The Aggregate Implications of Mergers and Acquisitions*

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Abstract

US merger activity exhibits a number of striking empirical patterns, regarding in particular, which firms form matches on the merger market and how the characteristics of transacting firms compare to the population of firms. A search and matching model exploits these facts to shed new light on the gains to firms from merger and their split across the transacting parties. Estimating the model in general equilibrium reveals a potentially significant beneficial impact of merger activity on aggregate economic performance, an important role for general equilibrium forces, and a key tradeoff between productivity gains and costly churn among unsuccessful new entrants.

Keywords: Mergers & Acquisitions, Search & Matching, Reallocation, Aggregate Productivity

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1 Introduction

Mergers and acquisitions (M&A) are a prominent feature of the US economy and account for huge flows of resources between firms.\(^1\) Moreover, an examination of the transacting firms reveals a number of striking empirical patterns in US merger activity, regarding in particular, which firms form matches on the merger market and how the characteristics of merging firms compare to the population of firms. Specifically, the key facts, documented here in some depth, are: (1) acquiring firms are generally larger and more profitable than their targets; (2) there is a large degree of positive assortative matching between transacting firms; and (3) acquirers tend to be the largest and most profitable firms, but targets are not the smallest or least profitable.

In this paper, I exploit these patterns to investigate the economic forces behind M&A as well as the consequences, both for the transacting firms and for the aggregate economy. In particular I ask: what are the gains to firms from merging; how are these gains generated and split; and finally, how do the aggregate resource flows occurring via M&A affect macroeconomic performance? To address these questions, I develop a dynamic general equilibrium model of merger activity, rich enough to capture the M&A decisions of individual firms, yet tractable enough to make explicit the mechanisms through which firm outcomes from M&A aggregate to impact the macroeconomy. I exploit a feature of the model linking merger gains and merger patterns to shed new light on exactly how gains from merger are generated and their split across the transacting firms. In turn, I use the general equilibrium framework to explore the channels through which M&A can affect the aggregate economy. My results suggest first, that merger activity can generate sizable gains to the transacting firms, that the gains are split with reasonable equity, and finally that merger activity can have a potentially significant beneficial impact on aggregate economic performance.

In describing the mechanics of the empirical merger market, the corporate finance literature generally highlights the importance of searching for profitable opportunities, often through costly intermediaries such as investment banks, of evaluating the potential gains from pursuing those opportunities, and of bargaining over the transaction price. In this light, I propose a search and matching model of the merger market in the spirit of Shimer and Smith (2000) and Shimer and Smith (2001a). Heterogeneous firms engage in costly and potentially time-consuming search for profitable merger partners. Upon meeting, the parties bargain over any gains that may be generated and decide whether to consummate the transaction or not. The set

\(^{1}\)To give just a sense of the size of this market, expenditures on M&A over the past few decades have averaged about 5% of GDP annually, reaching as high as 16% in the late 1990s, and about 42% of de novo business investment. The rate of capital reallocation via M&A has also averaged about 5%, accounting for an annual average of about two-thirds of total capital reallocation among large US firms, a figure that has grown steadily to over 80% by 2007 (Data are from SDC Platinum, Bureau of Economic Analysis, and for total reallocation, downloaded from Andrea Eisfeldt’s website at http://www.kellogg.northwestern.edu/faculty/eisfeldt).
of profitable merger partners is determined by a “merger technology” to which firms have access, determining the characteristics, and hence profitability, of the continuing entity as a function of those of the two pre-merger firms. The technology enables firms to combine the firm-specific asset over which they differ, be it interpreted as productivity, organization capital, or the quality of managers or products, and in this sense, mergers serve as a vehicle for knowledge or technologies to spread across transacting firms.

In addition to its realistic features, this framework proves powerful in generating a tight link between the way that merger gains are generated, that is, between the economic forces actually creating gains from merger, and the merger patterns predicted for the data. Specifically, for any particular way that merger gains are generated, the model has strong implications for the joint and marginal distributions over transacting firms, that is, for which firms form matches on the merger market and how the characteristics of merging firms compare to the population of firms. To illustrate this connection, I turn to several prevalent theories of merger activity that are nested by my framework and yield analytic characterizations of their predicted matching patterns. These include a theory of scale efficiencies through fixed cost savings, the q-theory of mergers for the transfer of resources from low to high productivity firms as outlined for example in Jovanovic and Rousseau (2002), and a theory of synergies through asset complementarities as in Rhodes-Kropf and Robinson (2008). In addition to making transparent the mapping between merger gains and merger patterns, it proves informative to explore some of the main drivers of M&A highlighted in previous work. I show that each is consistent with some aspects of observed merger activity, but none on its own can explain the full set of empirical patterns. Motivated by this finding, I propose a flexible technology that is able to jointly accommodate these theories, and a quantitative approach to pinning down its shape.

To analyze the interactions of M&A with the aggregate economy, I embed the merger market in an industry equilibrium setting in the spirit of Hopenhayn (1992) and Melitz (2003). Even as some firms grow through acquisition and others capitalize on their ideas by selling them and exiting the economy, firms compete in output markets and there is entry and exit by new entrepreneurs, jointly giving rise to a stationary equilibrium and firm size distribution. M&A influences aggregate performance both directly by reshaping the distribution of resources across firms, that is, through any productivity gains generated from merger transactions, as well as indirectly by changing the incentives for firm entry and exit. In particular, the existence of the merger market affects the entry decisions of entrepreneurs with new product ideas both through their individual merger prospects and by influencing the level of profitability upon entry, jointly affecting both the number and quality of successful new entrants.

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2Papers relying on versions of the q-theory also include Eisfeldt and Rampini (2006), Eisfeldt and Rampini (2008), Jovanovic and Rousseau (2008), and Faria (2008).
Importantly, the equilibrium framework I lay out is general enough to accommodate a number of alternative market structures. I put particular focus on two: monopolistic competition a la Melitz (2003) and limit pricing under head-to-head Bertrand competition a la Bernard et al. (2003). In both cases, productivity gains through merger increase the profitability of the transacting firms, though the precise mechanism differs: in the former, lower marginal costs lead to lower prices and hence higher revenues and profits, yet the markup of price over marginal cost is constant; in the latter, lower marginal costs do not result in lower prices, and hence translate into greater profitability through an increase in the markup. Despite this difference, however, the key equations of the model remain the same under constant or variable markups, an equivalence that holds through the model computation and estimation and leads to quantitative results that are quite similar across the two settings.

I structurally estimate the model to match key features of US M&A activity, exploiting the link between merger gains and merger patterns, as well as a relationship between the observed merger premium and the split of the gains across the transacting firms. The estimated model is able to match the target moments quite closely and performs well in replicating many non-targeted features of merger activity. The parameter estimates imply the potential for sizable gains to the transacting firms, with important roles for both merger synergies and productivity enhancements on transferred resources, and a reasonably equitable split of the gains across the transacting parties. In what would seem an appealing feature of my approach, the model estimation is completely independent of any particular assumption on market structure. The implications of the model regarding the gains to firms from merger and the split across transacting parties are thus the same no matter a constant or variable markup setting. Intuitively, the merger market parameters are identified by the merger patterns already described and their implications for the impact of merger on firm performance, but do not rely on the specific mechanism through which mergers may translate into changes in profitability.

Finally, I use the estimated model to assess the impact of M&A on the aggregate economy. In what seems a natural starting point, I first compare the aggregate outcomes in the estimated economy to a counterfactual economy with no merger activity. I next trace out how economic performance varies between these economies by changing the costs of engaging in merger activity. Lastly, I explore the potential effects of size-dependent policies hindering merger activity. The main results of these exercises are first, that M&A has the potential to greatly improve aggregate economic performance. Second, a significant portion of the gains come through general equilibrium channels that would be difficult to measure using only data on observed mergers. Finally, while barriers to merger generally reduce aggregate productivity and output, much of these losses are offset by an increase in consumption’s share of output due to a reduction in churn on the entry margin, i.e., less unsuccessful attempted entry, with
a corresponding reallocation of resources to the consumer. These findings hold across both the constant and variable markup settings, and indeed, are quantitatively quite similar, confirming that the key economic forces highlighted here are not reliant on a particular market structure, but rather, stem mainly from the features of the merger market itself, the primary focus of my analysis.

This paper relates to several branches of literature. Perhaps closest is the small number of papers already mentioned that have written models of M&A and take a stance on exactly how merger gains are generated, theories that are accommodated by my framework and which I explore in some depth. Additionally, there is a large number of empirical studies focusing on measuring the gains and split from merger using event-study analysis of financial market data. In contrast, I develop a structural model of M&A, allowing me first, to investigate more fully the economic forces actually generating gains from merger, and second, to explore the general equilibrium effects through which M&A can affect the aggregate economy. Moreover, I show that the quantitative implications of my model are well within the established range and additionally that the theory reconciles a puzzling empirical finding, the apparent disparity in the split of gains between acquirers and targets.

Considering M&A as spreading knowledge or technologies across heterogeneous productive units relates this paper to recent work exploring the effects of knowledge diffusion on aggregate performance, of which the primary examples are Lucas and Moll (2012) and Perla and Tonetti (2011). Where these papers consider knowledge as a public good that is freely imitated on contact, I consider knowledge as in some sense embodied in its user and model a market where firms actually transact over knowledge and bargain over any gains that are generated. Next, these papers explore informative but stylized variations on the learning technology, whereas I allow the data to speak directly to its shape. Lastly, I explicitly model the interactions of firm search and growth through knowledge acquisition with firm outcomes through competition in output markets, as well as with the innovation decisions of new entrepreneurs, channels not considered in these studies, but ones that play a key role here in assessing the general equilibrium effects of this type of market for knowledge.

Additionally, this paper connects with a literature exploring the role of resource reallocation in generating aggregate performance gains, examples of which include Hsieh and Klenow (2009) and Restuccia and Rogerson (2008). I model a particular vehicle for reallocation, the market that is formed for such activities, and use the empirical facts on observed reallocation activity to discipline the gains accruing to the transacting firms. By doing so, I highlight an additional mechanism through which reallocation can spur performance improvements and the potentially detrimental impact of different sorts of policy, those hindering M&A activity.

3For recent reviews see Andrade et al. (2001) and Betton et al. (2008).
Lastly, the search and matching model I develop builds on Shimer and Smith (2000) and Shimer and Smith (2001a), as well as an earlier predecessor, Lu and McAfee (1996). I extend the standard environment by allowing for repeat matching and for simultaneous search on the two sides of the market, two important features of observed merger activity. Additional contributions include highlighting a feature of this class of model enabling use of observed matching patterns to infer match surplus, an insight that should prove useful in future empirical work in search environments, as well as in the computation and estimation of the model, as I am unaware of any other work that has done so in this type of environment. Finally, this paper shows, I think, that the merger market represents an interesting and important new application of this framework.

The paper is organized as follows. Section 2 documents the key empirical patterns in US merger activity. Section 3 develops the model and presents analytic results. Section 4 describes the estimation of the model, and Section 5 quantitatively explores the aggregate implications of M&A. Section 6 concludes.

2 Empirical Patterns in US M&A

In this section, I document a number of stylized facts regarding observed patterns in US merger activity. The key empirical findings center around the joint and marginal distributions of transacting firms, i.e., which firms tend to form matches on the merger market, and how the characteristics of merging firms compare to the population of firms. Additionally, I document the prevalence of repeat acquisition, a distinctive feature of the merger market, and one that will play an important role in the theoretical framework.

2.1 The Data

To explore the empirical patterns in US M&A, I develop a sample of almost 58,000 domestic transactions announced between 1977 and 2009. Transaction-level data are from the Thomson Reuters SDC Platinum database (SDC). SDC is a comprehensive source of data on US M&A, covering all corporate transactions involving at least 5% of the ownership of a company where the transaction is valued at $1 million or more (after 1992, all deals are covered) or where the value of the transaction was undisclosed. SDC covers both public and private transactions. I restrict the sample to transactions valued over $1 million. The SDC data and sample construction are described in detail in the Appendix.

\[4\]
market value of the target. SDC contains a number of pre-transaction statistics on the merging parties including sales, employment, property, plant, and equipment (PP&E), earnings before interest, taxes, depreciation, and amortization (EBITDA), and market value, which are generally calculated for the 12 month period preceding the deal announcement.\(^5\) PP&E and EBITDA capture the size of the firm’s capital stock and level of profitability, respectively. I deflate all nominal variables to constant 2005 dollars using the CPI.

To compare the characteristics of transacting firms to the population of firms, I obtain the corresponding set of statistics for the universe of Compustat firms. To ensure comparability between the two sets of firms, I match the SDC database to Compustat and use Compustat operating statistics in calculations that involve industry aggregates, e.g., industry means or medians. I use the SDC statistics in calculations that do not, mainly because SDC provides wider coverage of private firms, greatly expanding the set of included firms, in particular targets. For example, sales are available for about 6,800 targets from Compustat and for 18,500 targets from SDC.\(^6\)

### 2.2 Transaction Summary Statistics

**Most deals are small, premia are large.** Table 1 reports summary statistics of transaction values and premia. The merger premium is defined as the percentage by which the purchase price exceeds the pre-merger market value of the target firm.\(^7\) The mean transaction value is quite modest at $267 million as is the median at only $31 million. The difference reflects a great deal of right-skewness in the distribution of transaction values, that is, the majority of mergers are very small, with some very large outliers. In contrast, merger premia tend to be substantial, with a mean premium of about 53% and a median of 39%.\(^8\) The typical transaction is thus characterized by a purchase price well above the pre-merger market value of the target firm: on average, the acquirer pays over 50% more than the standalone value of the firm it is purchasing.

### 2.3 Joint Distribution of Acquirers and Targets

Turning to the joint distribution over transacting firms, Figure 1 shows as an example the entire joint distribution over the profits (i.e., EBITDA) of acquirers and targets. Each point in the

\(^{5}\)Market values are calculated 4 weeks prior to announcement. Firm-level data are only available for a subset of transactions, in large part because many of the firms in the database are privately owned and are not required to report operating statistics to any regulatory agency.

\(^{6}\)I describe the Compustat data and matching process in more detail in the Appendix.

\(^{7}\)To avoid the known run-up in target share price once rumors of the merger begin to circulate, the premium is calculated using the market value of the target firm 4 weeks prior to the merger announcement.

\(^{8}\)I omit about 500 transactions with negative premia (less than 8% of transactions where premia are available).
Table 1: Transaction Values and Premia

<table>
<thead>
<tr>
<th></th>
<th>Trans. Val. ($M)</th>
<th>Premium (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>267.4</td>
<td>52.6</td>
</tr>
<tr>
<td>Median</td>
<td>31.0</td>
<td>39.0</td>
</tr>
<tr>
<td>SD</td>
<td>1,911.5</td>
<td>65.8</td>
</tr>
<tr>
<td>Max</td>
<td>186,824.1</td>
<td>1,937.0</td>
</tr>
<tr>
<td>Min</td>
<td>0.9</td>
<td>0.0</td>
</tr>
<tr>
<td>N</td>
<td>57,858</td>
<td>5,976</td>
</tr>
</tbody>
</table>

figure represents one transaction. In Panel A, I show profits as reported. In Panel B, I rescale the data by deviating the profits of each firm, i.e., acquirer and target, from the median in its respective industry. I make this adjustment to ensure that cross-industry differences between the transacting firms do not skew the results. Industries are defined at the 4-digit SIC level as reported in Compustat. There are about 5,100 transactions in the first panel and 3,500 in the second. The figure reveals two key properties of the joint distribution over transacting firms, each of which is robust to rescaling: first, there are significant and pervasive profitability differences between acquirers and targets, a fact that can be seen because the majority of transactions lie above the 45° line, as does the line of best fit. Second, there is a large degree of positive sorting between firms that form matches, a feature highlighted by the positive slope of the line of best fit, as well as by the absence of points in the northwest corner of the plot, showing that the most profitable acquirers do not match with the least profitable targets, and a similar absence in the southeast corner, showing that the least profitable acquirers do not match with the most profitable targets. This section explores these findings in more depth.

**Acquirers are larger and more profitable than their targets.** Figure 1 shows that acquiring firms tend be more profitable than their targets. To explore these differences in more detail, I report in Table 2 the mean and median log differences between matched acquirers and targets in profits, sales, employment, capital stock, and market value. Following the convention already discussed, I report statistics both as reported and after rescaling by industry medians. The first set of columns shows that across all metrics are acquirers generally larger and more profitable than their targets. The disparities are substantial, with a mean and median difference between acquirer and target of about 2 log points, a factor of almost 7.5. Moreover, the differences are pervasive, with the acquirer exceeding its target in size and profitability in

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9A similar figure obtains when using any other metric of firm size or profitability. I focus on profits as an example here as this will be the key driver of merger activity in the theory and the primary set of facts I aim to capture in the model estimation. As an example of the robustness of this figure, I show in the Appendix the analogous plot for firm sales.
about 90% of transactions. The second set of columns confirm that these facts are qualitatively robust to rescaling.\textsuperscript{10}

<table>
<thead>
<tr>
<th></th>
<th>Reported</th>
<th>Scaled by Industry Medians</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
</tr>
<tr>
<td>Profits</td>
<td>2.1</td>
<td>1.9</td>
</tr>
<tr>
<td>Sales</td>
<td>2.0</td>
<td>1.9</td>
</tr>
<tr>
<td>Employment</td>
<td>2.0</td>
<td>1.8</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>2.1</td>
<td>1.9</td>
</tr>
<tr>
<td>Market Value</td>
<td>2.3</td>
<td>2.1</td>
</tr>
</tbody>
</table>

**Positive assortative matching.** The second key fact revealed in Figure 1 is the pattern of positive sorting between acquirers and targets: more profitable acquirers tend to partner with more profitable targets, as in turn do the less profitable. To explore this finding in more detail, Table 3 reports the log correlations between the characteristics of acquirers and targets in individual matches across metrics of firm size and profitability. Again, I report the correlations from the data as reported and after rescaling. The first column shows a large positive correlation between acquirers and targets along all dimensions of the data, on the order of about 0.5, confirming the strength and ubiquity of the positive assortative matching between transacting firms. The second column confirms this pattern is robust to rescaling.\textsuperscript{11}

\textsuperscript{10}Similar differences using Tobin’s q, or market-to-book ratios, have been found in previous work, for example, Andrade et al. (2001).

\textsuperscript{11}Rhodes-Kropf and Robinson (2008) point out a similar sorting pattern in market-to-book ratios.
Table 3: Log Correlations Between Acquirers and Targets

<table>
<thead>
<tr>
<th></th>
<th>Reported</th>
<th>Scaled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profits</td>
<td>0.53</td>
<td>0.43</td>
</tr>
<tr>
<td>Sales</td>
<td>0.52</td>
<td>0.42</td>
</tr>
<tr>
<td>Employment</td>
<td>0.58</td>
<td>0.38</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>0.59</td>
<td>0.39</td>
</tr>
<tr>
<td>Market Value</td>
<td>0.54</td>
<td>0.48</td>
</tr>
</tbody>
</table>

2.4 Marginal Distributions of Transacting Firms

Acquirers are large and profitable, targets are median. Having examined the joint distribution over acquirers and targets, I now turn to the marginal distributions of these firms and ask how the set of transacting firms compares to the population of firms. As an example, I show in Figure 2 how the distributions of acquirer and target profits compare to the profit distribution of all firms in their respective industries. Specifically, I calculate the deciles of the firm size distribution measured in profits across all firms in each industry-year. I then count the proportion of acquirers and targets that fall into each decile. If transacting firms were distributed similarly to the population of firms, about 10% of transacting firms would fall into each decile, which is represented by the dashed line. Deciles above this line are overrepresented, in that they account for more than 10% of transacting firms, and deciles below the line are underrepresented. Panel A shows that acquirers predominantly come from the upper deciles of the firm size distribution. For example, only about 5% of acquirers come from the lowest decile of the distribution, with the proportion monotonically increasing to about 16% in the highest. The bottom 5 deciles are all underrepresented among acquiring firms while the top 5 are all overrepresented. This implies that the median acquirer is more profitable than the median firm in its industry. Turning to targets in Panel B reveals quite a different pattern. Targets predominantly come from the middle of the distribution, deciles 3 to 7, are just about proportionally represented in deciles 1-2 and underrepresented in deciles 8-10, severely so in the last. Targets then are not overrepresented in the lowest deciles, and indeed, are distributed approximately equally around the median firm so that the typical target is similar to the typical firm. The key takeaway then, is that while acquirers tend to be highly profitable, targets do not tend to be the least profitable.

To explore these facts in more detail, I report in Table 4 the mean and median log differences between acquirers and targets and the median firm in their respective industries across metrics of firm size and profitability. The table confirms the key implications of Figure 2: first, acquirers

\(^{12}\)Again, the patterns shown are qualitatively similar no matter the metric of size or profitability. The analogous figure for firm sales is shown in the Appendix.
tend to be larger and more profitable than the median firm in their industry along every
dimension. The mean and median differences are both substantial, hovering around 0.8 and
0.7 log points respectively, a factor of about 2, and the share of transactions with this feature
is about two-thirds. Turning to targets, the typical target is almost identical to the median
firm. Targets on average exceed the median firm in their industry on 3 out of 5 dimensions
and are smaller on the remaining two. The absolute size of these differences are quite small
across all metrics, especially as compared with the magnitudes by which acquirers differ from
the median and by which acquirers exceed targets. On all dimensions, targets are distributed
almost equally around the median firm so that the median difference between targets and the
median firm is essentially zero.

Table 4: Log Deviations from Industry Median

<table>
<thead>
<tr>
<th></th>
<th>Acquirer</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
</tr>
<tr>
<td>Profits</td>
<td>0.74</td>
<td>0.57</td>
</tr>
<tr>
<td>Sales</td>
<td>0.75</td>
<td>0.58</td>
</tr>
<tr>
<td>Employment</td>
<td>0.57</td>
<td>0.50</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>0.79</td>
<td>0.51</td>
</tr>
<tr>
<td>Market Value</td>
<td>1.01</td>
<td>0.86</td>
</tr>
</tbody>
</table>

2.5 Repeat Matching

Repeat acquisition is the rule. Before turning to the theory, I document the role of repeat
matching, an important feature of the merger market that stands in distinction to the standard
markets analyzed in the search literature. In the typical search market, such as the labor market
with firms and workers, or the marriage market with men and women, it is generally difficult
to form a new match without dissolving the existing one. The merger market represents a
significant departure from this paradigm. Once a firm has made an acquisition, it is free to reenter the merger market, either to pursue additional acquisitions, or perhaps seeking to sell itself. Table 5 shows that repeat acquisition is not the exception, but rather is the rule. In particular, the table shows the distribution of transactions by the number of purchases made by the acquirer during the sample period. In only about 19,000 transactions, or one-third the total, is the acquirer a one-time purchaser. This implies that in two-thirds of transactions, the acquirer is a repeat participant in the market. In about a quarter of transactions, the acquirer has made either 2 or 3 purchases, in another quarter between 4 and 10, and there is a long right tail, which I have truncated at 40, but which reaches to some acquirers that make over 100 purchases during the sample period. Serial acquisition is thus quite common, and the idea of repeat matching will play an important role in the theoretical framework, to which we are now ready to turn.

<table>
<thead>
<tr>
<th>Purchases</th>
<th>Firms</th>
<th>Transactions</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18,870</td>
<td>18,870</td>
<td>32.5</td>
</tr>
<tr>
<td>2-3</td>
<td>5,777</td>
<td>13,301</td>
<td>23.0</td>
</tr>
<tr>
<td>4-5</td>
<td>1,669</td>
<td>7,345</td>
<td>12.7</td>
</tr>
<tr>
<td>6-7</td>
<td>733</td>
<td>4,695</td>
<td>8.1</td>
</tr>
<tr>
<td>8-10</td>
<td>490</td>
<td>4,311</td>
<td>7.5</td>
</tr>
<tr>
<td>11-15</td>
<td>340</td>
<td>4,251</td>
<td>7.3</td>
</tr>
<tr>
<td>16-20</td>
<td>119</td>
<td>2,100</td>
<td>3.5</td>
</tr>
<tr>
<td>21-30</td>
<td>79</td>
<td>1,914</td>
<td>3.3</td>
</tr>
<tr>
<td>31-40</td>
<td>22</td>
<td>751</td>
<td>1.3</td>
</tr>
<tr>
<td>More than 40</td>
<td>6</td>
<td>320</td>
<td>0.5</td>
</tr>
<tr>
<td>Total</td>
<td>28,105</td>
<td>57,858</td>
<td>100.0</td>
</tr>
</tbody>
</table>

3 The Model

I present the model in stages starting with development of the merger market. I pause to illustrate the link the model provides between the way that merger gains are generated and the implications for the resulting merger patterns, i.e., for the joint and marginal distributions of transacting firms just described. I then close the model in a general equilibrium setting, making explicit the interactions of merger activity with the size distribution of firms, the product innovation decisions of new entrepreneurs, and the resulting impact on aggregate performance of the economy.
I consider an infinite-horizon economy set in continuous time. The agents of interest consist of a continuum of firms that are heterogeneous over some firm-specific asset $z$, which I will generally call productivity, but can alternatively be interpreted as organization capital, or the quality of the firm’s management or product offering. The firm’s $z$ may evolve through merger activity and is otherwise persistent. Labor is the only factor of production and the wage serves as numeraire. Market competition results in firm variable profits that are directly proportional to $z$, such that higher productivity firms are more profitable, an assumption clarified in Section 3.3. Importantly, the structure of the theoretical merger market, and the resulting insights of this section into the merger patterns documented above, are completely independent of the precise form of market competition and rely only on the existence of some intangible asset over which firms differ that delivers heterogenous levels of profitability. Denote with $\Pi z$ the variable profits of a firm of type $z$, where $\Pi$ is a factor of proportionality common across all firms. In order to produce and remain in operation, firms must pay a fixed cost of operation $c_f$ denominated in units of an outside good with price $P$. The level of firm profitability $\Pi$ and the price of the outside good $P$ can at present remain exogenous, but will later be endogenously determined in general equilibrium. The firm’s net profit flows are then given by

$$\pi(z) = \Pi z - Pc_f$$

### 3.1 The Merger Market

Firms participate in the merger market seeking value-increasing opportunities to combine their assets with a merger partner. Through this process, firms are able to influence their $z$’s, that is, to improve their productivity or quality, with an associated change in their level of profitability. Intuitively, we can interpret the merger process as enabling firms to enhance their intangible assets by growing their organization capital or productive know-how, or bundling products in some quality improving way, and in this sense, the view I take of merger activity is as a vehicle for knowledge or technologies to spread across productive units in the economy. Another interpretation would be of firms trading the blueprints or knowledge to produce some product, or simply exchanging the team of assembled labor with that particular expertise. The parsimonious nature of the framework here is able to capture any of these alternative notions of exactly which type of assets are transacted upon merger.

The merger process is mediated by search and matching frictions. In particular, firms engage in costly and potentially time consuming search for prospective merger partners. Upon meeting, the parties bargain over the gains, if any, determining the purchase price, and choose to consummate some transactions and reject others. After a merger takes place, the acquirer pays a one-time acquisition price to the target, evolves into a new continuing entity with a $z$
determined by the $z$’s of the two pre-merger firms, and continues on in production. Target firms receive the merger payment and exit the market.\textsuperscript{13}

Two features of the theory represent significant extensions of existing search models featuring two-sided ex-ante heterogeneity a la Shimer and Smith (2000). First, it allows for repeat acquisition, capturing the prevalence of this phenomenon highlighted in Table 5, and in particular, the firm’s type evolves with each transaction, capturing the dynamic nature of the merger decision and changing not only its flow profits, but its future search decisions and outcomes. Second, the merger market is not two-sided in the sense of having two distinct types seeking to match as in the labor market with firms and workers or the marriage market with men and women, nor is it a pure partnership market characterized by transactions between equals since virtually the entirety of “mergers” are truly acquisitions and there is no a priori reason to assume symmetry in these transactions. Rather, firms are searching for one another in well-defined but temporary roles and may end up on either side of a transaction, and indeed, one-time acquirers may themselves become targets. The data show that this is not anomalous, but rather, a significant number of one-time acquirers are later targets. To capture this idea, firms are able to search simultaneously on both sides of the market, although their activities on each side are endogenous and depend on their expected gains from each type of match.\textsuperscript{14} Figure 3 illustrates the sequence of steps on the merger market, which I now describe in some depth.

**Merger technology.** To detail the workings of the merger market, it seems natural to begin by outlining exactly how gains are generated from merger. Upon entering a meeting, a process described next, the gains from merger are determined by a “merger technology” to which firms have access. This technology determines the characteristics, i.e., the productivity and profitability, of the post-merger firm $z_m$ as a function of the characteristics of the two pre-merger

\textsuperscript{13}Recent work recognizing the potential importance of search frictions in this market include Rhodes-Kropf and Robinson (2008) and Martos-Vila (2008). DePamphilis (2009) describes the typical merger process and points out the important role of search via intermediaries such as investment banks or law firms, corporate databases, and directories, the screening of potential partners, and bargaining over the gains. Welch (2009) similarly details the key role of investment banks as intermediaries in this market.

\textsuperscript{14}An alternative search structure would be one where firms simply search for partners, and their respective roles are determined after observing each other’s types. Although attractive for its efficiency properties, this alternative approach is less consistent with how firms actually seem to approach the buy/sell decision as detailed, for example, in Boone and Mulherin (2007) and DePamphilis (2009), who describe a process in which firms outline a strategic plan and then search for appropriate partners, such that roles are well-defined prior to a meeting taking place. From a modeling standpoint, this alternative approach adds a layer of complexity, as it introduces an interaction between the firm’s search decisions on the two sides of the market. In particular, a firm’s search decision would depend not only on its expected surplus from a meeting in either role, but additionally the probability of becoming acquirer or target once a meeting takes place, a tradeoff that is not emphasized in descriptions of the empirical merger process. My analysis leaves as an open question then the particular friction that may prevent an efficient reversal of roles post-meeting.
Choose search intensities $\lambda$ and $\mu$.

**Target**
- Meet acquirer
  - Bargain
  - Purchase decision

**Acquirer**
- Meet target
  - Bargain
  - Purchase decision

- Search offer
  - Accept
    - Sell and exit
  - Reject
    - Continue searching
  - Do not acquire
    - z evolves
      - Choose new search intensities
      - Continue searching
  - Acquire

Figure 3: The Timing of Actions on the Merger Market

Firms, $z_a$ for the acquirer and $z_t$ for the target:

$$z_m = s(z_a, z_t)$$  \hspace{1cm} (1)

A useful functional form to keep in mind is a CES technology of the form

$$z_m = s(z_a, z_t) = A \left[ \alpha z_a^{\gamma} + (1 - \alpha) z_t^{\gamma} \right]^\frac{1}{\gamma}$$  \hspace{1cm} (2)

which is convenient in nesting several variations of particular interest. The shape of the merger technology will play a pivotal role in determining the gains from merger. Key to this approach is its parsimonious nature in accommodating a number of forces potentially creating merger gains and in providing sufficient flexibility to enable use of the data to infer exactly how gains are generated. Section 3.2 explores in depth the implications of some particular specifications corresponding to several prevalent theories of merger activity. Section 4 shows how we can use salient features of the transaction-level data to directly pin down the shape of the technology, and I defer further discussion until these sections.
**Bargaining.** The combined gains from merger are simply the value of the merged entity less the values of the two pre-merger firms as standalone entities:

$$\Sigma (z_a, z_t) = V(z_m) - V(z_a) - V(z_t)$$

(3)

The parties bargain over the gains according to the generalized Nash bargaining protocol, resulting in a commonly agreed upon purchase price $P(z_a, z_t)$. Denoting with $\beta$ the bargaining power of the acquirer and $1 - \beta$ that of the target, the purchase price satisfies

$$P(z_a, z_t) = V(z_t) + (1 - \beta) \Sigma (z_a, z_t)$$

The price depends on both $z_a$ and $z_t$ since the gains depend on the characteristics of both parties to the transaction, and reflects both the outside option of the target, which is to continue as a standalone entity with value $V(z_t)$, as well as the target’s share $1 - \beta$ of the combined gains. It is straightforward to derive the following expression for the merger premium, which we may recall is defined as the percentage by which the purchase price exceeds the pre-merger market value of the target:

$$\frac{P(z_a, z_t) - V(z_t)}{V(z_t)} = \frac{(1 - \beta) \Sigma (z_a, z_t)}{V(z_t)}$$

(4)

The premium then is equal to the target’s share of the combined gains divided by its pre-merger value. To preview the usefulness of this expression, equation (4) is used to pin down the Nash bargaining weights in the quantitative analysis of Section 4. Moreover, despite the simplicity of the bargaining structure, Section 4.2 documents how the outcome in (4) rationalizes a puzzling empirical finding, that is, movements in asset prices suggest that targets receive almost the entirety of merger gains.

To ease notation going forward, I define the individual gains from merger for an acquirer and target of a particular type $z$ partnering with a target of type $z_t$ or acquirer of type $z_a$ as

$$\Sigma_a (z, z_t) = \beta \Sigma (z, z_t), \quad \Sigma_t (z_a, z) = (1 - \beta) \Sigma (z_a, z)$$

(5)

**Search technology.** Turning to the search technology, firms choose search intensities $\lambda (z)$ of meeting a potential target and $\mu (z)$ of meeting a potential acquirer. To obtain these intensities, the firm must expend $C_\lambda (\lambda)$ and $C_\mu (\mu)$ units of the outside good respectively. $C_x (x)$ is convex and satisfies the standard properties $C_x (0) = 0, C'_x (x) > 0, C''_x (x) > 0, \lim_{x \to \infty} C_x (x) = \infty$ for $x = \lambda, \mu$. Denote by $dG(z)$ the stationary distribution of firm types in the market, which is kept exogenous at present, but will be endogenous and in part determined by merger activity in the general equilibrium analysis of Section 3.3. The aggregate meeting rate, which is the matching
function divided by the mass of searching firms, depends on the aggregate search intensities on the two sides of the market and takes the form

\[
\min \left\{ \int \lambda(z) \, dG(z), \int \mu(z) \, dG(z) \right\}
\]  

(6)

As a simple motivating example, consider firms sending out a number of investment bankers, asking some to search for targets and some for acquirers. The bankers on the two sides of the market then meet one another in a series of bilateral matches. If the number of bankers is not equal on the two sides of the market, the short side of the market then determines the number of matches, as shown in (6), and bankers on the long side are rationed, that is, some are left unmatched. The probability of match on the short side of the market is equal to one, and on the long side to the ratio of bankers on the short side to the long.

To obtain the matching rates on each side of the market, search intensities must then be scaled by these probabilities, which represent the effective meeting rates per unit of search and, it is worth pointing out, correspond to the standard notion of market tightness. Market tightness on the acquirer and target sides of the market is then defined as

\[
\theta_a = \min \left\{ \frac{\int \mu(z) \, dG(z)}{\int \lambda(z) \, dG(z)}, 1 \right\}, \quad \theta_t = \min \left\{ \frac{\int \lambda(z) \, dG(z)}{\int \mu(z) \, dG(z)}, 1 \right\}
\]  

(7)

The rates at which a type \( z_a \) acquirer meets a type \( z_t \) target, and in turn, a type \( z_t \) target meets a type \( z_a \) acquirer are given by

\[
\lambda(z_a) \theta_a \underbrace{\frac{\mu(z_t) \, dG(z_t)}{\Gamma(z_t)}}_{\Gamma(z_t)}, \quad \mu(z_t) \theta_t \underbrace{\frac{\lambda(z_a) \, dG(z_a)}{\Lambda(z_a)}}_{\Lambda(z_a)}
\]  

(8)

Examining the first expression, the product of its search intensity \( \lambda(z_a) \) and market tightness \( \theta_a \) represents the matching rate for this particular acquirer, i.e., the rate at which it meets some candidate target. Next, note that the presence of a type \( z_t \) target in the search market is equal to the product of its search intensity \( \mu(z_t) \) with its density in the firm type distribution \( dG(z_t) \). Conditional on entering a meeting, the probability of an acquirer meeting this particular target is then given by the ratio of this target’s presence in the search market to the aggregate search intensity on the target side of the market, the object defined by \( \Gamma(z_t) \). A similar interpretation holds for the rate at which a target of type \( z_t \) meets an acquirer of type \( z_a \).
**Value functions and decision rules.** We can now write the value of the firm in a stationary environment as

\[ rV(z) = \max_{\lambda(z), \mu(z)} \pi(z) - PC_\lambda (\lambda(z)) - PC_\mu (\mu(z)) + \lambda(z) \theta_a E [\Sigma_a (z, z_t)] + \mu(z) \theta_t E [\Sigma_t (z_a, z)] \]

(9)

where \( r \) represents the rate at which the firm discounts time.\(^{15}\) The flow value of a firm equals its flow profits \( \pi(z) \), which are net of fixed operating costs, less its expenditures on search in the merger market, which we should recall are in units of the outside good, plus its expected capital gains on the merger market, both as a potential acquirer and target. The expectations are with respect to the candidate partners the firm may meet.\(^{16}\) Intuitively, the firm’s value stems from both its current profit streams as well as its prospects on the merger market.

The firm then makes two types of decisions on the merger market. First, with what intensity to search for merger partners. Optimal search is governed by a pair of first order conditions:

\[ PC'_\lambda (\lambda(z)) = \theta_a E [\Sigma_a (z, z_t)], \quad PC'_\mu (\mu(z)) = \theta_t E [\Sigma_t (z_a, z)] \]

(10)

Firms equate the marginal costs of search to the marginal benefit, where the latter is composed of the incremental probability of a meeting multiplied by the expected gain.

Second, once the firm has met a candidate partner, it must choose whether to consummate the merger or proceed as a standalone entity. This decision is characterized by a pair of acceptance regions representing the set of partners with which the firm is willing to merge. For acquirers, this is the set of targets \( z_t \) with which a merger would create positive gains and similarly for targets over the set of acquirers \( z_a \). Formally, I define the acceptance regions of acquirers and targets by

\[ \Upsilon_a (z) = \{ z_t : \Sigma(z, z_t) \geq 0 \}, \quad \Upsilon_t (z) = \{ z_a : \Sigma(z_a, z) \geq 0 \} \]

There is a common acceptance set for acquirers and targets. Any meeting where a merger generates positive combined gains, i.e., where \( \Sigma(z_a, z_t) \geq 0 \), results in a consummated transaction. The acceptance regions generated by the search and matching structure are an important ingredient in the analysis as they provide a link between merger gains and the clear, though imperfect and noisy, matching patterns observed in the data. It is to this link that I now turn.

---

\(^{15}\)Conditions to maintain stationarity are detailed in the general equilibrium analysis of Section 3.3. The firm’s rate of time discount \( r \) equals the sum of the economy’s real interest rate and an exogenous rate of exit.

\(^{16}\)The firm’s expected gains as an acquirer are \( E[\Sigma_a (z, z_t)] = \int \max \{ \Sigma_a (z, z_t), 0 \} \Gamma(z_t) \), where \( \Sigma_a (z, z_t) \) is as defined in (5) and \( \Gamma(z_t) \) as in (8), and its expected gains as a target are similarly \( E[\Sigma_t (z_a, z)] = \int \max \{ \Sigma_t (z_a, z), 0 \} \Lambda(z_a) \).
3.2 Merger Gains and Merger Patterns: Analytic Examples

Having detailed the working of the merger market, this section illustrates the power of this framework to link the way merger gains are generated, that is, the economic forces creating gains from merger, with the empirical merger patterns documented in Section 2. Specifically, for any particular way that merger gains are generated, that is, any particular shape of the merger technology (1), the model has strong implications for the joint and marginal distributions of transacting firms. To illustrate this connection, I turn to several prevalent theories of merger activity that allow for analytic characterization of their predicted matching patterns. In addition to simply highlighting the mapping between merger gains and merger patterns, it proves useful to understand the implications of these theories: each is consistent with some aspects of observed matching behavior, but none on its own can explain the full set of empirical facts, a finding that motivates the use of a flexible technology that is able to jointly accommodate these theories, as well as a quantitative approach to pinning down its shape.

I begin with a theory of pure scale efficiencies, where firms merge only for fixed cost savings. Here there are no particular gains from combining any particular sets of firms but rather the merged firm is simply the sum of the two pre-merger firms, i.e., the merger technology is of the form $z_m = z_a + z_t$, a case that is nested in the CES example (2) by setting $\gamma = 1, \nu = 1, \alpha = \frac{1}{2}, A = 2$. Firms then merge only to save on the fixed cost of operation, which remains constant as the firm grows through acquisition, making this a theory of scale efficiency or capacity building. This additive technology seems a natural starting point to consider the implications of various technologies for merger patterns. Under this theory of the gains from merger, the following proposition, which I prove in the Appendix, holds:

**Proposition 1.** If the merger technology exhibits only scale efficiencies, i.e., $z_m = z_a + z_t$, then (i) the mean and median differences between acquirers and targets will be zero, (ii) the correlations between acquirers and targets will be zero, and (iii) the median acquirer and median target will be the same as the median firm.

The intuition is straightforward. Because the gains from merger are simply some function of the discounted value of the fixed operating cost, they are constant across all firms and all possible meetings. All firms then search with the same intensity on the merger market and will consummate a merger with any candidate partner, implying an essentially random assortment of mergers. The proposition is then immediate. The implications of a pure scale efficiency theory stand in contrast to the empirical patterns documented above, which reveal large and systematic differences between acquirers and targets, high correlation between acquirers and targets, and a mean and median acquirer that is considerably larger than the median firm.

In Figure 4 I take as an example the joint distribution over the profits of acquirers and
targets and directly contrast the observed distribution to that implied by this theory. In Panel A, I replicate the empirical distribution presented in Figure 1, i.e., the empirical matching set. In Panel B, I show the matching set that results from a pure scale efficiency theory. The horizontal arrow represents the acceptance region for acquirers, that is, an acquirer is willing to merge with any firm along the length of the arrow. The vertical line analogously shows the acceptance region for targets. The figure shows then that the entire domain becomes the matching set, that is, all firms are willing to merge with all others. Because search intensities are equal across firms, the entire domain will be populated by a random array of mergers. Clearly, this technology predicts merger patterns (or lack thereof) that are quite far from the observed empirical relationships.\footnote{Note that in an economy with no fixed cost, this technology would imply no merger activity. Merger surplus would be zero, and no firm would make an expenditure on search. For additional intuition, notice that if we posited exogenous search, that is, \( \lambda \) and \( \mu \) were free to the firm and exogenous, the model with this technology is analytically solvable and gives a capital gain from merger of \( \frac{P_{Cf}}{r+\lambda} \), that is, the gain is simply the discounted present value of the fixed cost.}

I next turn to a theory of purely synergistic mergers as described, for example, in Rhodes-Kropf and Robinson (2008). The idea here is that merger gains are generated from the bundling of complementarity assets and so by combining firms with similar characteristics, i.e., holding assets of similar quality, the post-merger firm will be more than the sum of its parts. This notion can be captured by any symmetric and supermodular specification of the merger technology (1). A natural case nested by the CES example (2) is 

\[
z_m = A(z_az_t)^\nu, \nu > 1
\]

which is obtained by setting \( \gamma = 0 \) and \( \alpha = \frac{1}{2} \), where in some abuse of notation, I have renormalized \( \nu = \frac{1}{2} \nu \). In this case, the technology features merger synergies through the complementarities in \( z \)'s. Under this theory, the following proposition emerges:

**Proposition 2.** If the merger technology exhibits pure synergies, i.e., is symmetric and supermodular, then the mean and median differences between acquirers and targets will be zero.
The symmetry in the technology renders irrelevant the identity of acquirer and target in any particular transaction, that is, the same gains are generated from a merger with firm \( z_1 \) as the acquirer and \( z_2 \) as the target as from the reverse transaction with the roles reversed. Firms then equate their effective search intensities on the two sides of the market.\(^\text{18}\) Together, these imply that every match a particular firm type makes as an acquirer will be reflected in equal weight by the reverse match with the roles reversed. It follows that in the aggregate, the mean and median differences between acquirers and targets will be zero. Again, this prediction stands in contrast to the data, which exhibit substantial and pervasive size and profitability differences between acquirers and targets.

Figure 5 compares the implied matching set to the empirical. The matching set more closely resembles the empirical one and captures the observed positive sorting, since as indicated by the acceptance regions, any meeting between the lines results in a consummated merger. However, in the absence of any asymmetry in the technology generating merger gains, the matching set is symmetric around the \( 45^\circ \) line as will be the intensity of matches. In contrast, the empirical matching set is centered 2 log points, a factor of over 7, above the \( 45^\circ \) line, with over 90% of transactions lying above the line. This suggests then that to capture the significant and pervasive differences between acquirers and targets, the theory must exhibit some asymmetry in the roles of acquirer and target in the technology creating gains from merger.

Lastly I turn to the \( q \)-theory of mergers as outlined, for example, in Jovanovic and Rousseau (2002), in which mergers serve as a vehicle for reallocating resources from less productive to more productive firms. The assumption here is that merger gains are increasing in the difference between the productivities of the transacting firms, i.e., \( \frac{\partial \Sigma(z_a, z_t)}{\partial (z_a - z_t)} > 0 \), since where these firms

\(^{18}\)Due to potential asymmetry in bargaining weights, this point is not obvious. All firms actually choose to search more intensively on the side of the market with greater bargaining power. In equilibrium, however, market tightness adjusts to offset the larger degree of aggregate search and equates the effective meeting rates on the two sides of the market.
are most distant is precisely where there is most room for productivity improvements for the transferred resource, and so gains to be generated. A natural case nested in the CES example (2) is $z_m = 2z_a$, which is derived for $\gamma = 1, \nu = 1, \alpha = 1, A = 2$, and implies that the productivity of the acquirer is fully extendable to the resources of the target, the case generally assumed in the literature. Under this theory, we can derive the following proposition:

**Proposition 3.** If the merger technology exhibits the $q$-theory, i.e., gains are increasing in $z_a - z_t$, then (i) low $z$ firms will be overrepresented in the set of targets and high $z$ firms in the set of acquirers, (ii) the median target will be below the median firm and the median acquirer above, and (iii) the highest rate of transaction will occur between low $z$ targets and high $z$ acquirers.

For any acquirer $z_a$, merger gains are decreasing in the type of its partner $z_t$, implying expected gains on the merger market for targets are a decreasing function of $z$. Lower type $z_t$'s then search for acquirers most intensively and are acquired most rapidly. Analogously, the most gains for any particular target $z_t$ come from being purchased by the highest $z$ acquirers, and it is this latter set of firms that search most aggressively for targets and make purchases most speedily. Because the rate of being acquired is decreasing in $z$, low $z$ firms must compose the majority of targets, driving the median target below the median firm. Similarly, because the rate of acquisition is increasing in $z$, high $z$ firms must compose the majority of acquirers. Finally, because the highest and lowest $z$ firms search most intensively and form an acceptable match, indeed, the match that generates the greatest combined gains, they will transact with one another at the highest rate. Again, these predictions stand in contrast to the data, which show that target firms are not predominantly low $z$, indeed that the median target is almost identical to the median firm, and that the highest $z$ acquirer and lowest $z$ target almost never transact.

Figure 6 illustrates the implied matching set. Any meeting of firms above the $45^\circ$ line, that is, where $z_a$ is greater than $z_t$, results in a consummated merger, capturing the pervasive differences between acquirers and targets. Interestingly, it also captures to some degree the positive sorting, since higher $z$ acquirers are willing to take on higher $z$ targets. However, because the most gains are generated from matching high $z$ acquirers with low $z$ targets, these firms transact at the most rapid rate, implying that the greatest density of matches should occur in the northwest corner of the domain, that is between the highest $z_a$ and lowest $z_t$, and moreover, that the density should be decreasing to the south and east from this corner. Turning to the empirical matching set, this match almost never occurs in the data, and the greatest density of matches is much more centered and decreases in the opposite direction, towards the

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19In addition to Jovanovic and Rousseau (2002), see the papers in footnote 2.
northwest and southeast corners of the domain.

Figure 6: A Q-theory of Mergers

3.3 General Equilibrium

Thus far I have outlined a theory of the merger market and illustrated a key feature of the framework enabling use of the data to make sharp inferences about the gains to firms from merging. I now embed the merger market in a general equilibrium setting, endogenizing the objects thus far taken as given, and allowing for explicit analysis of the interactions of M&A with the aggregate economy. In particular, I detail the mechanisms through which merger activity can in part determine aggregate outcomes, both directly through any productivity gains created through transaction, and indirectly by altering the degree and quality of product innovation by new entrepreneurs in the economy.

Preferences and production. A measure $L$ of identical consumers inelastically supply labor to firms and have time-separable and risk-neutral utility over consumption of a final good $C$. Denoting by $\rho$ the consumer’s rate of time discount, the focus on a stationary environment implies that $\rho$ is the constant real interest rate.

The final good is produced in a competitive sector from a continuum of differentiated intermediate goods indexed by $\omega$. Final good producers operate with a constant returns to scale CES production technology of the form:

$$Y = \left[ \int q(\omega) \frac{\sigma-1}{\sigma} d\omega \right] ^{\frac{1}{\sigma-1}}$$  \hspace{1cm} (11)
Standard arguments give the aggregate price index

\[ P = \left[ \int p(\omega)^{1-\sigma} \, d\omega \right]^{\frac{1}{1-\sigma}} \]  

(12)

and demand and expenditure functions for each product

\[ q(\omega) = Y \left[ \frac{p(\omega)}{P} \right]^{-\sigma}, \quad r(\omega) = R \left[ \frac{p(\omega)}{P} \right]^{1-\sigma} \]  

(13)

where \( R = PY \) denotes aggregate expenditure on the final good. In addition to serving as the consumption good for consumers, the final good is used to pay the resource costs in the economy and so corresponds to the outside good used in the partial equilibrium analysis thus far.

Intermediate goods are produced by a continuum of firms that are heterogeneous over productivity levels \( \tilde{z} \). Firms may offer multiple products as they add varieties through acquisition. For convenience, a firm is characterized by a single productivity level that is applicable to all of its products. We can then think of each firm as offering a single product portfolio composed of a bundle of individual varieties.\(^\text{20}\) The production technology exhibits constant returns to scale in labor and takes the form \( q = \tilde{z}^{\frac{1}{\sigma-1}} l \). I follow Atkeson and Burstein (2010) in rescaling \( \tilde{z} \) by the exponent \( \frac{1}{\sigma-1} \), a renormalization which will prove convenient in characterizing firm outcomes below.

**Market structure.** I consider two alternative market structures accommodated by this framework. Under monopolistic competition, a firm is the monopolist producer of each of the products in its portfolio. Standard arguments give the common output price set by the firm for each of its products as a constant markup over marginal cost, \( p(\tilde{z}) = \frac{\sigma}{\sigma-1} \left( \frac{1}{\tilde{z}} \right)^{\frac{1}{\sigma-1}} \), so that I alternately refer to this as the “constant markup” setting. Aggregating over products gives that the scale of the firm as measured by revenue, employment, and variable profits are all directly proportional to the product of the number of varieties it produces \( k \) and its physical productivity \( \tilde{z} \). That these two objects enter everywhere multiplicatively allows definition of an index \( z = k\tilde{z} \) that represents a sufficient statistic for firm outcomes. In particular, firm-level revenue, employment, and, most importantly, profits, are all directly proportional to its type \( z \), as was previously assumed, and can be written as

\[ r(z) = \sigma \Pi z, \quad l(z) = (\sigma - 1) \Pi z, \quad \pi(z) = \Pi z - P \sigma f \]  

(14)

\(^2\text{That the firm has a single productivity level for all products nests the case in which each product retains an individual productivity level in a straightforward manner.}\)
where $\Pi = \frac{1}{\sigma} \left( \frac{2-1}{\sigma} \right)^{\sigma-1} R P^{\sigma-1}$. Intuitively, firm outcomes are determined jointly by the size of its product suite and its level of productive efficiency. Henceforth, we can treat these multiproduct firms as if they offered only a single product with productivity $z$, which, in conjunction with the relevant economic aggregates, captures the state of the firm in a single dimension. Additionally, we will see that the aggregates themselves can be determined by the distribution of $z$, and hence do not require tracking the joint distribution of $k$ and $\tilde{z}$.

Alternatively, I consider head-to-head Bertrand competition along each product line, in the spirit of Bernard et al. (2003) and descendants. The elasticity of substitution between goods is now equal to 1, so that the final good aggregator takes on a Cobb-Douglas form. If this were not the case, markups would be bounded above by the monopolist markup, which is exactly that under monopolistic competition, and large mergers would not result in higher markups, rendering this additional exercise rather irrelevant. The renormalization of $\tilde{z}$ in the production function no longer plays a role, so a firm’s productive efficiency is simply $\tilde{z}$. There is a mass of “latent competitors,” or “followers,” freely endowed with a single product and a common productivity $\tilde{z}_f$. Additionally, each follower is able to freely enter an existing product line, that is, the introduction of each product is accompanied by a follower, so that though they are never active in equilibrium, their entry threat constrains the price of the “leader,” or active firm.\footnote{Intuitively, we can think of followers as low-efficiency imitators.}

Specifically, brands within a product line are perfect substitutes, which along with the CRS production technology, results in a limit pricing outcome: only the most productive firm, the leader, is active, i.e., produces and reaps profits, yet it is constrained to set its price to the marginal cost of the follower $\frac{1}{\tilde{z}_f}$, the highest price that can be charged while continuing to keep the follower inactive, giving an associated markup of price over marginal cost $\varphi = \frac{\tilde{z}}{\tilde{z}_f}$, which is increasing in $\tilde{z}$ (thus a “variable markup” setting). Notice that the price is independent of the active firm’s type and depends only on the follower, so that upon merger, which occurs only between active firms across product lines, productivity gains result in lower costs yet do not change the price of output. All gains from merger then are captured by the firm through a higher markup.

The Cobb-Douglas aggregator implies that expenditures, or revenues, are equalized across products, and we can then derive firm revenues, employment, and profits analogously to equation (14),

$$r(k) = \Pi k, \quad \ell(\varphi, k) = \frac{\Pi k}{\varphi}, \quad \pi(z) = \Pi z - P c_f$$

(15)

where a firm’s profit type is now defined by $z = k \left(1 - \frac{1}{\varphi} \right)$ and $\Pi = \frac{R}{M_v}$, where $M_v$ denotes the mass of available varieties. Revenue and employment depend on $k$ and $\varphi$ separately and cannot be characterized at the firm-level directly from the firm’s type $z$. However, because profits are
ultimately the driving force behind firm actions, just as in the constant markup setting, the
firm’s merger market and entry and exit decisions depend only on its single state \( z \) and do not
require tracking the joint distribution of \( k \) and \( \tilde{z} \), nor does characterizing the aggregate state
of the economy. That profits take on the same representation in (14) and (15), simply with a
different interpretation of \( z \), is a key result that ensures the dynamic equations of the model
remain the same across settings, as will the model computation and estimation.

In both environments then, profits are increasing in \( z \) and are proportional to a constant
that is determined in the economy’s general equilibrium. Under monopolistic competition with
constant markups, productivity-enhancing mergers allow the new lower-cost firm to reduce its
price and hence increase output, revenues, and profits. Under Bertrand competition with limit
pricing and variable markups, the post-merger firm has lower costs yet need not change its
price, and so garners greater profits through a higher markup. In both cases the underlying
motive for merger is the same: higher profits. Additionally, in both settings, the level of firm
profitability \( \Pi \) is determined in general equilibrium through changes in the aggregate price index
\( P \) or revenue per product \( \frac{R}{M} \). This plays an important role in the analysis, as it represents
the key mechanism through which aggregate outcomes on the merger market impact firm-level
performance.\(^{22}\)

**Entry and exit.** There is entry and exit into the economy. Incumbent firms are subject to an
exogenous exit shock that arrives at rate \( \delta \), common across all firms. Additionally, exit comes
when a firm is acquired. The rate of exit for an incumbent firm of type \( z \) is then given by

\[
\delta + \mu (z) \theta_t \int \Phi (\Sigma_t (z_a, z)) \Lambda (z_a)
\]

where the latter term is the rate at which this firm type is acquired, composed of the product of
its meeting rate as a target and the conditional probability that a transaction is consummated.
This last is the integral over the set of acquirers with which a merger generates positive gains,
weighted by the presence of firm types on the acquiring side of the market as set out in (8).
\( \Phi (\cdot) \) denotes the indicator function equal to 1 if its argument is greater than or equal to zero,

\(^{22}\) Although I focus on two alternative competitive structures each featuring differentiated products and imper-
fect competition, the model is even more general. In the Appendix, I show that the framework can additionally
accommodate a perfectly competitive market with a single homogenous good and decreasing returns to scale in
production as in the Lucas (1978) span of control model. This version of the model gives identical implications
as the constant markup setting and maintains the mapping to the partial equilibrium assumptions set forth
above, while eliminating the need to track the size of the firm’s product portfolio. The approach I take in the
paper is motivated by the goal of studying the interactions between merger activity and product innovation
in the economy, an idea that is more easily interpreted in the differentiated goods setting. Additionally, the
environment can be recast in a straightforward way to one where firms share a common marginal cost but differ
in the quality of their products.
else equal to zero. Despite the common exit shock $\delta$, exit rates will vary significantly and systematically across incumbent firm types to the extent that the rate of being acquired does.

There is a large pool of ex-ante identical potential entrants. To enter, entrepreneurs must expend $c_e$ units of the final good to obtain an initial $z$ draw from an exogenous distribution $F(z), z \in (z_{\text{min}}, \infty)$ with associated density function $dF(z)$. Free entry requires that in an equilibrium with positive entry, the case I focus on, the expected value of entry equal the cost:

$$\int V(z) dF(z) = P c_e \quad (17)$$

Once an entrant realizes its initial $z$, it may enter the market and begin operations or exit immediately, and will only choose to enter if it garners positive value from doing so. The presence of the fixed operating cost $c_f$ implies that some entrepreneurs may draw a low enough $z$ such that the value from entering is negative. The entry decision will thus be determined by a threshold $\hat{z}$ defined implicitly where the value of entry is equal to zero, i.e., where $V(\hat{z}) = 0$. Entrepreneurs drawing $z \geq \hat{z}$ will choose to enter while those with $z < \hat{z}$ will exit immediately.

Using the definition of firm value in (9), the threshold entrant must satisfy

$$\pi(\hat{z}) = - \{ \lambda(\hat{z}) \theta_a E[\sum_a (\hat{z}, z_t)] + \mu(\hat{z}) \theta_t E[\sum_t (z_a, \hat{z})] - P C_\lambda (\lambda(\hat{z})) - P C_\mu (\mu(\hat{z})) \} \quad (18)$$

Intuitively, the threshold is set where the firm’s net profit flows exactly equal the negative of the value it obtains from its M&A prospects, that is, its expected gains less costs. This value must be nonnegative, else the firm would optimally choose not to participate in the merger market at all. Equation (18) reveals then that there may be a set of firms that enter even while accruing negative profits and are only present in the economy hoping to take advantage of some value-increasing opportunity on the merger market. Without the possibility of merger, the threshold would be set where flow profits are exactly zero. This suggests that all else equal, by directly increasing the value from entry, M&A may induce less productive firms to enter the market, driving down the entry threshold. Seen another way, once an entrepreneur has paid the sunk cost of entry, the option value of participating in the merger market makes him more reluctant to exit.

In general equilibrium, however, it is not necessarily the case that the entry threshold is lower than in an economy without M&A. The entry decision also depends upon the level of firm profitability, which we should recall is a function of the economic aggregates, and in particular the aggregate price index or revenue per product. To the extent that merger activity generates a higher level of aggregate efficiency or a greater number of available products, the price index or revenue per product may fall, reducing firm profitability and making successful entry more
difficult, driving up the entry threshold and leading to increased selection on the entry margin. Thus, the effect of merger activity on the entry decisions of new entrepreneurs is ambiguous, and depends upon the rate at which profitability falls due to greater aggregate efficiency or product variety versus the rate at which the potential gains from merger add to the marginal firm’s value, i.e., whether the left hand side of (18) falls faster or slower than the absolute value of the right hand side increases.

**Stationary equilibrium.** In a stationary equilibrium, the aggregate variables remain constant through time, implying that the inflows and outflows of firms in the market must balance for all types. The stationary conditions for each type $z \geq \hat{z}$ take the form

$$M \int \lambda(z) \theta_a \left[ \int_{s^{-1}(z,z_a)} \Phi(\Sigma(z_a,z_t)) \Gamma(z_t) dG(z_a) + M_e dF(z) \right] = \lambda(z) \theta_a M dG(z) \int \Phi(\Sigma(z,z_t)) \Gamma(z_t)$$

$$+ \mu(z) \theta_t M dG(z) \int \Phi(\Sigma(z_a,z)) \Lambda(z_a) + \delta M dG(z)$$

where $s^{-1}(z,z_a) = \{z_t : s(z_a,z_t) = z\}$ denotes the inverse of the merger technology defined in (1), $M$ the mass of incumbent firms, and $M_e$ the mass of new entrants. For each type, firms flow in as the continuing entity from merger and through product innovation by new entrepreneurs. Firms flow out through participation in a merger, either as an acquirer or target, and through the realization of the exogenous exit shock. Integrating both sides gives the aggregate condition

$$[1 - F(\hat{z})] M_e = \left\{ \delta + \int \mu(z) \theta_t \left[ \int \Phi(\Sigma(z_a,z)) \Lambda(z_a) dG(z) \right] dG(z) \right\} M$$

which requires that the total flow of firms into the market (successful new entrants) must equal the total flow of firms out, where the latter is the integral over the exit rates defined in (16).

There are two feasibility constraints in the economy. First, labor market clearing requires that payments to labor equal the difference between aggregate revenue and variable profits:

$$L = R - \Pi_T$$

where after defining an index of average productivity across the set of operating firms,

$$\bar{Z} = \int_{\hat{z}}^{\infty} zdG(z)$$

an object to which we return when exploring the aggregate impact of M&A in Section 5, we
can write total variable profits as $\Pi_T = M \Pi Z$. For the final good, feasibility requires

$$Y = C + Y_s + Y_f + Y_e \quad (23)$$

where

$$Y_s = M \left[ \int C_\lambda (\lambda (z)) dG (z) + \int C_\mu (\mu (z)) dG (z) \right]$$

denotes the total resources devoted to search activities on the merger market, $Y_f = M c_f$ resources devoted to the fixed costs of production, and $Y_e = M e c_e$ resources devoted to new firm creation. Final production is thus allocated to final consumption and to payment of the various resource costs in the economy.

A stationary search equilibrium in this economy then consists of (1) aggregate variables $\{Y, P, C, M, M_e, dG (z)\}$, (2) intermediate good profits, entry threshold, and values $\{\pi (z), \hat{z}, V (z)\}$, and (3) firm search intensities and acceptance sets on the merger market $\{\lambda (z), \mu (z), \Upsilon a (z), \Upsilon t (z)\}$, such that (1) consumers maximize utility, (2) intermediate and final goods firms maximize expected discounted profits, (3) the labor market and final good market feasibility constraints are satisfied, and (4) the evolution of firm types is consistent with the stationary conditions.\textsuperscript{23}

4 Estimation and Model Fit

I begin the quantitative analysis by describing the estimation of the model and the resulting parameter values. Of particular interest are the parameters governing the gains from merger and their split across transacting firms, which I discuss at some length. I then assess the ability of the estimated model to replicate some salient features of the data, in particular, its fit of the empirical merger patterns documented above, as well as its implications for the gains and split from merger as measured in the empirical corporate finance literature. In an appealing feature of my approach, the estimation of the model is completely invariant to the precise form of market structure, that is, the values of the key parameters of the model remain the same under the constant or variable markup setting, as do the implications for the gains and split from merger. This may come as no surprise, given that the firm’s dynamic decisions simply depend on its profit type $z$, no matter the interpretation.

I first make several normalizations and assign values to a small number of parameters where

\textsuperscript{23} Unfortunately, proving general properties of equilibria in search and matching models with ex-ante heterogeneous agents is difficult, when possible at all, and would necessitate a protracted detour from the main focus of my analysis. See, for example, Shimer and Smith (2000), Shimer and Smith (2001a), and relatedly, Lucas and Moll (2012), for some discussions of the hurdles in doing so. Although such an exercise may be useful, I limit my analysis in this paper to the equilibrium at the estimated parameter values and in economically interesting counterfactual scenarios.
identification is not dependent on the structure of the model. A time period is assumed to be one year. I normalize the mass of consumers \( L \) to be 1 and the sunk cost of entry \( c_e \) to the same. The real interest rate \( \rho \) is set to 5\%. Equation (16) shows that the exit rate in the model is a combination of firm shutdown through realization of the exit shock and exit through acquisition. Below, I will target the aggregate rate of acquisition, and thus I am able to choose the exit shock \( \delta \) directly to match the empirical exit rate in the US. I follow Restuccia and Rogerson (2008) and set \( \delta \) so that the overall exit rate is 10\%, a figure that roughly coincides with the average rate of establishment exit in the US over the period 1980-2009 as reported by the Census Bureau.\(^{24}\) This results in a value for \( \delta \) of 0.063. As a useful benchmark, I set the entry distribution \( dF(z) \) such that the endogenous distribution \( dG(z) \) takes on a Pareto with shape parameter \( \xi \) and minimum observed value \( \hat{z} \). This is consistent with a large number of studies pointing out that the empirical firm size distribution closely approximates a Pareto.\(^{25}\) I then choose \( c_f \) such that \( \hat{z} \) is normalized to one. Following Atkeson and Burstein (2010), I set the Pareto shape parameter \( \xi = 1.2 \), which is also in line with a large number of studies finding shape parameters slightly exceeding 1. The mechanics of how I compute \( dG(z) \) are described in more detail in the Appendix.

We now come to the new parameters of the model to structurally estimate, which are those governing the merger market. I specify the merger technology as a Cobb Douglas aggregator over the two pre-merger firms:

\[
z_m = Az_a^\gamma z_t^\nu
\]

The Cobb-Douglas is of course nested in the CES example (2) and is appealing here for its simplicity both in giving parameters that are easily interpreted, and as we will see in a moment, in providing clear intuition for how we can use the data to quantitatively discipline them. The level parameter \( A \) captures any autonomous growth from merger independent of the particular characteristics of the transacting firms. The share parameters \( \gamma \) and \( \nu \) represent the relative weights of the acquirer and target in determining the performance of the post-merger firm. To the extent that these are different, and in particular that \( \gamma \) is greater than \( \nu \), this captures some degree of q-theory, that is, the acquirer may have more weight than the target in determining the performance of the post-merger entity. Next, the sum of \( \gamma \) and \( \nu \) plays an important role, as it captures the strength of merger synergies, that is, given the relative weights of the two pre-merger firms, what is the role of synergistic forces in generating performance improvements post-merger.

Section 3.2 gave a series of analytic examples highlighting how the theoretical framework links merger gains to merger patterns. Here, I give some intuition as to how this will work

\(^{24}\)Data obtained from http://www.census.gov/econ/susb/.
\(^{25}\)See, for example, Axtell (2001) and references thereto.
quantitatively in the estimation. Let us abstract for a moment from the dynamics of the model and consider the simple static case, where firms merge so long as it increases current period profits. For acquirers, this implies that their share of the new profit stream $z_m$ must be larger than their current profit stream $z_a$, and similarly for targets. This gives the two inequalities shown below, which can be rearranged to derive a set of log-linear inequalities representing upper and lower bounds on the resulting matching set:

\[
\begin{align*}
\beta A z_a^\gamma z_t^{\nu} & \geq z_a \Rightarrow \log z_a \leq \frac{1}{1-\gamma} \log (\beta A) + \frac{\nu}{1-\gamma} \log z_t \tag{24} \\
(1-\beta) A z_a^\gamma z_t^{\nu} & \geq z_t \Rightarrow \log z_a \geq -\frac{1}{\gamma} \log ((1-\beta) A) + \frac{1-\nu}{\gamma} \log z_t
\end{align*}
\]

For an acquirer, the log of its current $z_a$ must be less than or equal to a linear function of the log of the target $z_t$, with an intercept that depends on the Nash bargaining weight $\beta$ and the level term in the merger technology $A$, and a slope that depends on the share parameters $\gamma$ and $\nu$. The function is increasing in $z_t$ so long as $\gamma < 1$, which is the relevant case here. A similar expression holds for targets.

The two solid lines in Panel A of Figure 7 show an example matching set, where any meeting of firms between the lines results in a consummated merger. Upon increasing $\gamma$, the bounds rotate outward to become the dashed lines, expanding the matching set. A similar rotation occurs when increasing $\nu$. Intuitively then, I estimate $\gamma$ and $\nu$ to find the bounds that best approximate the matching observed in the data. Specifically, as $\gamma$ and $\nu$ in large part determine the rates of search and matching across the set of firms in the market and so the rate of transaction for each firm type, I estimate their values jointly to target the median deviation of the log of acquirer and target profits from the median in their industries. Table 4 shows these figures to be 0.57 and -0.01, respectively.

![Panel A. Inferring $\gamma$ and $\nu$](image1)

![Panel B. Inferring $A$](image2)

Figure 7: Intuitive Identification of Merger Gains
Now, notice that the example in Panel A excludes a number of mergers between small firms that fall to the southwest of the intersection of the two solid lines. Panel B shows that since A appears only in the intercepts of (24), an increase in A results in a parallel expansion of the matching set, increasing the number of small transactions. I estimate A then so that the model generates the number of mergers among small firms observed in the data. Specifically, I estimate A to match the percentage of targets that fall in the bottom decile of the firm size distribution, which Figure 2 shows to be 10.0%.

To pin down the split of the gains, i.e., the Nash bargaining weight \( \beta \), notice that equation (4) directly relates the merger premium to the bargaining shares, and in particular, shows that the premium reflects both the combined gains from merger, and the target’s share. Once we know the gains from merger, we can use (4) in conjunction with data on the premium to infer the target’s bargaining share \( 1 - \beta \). Hence, I estimate the bargaining parameter \( \beta \) to target the mean premium reported in Table 1 of 52.6%.

The costs of search are standard convex functions

\[
C_\lambda (\lambda) = \frac{B}{\eta} \lambda^\eta, C_\mu (\mu) = \frac{C}{\eta} \mu^\eta, \eta > 1
\]

(25)

where for simplicity I have assumed an equal degree of curvature on the two sides of the market. I first estimate \( B \) so that the aggregate merger rate in the model matches that in the data. From the combined SDC and Compustat data, I find that about 3.7% percent of Compustat firms are acquired annually over the sample period, and I estimate \( B \) to target this figure. This value is in line with evidence from Maksimovic and Phillips (2001) who report that an annual average of 3.9% of large manufacturing plants in the US changed ownership in the Longitudinal Research Database over the period 1974-1992. Intuitively, having used the observed matching patterns to infer the gains from merger, I am essentially finding the costs that reconcile these gains with the empirical merger rate.

To estimate the cost of search for targets \( C \), I use the fact that a set of values for \( B \) and market tightness imply a corresponding value for \( C \). Given the data at my disposal, I examine the number of bidders per target as a proxy for market tightness. If the number of bidders interested in each target were to significantly exceed one, this would serve as evidence that multiple simultaneous bidders is a measure zero event. However, for lack of direct data on market tightness, I interpret the number of bidders per target as an indicator of imbalance in the market. Robustness checks around the targeted level of market tightness confirm that the parameter estimates change very little in response to this moment and as mentioned below, the model itself contains forces pushing it towards this outcome. I leave then for future work a richer model of the negotiation process between firms, particularly in the presence of simultaneous bidders, and the resulting impact on objects such as the split of rents and merger consummation decisions.

\[\text{Due to the Poisson arrival process, there is not an identical model counterpart to this statistic, since multiple simultaneous bidders is a measure zero event. However, for lack of direct data on market tightness, I interpret the number of bidders per target as an indicator of imbalance in the market. Robustness checks around the targeted level of market tightness confirm that the parameter estimates change very little in response to this moment and as mentioned below, the model itself contains forces pushing it towards this outcome. I leave then for future work a richer model of the negotiation process between firms, particularly in the presence of simultaneous bidders, and the resulting impact on objects such as the split of rents and merger consummation decisions.} \]
of inequality on the two sides of the market. The data, however, do not show this. Across the almost 58,000 transactions in SDC, the average number of reported bidders per target is 1.01 with only about 1% of transactions exhibiting multiple bidders. Andrade et al. (2001) report a similar average of only 1.1 bidders per target over the period 1973-1998 as do Boone and Mulherin (2007) at 1.29.\footnote{Noting that much bidding may be non-public, Boone and Mulherin (2007) examine the sale process in detail for a sample of 400 acquisitions between 1989 and 1999. After accounting for private bids, they find a total of 1.29 bidders per target. In half of the acquisitions they investigate, the target firm only contacted a single potential buyer. In the other half, the target firm contacted an average of 21 buyers, but only received bids from an average of 1.57.} In the absence of compelling evidence of unbalanced search on the two sides of the market, I estimate $C$ in order to generate a market tightness equal to one and to ease notation, define $\theta = \theta_a = \theta_t = 1$.\footnote{Given the equilibrating forces suggested by the model, it is not surprising that market tightness is close to one. Firms would be hesitant to make additional expenditures on search knowing that there is a low incremental probability of a meeting, a reasoning that would tend to result in this outcome. For a simple example, see the note following the proof of Proposition 1 in the Appendix.}

To estimate the curvature parameter $\eta$, recall that I require the model to match the empirical rate of merger. $\eta$ governs exactly how this will occur by influencing the distribution of search intensities across firms. A high value of $\eta$ implies a fast-increasing cost of search and will push the economy towards spreading out search across the range of firms. A low value of $\eta$ implies the opposite, causing search intensities to be more concentrated within those firms with the most to gain from merger. In thus regulating the dispersion in search, I estimate $\eta$ to match the dispersion in the size of targets, measured by the coefficient of variation in target profits $\frac{\text{std}(\pi(z_t))}{\text{mean}(\pi(z_t))}$. This figure is about 3.9, reflecting the considerable heterogeneity in the size of targets.\footnote{To prevent undue influence of a few extreme outliers, I exclude 13 firms on each tail of the profit distribution, about 0.2% of the number of observations.}

I estimate the model using simulated method of moments. Although I believe that computing and estimating the equilibrium in this type of environment is an important contribution of the paper, I leave the details to the Appendix. In brief, the estimation follows a nested fixed point algorithm in which I guess a candidate parameter vector $\Theta = \{\gamma, \nu, A, \beta, \eta, B, C\}$, compute the equilibrium under this guess, and simulate merger market outcomes. I then construct the target moments described above from the simulated data $\Psi^s(\Theta)$ and compare them to the moments from the actual data $\Psi^d$. I iterate on the guess of $\Theta$ to minimize a distance criterion between the simulated and empirical moments.

Before turning to the estimation results and their implications, it worth noting that all parameters are estimated jointly. However, in a structural model such as the one here, it is useful to have a sense for which moments from the data are most informative for which parameters, and it is this goal that motivates the heuristic discussion relating moments to...
parameters above and the presentation of the results in Table 6 below.

4.1 Parameter Values: The Gains and Split from Merger

Table 6 lists the resulting parameter estimates, as well as the empirical and simulated moments. The model is able to jointly fit all seven target moments quite closely. Standard errors are bootstrapped and are generally small, a not surprising result given the number of observations in the sample. Turning first to the gains from merger, the share parameters of acquirers and targets in the merger technology $\gamma$ and $\nu$ take on values of about 0.91 and 0.53, respectively. There are two key takeaways here: first, this represents a significant departure from symmetry, that is, the acquirer holds substantially more weight than the target in determining the performance of the post-merger entity. The technology thus exhibits some degree of q-theory, that is, some capability of acquiring firms to improve on the performance of their targets. However, there is a limit to this ability: if the pre-merger target is too unproductive relative to the acquirer, the prospect of productivity gains on the transferred asset is trumped by losses suffered on existing assets. Next, the sum of $\gamma$ and $\nu$ is about 1.44, significantly greater than one, implying that sizable synergies can be realized from combining compatible sets of firms. There is an economic force through which the combination of firms with similar quality assets generates a post-merger entity with performance exceeding the sum of its parts. The data thus point to a technology that displays both q-theory and synergistic forces, and the theoretical framework has enabled us to quantitively infer their contributions to merger gains.$^{30}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>Target Moment</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.91</td>
<td>0.003</td>
<td>Median deviation of log acquirer profits</td>
<td>0.57</td>
<td>0.57</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.53</td>
<td>0.017</td>
<td>Median deviation of log target profits</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>$A$</td>
<td>1.05</td>
<td>0.006</td>
<td>Percent of targets in lowest decile</td>
<td>0.08</td>
<td>0.10</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.45</td>
<td>0.006</td>
<td>Mean merger premium</td>
<td>0.52</td>
<td>0.53</td>
</tr>
<tr>
<td>$\eta$</td>
<td>13.87</td>
<td>0.223</td>
<td>Coefficient of variation of target profits</td>
<td>3.92</td>
<td>3.92</td>
</tr>
<tr>
<td>$B$</td>
<td>1.25</td>
<td>0.889</td>
<td>Acquisition rate (x 10$^{11}$)</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>$C$</td>
<td>1.05</td>
<td>0.838</td>
<td>Bidders per target (x 10$^{12}$)</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

The value of $\beta$ implies that the gains from merger are split with reasonable equity, with

$^{30}$Related findings are in Maksimovic and Phillips (2001), who find that the productivity of transferred assets generally improves following an ownership change. Schoar (2002) finds similarly, but additionally that the productivity of acquirer assets generally falls. Bhagat et al. (2005) find that takeover gains tend to be higher when acquirer and target are closer in size, but conditional on the size of the target, are increasing in the size of the acquirer. Their results are exactly consistent with the estimated merger technology here, and they conclude with a similar interpretation that “these findings are consistent with the importance of both synergies and target-specific improvements such as removal of bad management.”
targets capturing about 55% of the gains and acquirers about 45%. Recall that the merger premium reflects both target bargaining power and the size of the combined gains. Given the pattern of merger gains, the model suggests that bargaining shares must be relatively balanced to imply premia on the order of magnitude observed in the data. Section 4.2 revisits the model’s quantitative implications for the gains and split from merger in light of some of the salient findings of the empirical corporate finance literature.

Finally, notice that at least in the constant markup setting, we can alternatively infer a firm’s type \( z \) from its sales or employment, in addition to its profitability. Exploiting this feature of the model to get a sense of the robustness of the parameter estimates, I report in the Appendix the results from estimating the model using sales as the identifier of firm type, rather than profits. The resulting parameter estimates are quite similar to those in Table 6.

### 4.2 Non-Targeted Moments

Table 6 shows that the model is capable of matching the target moments quite closely. This section explores the model’s performance on some other salient features of the data. I first turn back to the joint and marginal distributions presented in Section 2. I then detail how my results fit with existing empirical findings on the gains and split from merger.

**Joint and marginal distributions.** First, notice that I do not directly target moments of the joint distribution over transacting firms, that is, the degree of positive sorting and the prevalence of size and profitability differences observed in the data. Table 7 compares the log correlations of acquirer and target profits, sales, employment, and market values from the estimated model and the data, where the reader should keep in mind that the sales and employment figures are only valid in the constant markup setting. The model predicts a degree of sorting very close to that in the data. Additionally, I report the fraction of transactions in which the size and profitability of the acquirer exceeds that of the target from the model and data, where the latter is an average over the metrics in Table 2. Again, the model comes quite close to the value from the data. Jointly then, the model is capable of quantitatively fitting the two main features of the empirical joint distribution: prevalent profitability and size differences alongside positive sorting.

Next, I assess the model’s ability to replicate the marginal distributions of transacting firms as shown, for example, in Figure 2. Recall that the estimation relied only on the medians of these distributions and the share of targets in the lowest decile. Figure 8 compares the model generated marginal distributions to those observed in the data. The top row replicates Figure 2 in showing the empirical distributions and the bottom row the corresponding distributions generated by the model. The model replicates closely the entire marginal distributions of both
Table 7: Matching Patterns in M&A: Model and Data

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlations:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log of profits</td>
<td>0.58</td>
<td>0.53</td>
</tr>
<tr>
<td>Log of sales</td>
<td>0.58</td>
<td>0.52</td>
</tr>
<tr>
<td>Log of employment</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>Log of market value</td>
<td>0.71</td>
<td>0.54</td>
</tr>
<tr>
<td>Share of transactions with $z_a &gt; z_t$</td>
<td>0.86</td>
<td>0.90</td>
</tr>
</tbody>
</table>

acquirers and targets. Turning to acquirers in Panel A, the model generates the monotonic pattern observed in the data, with acquirers overrepresented in the top 5 deciles and underrepresented in the bottom 5. The model somewhat underpredicts the share of acquirers at the bottom and overpredicts the share at the top, but overall fits the marginal distribution of acquirers quite well and certainly captures the main features of the empirical distribution. Turning to targets in Panel B, the model generates the overrepresentation of targets in the middle deciles, and while slightly underpredicting the share of targets in the lowest deciles, fits closely the sharp drop off at the top.

Figure 8: Marginal Distributions of Transacting Firms: Data (top) vs Model (bottom)

Implications for asset prices. In this section, I compare the model’s quantitative asset pricing implications to some of the salient findings in the literature. I turn first to the model’s
predictions for the combined gains in value and show that the magnitudes are well within the established range. Second, I show that the theory rationalizes the large disparity in returns between targets and acquirers typically found in empirical work, and reconciles this feature of the data with the equitable split found here.

To measure the gains from merger, the empirical literature generally relies on an event study approach using the cumulative abnormal returns (CAR) to participant market values upon merger announcement. Using the full set of available transactions, i.e., those where both firms are publicly traded, typical estimates of the mean combined CAR are between 2% and 3%.

The model’s equivalent to the combined CAR is given by
\[
CAR = \frac{\sum(z_a, z_t)}{V(z_a) + V(z_t)}
\]
where the numerator represents the combined dollar gains upon merger and the denominator the combined pre-merger participant values. To get a clear sense of the model’s prediction for the mean CAR, recall from (4) that the merger premium is given by
\[
\text{prem} = (1 - \beta) \sum(z_a, z_t) V(z_t)
\]
which can be rearranged to find
\[
CAR = \frac{\text{prem} V(z_t)}{1 - \beta V(z_a) + V(z_t)}
\]
Using a mean premium of 53%, \(\beta\) of 0.45, and noting from Table 2 that the mean ratio of \(V(z_a)\) to \(V(z_t)\) is 10 \(e^{2.3}\) gives a model-implied mean combined CAR of about 9%.

The literature has pointed out several factors that may confound identification of merger gains using the CAR, and in particular, that may cause a downward bias in the measured gains to acquiring firms, obscuring the true value gains that are realized from the transaction. First, contemporaneous financial transactions related to the merger can have a confounding effect on share price movements. For example, Andrade et al. (2001) point out that stock-financed mergers are generally accompanied by a new equity issue, which are typically associated with negative CAR of 2% to 3%, even absent an accompanying merger. Removing stock-financed transactions, they find that the mean combined CAR increases substantially to between 4% and 5%. Bhagat et al. (2005) show that traditional estimates understate takeover gains due to truncation of the event window so that the estimates reflect considerable uncertainty regarding the final outcome of the bid. After adjusting for this bias, they find combined gains of about 7%.

Finally, revelation of private information upon merger announcement regarding the acquirer’s standalone prospects can cause negative price reactions.\(^{32}\) Recent empirical work attempting to distinguish the true value of the transaction from this “revelation bias” finds significant value gains attributable to merger, ranging from 9% in Wang (2012) to 13% in Bhagat et al. (2005) and as high as over 17% in Masulis et al. (2008). Empirical estimates of the combined gains

\(^{31}\)See, for example, Andrade et al. (2001), Betton et al. (2008), and Masulis et al. (2008).

\(^{32}\)For example, Braguinsky and Jovanovic (2004) develop a theoretical environment in which mergers always improve firm performance, yet the revelation of new information to shareholders upon merger announcement causes the acquirer’s value to fall, as can the combined values of the acquirer and target.
from merger thus range from 2% to over 17% when controlling for these confounding factors in the data that are not present in the model, placing the model’s prediction of about 9% slightly below the midpoint.\footnote{It is possible of course to introduce some of these elements into the model, but such extensions would come at considerable cost to tractability. In the present formulation, the proper comparison is to the estimated gains from the data after controlling for these factors.}

Turning to the split of the gains, the literature overwhelmingly finds that targets capture the lion’s share of the gains, an inference based on the fact that the abnormal returns to the target are generally substantial, while those to the acquirer are generally much smaller and actually negative in a significant number of transactions, with an overall mean of about 1%.\footnote{See, for example, Andrade et al. (2001), Betton et al. (2008), and Moeller et al. (2004). Addressing the significant fraction of acquiring firms that realize negative returns is largely the point of the literature assessing the role of the revelation effect in biasing these estimates. Moreover, the adjustment suggested by Andrade et al. (2001) to account for associated new equity issues is likely to play a role here as well.} In contrast, I find the split to be reasonably equitable, with the acquirer capturing a significant share of the gains. The theory outlined above is able to reconcile these findings, and in particular, the bargaining structure reveals that the observed disparity in abnormal returns is not necessarily indicative of a corresponding disparity in the true split of the gains.

The model’s equivalent to the target’s percentage CAR is $CAR_t = \frac{(1-\beta)\Sigma(z_a,z_t)}{V(z_t)}$, i.e., the dollar gain to the target divided by its pre-merger market value, which is simply the merger premium defined in (4). For acquirers, $CAR_a = \frac{\beta\Sigma(z_a,z_t)}{V(z_a)}$. The ratio of target to acquirer CAR is then

$$\frac{CAR_t}{CAR_a} = \frac{(1-\beta) V(z_a)}{\beta V(z_t)} \tag{26}$$

which depends not only on the ratio of bargaining shares, but importantly, on the inverse ratio of the pre-merger values. Equation (26) reveals that percentage gains are not necessarily representative of the true split: in dollar terms, a small percentage gain to a large firm may outweigh a large percentage gain to a small firm. Recalling from Table 2 that acquirers are typically much larger than their targets, with an average difference in market values being a factor of 10, it is straightforward to see how disparities in abnormal returns are in large part due to disparities in pre-merger size, rather than in bargaining shares.\footnote{As a simple example, setting $\beta = 0.5$ such that the split is exactly equal, (26) shows that the ratio of abnormal returns is exactly equal to the inverse of the pre-merger size ratio and so on average, $CAR_t$ should exceed $CAR_a$ by a factor of 10. Ahern (2012) similarly points out this difference between dollar and percentage gains and finds that targets gain only modestly more than acquirers on average, a result quite close to what I find here.}
5 The Aggregate Implications of M&A

I now use the estimated model to assess the implications of M&A for aggregate performance of the economy. To do so, I conduct a series of counterfactual experiments to explore and quantify the aggregate effects of M&A and to decompose them into a number of different channels, described next. In what seems a natural starting point, I begin by comparing performance in the estimated economy to an economy with no merger activity. I then trace out how economic outcomes vary between these economies by changing the costs of engaging in merger activity. Lastly, I analyze the potential effects of size-dependent policies hindering merger activity. To preview the common themes across counterfactuals, I find first that M&A has the potential to greatly improve aggregate economic performance; a significant portion of the gains come through general equilibrium channels that would be difficult to measure using only data on observed mergers; and finally, while barriers to merger activity generally reduce aggregate productivity and output, much of these losses are offset by an increase in consumption’s share of output due to a reduction in churn on the entry margin, i.e., less unsuccessful attempted entry, with a corresponding reallocation of resources to the consumer.

Aggregation and general equilibrium mechanisms. The model economy aggregates in a simple fashion, although the precise forms are now dependent on the assumed market structure. Beginning with the constant markup setting and recalling our index of average firm productivity $Z$ from (22), it is straightforward to show that aggregate output, productivity, and prices satisfy

$$Y = (M\bar{Z})^{\frac{1}{\sigma - 1}} L, \quad TFP = (M\bar{Z})^{\frac{1}{\sigma - 1}}, \quad P = \frac{\sigma}{\sigma - 1} (M\bar{Z})^{\frac{1}{1 - \sigma}} \quad (27)$$

such that the impact of M&A on aggregate performance can be summarized through two key statistics: the mass of operating firms $M$, which I alternatively refer to as the degree of product innovation in the economy, and the average productivity of those firms $\bar{Z}$. Notice that the sufficiency of $z$ in characterizing individual outcomes holds in the aggregate as well. As is standard in this class of model, once $M$ and $\bar{Z}$ are determined, the economy performs as one with a representative firm with productivity equal to $TFP$ as defined in (27). The existence of the merger market endogenizes the components of $TFP$ and impacts aggregate performance by affecting both the mass of firms as well as the productivity index, where the latter is influenced by (1) any direct productivity gains generated through merger and the associated effect on the distribution of resources across operating firms $dG(z)$, what I will call the intensive margin, and (2) through the general equilibrium effect on the threshold productivity level $\hat{z}$, what I will call the extensive margin. The effects of merger activity on $M$ and $\bar{Z}$ are ambiguous and depend on whether the incentives for additional entry are enough to offset the removal of firms
through acquisition, the magnitude of any productivity gains created directly through merger, and whether the entry threshold is moved up or down in equilibrium.

Written in logs, the change in output when moving to a counterfactual economy is then

$$\Delta \log Y = \frac{1}{\sigma - 1} \Delta \log M + \frac{1}{\sigma - 1} \Delta \log Z$$

(28)

where $\Delta \log X = \log X_{cf} - \log X$ denotes the log change in a variable when moving from the benchmark estimated economy (no subscript) to the counterfactual (subscript $cf$). Equation (28) decomposes the change in output into changes in the degree of product innovation and in average firm productivity. Next, we can decompose the change in $Z$ into its intensive and extensive margin components as

$$\Delta \log Z = \left[ \log \int_{\hat{z}}^{\infty} zd\tilde{G}_{cf}(z) - \log \int_{\hat{z}}^{\infty} zd\tilde{G}(z) \right]_{\text{intensive margin}} + \left[ \log \int_{\hat{z}_{cf}}^{\infty} zdG_{cf}(z) - \log \int_{\hat{z}}^{\infty} zd\tilde{G}_{cf}(z) \right]_{\text{extensive margin}}$$

(29)

where $d\tilde{G}_{cf}(z) = \frac{dG_{cf}(z)}{G_{cf}(\hat{z})}$. The first term captures the change on the intensive margin, that is, fixing the threshold productivity level, what is the effect of moving from the benchmark distribution $dG(z)$ to the counterfactual one $dG_{cf}(z)$. The second term captures the extensive margin: fixing the distribution, what is the impact of a change in the threshold productivity level from $\hat{z}$ to $\hat{z}_{cf}$. Scaling these terms by $\frac{1}{\sigma - 1}$ gives their contributions to changes in output.

Turning to the variable markup setting, aggregation delivers the following expressions for aggregate output (which equals $TFP$ since $L = 1$) and the aggregate price level:

$$Y = M_{V}^{M_{0}^{-1}}, \quad P = \frac{1}{\tilde{z}_{f}}$$

(30)

Notice the difference between these expressions and those in (27): under constant markups, the aggregates depend on both the mass of firms and their average productivity, and thus give a role for mergers to directly impact aggregate performance through any productivity gains generated upon transaction. In contrast, there is no such role here, precisely because of limit pricing: productivity gains upon transaction all accrue to the firm in the form of a higher markup. Mergers then only impact the aggregate economy through the equilibrium effects on first, the number of available varieties, and second, the identity of the latent competitor.

The impact of M&A on the number of varieties is similar to that in the constant markup setting and depends on the strength of the incentives for additional entry that mergers create. In contrast, the impact on the aggregate price level $P$ depends now on the relation between merger activity and the productivity of the latent competitor. In order to constrain the pricing
of the leader, the threat of follower entry must be credible, i.e., a price increase by the leader must induce entry by the follower, but the follower must have non-negative value from doing so, where the value from entry is endogenous and depends on both flow profits and expected merger market outcomes. Consider then an off-equilibrium path price increase by the leader immediately inducing entry by the follower, and recall that the follower is endowed with an initial product and productivity \( \tilde{z}_f \). If upon entry, the follower’s \( z \) would be greater than the marginal active firm \( \hat{z} \), the follower would immediately enter a product line where \( \tilde{z} \) was the leader, and thus, such a firm could not be the latent competitor. On the other hand, if such entry would leave the follower with \( z \) less than \( \hat{z} \), entry would result in negative value, and so the threat of such would not be credible. Thus, there is a relationship between the threshold active firm and the latent competitor, so that as M&A changes the level of \( \tilde{z} \), the identity of \( \tilde{z}_f \) changes as well. Intuitively, the more productive the marginal firm, the more productive must be the latent firm in order to credibly constrain prices. Assuming that the price of the follower after such off-equilibrium path entry is constrained by a generic firm with \( \tilde{z} \) normalized to 1 and respecting an integer constraint on the number of products a firm holds, the follower type must satisfy \( \tilde{z}_f = \frac{2}{2-\hat{z}} \), so that the aggregate price index is decreasing in \( \tilde{z} \) and satisfies \( P = 1 - \frac{\hat{z}}{\tilde{z}} \). Under variable markups then, M&A impacts the aggregate price level only through the equilibrium effect on the identity of the marginal firm \( \tilde{z} \).

Finally, recall that across market structures, consumer outcomes depend additionally on the allocation of output across its various uses, that is, given a level of output \( Y \), what share goes towards final consumption versus paying the resource costs in the economy outlined in (23). To the extent that the amount of resources absorbed by non-consumption activities changes due to M&A, consumer outcomes may move differently than industrial outcomes such as output and productivity, an idea that will play an important role below. A last useful decomposition comes then in assessing the change in consumption, which also serves as measure of welfare. Consider the identity \( C = Y \times \frac{C}{Y} \). Taking logs and differencing gives the change in consumption as a function of the change in output and in consumption’s share:

\[
\Delta \log C = \Delta \log Y + \Delta \log \frac{C}{Y}
\]

\[\text{(31)}\]

### 5.1 An Economy Without M&A

To assess the aggregate impact of M&A, I first compare the aggregate outcomes in the estimated economy to a counterfactual economy with no merger activity.\(^{36}\) Under constant markups, it remains to put a value on the elasticity of substitution in final good production \( \sigma \), which I

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\(^{36}\)The no-merger economy here is essentially the closed economy version of Melitz (2003), a model thoroughly explored in the literature and so one that serves as a natural reference point for this exercise.
set equal to 3, a common value in studies with CES demand and monopolistic competition, for example Hsieh and Klenow (2009) and Atkeson and Burstein (2008), and is close to the value in Bernard et al. (2003). The value of $\sigma$ determines the level of aggregate revenue in the economy, which along with the equilibrium solution, gives enough information to back out the other aggregates of interest.\textsuperscript{37} Under variable markups, the cost of entry $c_e$ determines total revenue in the economy. I set $c_e$ then to target $\frac{R}{wL}$, the inverse of labor’s share of income, which is also the average markup. I report results from two alternative values: the first is a value of 1.5, which delivers the same average markup as in the constant markup setting with $\sigma = 3$ and implies that labor’s share of total income is two-thirds, a standard value in the macro literature. On the other hand, recent studies (see, for example, Jaimovich and Floetotto (2008)) report that markup estimates among US firms range from 1.2 to 1.4 in value-added data and so in this light, I also report results with an average markup of 1.3, the midpoint from these findings and a value that coincides closely with that in other studies, for example, Atkeson and Burstein (2008). We will see that the quantitative implications of the model are quite robust both markup setting and level.

Table 8 reports the changes in final output and consumption when moving from the benchmark (estimated) economy to one with no M&A.\textsuperscript{38} I use the decompositions in (28), (29), and (31) to measure the relative contributions of the various channels driving these changes in the constant markup setting, although as we have seen, these are largely not relevant under variable markups. Due to the linear decompositions, the sum of the individual factors cumulate to the total changes. The table shows the potentially significant beneficial impact of M&A, with its removal leading to losses in aggregate productivity and output of between 21% and 26% and of between 9% and 12% in consumption.\textsuperscript{39} The results are strikingly similar across the alternative competitive settings.

Turning to the components of the output loss under constant markups, about 4% of the decline comes through a reduction in the mass of firms, that is, through decreased product innovation. The existence of the merger market induces a greater amount of product innovation by new entrepreneurs, more than offsetting the mechanical effect of mergers in removing firms from the market.\textsuperscript{40} The remaining 22% of the output loss comes through $Z$, that is, through a reduction in average firm productivity. About 13% is on the intensive margin, i.e., is attributable directly to the loss of the productivity gains created through mergers, while 8% is on the

\textsuperscript{37}See the Appendix for details.

\textsuperscript{38}I outline the details of the computational algorithm used to compute counterfactuals in the Appendix.

\textsuperscript{39}Note that these values represent a comparison between two stationary equilibria not accounting for the transition path, and so should be interpreted as capturing the potential long-run effects of M&A.

\textsuperscript{40}In a related finding, Phillips and Zhdanov (2012) show that M&A encourages R&D investment and innovation, particularly among small firms.
extensive margin, i.e., is due to reduced selection on the entry margin. The direct gains from M&A compose only about half of the total then, suggesting that relying only on observed M&A to gauge its contribution to aggregate performance may cause us to miss some quantitatively important channels operating in general equilibrium. This point is even more critical under variable markups, where all of the aggregate gains from M&A come through equilibrium effects on the amount of product innovation in the economy.

Next, although the output and consumption losses when moving to the no-merger economy are both quite large, the fall in consumption is considerably smaller, averaging about half that in output across settings. This is due to the increased resource costs imposed by M&A, which changes the allocation of output across its various uses. In particular, much of the fall in output in the no-merger economy is offset by a rise in consumption’s share of output. Interestingly, the wedge between output and consumption changes is not due to expenditures on search in the merger market, which account for less than 1% of output. Rather, the existence of the merger market generates a greater amount of unsuccessful attempted entry, what I will call churn on the entry margin. Intuitively, the additional value stemming from their M&A prospects induces a large increase in the number of entrepreneurs attempting to enter the economy. Because M&A simultaneously makes successful entry more difficult by pushing up the entry threshold, it follows that there is a greater degree of churn, that is, entrepreneurs who pay the entry cost, draw an initial productivity below the threshold, and exit immediately. This activity imposes large costs on the economy by absorbing a substantial share of output. A reduction in churn and a corresponding reallocation of resources to the consumer accounts for the majority of the gain in consumption’s share of output when moving to the no-merger economy. Thus, there is a key tradeoff in the economy between the gains in aggregate productivity and output generated
by merger activity and the increased resources going towards churn.\footnote{A natural example to have in mind here is the dot-com boom, which featured large rates of entry, anecdotal evidence suggests much of which was fueled by the potential to participate in the merger market, followed quite closely by large rates of exit. Though this may have led to gains in aggregate efficiency through greater amounts of successful entry and the reallocation flows occurring via M&A, a significant amount of resources were absorbed by the activities of the ultimately unsuccessful entrepreneurs.}

It is important to understand why the aggregate consequences of M&A are so similar whether constant or variable markups: the equilibrium forces at play are the same in the two environments. Removing the option to merge directly reduces firm values; yet free entry disciplines the extent of the fall. In equilibrium, the level of firm profitability must rise to compensate for the reduction in value from the absence of M&A, an effect that works through the aggregate price index. Under constant markups, the loss of productivity gains from merger along with a reduction in the mass of firms work to effectuate this increase; under variable markups, the productivity of the latent competitor falls, increasing the price index and delivering higher markups and hence profits to active firms. The last row of Table 8 shows that the average markup falls only between 1\% and 5\% despite the removal of M&A entirely, the modest nature of the reductions highlighting the offsetting equilibrium effect on the level of markups. Thus, the equilibrium forces at work are fundamentally the same across the two competitive settings and the model delivers results that are quite similar.

5.2 Varying the Costs of Merger Activity

I next explore the the economy’s performance under varying levels of the search costs $B$ and $C$. By representing the importance of search frictions, differences in these costs can capture disparities across countries, or even industries, in the functioning of financial markets and intermediaries, or other institutions, in enabling the matching process. An alternative interpretation would be one of government policies that impose additional costs on merger activity, for example so-called “bureaucratic red-tape.”

Panel A of Figure 9 shows the changes in aggregate productivity in the economy’s stationary equilibrium as search costs increase from their estimated level towards an infinitely high level, in which case the economy approaches the no-merger outcome. Here, I focus on the constant markup setting, as it allows a richer decomposition of the various effects at play, though given the results in Table 8 it should come as no surprise that the movements in $Y$ and $C$ are quite similar under variable markups. $TFP$ monotonically decreases in the costs of search, eventually converging to its level in the no-merger economy. Distinguishing the movements in $TFP$ stemming from changes in the the mass of firms $M$ and from changes in average firm productivity $Z$ reveals that its two components move non-monotonically and in opposite directions. The mass of firms initially falls, overshooting the drop to the no-merger economy...
before eventually rising to this level. In contrast, $Z$ shows an initial rise before falling to its level in the no-merger economy. To understand the intuition for these movements, it is useful to further decompose the changes in $Z$ into those on the intensive and extensive margins. Doing so reveals that the intensive margin is the primary driver of movements in $Z$: while gains on the extensive margin monotonically drift downward, initial increases in search costs actually lead to more efficient matching behavior on the merger market and so larger intensive margin gains. Cost increases then initially reduce incentives for entry and so the mass of firms $M$ both directly by reducing firms’ option value from merger participation and indirectly by actually increasing the productivity gains from merger, driving down the aggregate price level and the level of firm profitability. As search costs continue to rise, however, the merger market becomes less efficient, generating lower productivity gains and so raising prices and profitability. This latter effect eventually dominates the direct reductions in firm values, inducing more new entry and causing the mass of firms to rise.

![Panel A. TFP and Determinants](image1)

![Panel B. Consumption and Determinants](image2)

Figure 9: Increasing the Costs of Search

Although $TFP$ falls monotonically as search costs rise, the level of consumption does not. Panel B of Figure 9 shows that consumption initially increases and then decreases in the costs of search. Again, the key tension comes from balancing the fall in output from lower $TFP$ with a reduction in new firm creation due to reduced prospects on the merger market and the resulting reallocation of output towards consumption. Output declines are thus generally accompanied by increases in consumption’s share. The latter effect is large enough to initially offset the losses in output, leading to net increases in consumption. Eventually, the $TFP$ losses dominate and consumption begins to fall towards its level in the no-merger economy.

That increases in search costs can lead to increases in consumption and so welfare in stationary equilibrium suggests a role for policy to improve on equilibrium outcomes by changing the incentives for firms to engage in merger activity. Intuitively, firms do not take into account the effects of their merger decisions on the innovation incentives of new entrepreneurs, leading to an inefficient amount of new firm creation. An appropriate policy would alter the incentives
of firms on the merger market in order to optimally balance this tradeoff. While Figure 9 shows that a flat tax on all search activity is able to generate a higher level of welfare, given the ex- ante heterogeneity in the economy, it is likely that the optimal policy would involve a system of taxes and subsidies to not only induce the optimal amount of aggregate merger activity, but to actually induce the “right” amount of search and matching behavior across firm types, leading to even greater welfare improvements.42

5.3 Size-Dependent Merger Policies

As a last experiment, I analyze the potential effects of size-dependent policies hindering merger activity. Specifically, I consider policies under which proposed transactions are subject to a probability of being blocked by government action, where these probabilities vary with the size of the deal. Such policies are clearly in the spirit of antitrust enforcement and it will thus be informative to understand their potential impact in i) mechanically preventing efficiency-enhancing transactions; ii) changing firm incentives to search and match on the merger market; and iii) altering the resulting general equilibrium effects on the firm size distribution and particularly on the innovation decisions of new entrepreneurs. These latter two effects have typically been difficult to investigate in a systematic fashion as they require an environment with endogenous merger formation in a general equilibrium setting, a feature unique to my framework.

Merger enforcement activity in the US differs substantially for transactions of size below $1 billion from those above. For example, over the past 10 years, an average of 11.5% of transactions exceeding $1 billion were subject to further investigation by the antitrust authorities through a “second request” for further information, while the corresponding figure for transactions below $1 billion is only about 2.5%.43 In light of this disparity, I implement a stylized size-dependent policy in the model by differentially treating transactions above and below $1 billion in size.44 I consider three alternative policies that differ in the severity with which they treat large transactions. Under all three regimes, transactions less than $1 billion go unreg-

42See Shimer and Smith (2001a) for an example of this type of policy in a related environment. Given their results, it is likely that merger activity here is not efficient, even in the absence of entry considerations.
43Data from Hart-Scott-Rodino Annual Reports available at http://www.ftc.gov/bc/anncomreports.shtml. Transaction size is based on the total value of the acquired assets as estimated by the transacting firms and reported to the investigating authority.
44To follow as closely as possible the definition of transaction size used by the antitrust authorities, I use the pre-merger value of the target firm to capture the existing value of all transferred assets. To map dollar values between the model and the data, I assume the smallest transaction price generated by the benchmark model is $1 million, corresponding to the smallest price observed in the microdata used for the parameterization. I then calculate the size threshold as 1000x this price. To ease the computational burden, I set the threshold on $z$, rather than on $V$, as the latter is endogenous and will change under each policy regime, causing a great deal of difficulty in finding a fixed point in the economy. To do so, I find the $z$ with value corresponding to the size threshold in the benchmark economy and treat this as the threshold target across all policies, i.e., transactions with $z_t$ exceeding this $z$ are subject to challenge and those below are not.
ulated, i.e., are free from any barriers, while transactions exceeding $1 billion are subject in a “light” regime to an 11.5% probability of challenge, matching the empirical rate of second request for transactions in this group, a 50% probability of challenge in a “strict” regime, and an 80% probability of challenge in a “prohibitive” regime.

Table 9 reports changes from the benchmark economy under each of these policies. The top panel reports results from the constant markup setting and the bottom panel from the variable markup setting with average markup equal to 1.5. The results are quite similar across the two settings. Aggregate $TFP$ falls as merger barriers for large firms become more severe, ranging from a 3% decline under the light regime to 16% in the prohibitive regime. Interestingly, even in the constant markup setting, the majority of the fall comes through general equilibrium channels, rather than directly through reduced productivity gains from merger. In all regimes, the greatest single component of the productivity loss comes through a reduction in product innovation, which together with changes on the extensive margin, generally account for upwards of three-quarters of the productivity decline with constant markups, and the entirety of the decline under variable markups. Consumption stays fairly stable under the light regime before declining 2% under the strict regime and 5% under the prohibitive. Consumption losses are again significantly more mild than output losses, as the share of output going towards new entrants falls and consumption’s share rises. Policies targeting large transactions can thus have potentially significant detrimental effects on aggregate outcomes. General equilibrium forces play an important role, suggesting that these effects should be taken into account when considering the implications of these types of policies.

5.4 Constant or Variable Markups

Thus far, we have seen a theory of the merger market and used this structure along with observed merger patterns to make sharp inferences about the gains to firms from merging and their split across the transacting parties, insights completely independent of the particular assumption on market structure. Moreover, the fundamental economic forces at work in the theory are robust to a constant or variable markup setting, both qualitatively in terms of the tradeoffs involved with M&A activity, and quantitatively, as shown by the counterfactual exercises. Before concluding, however, I provide some evidence on which of these settings seems most applicable to aggregate merger activity, mainly to help hone in on the most relevant results, as well to perhaps guide future researchers in this area. To do so, I explore the key distinguishing feature of the two alternatives: the change in markup associated with merger.

Notice from (14) and (15) that common across both settings, the markup of price over marginal cost is equal to the ratio of sales to wage bill, i.e., $\varphi = \frac{r}{w}$, an object easily mea-
Table 9: Size-Dependent Merger Policies

<table>
<thead>
<tr>
<th></th>
<th>Change to Policy Regime (Percent)</th>
<th>‘Light’</th>
<th>‘Strict’</th>
<th>‘Prohibitive’</th>
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<tbody>
<tr>
<td>Constant Markup</td>
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<td></td>
<td></td>
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<tr>
<td>Output/TFP</td>
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<td>−9.3</td>
<td>−15.5</td>
<td></td>
</tr>
<tr>
<td>Product Innovation</td>
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<td>−3.8</td>
<td>−5.6</td>
<td></td>
</tr>
<tr>
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<td>−3.2</td>
<td>−5.6</td>
<td></td>
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<tr>
<td>Extensive Margin</td>
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<td>−2.3</td>
<td>−4.3</td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
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<td>−2.2</td>
<td>−5.5</td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>−2.6</td>
<td>−9.3</td>
<td>−15.5</td>
<td></td>
</tr>
<tr>
<td>Consumption Share of Output</td>
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<td>+7.0</td>
<td>+10.0</td>
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</tr>
<tr>
<td>Variable Markup</td>
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<td></td>
<td></td>
<td></td>
</tr>
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<td>Output/TFP</td>
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<td>−9.9</td>
<td>−15.9</td>
<td></td>
</tr>
<tr>
<td>Product Innovation</td>
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<td>−9.9</td>
<td>−15.9</td>
<td></td>
</tr>
<tr>
<td>Intensive Margin</td>
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<td>*</td>
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<tr>
<td>Extensive Margin</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>+0.5</td>
<td>−2.0</td>
<td>−4.9</td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>−2.9</td>
<td>−9.9</td>
<td>−15.9</td>
<td></td>
</tr>
<tr>
<td>Consumption Share of Output</td>
<td>+3.3</td>
<td>+7.9</td>
<td>+11.0</td>
<td></td>
</tr>
<tr>
<td>Average Markup</td>
<td>−1.6</td>
<td>−2.8</td>
<td>−3.7</td>
<td></td>
</tr>
</tbody>
</table>

Surely from the data. Constant markups imply that \( \varphi \) remain unchanged upon merger, as productivity gains are associated with both larger revenues and employment, leaving the ratio constant, whereas a variable markup setting implies that \( \varphi \) should increase upon transaction, as productivity gains translate directly into higher markups.45

For a small set of firms, Compustat directly reports the total wage bill \( w_l \). I then directly construct the markup \( \varphi \) for each firm as well as its value for that firm 1, 2, and 3 years forward. I calculate the log change in markups for each firm over each of these time horizons and compare the changes for merging firms relative to non-merging firms. To control for industry-time effects, I calculate the mean change for non-merging firms for each industry and year and deviate the change for each merging firm from this benchmark, that is, I consider the change in markups for merging firms relative to the mean of non-merging firms in the same industry-year cell. I report the results in the first panel of Table 10. Strikingly, the data show no significant increase in markups following merger. Both the mean and median change in markup for merging firms are

45It is worth noting that this measure of the markup is more general than just the setting here. For example, it holds under any market structure, including Bertrand or Cournot oligopoly models, where production is CRS in labor, that is, when marginal costs are constant. Even more generally, differences between firms in the log of \( w_l \), the object I examine below, are equal to differences in the log of markups no matter market structure, so long as production is iso-elastic in labor, for example, even if production is DRS.
almost identical to their industry peers who do not participate in a merger, suggesting that the constant markup setting may be most apt for analyzing the aggregate impact of M&A activity. I would emphasize that these results certainly do not imply that greater markups are not cause for concern in particular transactions and in particular industries, but rather that in the space of aggregate activity, the focus of the analysis here, the ability to raise markups does not seem to be the primary driver of merger activity.

Table 10: Relative Markup Changes for Merging Firms

<table>
<thead>
<tr>
<th></th>
<th>Sales over Wage Bill</th>
<th></th>
<th>Sales per Worker</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t + 1</td>
<td>t + 2</td>
<td>t + 3</td>
</tr>
<tr>
<td>Mean</td>
<td>0.001</td>
<td>0.004</td>
<td>-0.004</td>
</tr>
<tr>
<td>Median</td>
<td>0.001</td>
<td>0.002</td>
<td>0.000</td>
</tr>
<tr>
<td># of obs. (merging firms)</td>
<td>3,925</td>
<td>3,555</td>
<td>3,159</td>
</tr>
<tr>
<td># of obs. (non-merging firms)</td>
<td>27,053</td>
<td>23,308</td>
<td>20,133</td>
</tr>
</tbody>
</table>

Compustat reports employment $l$ for a much larger set of firms, and given that I am deviating merging firm changes from average changes, the assumption of a common wage implies that the wage cancels from the equation and we can infer differential movements in markups using only relative changes in sales per worker for merging and non-merging firms (again, at an industry-year level). I report these results in the second panel of Table 10, noting the large increase in the number of observations. Again, the data show no compelling evidence of significant increases in markups following a merger. Year $t + 1$ actually shows a decline in the mean relative markup, followed by slightly higher levels in the following 2 years, although the economic magnitudes are quite small. The median change in markups for merging firms relative to non-merging firms is actually slightly negative in all years, again suggesting that the majority of merging firms do not garner a relative increase in their markup.

6 Conclusion

This paper documents a number of empirical patterns in US M&A and develops a search and matching model that exploits these findings by linking merger gains and merger patterns. I illustrate this connection in a series of analytic examples and embed the merger market in a general equilibrium setting to quantitatively infer the gains from merger and their split across the transacting firms. The results suggest that merger activity can generate sizable gains to the transacting firms and that the gains are split with reasonable equity. A series of counterfactual experiments highlight the potentially significant beneficial impact of mergers on aggregate performance, the quantitative importance of general equilibrium channels in generating these
effects, and a key tradeoff between the gains in aggregate productivity generated by M&A and a corresponding reduction in consumption’s share of output as more resources are utilized in ultimately unsuccessful attempted entry. Importantly, these results hold in both a constant and variable markup setting, confirming that the key economic forces highlighted here are not reliant on a particular market structure, but rather stem mainly from the features of the merger market itself, the primary focus of my analysis.

The theory I construct is among the first to explore the implications of M&A in a general equilibrium setting. The framework features sufficient richness to capture the empirical patterns in observed M&A, while remaining tractable enough to explicitly analyze the interactions of merger activity with the aggregate economy, and particularly with its dynamic behavior through firm entry and exit, features that give new insight into the economic causes and consequences of M&A. Areas for future work include, first, providing explicit microfoundations for the merger technology determining the performance of a post-merger firm. Although I successfully discipline the shape of the technology in the sense of closely matching the empirical patterns in M&A, I do not model the underlying mechanisms through which firms actually combine assets that generate a technology of this form. A better understanding of this process would make clear exactly the economic forces leading to the observed technology and so the patterns of matching observed in the data.

The model focuses on the merger process and the interactions with the aggregate economy and so takes a fairly stylized view of the productivity evolution and growth process of firms. In reality, firms can grow through many channels, including random productivity improvements, those generated by internal R&D, through imitation of rivals, etc. While the approach here is meant to highlight some particular features of the merger process, it would be a useful exercise, and mathematically feasible within the structure of the framework I lay out, to investigate the interactions of M&A with these alternative growth channels and the resulting impact on the quantitative gains that I compute. For example, to the extent that M&A and R&D are substitute growth options, the results here may overstate the gains from M&A. On the other hand, Phillips and Zhdanov (2012), for example, find that an active M&A market encourages R&D and innovation, suggesting that there may be some complementarities between the two, similar to the interaction between product innovation and M&A found here. It is certainly worth future work to delve deeper into these interactions and their sources.

While the model highlights the long run benefits of M&A, the literature has long been interested in its higher frequency behavior in the form of merger waves and cyclical properties. Examples of the former include Jovanovic and Rousseau (2008) and Harford (2005) and of the latter, Eisfeldt and Rampini (2006). Moreover, M&A can play a major role in the life-cycle of an industry, in particular, the often-seen evolution of an industry from one with many firms and
dispersed production to one where production is concentrated in the hands of a few large firms. Search environments of the form here have rich dynamic implications, as shown for example, by Lu and McAfee (1996) and Shimer and Smith (2001b). Using this framework to analyze the short-run dynamic behavior of M&A holds some promise in generating new insights into merger waves and the cyclical behavior of merger activity, as well as into the role of M&A in the industry life-cycle and indeed, in specific industry experiences.

Finally, exploring in greater depth the role for policy to improve on performance in the economy studied here would be informative. Shimer and Smith (2001a) point out that search and matching behavior is generally inefficient in this type of environment due to standard search externalities. The precise nature of the externalities here, where repeat matching is feasible, agents additionally interact in output markets, and entry is costly and in part determined by matching behavior, are not obvious. The quantitative results illustrate that certain policies are able to improve long-run outcomes, but does not indicate the set of policies that replicates the social optimum, an analysis that would prove fruitful in further uncovering the link between outcomes on the merger market and the response of new entrepreneur entry. In a positive sense, the model provides a lens through which to further explore the effects of empirical policies that may hasten or hinder the merger process, for example through the functioning of financial markets or costly government regulation.

Appendix

A Data

In this Appendix, I describe in more detail the data used in the paper, beginning with SDC. I select from SDC all domestic transactions announced between 1977 and 2009 with a nominal deal value of at least $1 million. I include only completed transactions; those not classified as hostile (only about 300 transactions are classified as hostile takeovers); those in which the acquirer newly gains majority control of the target, i.e., the acquirer must own less than 50% of the target prior to the merger and over 50% after; and those with relevant ownership status (excluding, for example, government-owned entities). After this process, and eliminating several observations with obvious data entry errors, there are 57,858 transactions. For each transaction, I obtain the following data: transaction value (total value of consideration paid by the acquirer, excluding fees and expenses), premium (premium of offer price to target closing stock price 4 weeks prior to the original announcement date), and pre-merger performance variables including net sales, employment, PP&E, EBITDA, and market value. Data availability differs across the
SDC variables. In Table 11, I show the number of transactions with available data for acquirers, targets, and both, for each dimension of analysis.

<table>
<thead>
<tr>
<th></th>
<th>Acquirer</th>
<th>Target</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>31,736</td>
<td>18,541</td>
<td>12,251</td>
</tr>
<tr>
<td>Employment</td>
<td>28,050</td>
<td>6,138</td>
<td>3,957</td>
</tr>
<tr>
<td>PP&amp;E</td>
<td>28,792</td>
<td>10,095</td>
<td>6,672</td>
</tr>
<tr>
<td>EBITDA</td>
<td>26,424</td>
<td>8,208</td>
<td>5,080</td>
</tr>
<tr>
<td>Market Value</td>
<td>25,38</td>
<td>6,969</td>
<td>4,112</td>
</tr>
<tr>
<td>Premium</td>
<td>*</td>
<td>*</td>
<td>6,474</td>
</tr>
</tbody>
</table>

Turning to Compustat, I obtain data on the universe of firms contained in the CRSP/Compustat merged database (CCM) from 1977 to 2009. This yields a total of 210,275 observations. The SDC to Compustat match is not straightforward since the two databases use different company identifiers. The most specific identifier provided by SDC is the 6-digit CUSIP for both parties in each transaction. This is not sufficient for the match, however, because Compustat only records the most recent CUSIP rather than a CUSIP history. Because of this, matching on CUSIP may result in missed pairs and erroneous matches.

To perform the match, I use the CRSP translator to associate 6-digit CUSIPs from SDC with the CRSP company identifier. I then match this identifier with the CCM database, which already associates the CRSP identifier with the set of Compustat firms. I follow this process for both acquirers and targets. I associate transactions with the Compustat data for the fiscal year preceding the year of merger announcement. Not surprisingly, the set of successful matches corresponds quite closely to the set of firms classified as public in SDC. I obtain data on net sales, employees, PP&E (net of depreciation), EBITDA, and market value, where I calculate the latter as the product of common shares outstanding and the closing price at fiscal year end. Table 12 shows availability of the Compustat data.

<table>
<thead>
<tr>
<th></th>
<th>All Firms</th>
<th>Acquirers</th>
<th>Targets</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>191,992</td>
<td>30,453</td>
<td>6,828</td>
<td>4,465</td>
</tr>
<tr>
<td>Employment</td>
<td>179,787</td>
<td>28,597</td>
<td>6,186</td>
<td>3,862</td>
</tr>
<tr>
<td>PP&amp;E</td>
<td>187,270</td>
<td>28,368</td>
<td>6,608</td>
<td>4,284</td>
</tr>
<tr>
<td>EBITDA</td>
<td>152,671</td>
<td>25,472</td>
<td>5,525</td>
<td>3,478</td>
</tr>
<tr>
<td>Market Value</td>
<td>206,309</td>
<td>30,890</td>
<td>6,913</td>
<td>4,548</td>
</tr>
</tbody>
</table>

Macroeconomic data are obtained from standard sources. US GDP and stock of fixed assets...
are from the Bureau of Economic Analysis (http://www.bea.gov/). The CPI is from the Bureau of Labor Statistics (http://www.bls.gov/).

B  Empirical Patterns in US M&A - Sales

As an example of the robustness of Figures 1 and 2, I display here the analogous pictures using firm sales. Figure 10 shows the joint distribution over the sales of acquirers and targets and Figure 11 the marginal distributions. An examination shows that they resemble quite closely those in the text and that the key implications for the analysis are the same.

Figure 10: Joint Distribution of Acquirers and Targets - Sales

Figure 11: Marginal Distributions of Transacting Firms - Sales

C  Proofs

Proposition 1. Conjecture that the gains from merger are constant and strictly positive across all firm types, i.e., \( \Sigma (z_a, z_t) = \bar{\Sigma} > 0 \ \forall z_a, z_t \). Then all meetings will result in a completed
transaction and we can write the value function as

\[ rV(z) = \pi(z) - PC_\lambda(\lambda(z)) - PC_\mu(\mu(z)) + \lambda(z) \theta_a \beta \Sigma + \mu(z) \theta_t (1 - \beta) \Sigma \]

The first order conditions governing optimal search in (10) give

\[ PC'_\lambda(\lambda(z)) = \theta_a \beta \Sigma, \quad PC'_\mu(\mu(z)) = \theta_t (1 - \beta) \Sigma \]

which shows that the choice of \( \lambda \) and \( \mu \) are common across firms and independent of \( z \). Denoting these common search intensities as \( \bar{\lambda} \) and \( \bar{\mu} \), we can rewrite the value function as

\[ rV(z) = \pi(z) - PC_\lambda(\bar{\lambda}) - PC_\mu(\bar{\mu}) + \bar{\lambda} \theta_a \beta \Sigma + \bar{\mu} \theta_t (1 - \beta) \Sigma \quad (32) \]

Recall that the gains from merger between a type \( z_a \) acquirer and type \( z_t \) target are

\[ \Sigma(z_a, z_t) = V(z_m) - V(z_a) - V(z_t). \]

Using (32), it is straightforward to show that merger gains equal

\[ \frac{\pi(z_m) - \pi(z_a) - \pi(z_t)}{r} - \bar{\lambda} \theta_a \beta \Sigma - \bar{\mu} \theta_t (1 - \beta) \Sigma + PC_\lambda(\bar{\lambda}) + PC_\mu(\bar{\mu}) \]

Under the assumption that the merger technology displays no gains from bundling, we have

\[ \pi(z_m) - \pi(z_a) - \pi(z_t) = P_{cf} \]

that is, the only gain in flow profits from merging is a single fixed cost savings. Then,

\[ \Sigma(z_a, z_t) = \frac{P_{cf} - \bar{\lambda} \theta_a \beta \Sigma - \bar{\mu} \theta_t (1 - \beta) \Sigma + PC_\lambda(\bar{\lambda}) + PC_\mu(\bar{\mu})}{r} = \bar{\Sigma} > 0 \]

Thus, we have proved our initial conjecture that the gains from merger are positive and constant across all firm types such that every meeting will result in merger.

Because each firm searches with the same intensities, and the effective meeting rates on the two sides of the market must equate, each firm has an equal probability of meeting a particular partner as an acquirer or a target. That is, the rate at which acquirer \( z_1 \) meets target \( z_2 \) equals the rate at which acquirer \( z_2 \) meets target \( z_1 \). That meetings are random and all result in a completed transaction, that all firms choose the same search intensities, and that each transaction is reflected by the opposite transaction with the roles reversed in equal weight together imply that (i) the mean and median differences between acquirers and targets are zero, (ii) the correlations between acquirers and targets are zero, and (iii) the median acquirer and median target are the same as the median firm.

\[ \square \]
Before moving on to proposition 2, a few additional notes are in order. In particular, let us gain some intuition by parameterizing the search cost functions as in (25), and assuming, for example, that $\int \lambda(z) \, dG(z) > \int \mu(z) \, dG(z)$, i.e., $\theta_a < 1, \theta_t = 1$. In this case, we can derive

$$
\bar{\lambda} = \left[ \left( \frac{1 - \beta}{C} \right)^{\frac{1}{\eta}} \left( \frac{\beta}{B} \right)^{ \frac{n-1}{\eta} } \bar{\Sigma} \right]^{\frac{1}{n-1}}
$$

$$
\bar{\mu} = \left[ \frac{1 - \beta}{PC} \bar{\Sigma} \right]^{\frac{1}{n-1}}
$$

$$
\theta_a = \left[ \frac{1 - \beta B}{\beta C} \right]^{\frac{1}{\eta}}
$$

that is, the ratio of search on the target and acquirer sides of the market is independent of merger gains and depends only on the ratio of bargaining shares and real search costs on each side. Because targets are on the short side of the market, their search intensity only depends on their expected gains and costs of search. Because acquirers are on the long side of the market, their search intensity is increasing in their expected gains, decreasing in their costs of search, but is increasing (although at a slower rate) in the expected gains and decreasing in the costs of search for targets. This is because as these latter increase, targets will search more intensively, which increases the effective meeting rate for acquirers. From here, we can solve for the merger gains up to a single nonlinear equation in $\bar{\Sigma}$:

$$
\frac{n - 1}{n} \left( \frac{1 - \beta}{PC} \right)^{\frac{1}{n-1}} \bar{\Sigma}^{\frac{n}{n-1}} + r\bar{\Sigma} - Pc_f = 0
$$

If we had alternatively assumed that $\int \lambda(z) \, dG(z) < \int \mu(z) \, dG(z)$, we would have obtained analogous expressions. Notice that if $c_f = 0$, there is no solution such that $\bar{\Sigma} > 0$ and there will be no mergers. If the merger technology displays no gains from bundling and there is no fixed cost of production, there are no gains to merging, and no firms will expend any resources to do so.

**Proposition 2.** The symmetry of the merger technology along with the definition of the combined gains in (3) immediately imply that $\Sigma(z_1, z_2) = \Sigma(z_2, z_1)$ and that matching sets are symmetric around the $45^\circ$ line. Conjecture that $\lambda(z) = K\mu(z), K > 1$, that is, for each firm, search intensity on the acquiring side of the market is some constant proportion of search intensity on the the target side. Then, $\int \lambda(z) \, dG(z) = K \int \mu(z) \, dG(z)$, that is, the aggregate search intensity of acquirers is the same proportion of the aggregate search intensity of targets. This implies $\theta_a < 1$ and $\theta_t = 1$. Expected gains for a type $z$ target conditional on meeting a
prospective buyer are

\[ E[\Sigma_t(z_a, z)] = (1 - \beta) \int \max \{\Sigma(z_a, z), 0\} \Lambda(z_a) \]

\[ = (1 - \beta) \int \max \{\Sigma(z_a, z), 0\} \frac{\lambda(z_a) dG(z_a)}{\int \lambda(z) dG(z)} \]

\[ = (1 - \beta) \int \max \{\Sigma(z, z_a), 0\} \frac{K \mu(z_a) dG(z_a)}{K \int \mu(z) dG(z)} \]

\[ = (1 - \beta) \int \max \{\Sigma(z, z_a), 0\} \frac{\mu(z_a) dG(z_a)}{\int \mu(z) dG(z)} \]

\[ = (1 - \beta) \int \max \{\Sigma(z, z_a), 0\} \Gamma(z_a) \]

\[ = \frac{1 - \beta}{\beta} E[\Sigma_a(z, z_a)] \]

That is, the expected gains conditional on meeting a prospective buyer are a constant multiple of the expected gains conditional on meeting a prospective target, and simply depends on the ratio of bargaining powers. Note that in the third line, I have used the symmetry assumption on the technology, as well as the initial conjecture that search intensities are in constant proportion. From the first order conditions governing optimal search (10), we can see that if expected gains are in constant proportion, than search intensities are as well, verifying our initial conjecture. Similar reasoning holds for the cases of \( K < 1 \) and \( K = 1 \).

From (8), the rates at which a type \( z_1 \) acquirer meets a type \( z_2 \) target, and in reverse, a type \( z_1 \) target meets a type \( z_2 \) acquirer are equal to

\[ \lambda(z_1) \theta_a \frac{\mu(z_2) dG(z_2)}{\int \mu(z) dG(z)}, \quad \mu(z_1) \theta_t \frac{\lambda(z_2) dG(z_2)}{\int \lambda(z) dG(z)} \]

Substituting \( \lambda(z) = K \mu(z) \) in both expressions, we obtain

\[ K \mu(z_1) \theta_a \frac{\mu(z_2) dG(z_2)}{\int \mu(z) dG(z)}, \quad \mu(z_1) \theta_t \frac{\mu(z_2) dG(z_2)}{\int \mu(z) dG(z)} \]

which are equivalent since \( \theta_t = 1 \) and \( K \theta_a = 1 \). Thus, each transaction is reflected in equal weight by its counterpoint transaction with the roles reversed. It is then immediate that the mean and median differences between acquirers and targets are zero.

Proposition 3. Assume that merger gains \( \Sigma(z_a, z_t) \) are increasing in the difference between the acquirer and target \( z_a - z_t \). Then for a given \( z_a \), gains are decreasing in \( z_t \), and so the set of acceptable targets \( \Upsilon_a(z_a) \) is characterized by an upper threshold \( z_t^* \) such that \( \Sigma(z_a, z_t^*) = 0 \). That is, acquirer \( z_a \) will be willing to purchase any targets with \( z \leq z_t^* \). It is straightforward to
establish an analogous result for targets, that is, \( \Upsilon_t(z_t) \) is characterized by a lower threshold \( z^*_a \) such that target \( z_t \) will sell itself to any acquirer with \( z \geq z^*_a \). Together these imply low \( z \) targets and high \( z \) acquirers are in a greater share of matching sets. That gains are decreasing in \( z_t \) and increasing in \( z_a \) implies that expected gains conditional on meeting a candidate purchaser or target are also respectively decreasing and increasing in \( z \). From the first order conditions governing optimal search (10), we see that search intensities \( \mu(z) \) and \( \lambda(z) \) must be decreasing and increasing in \( z \) respectively, that is, low \( z \) targets and high \( z \) acquirers search most intensively for partners. The fact that low \( z \) targets and high \( z \) acquirers are in a greater share of matching sets and search most intensively together imply that the rate at which firms are acquired \( \mu(z) \theta_t \int \Phi(\Sigma_t(z_a, z)) \Lambda(z_a) \) is decreasing in \( z \) and similarly the rate at which they make acquisitions \( \lambda(z) \theta_a \int \Phi(\Sigma_a(z, z_t)) \Gamma(z_t) \) is increasing in \( z \). It is then immediate that (i) low \( z \) firms are overrepresented in the set of targets and high \( z \) firms in the set of acquirers and that (ii) the median target is below the median firm and the median acquirer above. Finally, the greatest rate of meeting take place between the highest \( z \) acquirer and the lowest \( z \) target, which is an acceptable match, giving that (iii) the highest rate of transaction occurs between low \( z \) targets and high \( z \) acquirers.

\[ \square \]

**D Span of Control**

In this Appendix, I outline an additional alternative market structure accommodated by the model: one with a single homogenous good, perfect competition, and diminishing returns (increasing marginal costs) in production, as in the Lucas (1978) span of control model.

There is a single homogenous good produced using labor. The production function exhibits diminishing returns to labor and takes the form \( q = z^{1-\zeta} \) where the normalization of \( z \) plays a similar role as in the text. Competitive firms take the market price \( P \) as given and choose labor to maximize profits, \( Pz^{1-\zeta} - l \) where I have normalized the wage \( w = 1 \) to be numeraire. Standard arguments give revenue, employment, and variable profits as

\[ r(z) = \frac{1}{1-\zeta} \Pi z, \quad l(z) = \frac{\zeta}{1-\zeta} \Pi z, \quad \pi(z) = \Pi z \]

where \( \Pi = (1-\zeta) P^{\frac{1}{1-\zeta}} \) and so as above, firm product market outcomes are proportional to \( z \) and depend on parameters and industry aggregates that are common across all firms, here the span of control parameter \( \zeta \) and the competitive price of output \( P \).

The outcomes here are thus clearly isomorphic to those described in the text. Mergers enable firms to improve their productivity \( z \). The merger technology combines the \( z \)'s of the acquirer and target and produces a firm with a new \( z \). Because there is only a single homogenous
good, the firm’s $z$ represents only its physical productivity, eliminating the notion of a product portfolio held by the firm. All the dynamic equations of the model are then as in the text.

E An Alternative Estimation Approach

In this Appendix, I present results from an alternative estimation to that in the text in which I rely on firm sales to identify firm type. The key finding here is that the parameter estimates and resulting fit of the model are robust to this alternative approach, although the reader should keep in mind that the use of sales as an identifying characteristic of a firm’s type $z$ is only valid in a constant markup setting; no such alternative is available under variable markups.

The target moments for the estimation here are analogous to those in Table 6, I simply replace each moment in profits with the equivalent moment in sales. These include the median deviation of the log of acquirer and target sales from the median in their industries, 0.58 and 0.00, respectively, the percent of targets that fall in the lowest decile of the firm size distribution measured by sales, 6.9%, and lastly the coefficient of variation in target sales, 3.96. The remaining moments remain the same, although the parameter estimates may of course change, as they are pinned down jointly. I report the results in Table 13 (suppressing standard errors) and simply note the closeness of the parameter estimates to those in Table 6.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Target Moment</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.91</td>
<td>Median deviation of log acquirer sales</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.53</td>
<td>Median deviation of log target sales</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>$A$</td>
<td>1.05</td>
<td>Percent of targets in lowest decile</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.43</td>
<td>Mean merger premium</td>
<td>0.53</td>
<td>0.53</td>
</tr>
<tr>
<td>$\eta$</td>
<td>13.28</td>
<td>Coefficient of variation of target sales</td>
<td>3.96</td>
<td>3.96</td>
</tr>
<tr>
<td>$B$</td>
<td>2.75</td>
<td>Acquisition rate ($x 10^{10}$)</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>$C$</td>
<td>2.58</td>
<td>Bidders per target ($x 10^{11}$)</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Next, I examine the same non-targeted moments over the joint and marginal distributions of transacting firms as in the text, i.e., the results in Table 7 and Figure 8. I replicate each of these using the parameter estimates in Table 13. Table 14 reports the correlations and size differences between transacting firms, and Figure 12 displays their marginal distributions. As in the text, the estimated model performs quite well in predicting the positive sorting observed in the data as well as the prevalence of size and profitability differences between acquirers and targets. Additionally, the model continues to capture quite closely the marginal distributions of transacting firms, displaying the monotonically increasing pattern of acquirers over the firm.

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size distribution, and almost exactly replicating the marginal distribution over target sales.

Table 14: Matching Patterns in M&A: Model and Data - Sales

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of profits</td>
<td>0.58</td>
<td>0.53</td>
</tr>
<tr>
<td>Log of sales</td>
<td>0.58</td>
<td>0.52</td>
</tr>
<tr>
<td>Log of employment</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>Log of market value</td>
<td>0.71</td>
<td>0.54</td>
</tr>
<tr>
<td>Share of transactions with ( z_a &gt; z_t )</td>
<td>0.86</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Figure 12: Marginal Sales Distributions of Transacting Firms: Data (top) vs Model (bottom)

F Computational Algorithms

In this Appendix, I describe the computational algorithms used to compute and estimate the model, to compute equilibria in the counterfactual economies, and the procedure used to back out the economic aggregates from the computed equilibria.

**Estimation.** To estimate the model, I use a simulated method of moments estimator with a minimum distance criterion to find the parameter values. There are seven parameters to
pin down in this way, which I collect in the vector \( \Theta = \{ \gamma, \nu, A, \beta, \eta, B, C \} \). Formally, the parameter vector \( \Theta^* \) is chosen to solve

\[
\Theta^* = \arg \min \left( \Psi^s(\Theta) - \Psi^d \right) I (\Psi^s(\Theta) - \Psi^d)^\prime
\]

where \( I \) is the identity matrix, \( \Psi^s(\Theta) \) the simulated moments for a candidate value of \( \Theta \) and \( \Psi^d \) the empirical moments.

I outline the computational algorithm in Table 15. It is worth pointing out three particularly noteworthy features: first, to see that computation of the model and hence estimation is independent of market structure, notice that the firm’s merger market and entry and exit decisions depend only on \( D = \frac{\Pi}{Pc} \), which is common across market structures, and not on any of these objects individually. For intuition here, notice that renormalizing the free entry condition (17) in this way gives a constant value of 1 on the RHS, no matter the assumed market structure. The model solution and estimated parameters thus do not depend on the value of \( P \) separately from \( \frac{\Pi}{Pc} \) and so for computation purposes, it suffices to know the latter and normalize the value of the former to 1, although we will back out the true value of \( P \) when imposing a specific market structure below. Second, it proves convenient to iterate over a candidate aggregate search intensity \( \mu^c = \int \mu(z) dG(z) \) and impose the target value of \( \theta \), rather than loop directly over the cost parameters \( B \) and \( C \). Given values for these endogenous objects, it is straightforward to rearrange the first order conditions governing optimal search in (10) to infer the corresponding values for \( B \) and \( C \). Third, rather than iterate on the entry distribution \( dF(z) \), I directly impose the target distribution \( dG(z) \). This entails directly constructing both the density at each \( z \) as well as the entry threshold \( \hat{z} \). I then use the stationary conditions in (19) and the entry threshold condition (18) to infer the exogenous entry distribution \( dF(z) \) and the level of the fixed cost \( c_f \) that in equilibrium give rise to the target \( dG(z) \) and \( \hat{z} \). Given these features, the estimation and model solution takes the recursive structure outlined in Table 15.

I now describe the mechanics of the numerical algorithm in more detail. I discretize the productivity distribution over \( z \) into 500 points from a \( z \) of 1, which corresponds to the normalization of \( \hat{z} \) described above, up to a \( z \) of 10,000. Recalling from (14) that the ratio of the size of two firms is equal to the ratio of their \( z \)'s, I follow Restuccia and Rogerson (2008) in constructing a grid such that the largest operating firm will be 10,000 times the size of the smallest, and additionally in log-spacing the grid to ensure greater accuracy over the lower tail of the distribution, where most firms reside. I then construct the endogenous distribution \( dG(z) \) over this grid such that \( dG(z) \) takes on a Pareto with shape parameter \( \xi \). Next, I guess a candidate value of \( \Theta^c = \{ \gamma, \nu, A, \beta, \eta, \mu^c, \theta \} \). With the candidate values of \( A, \gamma, \) and \( \nu \), I can construct a “merger matrix” which represents the \( z_m \) resulting from each combination of
Table 15: Estimation Algorithm

1. Construct \( z, dG(z) \) and set direct parameters.
2. Guess candidate vector \( \Theta^c = \{\gamma, \nu, A, \beta, \eta, \mu^c, \theta\} \).
3. Construct merger matrix \( s(z, z') \).
4. Guess candidate \( D = \frac{P}{Pc} \) and normalize \( P = 1 \). Compute \( \pi(z) \).
5. Guess candidate \( V(z) \). Evaluate merger matrix.
6. Solve for \( B, \lambda(z), C, \) and new \( \mu(z) \). Iterate on \( \mu(z) \).
7. Guess candidate \( \mu(z) \) such that \( \int \mu(z) dG(z) = \mu^c \). Evaluate merger matrix.
8. Solve for \( c_f \) s.t. \( V(\hat{z}) = 0 \) and construct new \( V(z) \). Iterate on \( V(z) \).
9. Construct \( dF(z) \) using (19) and check FE condition. Update \( D \) until FE satisfied.
10. Simulate data and construct target moments.
11. Compute objective function in (33) and iterate on \( \Theta^c \) until minimized.

\( z_a \) and \( z_t \), where the two pre-merger firms are drawn from the entire set of \( z \)'s. That is, the merger matrix contains the productivity of a merged entity formed by the merger of all possible combinations of \( z \)'s.

Computation of the equilibrium begins by guessing the industry aggregate \( D = \frac{P}{Pc} \) and normalizing \( P = 1 \). I perform value function iteration to find \( V(z), \lambda(z), \mu(z), \Upsilon_a(z), \Upsilon_t(z) \).

For a candidate \( V(z) \), I use the merger matrix to compute the value of each potential transaction on the merger market and in particular, to find those generating positive gains. I then use an iterative procedure to construct optimal search intensities, by which I guess a candidate vector \( \mu(z) \), solve for \( \lambda(z) \) and recompute \( \mu(z) \). Recall that \( \Theta^c \) contains a candidate \( \mu^c = \int \mu(z) dG(z) \), from which, in conjunction with the value of \( \theta \), it is straightforward to compute aggregate search on the opposing side of the market \( \int \lambda(z) dG(z) \). Given a feasible vector \( \mu(z) \), a straightforward manipulation of the first order condition (10) along with the parameterization (25) gives an expression for \( B \) that is independent of the individual values of \( \lambda(z) \):

\[
B = \left( \frac{1}{\int \lambda(z) dG(z)} \right)^{\eta-1} \left( \int \left\{ E[\Sigma_a(z, z_t)] \right\}^{\frac{1}{\eta-1}} dG(z) \right)^{\eta-1}
\]

where the expected gains to making an acquisition \( E[\Sigma_a(z, z_t)] \) depends on objects that are known (for this candidate parameter vector). With this value of \( B \), I can construct \( \lambda(z) \). An analogous procedure gives \( C \). Finally, I compute a new value of \( \mu(z) \) as a function of \( \lambda(z) \) and the inferred values of \( B \) and \( C \). I iterate on this process until \( \mu(z) \) converges.

It is now straightforward to construct new values of \( V(z) \) in accordance with (9). In doing so, I compute the fixed cost \( c_f \) that is consistent with this equilibrium by solving \( V(\hat{z}) = 0 \). Next, I use the firm search and matching decisions to construct the flows in (19) and in conjunction with the distribution \( dG(z) \), I infer the entry distribution \( dF(z) \). Here, I must make a normalization
of the minimum possible draw of \( z \), \( z_{\text{min}} \), which I set to 0.3. Figure 13 displays the CDFs of the endogenous distribution \( dG(z) \) and the estimated entry distribution \( dF(z) \). Finally, I use \( V(z) \) and \( dF(z) \) to construct the free entry condition (17) and iterate on the candidate value of \( D \) until the free entry condition is satisfied.

![Figure 13: Entry and Endogenous Productivity Distributions](image)

To simulate the economy, I draw 1 million firms from the stationary distribution \( dG(z) \). Standard arguments show that each acquirer has a probability of meeting a target in a single period equal to \( 1 - e^{-\lambda(z)} \). Using these probabilities, I calculate the set of potential acquirers and match them to a set of potential targets who are drawn randomly according to their meeting probabilities \( 1 - e^{-\mu(z)} \). Elimination of matches that generate negative gains gives a simulated merger dataset with matched acquirers and targets analogous to the actual data described above. It is then straightforward to calculate the target moments and compute the value of the objective function in (33). I iterate on the guess of \( \Theta^c \) until this function is minimized.

**Equilibria in counterfactual economies.** Computation of the equilibrium in the counterfactual economies follows a fairly similar process to that just described. I again begin by guessing the aggregate level of profits \( D \) and normalizing \( P = 1 \). I then guess a candidate threshold entrant \( \tilde{z} \), value function \( V(z) \) for \( z \geq \tilde{z} \), productivity distribution \( dG(z) \), and target search intensities \( \mu(z) \). For this guess of \( V(z) \), I evaluate the merger matrix and compute the gains from all potential transactions. Together, the guess of \( \mu(z) \), \( dG(z) \), and the expected gains from merger imply values of acquirer search intensities \( \lambda(z) \) using (10). In turn, \( \lambda(z) \) imply new values of \( \mu(z) \). I iterate on the guess of \( \mu(z) \) until convergence, delivering the equilibrium values of \( \mu(z) \) and \( \lambda(z) \) for this particular candidate \( V(z) \) and \( dG(z) \). Next, I use the stationary conditions in (19) to construct the flows into and out of each firm type, from which we can compute a new distribution \( dG(z) \). I iterate on \( dG(z) \) until convergence, yielding the counterfactual productivity distribution. Figure 14 displays the CDFs of the endogenous productivity distribution \( dG(z) \) and an example counterfactual distribution \( dG_{cf}(z) \), where the
latter is calculated using the midpoint of the search costs from the counterfactual exercise in Section 5.2. The solved values of \( \mu(z) \), \( \lambda(z) \), and \( dG(z) \) imply a new value function \( V(z) \) and I iterate on the value function until convergence. I then check the guess of the threshold entrant by asking if a unilateral deviation by the next best firm would be optimal, that is, I compute the value of entry by the next best firm, and if it is positive, I set this firm to \( \hat{z} \) and reperform all the calculations just described. I continue in this way until I have found the true marginal entrant, that is, the firm with positive value from entry where the next best firm would have negative value. Finally, it remains to check the free entry condition (17), and iterate on the initial guess of \( D \) until it is satisfied.

![Figure 14: Endogenous and Example Counterfactual Productivity Distributions](image)

**Aggregate outcomes.** The computed equilibrium in either the estimated or counterfactual economy gives the value of the normalized level of profits \( \frac{\Pi}{P_{cc}} \), the cutoff entrant \( \hat{z} \), the average productivity level \( \bar{Z} \), and the average costs of search \( Y_s \). With these objects in hand, it is straightforward to back out the value of the aggregate variables in the economy. In the constant markup setting, \( \frac{\Pi}{P_{cc}} = \frac{1}{c_{cc}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{1}{\sigma - 1}} R P^{\sigma - 2} \), from which, using the fact that \( R = \frac{\sigma - 1}{\sigma} L \), we can derive \( P \) and so \( Y \) and \( \Pi \). Similarly, in the variable markup setting, \( \frac{\Pi}{P_{cc}} = \frac{1}{c_{cc}} M_{v}^{\frac{\sigma - 1}{\sigma - 2}} \), from which we can solve for \( M_v \) and so \( Y \) and \( R \). In both cases, aggregate variable profits \( \Pi_T \) are calculated as \( R - L \) and average variable profits as \( \Pi \bar{Z} \). Division of the former by the latter gives \( M \), and using this value, along with (20) and the value of \( \hat{z} \), gives \( M_e \). Finally, we can use \( M_e \) and \( M \) along with the value of \( Y_s \) to obtain consumption \( C \).

**References**


