (1-\theta)[F(k^p) - F(\frac{z}{\theta})] \end{bmatrix} + 1.
\end{aligned}$$

Proof of Proposition 7 The optimal retention rate maximizes the welfare measure (35). Define $\hat{z} = \frac{z}{\theta}$. First, we can derive the following condition from Lemma 1.

$$\frac{\partial k^p}{\partial \theta} = \frac{\partial (k^T - z + \hat{z})}{\partial \theta} = \frac{\partial k^T}{\partial m}\frac{\partial m}{\partial \theta} + \frac{\partial \hat{z}}{\partial \theta} \tag{37}$$

We also derive the following condition from the bank's binding participation condition under policy.

$$\frac{\partial y^*}{\partial \theta} = \frac{\partial y^*}{\partial (m+\hat{z})}\frac{\partial (m+\hat{z})}{\partial \theta} = -\frac{\partial y^*}{\partial k}(\frac{\partial m}{\partial \theta} + \frac{\partial \hat{z}}{\partial \theta}). \tag{38}$$

With these two conditions, the first-order condition is given by

$$\begin{aligned}
\frac{\partial \Omega}{\partial \theta} &= \alpha \left\{ \begin{array}{l} (1-\phi)(\mu_\omega - m) f(m) \frac{\partial m}{\partial \theta} + \\ \phi \left[(\mu_\omega - k^p) f(k^p) \frac{\partial k^p}{\partial \theta} - \gamma G(y^\dagger) f(k^p) \frac{\partial k^p}{\partial \theta} - \int_{m+\hat{z}}^{k^p} \gamma g(y^*) \frac{\partial y^*}{\partial \theta} f(k) dk \right] \end{array} \right\} \\
&= \alpha \left\{ \begin{array}{l} (1-\phi)(\mu_\omega - m) f(m) \frac{\partial m}{\partial \theta} + \\ \phi \left[(\mu_\omega - \gamma G(y^\dagger) - k^p) f(k^p) (\frac{\partial k^T}{\partial m}\frac{\partial m}{\partial \theta} + \frac{\partial \hat{z}}{\partial \theta}) + \int_{m+\hat{z}}^{k^p} \gamma g(y^*) \frac{\partial y^*}{\partial k} (\frac{\partial m}{\partial \theta} + \frac{\partial \hat{z}}{\partial \theta}) f(k) dk \right] \end{array} \right\} \\
&= \alpha \left\{ \begin{array}{l} \left\{ (1-\phi)(\mu_\omega - m) f(m) + \phi \left[(\mu_\omega - \gamma G(y^\dagger) - k^p) f(k^p) \frac{\partial k^T}{\partial m} + \underbrace{\int_{m+\hat{z}}^{k^p} \gamma g(y^*) \frac{\partial y^*}{\partial k} f(k) dk}_{A} \right] \right\} \frac{\partial m}{\partial \theta} + \\ \phi \left[(\mu_\omega - \gamma G(y^\dagger) - k^p) f(k^p) + \underbrace{\int_{m+\hat{z}}^{k^p} \gamma g(y^*) \frac{\partial y^*}{\partial k} f(k) dk}_{A} \right] \frac{\partial \hat{z}}{\partial \theta} \end{array} \right\} \\
&= \alpha \left\{ \begin{array}{l} \left\{ (1-\phi)(\mu_\omega - m) f(m) + \phi \left[\begin{array}{l} (\mu_\omega - k^p) f(k^p) \frac{\partial k^T}{\partial m} - \gamma G(y^\dagger) f(k^p) (\frac{\partial k^T}{\partial m} - 1) \\ - \int_{m+\hat{z}}^{k^p} \gamma G(y^*) f'(k) dk \end{array} \right] \right\} \frac{\partial m}{\partial \theta} + \\ \phi \left[(\mu_\omega - k^p) f(k^p) - \int_{m+\hat{z}}^{k^p} \gamma G(y^*) f'(k) dk \right] \frac{\partial \hat{z}}{\partial \theta} \end{array} \right\}
\end{aligned} \tag{39}$$

where $y^\dagger$ is the repayment when $k = k^p$ and $A = \gamma G(y^\dagger) f(k^p) - \int_{m+\hat{z}}^{k^p} \gamma G(y^*) f'(k) dk$ via applying integration by part. Notice that k^T is the investment threshold without policy when entrepreneurs hold portfolio $(m,\ z)$.

Given Lemma 1 and uniform distribution assumption, i.e. $f'(k) = 0$ and $f(k^p) = f(k^T)$, we rewrite (31) as

$$\frac{\sigma}{\beta} = \alpha \left\{ (1-\phi)(\mu_\omega - m) f(m) + \phi(\mu_\omega - k^T) f(k^T) \right\} + 1,$$

which is the optimal choice of real balances under the policy.

Therefore, the decision of money holding is unaffected by the policy because the marginal benefit of money remains the same, which implies

$$\frac{\partial m}{\partial \theta} = 0.$$

Substituting $\frac{\partial m}{\partial \theta} = 0$ into (39), we have

$$\frac{\partial \Omega}{\partial \theta} = \alpha\phi(\mu_\omega - k^p) f(k^p) \frac{\partial \hat{z}}{\partial \theta} = -\frac{\alpha\phi z}{\theta^2}(\mu_\omega - k^p) f(k^p),$$

which suggests that the welfare measure reaches maximum when $k^p = \mu_\omega$ or $k^p > \overline{k}$. We focus on the non-trivial case with $k^p = \mu_\omega$ in the analysis. Then, the optimal risk retention rate satisfies the following condition

$$\begin{aligned} \mu_\omega &= k^p = k^T + \frac{1-\theta^*}{\theta^*} z \\ &\Rightarrow \theta^* = \frac{z}{\mu_\omega - k^T + z}. \end{aligned}$$

We need to check the second-order condition:

$$\frac{\partial^2 \Omega}{\partial^2 \theta} = -\frac{\partial \frac{\alpha\phi z}{\epsilon_k \theta^2}(\mu_\omega - k^p)}{\partial \theta} = 2\frac{\alpha\phi z}{\epsilon_k \theta^3}(\mu_\omega - k^p) + \frac{\alpha\phi z}{\epsilon_k \theta^2}\frac{\partial k^p}{\partial \theta} \tag{40}$$

Substituting θ^* into equation (40), we have $\frac{\partial^2 \Omega}{\partial^2 \theta} \;|_{\theta=\theta^*} < 0$, which implies the sufficient condition holds.

Now we investigate how risk affects the optimal risk retention rate. We know that $\frac{\partial \theta^*}{\partial k^T} > 0$, and from the results of Proposition 3, we derive $\frac{\partial k^T}{\partial \alpha} > 0$, $\frac{\partial k^T}{\partial \epsilon_\omega} < 0$, and $\frac{\partial k^T}{\partial \epsilon_k} < 0$. Hence, we conclude

$$\begin{cases} \dfrac{\partial \theta^*}{\partial \alpha} = \dfrac{\partial \theta^*}{\partial k^T}\dfrac{\partial k^T}{\partial \alpha} > 0 \\ \dfrac{\partial \theta^*}{\partial \epsilon_\omega} = \dfrac{\partial \theta^*}{\partial k^T}\dfrac{\partial k^T}{\partial \epsilon_\omega} < 0 \\ \dfrac{\partial \theta^*}{\partial \epsilon_k} = \dfrac{\partial \theta^*}{\partial k^T}\dfrac{\partial k^T}{\partial \epsilon_k} < 0. \end{cases}$$

Derivation.

Case 1. Credit rationing.

$$k^T = \mu_\omega + (1-\phi)(z + \bar{\ell}_{yb}) - \frac{\sigma - \beta}{\alpha\beta}\epsilon_k.$$

Thus,

$$\begin{aligned}
\frac{\partial k^T}{\partial \alpha} &= \frac{\sigma - \beta}{\alpha^2 \beta}\epsilon_k > 0, \\
\frac{\partial k^T}{\partial \epsilon_\omega} &= (1-\phi)\frac{\partial \bar{\ell}_{yb}}{\partial \epsilon_\omega} < 0, \\
\frac{\partial k^T}{\partial \epsilon_k} &= -\frac{\sigma - \beta}{\alpha\beta} < 0.
\end{aligned}$$

Case 2. Non-credit rationing if $\sigma > \beta$.

$$\begin{aligned}
m \quad & solves \quad (1-\phi)(\mu_\omega - m) + \phi\gamma\left(1 - \sqrt{\frac{2(m+z)}{\epsilon_\omega}}\right) - \frac{(\sigma - \beta)}{\alpha\beta}\epsilon_k = 0, \\
k^T \quad & = \quad \mu_\omega - \gamma + \gamma\sqrt{\frac{2(m+z)}{\epsilon_\omega}}.
\end{aligned}$$

We have

$$\frac{\partial m}{\partial \alpha} > 0, \quad \frac{\partial m}{\partial \epsilon_k} < 0, \quad \frac{\partial m}{\partial \epsilon_\omega} > 0.$$

The last condition $\frac{\partial m}{\partial \epsilon_\omega} > 0$ implies $\partial\sqrt{\frac{2(m+z)}{\epsilon_\omega}}/\partial\epsilon_\omega < 0$. Hence,

$$\begin{aligned}
\frac{\partial k^T}{\partial \alpha} &= \frac{\gamma}{\epsilon_\omega}\left(\frac{2(m+z)}{\epsilon_\omega}\right)^{-\frac{1}{2}} \frac{\partial m}{\partial \alpha} > 0, \\
\frac{\partial k^T}{\partial \epsilon_k} &= \frac{\gamma}{\epsilon_\omega}\left(\frac{2(m+z)}{\epsilon_\omega}\right)^{-\frac{1}{2}} \frac{\partial m}{\partial \epsilon_k} < 0, \\
\frac{\partial k^T}{\partial \epsilon_\omega} &= \frac{\gamma}{\epsilon_\omega}\left(\frac{2(m+z)}{\epsilon_\omega}\right)^{-\frac{1}{2}} \left[\partial\sqrt{\frac{2(m+z)}{\epsilon_\omega}}/\partial\epsilon_\omega\right] < 0.
\end{aligned}$$