Playing Hard to Get: Strategic Misrepresentation in Marriage Markets

Daniel Simundza*†

July 14, 2015

Abstract

This paper presents a study of strategic interaction in marriage markets, specifically the strategy known colloquially as "playing hard to get." Even though the goal of playing hard to get is to misrepresent oneself, I find that the equilibrium response to this behavior actually results in a more efficient mating. In the model, agents observe noisy signals of potential mates' types and have *two* chances to accept or reject a marriage. This modeling choice enables an agent to make initial strategic rejections in the hopes of improving her partner's perception of her type. Previous analyses of matching markets have not accounted for this behavior. I prove the existence of an equilibrium in which some low quality agents play hard to get by initially rejecting high signals. Despite this attempt at strategic misrepresentation, I show that, relative to an equilibrium without any strategic behavior, mating is more efficient in the equilibrium with playing hard to get.

Keywords: Matching; Misrepresentation; Search; Adverse selection
JEL Classification: D82, D83

*I thank Hector Chade, Hans Haller, Ofer Setty, and Lones Smith for insightful discussions and comments.

†Contact: 3118 Pamplin Hall (0316), Blacksburg VA 24061, dan.simundza@gmail.com, (540) 231 - 5764 Virginia Tech

Rule #5: Don't call him and rarely return his calls.
– *The Rules: Time-Tested Secrets for Capturing the Heart of Mr. Right* by Ellen Fein & Sherrie Schneider

1 Introduction

In matching markets with private information over individual attributes, the decision of whether to form a partnership may take place under significant uncertainty. Strategic behavior in such markets is therefore pervasive as agents attempt to exploit their informational edge. This paper formalizes one particular strategy, colloquially known as "playing hard to get," in which agents initially conceal their interest in desirable partners in order to appear to be of a more desirable type. I prove the existence of an equilibrium involving playing hard to get and show that mating remains positively assortative (i.e. people marry others of similar type), a benchmark first established in Becker [1973], and recently generalized to environments of incomplete information in Chade [2006]. Moreover, I show that this sort of strategic behavior improves the sorting of mates.

Previous frictional matching papers have modeled the "marry or not" decision as a one-shot game, and thus not accounted for strategic considerations within a relationship. In my model, agents have two chances to accept or reject a potential mate. This modeling choice enables an agent to make initial strategic rejections in the hopes of improving her potential partner's perception of her type.

In deciding whether to marry, agents judge the attractiveness of their current partner against the outside option of continuing to search for a better mate.[1] When types are private information, agents do what they can to learn about potential mates' quality. Learning in relationships occurs not just through the observation of noisy signals, but also by inferring type from behavior. And while it may not be possible to alter one's appearance or conversational ability, it is possible for the ordinary to behave as if they were extraordinary by playing hard to get.

In order to model this strategic behavior, I add dynamics to the pre-marriage relationship. Specifically, matched agents observe noisy signals of each other's true type and then have two "rounds" in which they can accept or reject a marriage. In equilibrium, agents' first round announcements convey information about their type. Match partners must take this information into account when determining their second round announcement. Embedding this dynamic game

[1] Since my model is one in which utility is non-transferable (i.e. the quality of one's match partner completely determines payoffs), I focus on social matching and the marriage market. If, alternatively, utility was transferable, all divorces would be mutually agreeable, which is clearly not the case. While strategic interaction is also likely to be important in the job search setting and other situations in which the transferable utility assumption is reasonable, my assumption that utility is non-transferable rules out the possibility of side payments (e.g. wages) to facilitate matches. See McNamara and Collins [1990] and Moscarini [2005] for more on search in labor markets.

in a marriage model allows low types to misrepresent themselves as high types in hopes of marrying a better mate.

I show a *Socially Strategic* equilibrium exists in which some low type agents play hard to get by declining a match after observing an attractive signal in the first round. If the agents do not form a marriage in the first round, they update their beliefs as to their match partner's type based on their first round announcements. In the Socially Strategic equilibrium, a positive fraction of jilted high types who rejected their initially undesired partner change their announcement and optimally accept in the second round.

This behavior by the high types improves sorting by increasing the chances that a pair of high types marry in the event that they both send low signals. While the low types' goal in playing hard to get is to increase mismatch, it is ineffective in equilibrium. I show that the Socially Strategic equilibrium increases sorting of types relative to an equilibrium without any strategic behavior.[2]

Despite the low types' attempts at strategic misrepresentation, mating remains positively assortative in the Socially Strategic equilibrium. While high types who receive low signals mate with low types who receive high signals, the expected partner of a high type agent is better than the expected partner of a low type agent.

In addition to establishing these results on the sorting and mating, I provide comparative statics results. I first focus on how the accuracy of the signaling technology, the fraction of high types in the population, the discount rate, the baseline value of forming a match relative to remaining single, and the added benefit of marrying a high type affect the equilibrium behavior of the agents. It is then straightforward to determine the effect of changes in these variables on the set of matches that occur in equilibrium.

2 Related literature

In an early frictionless model, Becker [1973] established the allocative benchmark of Positive Assortative Mating (PAM). He proved two results relevant for the present study: first, PAM maximizes total output when types are productive complements, and second, PAM obtains in equilibrium when utility is non-transferable and higher-type partners are more preferred.[3] Smith [2006] extended this framework to incorporate search frictions, and found that Becker's simple requirement that "higher is better" is insufficient to generate PAM when search takes time and agents are impatient. Instead, he showed that PAM obtains if the proportionate gains from mating with better partners are greater for high types than low.

[2]Damiano et al. [2005] study the allocative efficiency of dynamic matching markets in both the presence and absence of participation costs. They show that, when there is no cost of participation, the allocation achieved by the matching mechanism improves upon purely random matching because high types match only with high types in early rounds. In contrast, my analysis takes place entirely in the steady state.

[3]See Consuegra et al. [2013] for a discussion of the necessity of the "higher is better" condition to generate PAM.

The first paper to incorporate information frictions into the searching and matching framework was Chade [2006]. In his model, agents observe noisy signals of their match-partner's (private) type before deciding whether to marry.[4] Chade [2006] proves the existence of an equilibrium in which "reservation signals" characterize agents' strategies, wherein agents accept only if the observed signal exceeds a threshold which is increasing in the agent's own type. Given such strategies, equilibrium matching is assortative on signals and, moreover, acceptance conveys bad news, a result the author terms the "acceptance curse."

But if acceptance conveys bad news, then rejection makes the heart grow fonder. To explain this point and to help motivate my model, consider a high type man dating a high type woman who each, by chance, observe low signals. Further, suppose that the equilibrium strategies call for low types to accept all signals and for high types to accept high signals and to reject low signals.[5] After being rejected, our high type man and high type woman are each sure their match parter was a high type, and would like to seek the other out and mate based on this revelation.

By adding within-match dynamics to the model, I not only allow agents to pursue missed chances, I also allow others to take advantage of those who do. For if the high types in the above scenario take advantage of a second chance to mate, then low types observing high signals can guarantee themselves a marriage with all partners by initially rejecting and then accepting.

My paper also contributes to a growing literature on matching markets with asymmetric information. One strand in this literature addresses moral hazard in matching markets (see for example Wright [2004], Franco et al. [2011], and Serfes [2008]). Unlike the current paper, these studies describe types as commonly observable and actions as predictable given equilibrium incentives, so agents do not have uncertainty over payoffs or employ strategic behavior to improve payoffs. The second strand is concerned with adverse selection, and includes Inderst [2005] and Hopkins [2012]. While types are private information in these papers, the equilibria are "separating" – different types take different actions, and agents have no *ex post* uncertainty over type. I add to this literature by analyzing a situation in which agents attempt to exploit their asymmetric information.

Psychologists, perhaps more than economists, have studied how and why agents engage in strategic behavior while mating. Jonason and Li [2013] present experimental results on the tactics that agents use to play hard to get, how often men and women use these tactics, and why they use them. One of their most commonly reported answers to the important question of "Why do you play hard to get?" was to increase demand.[6] In fact, my model shows that while

[4] Anderson and Smith [2010] study matching with symmetric incomplete information about agent types and publicly observable joint production. They find conditions such that assortative matching on public reputation fails despite complementarities of production.

[5] Such a strategy profile is an equilibrium for intermediate discount rates in the binary type and signal specialization in Section 4 of Chade [2006].

[6] The other most commonly reported answer was to test a potential mate's level of commitment. While this is an interesting application of the strategy of playing hard to get, it is

playing hard to get can increase demand when one's potential partner is initially not interested, it also derails some marriages with initially desirable and desiring partners. The benefits of playing hard to get do not come without costs.

Strategic misrepresentation in stable matching mechanisms reflects a similar trade-off. Coles and Shorrer [2013] show that agents have incentives to truncate the preference list they submit to the Deferred Acceptance Algorithm in order to increase the expected value of being matched. Similar to the present paper, their study shows that truncation increases payoffs conditional on a match occurring, but truncation also prevents the formation of some mutually beneficial matches.

The literature on "bluffing" in poker (Bellman and Blackwell [1949], Von Neumann and Morgenstern [1964]), wherein a player with a bad hand bids high, also relates to the present study. Von Neumann and Morgenstern [1964] cite two benefits of bluffing. The first is to induce opponents to believe one has a strong hand when it is in fact weak, thereby causing them to drop out. The second is to generate uncertainty, causing the opponent to stay in against a strong hand at other points in the game. Only inducing opponents to believe a weak hand is actually strong relates to the motivation to play hard to get; the second benefit plays no role in my model because high types do not benefit from the added uncertainty. Moreover, my focus is not just on why agents play hard to get, but also on how this behavior affects mating.

3 Model

The main characteristics of the model are described below.

Time: Time is discrete and is divided into periods of length equal to one.

Agents: There is a continuum of male agents and a continuum of female agents. The measure of each population is normalized to one. Each female is characterized by a type $x \in \{L, H\}$, and each male is characterized by a type $y \in \{L, H\}$, with $0 < L < H$.[7] The distribution of types is described by the parameter λ, where $\text{Prob}(x = H) = \text{Prob}(y = H) = \lambda$.

Information structure: Agents know their own types. Agents observe only a noisy signal of their match partner's type. Men observe signals $\theta \in \{l, h\}$ and women observe signals $\omega \in \{l, h\}$. Signals are "accurate" with probability $\epsilon > 1/2$. That is, the probability a man partnered with a high type woman observes h is ϵ. The probability a man partnered with a low type woman observes h is $1 - \epsilon$. The distribution of signals observed by women is identical.

Meeting technology: In each period, unmarried agents from opposite populations randomly meet in pairs.

Marriage game: Upon meeting, the man privately observes θ and the woman privately observes ω. There are two rounds in each period. In the first round, agents simultaneously announce accept (A) or reject (R) . If both announce A,

beyond the scope of the present paper.

[7]In the comparative statics analysis in Section 6 below I write $H = L + G$ in order to differentiate between the effects of increasing the benefit to marrying in general, and the effects of increasing the benefit to marrying a high type specifically.

they marry and leave the market. Otherwise, they proceed to the second round at which point agents again simultaneously announce A or R. If both announce A, they marry and leave the market. Otherwise, they go back to the pool of singles and continue searching.

Replenishment: In order to keep the distribution of types invariant over time, agents with the same types as the departing ones replace any pair exiting the market.

Match payoffs: A single agent's per-period utility is zero. If a woman of type x marries a man of type y, each receives per-period utility of xy.[8] Agents discount future payoffs with the common discount factor $0 < \delta < 1$.

Strategies: A stationary strategy for a woman of type x specifies a probability of acceptance in the first round after observing her potential match partner's signal ω, and a probability of acceptance in the second round after observing ω and the vector of first round announcements, so long as there was at least one rejection. Specifically, let $\sigma_x^1 : \{l, h\} \mapsto [0, 1]$ be a woman of type x's announcement in round one and $\sigma_x^2 : \{l, h\} \times \{A, R\} \times \{A, R\} \setminus (\omega, A, A) \mapsto [0, 1]$ be a woman of type x's announcement in round two, where the range of each mapping refers to the probability of acceptance. Similarly, a stationary strategy for a man of type y consists of a mapping $\sigma_y^1 : \{l, h\} \mapsto [0, 1]$ and a mapping $\sigma_y^2 : \{l, h\} \times \{A, R\} \times \{A, R\} \setminus (\theta, A, A) \mapsto [0, 1]$.

Equilibrium: I focus on Perfect Bayesian Equilibria (PBE) of the marriage game because this equilibrium concept requires that agents have reasonable beliefs (i.e. they are determined by Bayes' rule and players' equilibrium strategies) and act in a dynamically consistent way. A strategy profile and belief profile constitute a PBE if the beliefs are determined by Bayes' rule and the equilibrium strategies wherever possible, and the strategies are optimal given these beliefs at each decision node.

4 Strategies

Depending on the parameter values, there may be multiple equilibria of the game presented in Section 3. In this section I first describe an equilibrium in which some low type agents play hard to get (HTG). I then describe other equilibria of this game and argue that the equilibrium in which some agents play HTG is more reasonable.

Definition The *Socially Strategic* (*SS*) strategy profile consists of the following strategies for low- and high types.
Low types:

[8] This specification, which exhibits complementarities in marital production, is used in order to measure the welfare benefits of equilibrium sorting. All of the positive results of the paper would continue to hold if there were neither productive complements nor substitutes (e.g. utility is equal to the type of one's mate), but all allocations of men to women would result in the same social welfare.

- With probability α play *Hard to Get* by accepting low signals and rejecting high signals in the first round. In the second round, agents playing HTG accept regardless of the relationship history.
- With probability $1 - \alpha$ play *Eager* by always accepting.

High types:

- With probability β play *Flip-Flop*, which entails accepting high signals and rejecting low signals in the first round. In the second round, Flip-Floppers accept match partners who they initially rejected (i.e. the Flip-Flopper observed a low signal in round one) if the parter initially rejected them; otherwise, Flip-Floppers reject in round two.
- With probability $1 - \beta$ play *Resolute* by accepting high signals and rejecting low signals in the first round. In the second round, Resolute high types always reject.

I will show that the SS strategy profile is part of a PBE for certain parameter values. Since all information sets are reached with positive probability in equilibrium (i.e. all information sets lie along the equilibrium path), beliefs are determined using Bayes rule at all times. This completes the description of a PBE.

The important difference between the HTG and Eager strategies for the low type occurs in the first round when observing the high signal. In this case, HTG low types reject, hoping to make their match partners think more highly of them, while the Eager low types accept, hoping for an immediate marriage.

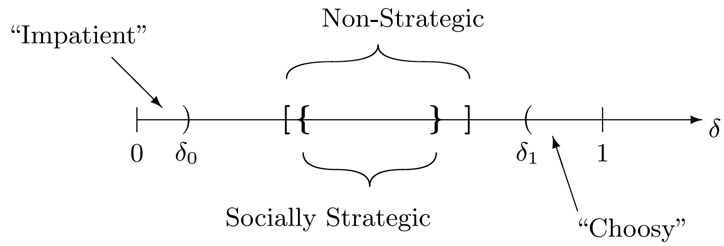
The important difference between the Flip-Flopping and Resolute strategies for the high type occurs in the second round after observing the low signal, rejecting, and being rejected in the first round. In this case the Resolute high type rejects, while the Flip-Flopping high type reverses his or her initial announcement and accepts in the second round.

Other equilibria of this model certainly exist.[9] When players are sufficiently patient, there is a "choosy" PBE in which all agents accept high signals in round one and reject at all other relationship histories. Since all information sets are on the equilibrium path, beliefs can be computed using Bayes rule and the equilibrium strategies. On the other extreme, when time is of the essence, there is an "impatient" PBE in which all agents accept at all histories. In this case, the information set in which one's match partner rejects is not on the equilibrium path. Beliefs at this information set are not important in the sense that, for small enough δ, acceptance is always optimal even if one's beliefs assign probability one to the event that the match partner is of low type. Figure 1 illustrates the intervals over which these equilibria exist.

While the above strategies can be equilibria, they are not in any sense competing with the SS strategy profile because they exist in different environments.

[9] An extreme example involves the "always reject" strategy profile in which all agents always reject. This strategy can be part of a PBE because, if all other players always reject, there is no strict incentive for any individual to deviate and accept.

Figure 1: Restrictions on discount rates for various equilibria



Notes: The figure shows intervals of discount rates over which different equilibria exist. When $\delta \leq \delta_0$, the "impatient" equilibrium exists. When $\delta \geq \delta_1$, the "choosy" equilibrium exists. When δ lies in the interval between the curly brackets ,{ }, the Socially Strategic equilibrium exists. When δ lies in the interval between the square brackets, [ ], the Non-Strategic equilibrium exists.

In order to better motivate the SS strategy profile, I will contrast it with an alternative strategy profile which can be an equilibrium in the same environment as the SS strategies. Consider the following *Non-Strategic* (*NS*) strategy profile: low types always accept, and high types accept high-signals in the first round and reject otherwise. All information sets lie along the equilibrium path, and so Bayes' rule can be used to calculate agents' beliefs. Figure 1 shows that there are discount rates such that both the SS and NS strategy profiles are equilibria.[10]

To see the problem with the NS equilibrium, consider a high type man in the second round who saw a low signal, rejected a match, and was himself rejected in the first round. Since his partner rejected him, he is certain that she is a high type, and, moreover, that he must have sent a low signal. The equilibrium calls for each to reject in round two, and indeed this is a weak best response given that both will be rejected. But it is not reasonable for them to reject because it is common knowledge that both agents know their partner is a high type, and rejecting is a weakly dominated strategy.[11]

But if high types in the above situation change their strategy and accept in round two, this creates incentives for low types to deviate from their strategy and instead reject high-signals in the first round in hopes of "tricking" a high type match partner into marrying them. That is, low types would face incentives to play hard to get. So trying to correct for this unreasonable behavior in fact creates incentives for the exact strategy on which I focus.

[10] Appendix A.3 proves the existence of the NS equilibrium and establishes the ordering of discount rates depicted in Figure 1.

[11] It is easy to show that, while this strategy profile is a PBE, it cannot be part of an Extensive Form Trembling Hand Perfect Equilibrium. It is well known that the trembling hand refinement rules out weakly dominated strategies. Proving that the SS strategy profile is an extensive form trembling hand perfect equilibrium is tedious and is not included in this paper.

5 Analysis

5.1 Inference

Let $a^1(\theta, y)$ be the probability a man of type y is accepted by a woman who sent signal θ in round one. Let $a^2(\theta, y, (s_i, s_j))$ be the probability a man of type y is accepted in round two by a woman who sent signal θ, and the man announced s_i and the woman announced s_j in round one. Let $\gamma^1(\theta, y)$ be the expected value to the man of type y of forming a match in round one with a woman who sent signal θ conditional on the match forming. Let $\gamma^2(\theta, y, (s_i, s_j))$ be the expected value to the man of type y of forming a match in round two with a woman who sent signal θ in round one, and the realized announcements were (s_i, s_j). Again, the expectation in $\gamma^2(\cdot)$ is taken conditional on the event that a match is formed. Lastly, I write the relationship history for an individual i who observed signal θ, announced s_i, and whose match partner j announced s_j in the first round as $\theta \cap (s_i, s_j)$.

Inference in the first round is relatively straightforward. Bayes' rule can be used to compute the posterior probability that the woman is a high type after conditioning on signal $\theta \in \{\ell, h\}$. [12]

Inference in the second round is more complicated because both signals and first round behavior must be considered; the appendix includes Table 2, which will assist the reader in performing these calculations. Here I will analyze and conduct inference for one possible second round relationship history as an example. Consider a low type man using the Eager strategy who received signal l. His strategy calls for him to accept in the first round. Suppose he was in turn rejected in the first round (of course, if he was accepted there would be no decision for him to make in the second round because he would be married). If his potential match partner is a low type, she must be playing hard to get and he must have sent a high signal. Low types comprise fraction $1 - \lambda$ of the population, and, of those, fraction α play HTG. The probability she sent signal ℓ and he sent signal h is $\epsilon(1 - \epsilon)$. The equilibrium strategies say that she will accept him for sure in round two.

If his potential match partner is a high type, he must have sent a low signal. High types comprise fraction λ of the population. The probability she sent signal ℓ and he sent signal ℓ is $(1 - \epsilon)\epsilon$. The equilibrium strategies say that she will reject him for sure after he accepted in round one.

We can then write the probability that an Eager low type forms a match if he accepts in round two given history $\underline{\theta} \cap (A, R)$ is

$$a^2(\ell, L, (A, R)) = \frac{(1 - \lambda)\epsilon(1 - \epsilon)\alpha}{(1 - \lambda)\epsilon(1 - \epsilon)\alpha + \lambda(1 - \epsilon)\epsilon}.$$

Since the only constellation in which the man is accepted in round two is when his partner is a low type playing HTG, his expected payoff of accepting conditional on a match occurring is $\gamma^2(\ell, L, (A, R)) = L^2/(1 - \delta)$.

[12] The posterior probability that the woman is a high type after conditioning on signal $\theta \in \{\ell, h\}$ is $\Pr(x = H|\theta = \ell) = \frac{\lambda(1-\epsilon)}{(1-\lambda)\epsilon+\lambda(1-\epsilon)}$ and $\Pr(x = H|\theta = h) = \frac{\lambda\epsilon}{(1-\lambda)(1-\epsilon)+\lambda\epsilon}$.

5.2 Values

In order to prove the SS strategy profile constitutes an equilibrium, I first compute the expected value of each type of unmatched agent within the equilibrium. These values are needed to evaluate each agent's optimal choice given a relationship history.

Let v_y be the expected value of man of type y who is not yet matched (i.e. the expectation is taken over the possible types of the woman).[13] I compute v_L and v_H using the SS strategy profile and the distribution of types in the population.

Both Eager and HTG low types must earn the same payoff in expectation, otherwise both could not optimally be played with positive probability. An Eager low type matches for sure with other low types (both strategies of the low type accept in round two regardless of the history). The only way an Eager low type man matches with a high type woman is if he sends a high signal in round one. In that case, the woman will accept, the Eager low type man accepts, and a match is consummated. The expected value of an eager low type can then be written

$$\begin{aligned} v_L &= (1-\lambda)\frac{L^2}{1-\delta} + \lambda\left((1-\epsilon)\frac{LH}{1-\delta} + \epsilon\delta v_L\right) \\ \Rightarrow v_L &= \frac{(1-\lambda)\frac{L^2}{1-\delta} + \lambda(1-\epsilon)\frac{LH}{1-\delta}}{1-\lambda\epsilon\delta}. \end{aligned} \tag{1}$$

In order to optimally play both strategies, Flip-Flopping and Resolute high types must earn the same payoff in expectation. The only instance in which Resolute high types accept is when they see the high signal from their partner; they reject at all other times. Using similar techniques as above gives the expected value of a high type as

$$v_H = \frac{(1-\lambda)(1-\epsilon)(1-\alpha\epsilon)\frac{LH}{1-\delta} + \lambda\epsilon^2\frac{H^2}{1-\delta}}{1-\delta\left[(1-\lambda)\epsilon(1-\alpha\epsilon+\alpha) + \lambda(1-\epsilon^2)\right]}. \tag{2}$$

5.3 Equilibrium

The following proposition asserts the existence of an equilibrium in which low type agents play HTG and high type agents optimally Flip-Flop. The proof is straightforward but lengthy and appears in the appendix.

Proposition 1. *The SS strategy profile and corresponding beliefs computed using Bayes' rule are a PBE if the signaling technology is sufficiently informative,*

[13] Denote by $v(\theta, y)$ the expected value of a man of type y in the first round who receives signal θ. Then given the signaling technology,

$$\begin{aligned} v_y &= \Pr(\theta = \ell)v(\ell, y) + \Pr(\theta = h)v(h, y) \\ &= [(1-\lambda)\epsilon + \lambda(1-\epsilon)]v(\ell, y) + [(1-\lambda)(1-\epsilon) + \lambda\epsilon]v(h, y) \end{aligned}$$

and the fraction of high types in the population and the discount rate are neither too large nor too small.

Proof. Please see Appendix A.1 □

The proof proceeds on a case-by-case basis, determining optimal actions for each type of agent in each relationship history. The following conditions must hold in an SS equilibrium:

$$\delta v_L \leq \frac{L^2}{1-\delta} \tag{3}$$

$$\frac{LH}{1-\delta} \leq \delta v_H \tag{4}$$

$$\beta = \frac{1-\epsilon}{\epsilon} \tag{5}$$

$$\alpha \frac{\delta v_H - \frac{LH}{1-\delta}}{\frac{H^2}{1-\delta} - \delta v_H} = \beta \frac{\lambda}{1-\lambda} \left(\frac{1-\epsilon}{\epsilon} \right)^2 \tag{6}$$

$$\alpha \leq \frac{(1-\epsilon)^2}{\epsilon^3 + \epsilon(1-\epsilon)^2} \equiv g(\epsilon) \tag{7}$$

$$\alpha \geq \frac{(1-\epsilon)^4}{(2\epsilon-1)\epsilon^3 + 2\epsilon(1-\epsilon)^4} \equiv h(\epsilon) \tag{8}$$

I will now interpret these conditions, and describe how they restrict behavior in an SS equilibrium. The inequality in (3) ensures that low types prefer to match with any partner rather than return to the pool of the unmatched. The inequality in (4) ensures that high types prefer to keep searching than to match with a known low type. These conditions can be combined with equations 1 and 2 to derive the following lower-bounds on the accuracy of the signaling system:

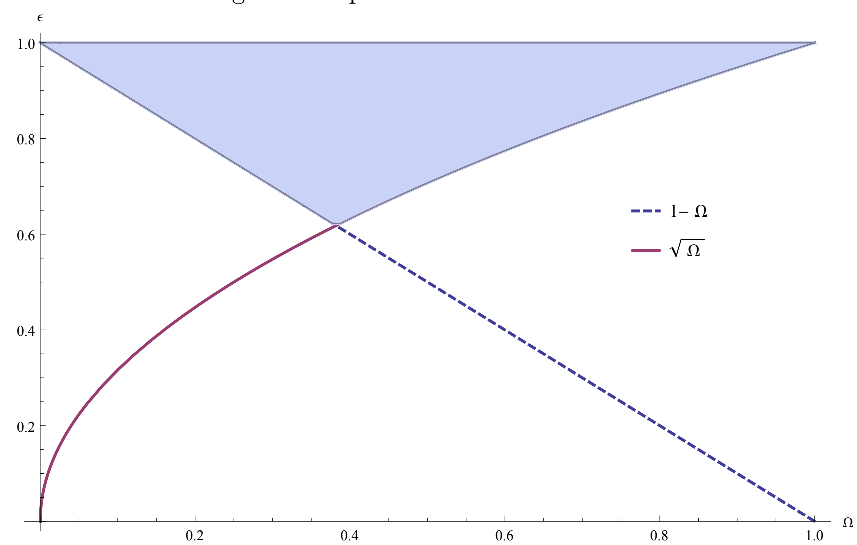
$$\epsilon \geq 1 - \frac{L(1-\delta)}{(H-L)\lambda\delta} \quad ; \quad \epsilon \geq \sqrt{\frac{L(1-\delta)}{(H-L)\lambda\delta}}$$

While the lower bound of ϵ depends on the parameters λ, δ, L, and H, the shaded area of Figure 2 shows that, in any SS equilibrium, it must be the case that $\epsilon \geq (\sqrt{5}-1)/2 \equiv 0.618$. That is, irrespective of the other parameter values, there is a lower-bound on the informativeness of the signaling technology.

In addition, since ϵ cannot be larger than 1, Figure 2 shows that there can be no SS equilibrium when $L(1-\delta) > \lambda\delta(H-L)$. This occurs when the benefit to marrying in general is large relative to the additional benefit of marrying a high type in particular, and the discount factor and the proportion of high types in the population are small. In such a case, the required inequality in (4) would not hold - high types would optimally accept a known low type partner because the expected additional benefit of waiting for a more promising match is small.

The probability that a high type will Flip-Flop is stated in (5) explicitly. This value is such that low types will be indifferent between playing HTG and Eager when they observe a high signal in round one. This probability is only

Figure 2: Equilibrium Restrictions on ϵ



Notes: The horizontal axis measures $\Omega = \frac{L(1-\delta)}{\lambda\delta(H-L)}$. The shaded region corresponds to the intersection of $\epsilon \geq 1 - \Omega$ and $\epsilon \geq \sqrt{\Omega}$

a function of the accuracy of the signaling technology, and does not depend on the fraction of high types in the population, the discount rate, or the value obtained from matching with a high type or low type. This is because when low types make a decision to play HTG or Eager (i.e. accept or reject h in round one), their information set consists solely of the signal, the realization of which depends probabilistically on parameter ϵ.

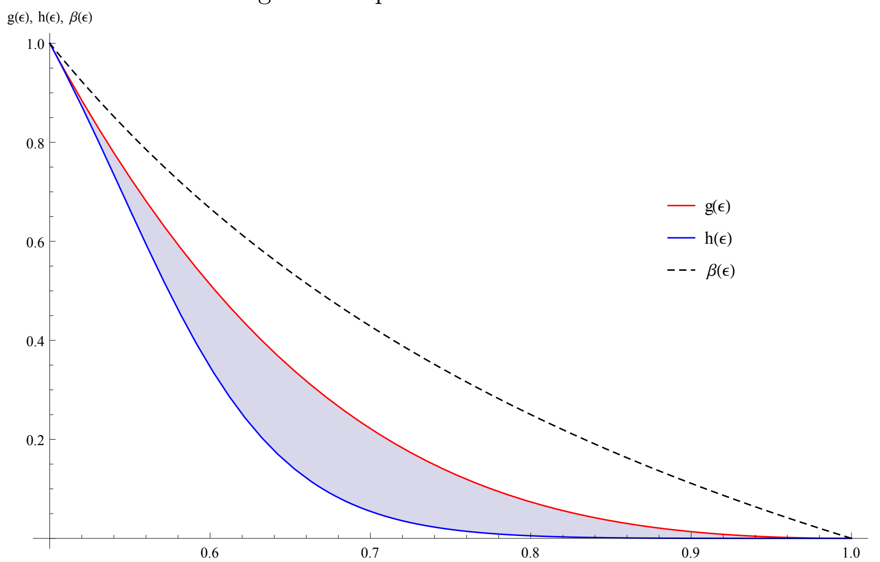
Equation 6 involves two endogenous variables: α and v_H. This equation gives a condition on α and v_H such that high types are indifferent between playing Resolute and Flip-Flopping. The values of α and v_H can be solved for explicitly using the expression for v_H derived earlier in (2).

Equations 7 and 8 provide additional restrictions on α such that the SS strategy profile constitutes an equilibrium. It is important to note that α is determined such that a high type is indifferent between playing Resolute and Flip-Flopping, but that value may nevertheless be inconsistent with an equilibrium. The function on the right hand side of (7) is greater than or equal to the function on the right hand side of (8), so it is mathematically possible for α to satisfy both of these inequalities simultaneously. The graph in Figure 3 shows that these restrictions placed on α by the high type's optimal behavior are not innocuous.

The dashed line in the figure is equal to $\beta = (1-\epsilon)/\epsilon$. This line is everywhere above the shaded area, indicating that the fraction of high types who Flip-Flop is always greater than the fraction of low types who play HTG.

Equation 7 says that the endogenously determined fraction of low types

Figure 3: Equilibrium Restrictions on α



playing HTG must be sufficiently low. When the fraction of low types playing HTG becomes too large, high types prefer to accept low signals (rather than reject, as is called for in the SS equilibrium) because the costs of doing so are sufficiently low. That is, when α is large enough, accepting low signals in round one becomes optimal not because it is more likely that the low signal came from a high type, but rather because the probability of actually forming a match with a low type is sufficiently low.

Equation 8 says that the endogenously determined fraction of low types playing HTG must be also be sufficiently high. If α is too low, then high types will not find it optimal to accept high-signals in round one. To understand why this condition must be met, consider the extreme case where $\alpha = 0$. Since a low type woman always accepts in round one in this case, high type men can ensure that they never match with low type women by rejecting in round one. As the signaling technology becomes more accurate (i.e. ϵ increases towards one), this lower bound on α falls towards 0.

6 Comparative statics

I now present results describing how equilibrium behavior and values depend on the parameters of the model. Ultimately, I am most interested in how playing HTG affects the set of matches which occur in equilibrium. Towards that end, Proposition 2 below shows how the equilibrium strategies depend on the parameters of the model. This result will be especially useful in Section 7 when analyzing equilibrium mating.

Proposition 2. *The equilibrium fraction of low types playing HTG*

- *decreases in the accuracy of the signaling technology, the discount rate, and the added benefit of mating with a high type;*
- *increases in the baseline value of mating relative to remaining single, and;*
- *can increase or decrease in the fraction of high types in the population.*

The equilibrium fraction of high types Flip-Flopping is decreasing in the accuracy of the signaling technology and does not depend on the other parameters of the model.

Proof. Please see Appendix A.2. □

In addition to studying how equilibrium behavior depends on the model's parameters, one might also be interested in the agents' welfare in equilibrium. Proposition 3 details how low- and high types' equilibrium values depend on the parameters.

Proposition 3. *The low type's value*

- *increases in the discount rate, the baseline value of mating relative to remaining single, and the added benefit of mating with a high type;*
- *decreases in the accuracy of the signaling technology, and;*
- *can increase or decrease in the fraction of high types in the population.*

The high type's value increases in the fraction of high types in the population, the discount rate, the baseline value of mating relative to remaining single, and the added benefit of mating with a high type.

Proof. Please see Appendix A.2. □

Section 8 develops a numerical example of an environment in which the SS strategy profile is an equilibrium. The example helps to illustrate the magnitude of the comparative statics results. In the remainder of this section I will discuss the intuition behind some of these results.

Signal Accuracy: Consider an increase in the accuracy of the signaling technology. Practically, such a change could occur when an institution or technology evolves to allow agents better signals of their match-partners' true underlying type. As examples, the advent and increased usage of social networking websites such as Facebook and LinkedIn make it easier for agents to learn more about their potential match partners' interests, abilities, and skills. Proposition 2 says that such changes lead to less within-match strategic behavior.

As the signaling technology becomes more informative, high types Flip-Flop less frequently. If this was not true, low types would have a strict incentive to play HTG after observing a high signal because, the more accurate the signal, the more likely their partner is a high type who received a low signal.

Recall that the fraction of low types playing HTG is determined such that jilted high types are indifferent between accepting and rejecting in round two a low signal who rejected them in round one. As the accuracy of the signaling technology increases, holding all other variables constant, the probability that one's match-partner is a high type given the relationship history $\ell \cap (R, R)$ decreases, and so high types are less inclined to accept. In order to keep high types indifferent between accepting and rejecting in this case, fewer low types play HTG.

As the accuracy of the signaling technology increases, the equilibrium value to a low type falls because the likelihood of sending a low signal increases, low types play HTG less often, and they therefore match with high types less frequently.

The Baseline Value of Mating: In order to differentiate between the baseline value of mating relative to remaining single and the added benefit of mating with a high type, write $H = L+G$. Then increasing L makes mating with any partner more attractive, while increasing G raises the benefit to mating specifically with a high type. As the baseline value of mating increases, the relative benefit to a high type with history $\ell \cap (R, R)$ of waiting for a more promising partner decreases. In equilibrium, low types respond by playing HTG more often in order to keep the high types in this situation indifferent between Flip-Flopping and playing Resolute.

The Added Benefit of Mating with a High type: If the value of mating with a high type increases but the fraction of low types playing HTG remains constant, high types in a relationship with history $\ell \cap (R, R)$ have a strict incentive to hold out and wait for a more promising partner next period. Therefore, low types respond by playing HTG less often in order to preserve high types' indifference.

This result may be surprising in that one may initially think that as the added benefit of mating with a high type increases, low types face greater incentives to play HTG in order to increase their chances of marrying one of these more valuable partners. Absent from this sort of reasoning, however, are the moderating forces of equilibrium behavior. If low types played HTG more often as G increased, high types would stop Flip-Flopping and would instead play Resolute. That is, high types would no longer give second chances to match partners who sent low signals and rejected, thereby removing any incentive to play HTG at all.

Discounting: The fraction of low types playing HTG decreases in the discount rate. As the discount rate increases, men in relationships with history $\ell \cap (R, R)$ are more willing to wait until next period in hopes of landing a more promising match partner. In equilibrium, women must respond by playing HTG less often in order to keep these high type men indifferent between accepting and rejecting.

High types' equilibrium value increases in the discount rate because not only does the value of a marriage with either type increase in the discount rate, but high types are more likely to mate with a fellow high type because the fraction of low types playing HTG decreases in the discount rate. To see that the low type's value also increases in the discount rate, consider the Eager low type.

Increasing the discount rate does not affect the partners the Eager low type marries, but it does increase the value of a marriage and decrease the cost of waiting until the next period. An increase in δ then raises the Eager low type's value, and since low types must be indifferent between playing HTG and Eager, the same holds for HTG low types.

Population Composition: The low type's equilibrium value can increase or decrease in the fraction of high types in the population. To see how the low type's value could decrease in λ, recall that low types *always* mate with other low types. When the proportion of high types increases, even though the expected payoff conditional on a match occurring increases, low types are less likely to mate because their match partners are more likely to be a discriminating high type. This effect is most pronounced when agents are impatient (i.e. δ is small).

The high type's equilibrium value always increases in λ. This is true regardless of whether the fraction of low types playing HTG increases or decreases because increasing λ increases the chances that a high type matches with another high type.

The fraction of low types playing HTG can either increase or decrease as λ increases. Recall that α is determined so that high types are indifferent between accepting and rejecting when their relationship history is $\ell \cap (R, R)$. As λ increases and the high type's continuation value of δv_H increases, α may need to increase or decrease to keep high types indifferent between accepting and rejecting.

7 Mating

Since types are private information and signals are noisy, it is inevitable that high types mate with low types in equilibrium. As might be expected, this can occur when agents send inaccurate signals and the low type is using the Eager strategy. But this type of mismatch can also occur when both agents send accurate signals and the low type is playing HTG and the high type Flip-Flops.

I therefore use a stochastic notion of assortative mating closely related to that in Chade [2006] – mating is positively assortative if the expected type of one's partner increases in one own's type.[14] Proposition 4 below formalizes this concept.

Proposition 4. *Mating in the SS equilibrium is positively assortative – the expected type of mate for a high type is greater than the expected type of mate for a low type.*

Proof. The Eager low type man marries a low type woman for sure and marries a high type woman only if he sends a high signal. Write the low type man's expected type of mate as

$$\bar{x}_L = \frac{(1-\lambda)L + \lambda(1-\epsilon)H}{(1-\lambda) + \lambda(1-\epsilon)}$$

[14] This definition of PAM originates in Shimer and Smith [2000], footnote 8.

The Resolute high type man marries a low type woman in one of two ways – first, she plays Eager and he sees a high signal, and second, she plays hard to get, he sees a high-signal, and she sees a low signal. The only way a Resolute high type man marries a high type woman is if both send high-signals. The high type man's expected type of mate is

$$\bar{x}_H = \frac{(1-\lambda)(1-\epsilon)(1-\alpha\epsilon)L + \lambda\epsilon^2 H}{(1-\lambda)(1-\epsilon)(1-\alpha\epsilon) + \lambda\epsilon^2}$$

Both $\bar{x}_L$ and $\bar{x}_H$ are weighted averages of L and H. Then $\bar{x}_H > \bar{x}_L$ because $\bar{x}_H$ places relatively less weight on L and relatively more weight on H than $\bar{x}_L$. This is because $\epsilon^2 > 1-\epsilon$ so long as $\epsilon > (\sqrt{5}-1)/2$, which Figure 2 shows holds in any SS equilibrium. This proves that high types have higher expected types of partners than low types, and mating is therefore positively assortative. □

PAM obtains in the SS equilibrium despite the low type agents' attempts at strategic misrepresentation. It is worth noting, however, that mating is not *strictly* assortative on signals – low type agents who receive high-signals mate with high type agents who receive low signals in equilibrium.

I next show that the strategic behavior studied in this paper improves sorting. Consider, as a benchmark, the NS strategy profile discussed in Section 4: low types always accept, and high types accept high signals in the first round, and reject otherwise. In the appendix, I show that this strategy profile, along with beliefs derived using Bayes' rule, can constitute a PBE.[15] Moreover, the appendix shows that the SS and NS equilibria both exist over a range of parameter values. Here in the text, I will focus on the efficiency gain from strategic behavior.

Denote the present discounted match value created each period in the SS and NS equilibria as V^{SS} and V^{NS}, respectively. Proposition 5 shows that the strategic behavior in the SS equilibrium leads to more marriage value being created than in the NS equilibrium. The proof shows that this increase in sorting is due to high types Flip-Flopping, and happens despite low types playing HTG.

Proposition 5. *The steady state value of marriages created is always greater in the SS equilibrium than in the NS equilibrium.*

Proof. SS: Low types always marry other low types. Eager low types only marry high types if the Eager low type sends a high signal. HTG low types marry high types in two ways: first if both send inaccurate signals, and second if both send accurate signals and the high type Flip-Flops. High types marry other high types in two ways: first if both send the "correct" signals, and second if both send the "wrong" signals and both Flip-Flop. Since the probability high types

[15]The high types' rejection in the second round after being rejected themselves is a weak best response. While the NS strategy profile can be part of a PBE, it would not survive a "trembling hand" refinement.

Flip-Flop is $\beta = (1-\epsilon)/\epsilon$, this gives

$$V^{SS} = 2(1-\lambda)^2 \frac{L^2}{1-\delta} + 4(1-\lambda)\lambda \left[(1-\alpha)(1-\epsilon) + \alpha\left((1-\epsilon)^2 + \epsilon^2 \frac{1-\epsilon}{\epsilon}\right)\right] \frac{LH}{1-\delta}$$
$$+ 2\lambda^2 \left(\epsilon^2 + (1-\epsilon)^2 \left(\frac{1-\epsilon}{\epsilon}\right)^2\right) \frac{H^2}{1-\delta}$$

NS: Low types always marry other low types. Low types and high types marry only if the low type sends the high signal. High types marry if both send high-signals. This gives

$$V^{NS} = 2(1-\lambda)^2 \frac{L^2}{1-\delta} + 4\lambda(1-\lambda)(1-\epsilon)\frac{LH}{1-\delta} + 2\lambda^2\epsilon^2 \frac{H^2}{1-\delta}$$

To see that $V^{SS} > V^{NS}$, note that the term in square brackets in V^{SS} simplifies to $1-\epsilon$. Then

$$V^{SS} - V^{NS} = 2\lambda^2 \frac{(1-\epsilon)^4}{\epsilon^2} \frac{H^2}{1-\delta} > 0$$

□

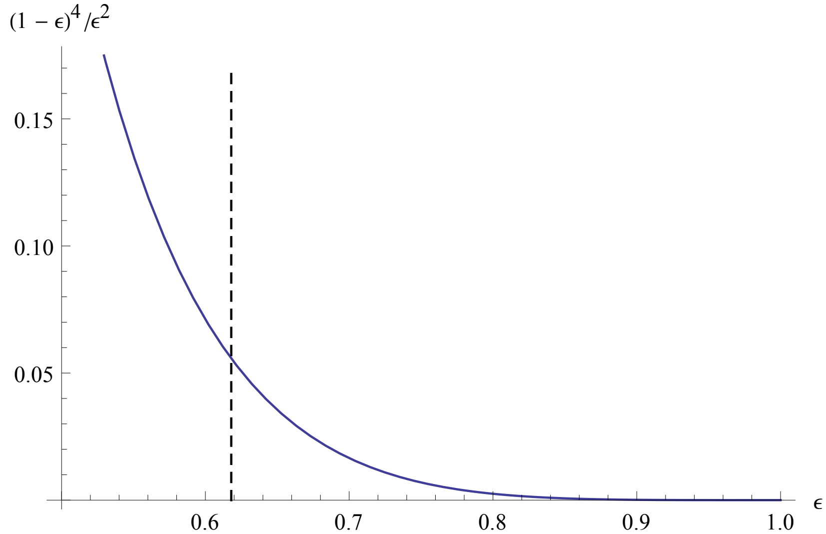
Proposition 5 shows that the sorting gains in the SS equilibrium come from the high types Flip-Flopping. Low types attempt to increase mismatch using the HTG strategy, but this is ineffective in equilibrium. The probability a low type and high type marry is the same in each equilibrium, but the probability two high types marry is greater by the term $(1-\epsilon)^4/\epsilon^2$ in the SS equilibrium because high types can marry even if both initially send low signals.

To be certain, the sorting gains are relatively modest. Figure 4 plots $(1-\epsilon)^4/\epsilon^2$, which is the probability two Flip-Flopping high types send the low signals. This figure shows that the probability that a matched pair of high type agents will both play Flip-Flop and both send low- signals is relatively low – not much greater than 0.05, and likely much less over the relevant range of $\epsilon > 0.618$. The increase in efficiency due to strategic behavior increases in the payoff to marrying a high type and the degree of complementarity in marital production.

8 Numerical example

In order to illustrate the existence and to give a sense of the magnitude of some of the qualitative results on behavior, mating, and welfare, this section provides a numerical example of the model. Fix the parameters of the model at $\epsilon = 0.75$, $\delta = 0.8$, $L = 1$, $H = 3$, and $\lambda = 0.4$. Then solving the system of equations given by (2) and (6) gives $\alpha = 0.126$ and $v_H = 24.9$. Low types prefer will accept a known low type because $v_L = 5.92$, and so $L^2/(1-\delta) > \delta v_L$. High types will not accept a known low type because $\delta v_2 > HL/(1-\delta)$. High types

Figure 4: Limits to sorting gains of strategic behavior



Notes: The horizontal axis measures ϵ. The vertical axis measures $(1-\epsilon)^4/\epsilon^2$, which is the probability a matched pair of high types both Flip-Flop and send low signals. The dashed vertical line occurs at $\epsilon = 0.618$; there can be no SS equilibria with lower ϵ.

play Flip-Flop with probability 0.33. Lastly, α satisfies the innequalities in (7) and (8) because $h(\epsilon) = 0.018$ and $g(\epsilon) = 0.133$. So all of the conditions for the Socially Strategic strategy profile to be an equilibrium are met. Table 1 illustrates the effects of independently increasing each parameter by 1% on the agents' behavior and values.

The Non-Strategic strategy profile is also an equilibrium for these parameter values. The steady state value of marriages in the NS equilibrium is $V^{NS} = 15.3$, while in the SS equilibrium it is $V^{SS} = 15.4$.

Table 1: Comparative statics

1% increase in:	$\Delta\alpha$	$\Delta\beta$	Δv_H	Δv_L
ϵ	- 0.020	- 0.01	+ 0.25	- 0.04
δ	- 0.011	No change	+ 1.30	+ 0.31
L	+ 0.003	No change	+ 0.04	+ 0.08
H	- 0.003	No change	+ 0.46	+ 0.06
λ	- 0.001	No change	+ 0.12	+ 0.05

Notes: Any?

9 Conclusion

This paper shows that, in addition to productive complementarities, the ability to transfer utility between partners, and the existence of private information over types, within-match strategic interaction matters in determining the set of equilibrium marriages. Perhaps contrary to intuition, this within-match strategic behavior actually improves the efficiency of sorting.

I established the existence of a Socially Strategic equilibrium in the marriage market in which low type agents play hard to get in order to appear more desirable to their partners. Using this strategy is not without costs, however, as marriages with some initially willing high types are derailed. In equilibrium, high type agents Flip-Flop with positive probability and accept in round two marriages they initially found undesirable. I showed that mating in the Socially Strategic equilibrium is stochastically positively assortative in that the expected mate of a high type is better than the expected mate of a low type.

Moreover, the strategic behavior in the Socially Strategic equilibrium increases the sorting of mates relative to the Non-Strategic equilibrium. While low types' goal of playing hard to get is to increase mismatch, it is ineffective in equilibrium. The increase in sorting efficiency is due entirely to the Flip-Flopping behavior of high types.

References

A. Anderson and L. Smith. Dynamic matching and evolving reputations. *Review of Economic Studies*, 77:3 – 29, 2010.

G. Becker. A theory of marriage: Part i. *Journal of Political Economy*, 81(4): 813–846, 1973.

R. Bellman and D. Blackwell. Some two-person games involving bluffing. *Proceedings of the National Academy of Sciences of the United States of America*, 35:600 – 605, 1949.

H. Chade. Matching with noise and the acceptance curse. *Journal of Economic Theory*, 129:81–113, 2006.

P. A. Coles and R. Shorrer. Optimal truncation in matching markets. *Mimeo*, 2013.

M. E. Consuegra, R. Kumar, and G. Narasimhan. Comment on "On the uniqueness of stable marriage matchings". *Economics Letters*, 121(3):468, 2013.

E. Damiano, L. Hao, and W. Suen. Unravelling of dynamic sorting. *Review of Economic Studies*, 72:1057–1076, 2005.

A. M. Franco, M. Mitchell, and G. Vereshchagina. Incentives and the structure of teams. *Journal of Economic Theory*, 146:2307–2332, 2011.

E. Hopkins. Job market signaling of relative position, or becker married to spence. *Journal of the European Economic Association*, 10(2):290–322, 2012.

R. Inderst. Matching markets with adverse selection. *Journal of Economic Theory*, 121:145–166, 2005.

P. K. Jonason and N. P. Li. Playing hard-to-get: Manipulating one's perceived availiability as a mate. *European Journal of Personality*, 27(5):458 – 469, 2013.

J. M. McNamara and E. J. Collins. The job search problem as an employer-candidate game. *Journal of Applied Probability*, 27(4):815–827, 1990.

G. Moscarini. Job matching and the wage distribution. *Econometrica*, 73(2): 481 – 516, 2005.

K. Serfes. Endogenous matching in a market with heterogeneous principals and agents. *International Journal of Game Thoery*, 36:587–619, 2008.

R. Shimer and L. Smith. Assortative matching and search. *Econometrica*, 68: 343–369, 2000.

L. Smith. The marriage model with search frictions. *Journal of Political Economy*, 114(6):1124 – 1144, 2006.

J. Von Neumann and O. Morgenstern. *Theory of Games and Economic Behavior*. Princeton University Press, Princeton, 1964.

D. J. Wright. The risk and incentives trade-off in the presence of heterogeneous managers. *Journal of Economics*, 83(3):209–223, 2004.

A Appendix

A.1 Proof of Proposition 1

Proof. The proof proceeds by deriving conditions on the model's parameters such that agents optimally use the SS strategy profile (i.e. they have no profitable deviation). I begin in round two because some round one decisions will depend on round two outcomes.

A.1.1 Low types in round two

The SS strategy profile requires both HTG and Eager low types to accept in round two, regardless of their relationhsip history. A condition sufficient to guarantee the optimality of this choice is that low types accept even if they are sure their partner is a low type. This occurs if $L^2/(1-\delta) \geq \delta v_L$. If this inequality holds, then low types will also optimally accept when there is a chance their partner is a high type (which occurs when the low type is playing HTG, observed a high signal and rejected in round one, and was rejected by their partner). Using (1), this inequality can be re-written to find that low types prefer to accept known low types so long as $\epsilon \geq 1 - (1-\delta)L/\lambda\delta(H-L)$. This inequality can hold for $\epsilon \in (1/2, 1)$ so long as λ, δ, and H are small enough and ϵ and L are large enough.

A.1.2 High types in round two

Suppose the relationship history is $\ell \cap (R,R)$ – the high type agent observed a low signal in round one, rejected a match, and was himself rejected. The SS strategy profile calls for Flip-Floppers to accept in round two, but for those playing Resolute to reject. High types must therefore be indifferent between announcing A or R in this situation. Setting the expected value of A equal to the expected value of R gives:

$$a^2(\ell, H, (R,R))\gamma^2(\ell, H, (R,R)) + (1 - a^2(\ell, H, (R,R)))\delta v_H = \delta v_H$$
$$\Leftrightarrow \gamma^2(\ell, H, (R,R)) = \delta v_H$$

This equality can be simplified using the expressions in Table 2 as

$$\frac{(1-\lambda)\epsilon^2\alpha\frac{LH}{1-\delta} + \lambda(1-\epsilon)^2\beta\frac{H^2}{1-\delta}}{(1-\lambda)\epsilon^2\alpha + \lambda(1-\epsilon)^2\beta} = \delta v_H \Rightarrow$$
$$\alpha\frac{\delta v_H - \frac{LH}{1-\delta}}{\frac{H^2}{1-\delta} - \delta v_H} = \beta\frac{\lambda}{1-\lambda}\left(\frac{1-\epsilon}{\epsilon}\right)^2 \tag{9}$$

When the above expression holds, high types are indifferent between playing Resolute and Flip-Flopping, as they must be in order for $\beta \in (0,1)$.

In all other possible second round situations, the SS strategy profile calls for high types to reject. In these situations, the high type does not know her partner's type, but she does know that only a low type agent would accept in these situations in the second round. The expected match value, conditional on a match occurring, is then $LH/(1-\delta)$. The high type agent then optimally follows the SS strategy profile by rejecting if the continuation value of δv_H is greater than the expected value of accepting, or:

$$\delta v_H \geq a^2(\theta, H, (S_i, S_j))\frac{LH}{1-\delta} + (1 - a^2(\theta, H, (S_i, S_j)))\delta v_H$$
$$\Rightarrow \delta v_H \geq \frac{LH}{1-\delta}$$

After substituting in for v_H using (2), simple algebra shows that the above holds when $\epsilon \geq \sqrt{(1-\delta)L/\lambda\delta(H-L)}$. So long as δ, λ, ϵ, and H are sufficiently large and L is sufficiently small, high types prefer to reject and re-enter the pool of the unmatched than to accept and marry an agent who for sure a low type.

A.1.3 Low types in round one

The SS strategy profile specifies different actions for low types when they observe high signals – Eager low types accept, while those playing HTG reject. In order for these different actions to be part of an equilibrium, the expected payoff of playing each action must be the same.

All low types, regardless of whether they are playing HTG or Eager, optimally accept in round two. This means low types will always match with other low types. Then any differences in the expected payoff calculations for the HTG and Eager low types confronted with a high signal in round one must derive from their chances of matching with a high type.

By accepting, Eager low types can match with a high type in the first round in the event the low type sends the "wrong" (i.e. high) signal. This occurs with probability $\lambda\epsilon(1-\epsilon)$, and the payoff of matching with a high type is $LH/(1-\delta)$.

By rejecting, low types playing HTG do not match in round one, but might match with a high type in round two if their partner is playing Flip-Flop. This occurs with probability $\lambda\epsilon^2\beta$, and again the payoff of matching with a high type is $LH/(1-\delta)$.

Since low types must be indifferent between playing HTG and Eager, one can solve for the probability with which high types Flip-Flop as follows:

$$\begin{aligned}\lambda\epsilon(1-\epsilon)\frac{LH}{1-\delta} &= \lambda\epsilon^2\beta\frac{LH}{1-\delta} \\ \Rightarrow \beta &= \frac{1-\epsilon}{\epsilon} \qquad (10)\end{aligned}$$

Next I derive conditions such that low types optimally accept low signals in the first round. Low types optimally accept in the second round because $L^2/(1-\delta) > \delta v_L$, as in Subsection A.1.1.[16]

Again, low types always marry other low types, so any difference in expected utility from accepting versus rejecting must derive from potential matches with high types. By accepting, the low type can match with a high type in the first round in the event the low type sent the "wrong" (i.e. high) signal. This occurs with probability $\lambda(1-\epsilon)^2$, and the payoff to matching with a high type is $LH/(1-\delta)$.

By rejecting, the low type will not match in round one, but might match with a Flip-Flopping high type in round two. This occurs with probability $\lambda\epsilon(1-\epsilon)\beta$, and again the payoff to matching with a high type is $LH/(1-\delta)$.

Low types will therefore optimally follow the SS strategy profile by accepting low signals in round one if

$$\lambda(1-\epsilon)^2\frac{LH}{1-\delta} \geq \lambda\epsilon(1-\epsilon)\beta\frac{LH}{1-\delta} \Rightarrow 1-\epsilon \geq \epsilon\beta$$

[16]Note that a low type rejecting a low signal in round one is not on the equilibrium path, and hence is not covered in Subsection A.1.1. Nevertheless, the condition which generates unconditional acceptance in the second round applies.

But since $\beta = (1-\epsilon)/\epsilon$ in order for low types to be indifferent between playing HTG and Eager, the above expression holds with equality and low types (weakly) prefer to accept high-signals in round one.

A.1.4 High types in round one

Both strategies of the high type man call for the same action conditional on the woman's signal. Suppose initially the man observes signal ℓ in round one. I now verify that it is indeed optimal for the man to reject as is called for by the SS strategy profile.

If a high type man rejects a low signal in round one, he proceeds to the second round for sure. Resolute high types then reject for sure, guaranteeing themselves a continuation value of δv_H. Of course, high types must be indifferent between playing Resolute and Flip-Flopping, and so the Flip-Flopping high type must receive an expected payoff of δv_H as well.

If a high type man deviates from the proposed equilibrium strategy and accepts a low signal in round one, the probability he would consummate a marriage and the expected utility conditional on mutual acceptance is

$$a(\ell, H) = \frac{(1-\lambda)\epsilon(1-\alpha\epsilon) + \lambda(1-\epsilon)\epsilon}{(1-\lambda)\epsilon + \lambda(1-\epsilon)}$$
$$\gamma(\ell, H) = \frac{(1-\lambda)\epsilon(1-\alpha\epsilon)\frac{LH}{1-\delta} + \lambda(1-\epsilon)\epsilon\frac{H^2}{1-\delta}}{(1-\lambda)\epsilon(1-\alpha\epsilon) + \lambda(1-\epsilon)\epsilon}$$

If a match does not occur in round one, however, it will not happen in round two. This is because only low types playing HTG will accept in the second round a partner who accepted in the first round, and high types prefer to re-enter the pool of the unmatched to marrying a low type.

The high type man then finds it optimal to play by the proposed equilibrium strategy and to reject low signals if

$$\delta v_H \geq a(\ell, H)\gamma(\ell, H) + (1 - a(\ell, H))\delta v_H \Leftrightarrow \delta v_H \geq \gamma(\ell, H)$$

Substituting in for $\gamma(\ell, H)$ and some minor manipulations allow the above condition to be re-written as

$$\frac{\delta v_H - \frac{LH}{1-\delta}}{\frac{H^2}{1-\delta} - \delta v_H} \geq \frac{\lambda}{1-\lambda}\frac{1-\epsilon}{1-\alpha\epsilon}$$

Substituting in from (9), the condition guaranteeing high types are indifferent between Flip-Flopping and playing Resolute, and the formula for β given in (10), followed by some simple manipulations gives

$$\alpha \leq \frac{(1-\epsilon)^2}{\epsilon^3 + \epsilon(1-\epsilon)^2} \equiv g(\epsilon) \tag{11}$$

So long as α satisfies the above condition, high types find it optimal to reject low signals in round one.

Next, suppose the high type man observes h in round one. The proposed equilibrium strategy calls for him to accept. The probability a match is consummated if he accepts and the expected payoff conditional on a match occurring are

$$a(h,H) = \frac{(1-\lambda)(1-\epsilon)(1-\alpha\epsilon) + \lambda\epsilon^2}{(1-\lambda)(1-\epsilon)+\lambda\epsilon}$$
$$\gamma(h,H) = \frac{(1-\lambda)(1-\epsilon)(1-\alpha\epsilon)\frac{LH}{1-\delta} + \lambda\epsilon^2\frac{H^2}{1-\delta}}{(1-\lambda)(1-\epsilon)(1-\alpha\epsilon)+\lambda\epsilon^2}$$

Then a high type man's expected utility of following the equilibrium strategy in round one when seeing h is

$$v(h,H|s_i = A) = \frac{(1-\lambda)(1-\epsilon)(1-\alpha\epsilon)\frac{LH}{1-\delta} + \lambda\epsilon^2\frac{H^2}{1-\delta}}{(1-\lambda)(1-\epsilon)+\lambda\epsilon} + \frac{(1-\lambda)(1-\epsilon)\epsilon\alpha + \lambda\epsilon(1-\epsilon)}{(1-\lambda)(1-\epsilon)+\lambda\epsilon}\delta v_H \tag{12}$$

If the high type man rejects, he proceeds to round two for sure. The probability with which a high type man who observes h is accepted or rejected in round one is

$$Pr(S_j = R|h,H) = \frac{(1-\lambda)(1-\epsilon)\epsilon\alpha + \lambda\epsilon(1-\epsilon)}{(1-\lambda)(1-\epsilon)+\lambda\epsilon}$$
$$Pr(S_j = A|h,H) = \frac{(1-\lambda)(1-\epsilon)(1-\alpha+\alpha(1-\epsilon)) + \lambda\epsilon^2}{(1-\lambda)(1-\epsilon)+\lambda\epsilon}$$

According to the proposed equilibrium strategies, only low type women playing HTG and high type women reject in round one. The man will be accepted in round two if she is a low type playing HTG to whom he sent a high-signal or a Flip-Flopping high type to whom he sent a low signal. The probability he is accepted and the expected payoff conditional on a match occurring are

$$a^2(h,H,(R,R)) = \frac{(1-\lambda)(1-\epsilon)\epsilon\alpha + \lambda\epsilon(1-\epsilon)\beta}{(1-\lambda)(1-\epsilon)\epsilon\alpha + \lambda\epsilon(1-\epsilon)}$$
$$\gamma^2(h,H,(R,R)) = \frac{(1-\lambda)(1-\epsilon)\epsilon\alpha\frac{LH}{1-\delta} + \lambda\epsilon(1-\epsilon)\beta\frac{H^2}{1-\delta}}{(1-\lambda)(1-\epsilon)\epsilon\alpha + \lambda\epsilon(1-\epsilon)\beta}$$

A high type man in a relationship with history $h \cap (R,R)$ will accept in round two if $\gamma^2(h,H,(R,R)) \geq \delta v_H$. Simple manipulations show that the high type man accepts if

$$\lambda\epsilon(1-\epsilon)\beta\left(\frac{H^2}{1-\delta} - \delta v_H\right) \geq (1-\lambda)(1-\epsilon)\epsilon\alpha\left(\delta v_H - \frac{LH}{1-\delta}\right)$$

Note that the expression in Equation 9 implies that the above inequality always holds, so that deviant high types in $h \cap (R,R)$ do in fact accept in round two. Expected utility in this case is

$$v^2(h,H,(R,R)) = \frac{(1-\lambda)(1-\epsilon)\epsilon\alpha\frac{LH}{1-\delta} + \lambda\epsilon(1-\epsilon)\beta\frac{H^2}{1-\delta} + \lambda\epsilon(1-\epsilon)(1-\beta)\delta v_H}{(1-\lambda)(1-\epsilon)\epsilon\alpha + \lambda\epsilon(1-\epsilon)}$$

I now compute the expected utility of a high type man in a relationship with the history $h \cap (R, A)$. Since the man was accepted in round one, his partner could be an Eager low type, a HTG low type to whom he sent ℓ, or a high type to whom he sent h. According to the SS strategy profile, only the low type women would ever accept in round two after they accepted in round two. The probability he is accepted is then

$$a^2(h, H, (R, A)) = \frac{(1-\lambda)(1-\epsilon)(1-\alpha+\alpha(1-\epsilon))}{(1-\lambda)(1-\epsilon)(1-\alpha+\alpha(1-\epsilon))+\lambda\epsilon^2}$$

The expected utility conditional on acceptance is $\gamma^2(h, H, (R, A)) = LH/1-\delta$. The high type man in a relationship with history $h \cap (R, A)$ therefore finds it optimal to reject and get a payoff of δv_H, which is greater than $LH/(1-\delta)$.

Then the expected utility of deviating in round one for the high type man who observes h can be written

$$v(h, H|S_i = R) = \frac{(1-\lambda)(1-\epsilon)\epsilon\alpha\frac{LH}{1-\delta} + \lambda\epsilon(1-\epsilon)\beta\frac{H^2}{1-\delta} + \lambda\epsilon(1-\epsilon)(1-\beta)\delta v_H}{(1-\lambda)(1-\epsilon)+\lambda\epsilon} + \frac{(1-\lambda)(1-\epsilon)(1-\alpha\epsilon)+\lambda\epsilon^2}{(1-\lambda)(1-\epsilon)+\lambda\epsilon}\delta v_H \tag{13}$$

In order for the high type man to play according to the SS strategy profile, the difference between (12) and (13) must be positive. The denominator of this expression is always positive, so the sign is determined by the numerator. Easy manipulations allow the numerator of this difference to be written as

$$\begin{aligned} v(h, H|S_i = A) - v(h, H|S_i = R) = \\ &(1-\lambda)(1-\epsilon)(1-\alpha\epsilon)\left(\frac{LH}{1-\delta} - \delta v_H\right) + \lambda\epsilon^2\left(\frac{H^2}{1-\delta} - \delta v_H\right) \\ &+ (1-\lambda)(1-\epsilon)\alpha\epsilon\left(\delta v_H - \frac{LH}{1-\delta}\right) + \lambda(1-\epsilon)\epsilon\beta\left(\delta v_H - \frac{H^2}{1-\delta}\right) \end{aligned}$$

Combining terms gives

$$\begin{aligned} v(h, H|S_i = A) - v(h, H|S_i = R) = \\ &(1-\lambda)(1-\epsilon)(2\alpha\epsilon-1)\left(\delta v_H - \frac{LH}{1-\delta}\right) + \lambda\epsilon(\epsilon - \beta(1-\epsilon)\left(\frac{H^2}{1-\delta} - \delta v_H\right) \end{aligned}$$

Simple manipulations show that, since $\alpha \le g(\epsilon)$ implies $1 - 2\alpha\epsilon > 0$, the above is positive whenever

$$\frac{\lambda\epsilon(\epsilon-\beta(1-\epsilon))}{(1-\lambda)(1-\epsilon)(1-2\alpha\epsilon)} \ge \frac{\delta v_H - \frac{LH}{1-\delta}}{\frac{H^2}{1-\delta} - \delta v_H}$$

Substituting in the value of β from (10) gives

$$\frac{\lambda(2\epsilon-1)}{(1-\lambda)(1-\epsilon)(1-2\alpha\epsilon)} \geq \frac{\delta v_H - \frac{LH}{1-\delta}}{\frac{H^2}{1-\delta} - \delta v_H}$$

The equality in (9), the equation which specifies the value of α which makes high types indifferent between playing Resolute and Flip-Flopping, can be substituted into the RHS of the above expression to get

$$\frac{\lambda(2\epsilon-1)}{(1-\lambda)(1-\epsilon)(1-2\alpha\epsilon)} \geq \frac{\lambda(1-\epsilon)^2\beta}{(1-\lambda)\epsilon^2\alpha}$$

The λ and $1-\lambda$ terms can be canceled. Again, since $\alpha < g(\epsilon)$ implies $1-2\alpha\epsilon > 0$, cross multiplication gives

$$\alpha(2\epsilon-1)\epsilon^2 \geq (1-\epsilon)^3(1-2\alpha\epsilon)\beta$$

Substituting in $\beta = (1-\epsilon)/\epsilon$ and rearranging to get α on the LHS and a function of ϵ on the RHS gives

$$\alpha \geq \frac{(1-\epsilon)^4}{(2\epsilon-1)\epsilon^3 + 2\epsilon(1-\epsilon)^4} \equiv h(\epsilon) \tag{14}$$

If the above inequality holds, high types find it optimal to play according to the SS strategy profile and accept high signals in round one. □

A.2 Proof of Propositions 2 and 3

Since the techniques are similar, I combine the proofs to limit repetition. The comparative static results on strategies and values are proved using a combination of the Implicit Function Theorem and explicit differentiation. To ease notation, let $\chi_{xy} = xy/(1-\delta)$ denote each agent's present discounted value when a woman of type x marries a man of type y. The two endogenous variables are α, the fraction of low types playing HTG, and v_H, the high type's expected value. To use the IFT, I define a two-dimensional system of equations $g(\alpha, v_H; t) = 0$ as

$$\begin{aligned} g_1(\alpha, v_H; t): &\quad \alpha\frac{\delta v_H - \chi_{LH}}{\chi_{HH} - \delta v_H} - \frac{\lambda}{1-\lambda}\left(\frac{1-\epsilon}{\epsilon}\right)^3 = 0 \quad ; \\ g_2(\alpha, v_H; t): &\quad v_H - (1-\lambda)\big(((1-\epsilon)^2 + \epsilon(1-\epsilon)(1-\alpha))\chi_{LH} + (\epsilon(1-\epsilon)\alpha + \epsilon)\delta v_H\big) \\ &\qquad - \lambda\big((1-\epsilon^2)\delta v_H + \epsilon^2\chi_{HH}\big) = 0 \quad , \end{aligned} \tag{15}$$

where t represents a generic parameter. Let $a = (\alpha, v_H)$. Then the Jacobian of $g(a;t)$ is

$$\frac{dg(a;t)}{da} = \begin{pmatrix} \frac{\delta v_H - \chi_{LH}}{\chi_{HH} - \delta v_H} & \alpha\delta\frac{\chi_{HH} - \chi_{LH}}{(\chi_{HH} - \delta v_H)^2} \\ -\epsilon(1-\epsilon)(1-\lambda)(\delta v_H - \chi_{LH}) & 1 - \delta[(1-\lambda)\epsilon(1 + \alpha - \alpha\epsilon) + \lambda(1-\epsilon^2)] \end{pmatrix}$$

This matrix is invertible because the determinant is strictly positive since all terms are positive except the lower-left, which is negative. I do not explicitly write the $-\left[dg(a;t)/da\right]^{-1}$ matrix here because it is quite cumbersome, but denote the terms in this matrix by

$$-\left[\frac{dg(a;t)}{da}\right]^{-1} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix},$$

where Z_{11}, Z_{21}, and Z_{22} are negative and Z_{12} is positive.

A.2.1 Signal accuracy (ϵ)

Straight-forward differentiation shows that $\beta = (1-\epsilon)/\epsilon$ is decreasing in ϵ. To show $d\alpha/d\epsilon < 0$ I solve for α explicitly using (2) and (9) to get

$$\alpha = \frac{(1-\epsilon)^3\lambda((1-\epsilon)(1-\lambda)\delta L - (1-\delta(\epsilon+\lambda-\lambda\epsilon))H)}{\epsilon(1-\lambda)(\epsilon^2(1-\delta)L-(H-L)\lambda\delta(1-2\epsilon(1-\epsilon)(2-\epsilon(1-\epsilon))))}$$

It is straightforward to show that the numerator is decreasing in ϵ. The derivative of the denominator with respect to ϵ is

$$-(1-\lambda)\lambda\delta(H-L)\left[3\epsilon^2\frac{L(1-\delta)}{\lambda\delta(H-L)} - (1-8\epsilon+18\epsilon^2-16\epsilon^3+10\epsilon^4)\right]$$

To see that this term is positive, recall from Subsection A.1.2 that $\epsilon > \sqrt{L(1-\delta)/\lambda\delta(H-L)}$, and so the term in square brackets is greater than $-1+8\epsilon-18\epsilon^2+16\epsilon^3-7\epsilon^4$, which is negative over the relevant range of ϵ. This implies that the denominator of α is increasing in ϵ, and so overall α decreases in ϵ.

Differentiation of (1) with respect to ϵ immediately gives the result that v_1 decreases in ϵ.

A.2.2 Fraction of high types (λ)

To see that α can increase or decrease in λ, one can compute the derivative and evaluate it in the following situations. Fix $\delta = 0.85, L = 1, H = 2$, and $\lambda = 0.5$. When $\epsilon = 0.85$, $d\alpha/d\lambda < 0$, but when $\epsilon = 0.90$, $d\alpha/d\lambda > 0$.

To see that v_L can increase or decrease in λ, differentiate as follows:

$$\frac{d}{d\lambda}\left[\frac{(1-\lambda)\frac{L^2}{1-\delta}+\lambda(1-\epsilon)\frac{LH}{1-\delta}}{1-\lambda\epsilon\delta}\right] = \frac{(1-\epsilon)\frac{LH}{1-\delta}-(1-\delta\epsilon)\frac{L^2}{1-\delta}}{(1-\lambda\epsilon\delta)^2}$$

Intuitively, when δ is large this expression is positive, but when δ is small enough, it can be negative.

I use the IFT to show that $dv_H/d\lambda > 0$. The vector of derivatives with respect to λ is

$$\frac{dg(a;\lambda)}{d\lambda} = \begin{pmatrix} \frac{-(1-\epsilon)^3}{\epsilon^3(1-\lambda)^2} \\ -A(\chi_{HH}-\chi_{LH}) - B(\delta v_H - \chi_{LH}) \end{pmatrix}$$

where $A = \epsilon^2$ and $B = 1-\epsilon(1+\alpha)-\epsilon^2(1-\alpha)$. The top term is clearly negative. To sign the bottom term, there are two cases to consider:

Case 1: $A > 0, B > 0$; the expression is negative because $\chi_{HH} - \chi_{LH}$ and $\delta v_H - \chi_{LH}$ are positive.

Case 2: $A > 0, B \leq 0$; the expression is negative because $A+B = (1-\epsilon)(1-\alpha\epsilon) > 0$ implies $|A| > |B|$, and $\chi_{HH} > \delta v_H$ implies $\chi_{HH} - \chi_{LH} > \delta v_H - \chi_{LH}$.

This shows that the bottom term of $\frac{dg}{d\lambda}$ is negative, and since Z_{21} and Z_{22} are also negative, $\frac{dv_H}{d\lambda} > 0$.

A.2.3 Discount rate (δ)

To show $d\alpha/d\delta < 0$ I solve for α explicitly using (2) and (9) to get

$$\alpha = \frac{(1-\epsilon)^3\lambda((1-\epsilon)(1-\lambda)\delta L - (1-\delta(\epsilon+\lambda-\lambda\epsilon))H)}{\epsilon(1-\lambda)(\epsilon^2(1-\delta)L - (H-L)\lambda\delta(1-2\epsilon(1-\epsilon)(2-\epsilon(1-\epsilon))))}$$

It is straightforward to show that the derivative with respect to δ is negative.

That v_L is increasing in δ is easy to see given (1). The terms in the numerator increase in δ while the denominator decreases, and so the overall expression for the low type's value is increasing in δ.

That v_H increases in δ can be seen by solving explicitly for v_H using (2) and (9) to get

$$v_H = \frac{(1-\lambda)\epsilon^2(1-\epsilon)LH + \lambda H^2\left[1-2\epsilon(1-\epsilon)(2-\epsilon(1-\epsilon))\right]}{(1-\delta)(\epsilon^2 - \delta\epsilon^3 + \delta(2\epsilon-1)(-1+\lambda\epsilon(2-\epsilon(1-\epsilon))))}$$

Differentiation with respect to δ and some simplifications show that $dv_H/d\delta > 0$.

A.2.4 Added value of marrying a high type (G)

Letting $H = L + G$, the fraction of low types playing HTG can be explicitly written as

$$\alpha = \frac{(1-\epsilon)^3\lambda(G + L(1-\delta) - G\delta\epsilon - G\delta\lambda(1-\epsilon))}{\epsilon(1-\lambda)(-\epsilon^2(1-\delta)L + G\delta\lambda(1-2\epsilon(1-\epsilon)(2-\epsilon(1-\epsilon))))}$$

Then the derivative in G is

$$\frac{d\alpha}{dG} = -\frac{L(1-\delta)(1-\epsilon)^3\lambda\left[\epsilon^2(1-\delta\epsilon) + \lambda\delta(2\epsilon-1)(2\epsilon-1-\epsilon^2+\epsilon^3)\right]}{\epsilon(1-\lambda)\left(-\epsilon^2(1-\delta)L + G\delta\lambda(1-2\epsilon(1-\epsilon)(2-\epsilon(1-\epsilon)))\right)^2}$$

Then the sign of the derivative is opposite of the sign of the bracketed term in the numerator. To see that the bracketed term is positive, note that

$$\begin{aligned}\epsilon^2(1-\delta\epsilon) + \lambda\delta(2\epsilon-1)(2\epsilon-1-\epsilon^2+\epsilon^3) &> \\ \epsilon^2(1-\delta\epsilon) + \lambda\delta(2\epsilon-1)(-\epsilon^2+\epsilon^3) &= \\ \epsilon^2\left[(1-\delta\epsilon) - \lambda\delta(2\epsilon-1)(1-\epsilon)\right] &> 0\end{aligned}$$

The first inequality holds because $\epsilon > 1/2$, and the second inequality holds because $1 - \delta\epsilon > 1 - \epsilon$ and $\lambda\delta(2\epsilon - 1) < 1$. Then the sign of $d\alpha/dG$ is negative.

Straight-forward differentiation of (1) shows that

$$\frac{dv_L}{dG} = \frac{\lambda(1-\epsilon)}{(1-\delta)(1-\lambda\epsilon\delta)} > 0$$

I use the IFT to show that $dv_H/dG > 0$. It is easy to show that both terms in the $dg(a; G)/dG$ matrix are negative. Since Z_{21} and Z_{22} are also negative, this implies $dv_H/dG > 0$.

A.2.5 Baseline value of marrying (L)

Differentiating the explicit expression of α with respect to L gives

$$\frac{d\alpha}{dL} = \frac{G(1-\delta)(1-\epsilon)^3\lambda\left[\epsilon^2(1-\delta\epsilon) + \lambda\delta(2\epsilon-1)(2\epsilon-1-\epsilon^2+\epsilon^3)\right]}{\epsilon(1-\lambda)\left(-\epsilon^2(1-\delta)L + G\delta\lambda(1-2\epsilon(1-\epsilon)(2-\epsilon(1-\epsilon)))\right)^2}$$

The bracketed term is the same as in $d\alpha/dG$, which was determined to be positive in Section A.2.4. Since all other terms are also positive, $d\alpha/dL > 0$.

Straightforward differentiation of (1) shows that

$$\frac{dv_L}{dL} = \frac{1-\epsilon\lambda}{(1-\delta)(1-\lambda\epsilon\delta)} > 0$$

I use the IFT to show that v_H is increasing in L. It is easy to show that both terms in the $dg(a; L)/dL$ vector are negative. Since Z_{21} and Z_{22} are also negative, this implies $dv_H/dL > 0$.

A.3 The Non-Strategic strategy profile

In this subsection I prove that the Non-Strategic (NS) strategy profile can be part of a PBE and prove that the order of the discount rates is as depicted in Figure 1. Under the NS strategy profile, low types always accept, and high types accept high-signals in round one and otherwise reject. All information sets are reached with positive probability on the equilibrium path, so beliefs can be computed using Bayes' rule. I will show there are constellations of parameters such that this strategy profile constitutes a PBE.

Low types: It is sufficient to show that low types prefer to follow the NS strategy rather than rejecting low-signals in both rounds. If low types optimally accept low signals in the first round, then it is also optimal to accept high signals. Moreover, rejecting only in the first round rules out the possibility of marrying a high type. The probability of acceptance and the expected utility conditional on acceptance are

$$a(\ell, L) = \frac{(1-\lambda)\epsilon + \lambda(1-\epsilon)^2}{(1-\lambda)\epsilon + \lambda(1-\epsilon)} \ ; \ \gamma(\ell, L) = \frac{(1-\lambda)\epsilon\frac{L^2}{1-\delta} + \lambda(1-\epsilon)^2\frac{LH}{1-\delta}}{(1-\lambda)\epsilon + \lambda(1-\epsilon)^2}$$

The expected utility of playing according to the NS strategy profile and accepting is $v(\ell, L|s_i = A) = a(\ell, L)\gamma(\ell, L) + (1 - a(\ell, L))v_L$, where v_L is the same as in (1). The expected utility of rejecting in both rounds is δv_L. Low types optimally accept low signals if $v(\ell, L|s_i = A) \geq \delta v_L$. Simple algebra allows this inequality to be re-written

$$\delta \leq \frac{(1-\lambda)\epsilon L + \lambda(1-\epsilon)^2 H}{(1-\lambda)(\epsilon + \lambda + \lambda\epsilon(2\epsilon - 3))L + \lambda(1-\epsilon)(\lambda - \epsilon(2\lambda - 1))H} \equiv \bar{\delta}^{NS}$$

Subtracting the numerator from the denominator shows that $\bar{\delta}^{NS} < 1$.

High types: It is sufficient to show that high types prefer to follow the NS strategy rather than accepting low-signals in round one. If high types optimally reject low signals in round one, then it is also optimal to reject in round two. The probability of acceptance and the expected utility conditional on acceptance are

$$a(\ell, H) = \frac{(1-\lambda)\epsilon + \lambda(1-\epsilon)\epsilon}{(1-\lambda)\epsilon + \lambda(1-\epsilon)} \;;\; \gamma(\ell, H) = \frac{(1-\lambda)\epsilon \frac{LH}{1-\delta} + \lambda(1-\epsilon)\epsilon \frac{H^2}{1-\delta}}{(1-\lambda)\epsilon + \lambda(1-\epsilon)\epsilon}$$

The expected utility of accepting a low signal is $v(\ell, H|s_i = A) = a(\ell, H)\gamma(\ell, H) + (1 - a(\ell, H))\delta v_H$, where

$$v_H = \frac{(1-\lambda)(1-\epsilon)\frac{LH}{1-\delta} + \lambda\epsilon^2 \frac{H^2}{1-\delta}}{1 - (1-\lambda)\epsilon\delta - \lambda(1-\epsilon^2)\delta}$$

The expected utility of rejecting is δv_H. High types optimally reject if $\delta v_H \geq v(\ell, H|s_i = A)$. Simple algebra allows this inequality to be re-written

$$\delta \geq \frac{L - \lambda(L - (1-\epsilon)H)}{L + \epsilon\lambda(H - 2L) - (H - L)(2\epsilon - 1)\lambda^2} \equiv \underline{\delta}^{NS}$$

So long as $\delta \in (\underline{\delta}^{NS}, \bar{\delta}^{NS})$, the NS strategy profile and beliefs derived using Bayes' rule constitute a PBE.

But note this equilibrium involves a weakly dominated strategy – when two high types are in a relationship in which both saw low signals and both rejected a marriage, they believe with probability equal to one that their match partner is a high type. Nevertheless, rejecting is a weak best response given that the partner follows the equilibrium strategy and rejects. So while this strategy profile is a PBE, it involves weakly dominated strategies and is not a Trembling Hand Perfect equilibrium.

It is possible to order the discount rates for which the SS and NS strategy profiles are PBE. Let $\underline{\delta}^{SS}$ and $\bar{\delta}^{SS}$ be the lowest and highest discount rate, respectively, such that a SS equilibrium exists.

Proposition 6. $\underline{\delta}^{NS} < \underline{\delta}^{SS} < \bar{\delta}^{SS} < \bar{\delta}^{NS}$

Proof. Note that $\bar{\delta}^{SS}$ comes from $L^2/(1-\delta) \geq \delta v_L$, which can be rearranged to get

$$\delta \leq \frac{L}{(1 - \lambda(1-\epsilon))L + \lambda(1-\epsilon)H} \equiv \bar{\delta}^{SS}$$

To see that $\bar{\delta}^{SS} < \bar{\delta}^{NS}$, write $\bar{\delta}^{NS} = (AL + BH)/(CL + DH)$, where

$$A = \frac{(1-\lambda)\epsilon}{\epsilon + \lambda - \lambda\epsilon(3-\epsilon)} \;\; ; \;\; B = \frac{\lambda(1-\epsilon)^2}{\epsilon + \lambda - \lambda\epsilon(3-\epsilon)}$$

$$C = \frac{(1-\lambda)(\epsilon + \lambda + \lambda\epsilon(2\epsilon - 3))}{\epsilon + \lambda - \lambda\epsilon(3-\epsilon)} \;\; ; \;\; D = \frac{\lambda(1-\epsilon)(\lambda - 2\lambda\epsilon + \epsilon)}{\epsilon + \lambda - \lambda\epsilon(3-\epsilon)}$$

Rewrite $\bar{\delta}^{SS} = L/[(1-E)L + EH]$, where $E = \lambda(1-\epsilon)$.

$$AL + BH - L = \frac{\lambda(1-\epsilon)^2(H-L)}{\epsilon + \lambda - \lambda\epsilon(3-\epsilon)} \equiv Z > 0$$

$$CL + DH - [(1-E)L + EH] = \frac{\lambda^2\epsilon(1-\epsilon)^2(H-L)}{\epsilon + \lambda - \lambda\epsilon(3-\epsilon)} \equiv \lambda\epsilon Z > 0$$

Then

$$\begin{aligned} \bar{\delta}^{NS} - \bar{\delta}^{SS} &= \frac{AL+BH}{CL+DH} - \frac{AL+BH-Z}{CL+DH-\lambda\epsilon Z} \\ &= Z\frac{1}{CL+DH}\,\frac{1}{CL+DH-\lambda\epsilon Z}\Big((CL+DH) - \lambda\epsilon(AL+BH)\Big) > 0 \;, \end{aligned}$$

where the last inequality holds because $\bar{\delta}^{NS} < 1$ and $\lambda\epsilon < 1$.

I next show that $\underline{\delta}^{SS} > \underline{\delta}^{NS}$. It is possible to solve explicityly for α, the equilibrium fraction of low types playing HTG, while imposing $\delta = \underline{\delta}^{NS}$. Then, subtracting the upper bound for α, $g(\epsilon) = (1-\epsilon)^2/(\epsilon^3 + \epsilon(1-\epsilon)^2)$ from the result gives

$$\frac{H(1-\epsilon)^4(2\epsilon-1)}{\epsilon(1-2\epsilon(1-\epsilon))[(1-\lambda)(1-\epsilon)L(1-2\epsilon(1-\epsilon)) + \lambda H(1-2\epsilon(1-\epsilon)(2-\epsilon(1-\epsilon)))]}$$

This expression is positive for all $\epsilon \in (1/2, 1)$, which shows that optimal behavior with $\delta = \underline{\delta}^{NS}$ is not consistent with the SS equilibrium. □

Since α is too large, and Proposition 2 shows that α is decreasing in δ, we must have $\underline{\delta}^{SS} > \underline{\delta}^{NS}$.

A.4 Updating table

Table 2 below is included to assist the reader in performing second round inference calculations. The first column lists the two different types, and the second column lists each type's possible strategies. The third column lists all possible histories in which a man could find himself in round two. The description of the history involves signal θ and realized round one announcements (s_i, s_j), where s_i is the man's announcement and s_j is the woman's announcement. A crossed-out history indicates it does not occur in equilibrium. The fourth column of the table lists the action called for by the SS strategy profile. The fifth column lists $a^2(\theta, y, (s_i, s_j))$. These probabilities are computed by combining the Bayesian updating given the relationship history in the third column with the SS strategy profile. Lastly, the sixth column lists the expected payoff of accepting conditional on a match occurring.

Table 2: Relationship Histories, Updating, and Acceptance Probabilities

Type	Strategy	History	s_i	$a^2(\theta, y, (s_i, s_j))$	$\gamma^2(\theta, y, (s_i, s_j))$
Low	Eager	$\ell \cap (A, R)$	A	$\frac{(1-\lambda)(\epsilon)(1-\epsilon)\alpha}{(1-\lambda)\epsilon(1-\epsilon)\alpha+\lambda(1-\epsilon)\epsilon}$	$\frac{L^2}{1-\delta}$
		~~$\ell \cap (A, A)$~~			
		$h \cap (A, R)$	A	$\frac{(1-\lambda)(1-\epsilon)^2\alpha}{(1-\lambda)(1-\epsilon)^2\alpha+\lambda\epsilon^2}$	$\frac{L^2}{1-\delta}$
		~~$h \cap (A, A)$~~			
	HTG	$\ell \cap (A, R)$	A	$\frac{(1-\lambda)\epsilon(1-\epsilon)\alpha}{(1-\lambda)\epsilon(1-\epsilon)\alpha+\lambda(1-\epsilon)\epsilon}$	$\frac{L^2}{1-\delta}$
		~~$\ell \cap (A, A)$~~			
		$h \cap (R, R)$	A	$\frac{(1-\lambda)(1-\epsilon)^2\alpha+\lambda\epsilon^2\beta}{(1-\lambda)(1-\epsilon)^2\alpha+\lambda\epsilon^2}$	$\frac{(1-\lambda)(1-\epsilon)^2\alpha\frac{L^2}{1-\delta}+\lambda\epsilon^2\beta\frac{LH}{1-\delta}}{(1-\lambda)(1-\epsilon)^2\alpha+\lambda\epsilon^2\beta}$
		$h \cap (R, A)$	A	$\frac{(1-\lambda)(1-\epsilon)(1-\alpha+\alpha\epsilon)}{(1-\lambda)(1-\epsilon)(1-\alpha+\alpha\epsilon)+\lambda\epsilon(1-\epsilon)}$	$\frac{L^2}{1-\delta}$
High	F-F	$\ell \cap (R, A)$	R	$\frac{(1-\lambda)\epsilon(1-\alpha+\alpha(1-\epsilon))}{(1-\lambda)\epsilon(1-\alpha+\alpha(1-\epsilon))+\lambda(1-\epsilon)\epsilon}$	$\frac{LH}{1-\delta}$
		$\ell \cap (R, R)$	A	$\frac{(1-\lambda)\epsilon^2\alpha+\lambda(1-\epsilon)^2\beta}{(1-\lambda)\epsilon^2\alpha+\lambda(1-\epsilon)^2}$	$\frac{(1-\lambda)\epsilon^2\alpha\frac{LH}{1-\delta}+\lambda(1-\epsilon)^2\beta\frac{H^2}{1-\delta}}{(1-\lambda)\epsilon^2\alpha+\lambda(1-\epsilon)^2\beta}$
		$h \cap (A, R)$	R	$\frac{(1-\lambda)(1-\epsilon)\epsilon\alpha}{(1-\lambda)(1-\epsilon)\epsilon\alpha+\lambda\epsilon(1-\epsilon)}$	$\frac{LH}{1-\delta}$
		~~$h \cap (A, A)$~~			
	Resolute	$\ell \cap (R, A)$	R	$\frac{(1-\lambda)\epsilon(1-\alpha+\alpha(1-\epsilon))}{(1-\lambda)\epsilon(1-\alpha+\alpha(1-\epsilon))+\lambda(1-\epsilon)\epsilon}$	$\frac{LH}{1-\delta}$
		$\ell \cap (R, R)$	R	$\frac{(1-\lambda)\epsilon^2\alpha+\lambda(1-\epsilon)^2\beta}{(1-\lambda)\epsilon^2\alpha+\lambda(1-\epsilon)^2}$	$\frac{(1-\lambda)\epsilon^2\alpha\frac{LH}{1-\delta}+\lambda(1-\epsilon)^2\beta\frac{H^2}{1-\delta}}{(1-\lambda)\epsilon^2\alpha+\lambda(1-\epsilon)^2\beta}$
		$h \cap (A, R)$	R	$\frac{(1-\lambda)(1-\epsilon)\epsilon\alpha}{(1-\lambda)(1-\epsilon)\epsilon\alpha+\lambda\epsilon(1-\epsilon)}$	$\frac{LH}{1-\delta}$
		~~$h \cap (A, A)$~~			