Credit Markets, Limited Commitment, and Government Debt

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Abstract

A dynamic model with credit under limited commitment is constructed, in which limited memory can weaken the effects of punishment for default. This creates an endogenous role for government debt in credit markets. Default can occur in equilibrium, and government debt essentially plays a role as collateral and thus improves borrowers’ incentives. The optimal provision of government debt acts to discourage default, whether default occurs in equilibrium or not.
1 Introduction

Though approaches to explaining the role of money in the economy differ, it is now well-recognized that monetary phenomena have historical importance for financial crises. The pre-Federal Reserve System banking panics in the United States, and the 1933 banking crisis can be viewed as currency shortages brought on my problems in the financial intermediation sector. However, in the recent global financial crisis, the key liquidity shortage was not a scarcity of currency, but of the safe assets that are needed in exchange and as collateral in large-value financial arrangements.

Roughly, the total stock of safe assets is the sum of government debt and safe assets that can be “produced” by private financial intermediaries. A scarcity of safe assets could arise for at least two reasons. First, during the financial crisis, the ability of the private sector to produce safe assets, particularly in the form of asset-backed securities, was impaired. Second, safe assets can sometimes substitute for credit in financial markets, in that large-value financial transactions can be executed either through credit arrangements or with safe assets on one side of the transaction. If credit frictions become more severe, as we could argue happened during the financial crisis, then the demand rises for the safe assets that are useful in executing transactions.

The role of assets in exchange, and the substitution between public and private assets in exchange has been studied by Lagos and Rocheteau (2008), Rocheteau (2011), Lester, Postlewaite, and Wright (2012), and Williamson (2012). As well, Kiyotaki and Moore (2008) and Kocherlakota (2008) study the role of safe-asset “bubbles” arising from the illiquidity of other assets. The purpose of this paper is to explore the role of safe assets in credit markets, and how credit market frictions arising from limited commitment can make government debt useful.

This paper makes three key contributions. First, we show how, when incentive constraints arising from limited commitment constraints bind, government debt can act to relax these constraints, increasing the quantity of exchange and economic welfare. Government debt acts to make defaulting on credit contracts more costly, and this effect acts to relax incentive constraints. One can interpret this as a role for government debt as collateral in credit contracts. In acquiring government debt to secure a credit contract, the potential costs of default are transferred from private lenders to the government. In spite of the fact that we assume in our model that the government is no better at collecting on its tax liabilities than private lenders are at collecting on their debts, this transfer acts to increase trade in equilibrium and increase social welfare.

Second, in our model borrowers may default in equilibrium. Typically, in models with limited commitment, for example Kehoe and Levine (1993), Kocherlakota (1996), or Sanches and Williamson (2010), there is only potential default. In equilibrium in this class of models, credit is typically supported by the threat of off-equilibrium punishments in the event of default, and no defaults occur in equilibrium. This is of course problematic if we want to use these models to address quantitative phenomena. The existence of default, and
regularities in default behavior are features that we would like to explain. In our model, we exploit limited memory/recordkeeping, in order to support default in equilibrium, using ideas from the monetary economics literature (e.g. Kocherlakota 1998). In the model, it is assumed that lenders sometimes do not have access to a would-be borrower’s credit history. Thus, it may be the case that a lender faces an adverse selection problem – he or she cannot tell the difference between a would-be borrower who has defaulted in the past, and one who has not. In equilibrium, among a group of identical borrowers, some default and some do not. Those who default do not have enough to lose from defaulting, as lenders who know their credit history will not lend to them in the future. However, some borrowers have enough to lose from defaulting that they will never do it. Effectively, default behavior is a self-fulfilling phenomenon.

Third, when there is default in equilibrium there is an additional role for government debt in exchange. If would-be borrowers can exchange government debt rather than engage in credit contracts, this can eliminate the adverse selection problem, though it need not eliminate default, as agents can still default on their tax liabilities. This can be interpreted as another role for collateral. Private lenders who have no access to credit histories can require that government debt be posted to secure credit contracts, and this generates more exchange, even if incentive constraints are not binding in the absence of government debt. Government debt can also act to eliminate default by ruling out equilibria with a large number of defaulting borrowers.

A key feature of the model is that Ricardian equivalence does not general hold, in general, since changing the quantity of government debt in existence matters for the equilibrium allocation. However, the extent to which Ricardian equivalence is violated depends on the equilibrium behavior of the economic agents in the model. In an equilibrium where there are global punishments, in that all economic agents are punished should any individual default, Ricardian equivalence holds. In such an equilibrium, changing the quantity of government debt has no consequences. However, in an equilibrium with individual punishments – only a borrower who defaults is punished for his or her bad behavior – government debt matters in general, though for some parameter values Ricardian equivalence will always hold. In particular, if the discount factor is sufficiently high (so that economic agents care sufficiently about what they lose from default – some future access to credit markets) and if access to credit histories by lenders is sufficiently good, then government debt is neutral.

The basic model we work with is closely related to the one in Sanches and Williamson (2010), which in turn builds on Lagos and Wright (2005). The Lagos-Wright structure, which incorporates centralized markets and decentralized trade, makes it convenient to integrate market trade in government debt with a strategic approach in a dynamic setting. A structure for incorporating strategic behavior is useful for handling aspects of the limited commitment problem. Sanches-Williamson focuses exclusively on the interaction between money and credit, and on monetary policy, while our interest in this paper is on the implications of government debt policy for default behavior and exchange.

This model is potentially of interest for explaining behavior that we observe
during financial crises. In particular, there can be an endogenous breakdown in credit relationships in the model, in that default behavior can be a self-fulfilling phenomenon. There can be equilibria in which lenders will not lend to particular borrowers because those borrowers are expected to default, and those borrowers indeed default, because of their limited access to credit markets. The exchange of government debt can mitigate or eliminate this problem.

The paper proceeds as follows. The model is constructed in the second section. In the third section, the properties of equilibria with global punishments are studied, and then symmetric equilibria with individual punishments, and asymmetric equilibria with individual punishments, respectively, are examined in Sections four and five. Finally, Section six concludes.

2 The Model

Time is indexed by \( t = 1, 2, 3, \ldots \), and each period consists of two subperiods, in which trade occurs, respectively, in a centralized market (\( CM \)) and a decentralized market (\( DM \)). There is a continuum of agents with mass 2, half of whom are buyers, with the other half being sellers. Each buyer has preferences given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t)]
\]

where \( H_t \) is labor supply minus consumption during the \( CM \), \( x_t \) is consumption in the \( DM \), and \( 0 < \beta < 1 \). Assume that \( u(\cdot) \) is strictly concave, strictly increasing, and twice continuously differentiable with \( u(0) = 0, u'(0) = \infty, u'(\infty) = 0, \) and \( -\frac{x u''(x)}{u'(x)} < 1 \). A seller has preferences given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t (X_t - h_t),
\]

where \( X_t \) is consumption in the \( CM \), and \( h_t \) is labor supply in the \( DM \). Buyers can produce only in the \( CM \), and sellers produce only in the \( DM \). When productive, an agent has access to a technology which permits the production of one unit of the perishable consumption good for each unit of labor input.

The government can tax buyers lump-sum in the \( CM \), and can issue one-period government bonds. In the \( CM \), agents first meet in a centralized location, where debts from the previous \( DM \) are settled, taxes are paid to the government, and the government makes the payoffs on the government bonds issued in the previous period. Then, in the latter part of the \( CM \), government bonds are sold on a Walrasian market in which exchange is anonymous. A key assumption is limited commitment, i.e. all exchange is voluntary. In particular, buyers cannot be forced to pay their taxes.

During the \( DM \), each buyer is randomly matched with a seller. A fraction \( \rho \) of \( DM \) meetings are limited-information meetings, where the seller does not have access to the buyer’s history. Even though there is limited information...
in this sense, the interaction between the buyer and seller in the meeting will be publicly recorded. The remaining fraction $1 - \rho$ of $DM$ meetings are full-information meetings, where the seller has access to the public record and the interaction between buyer and seller is recorded.

As a visual aid, Figure 1 shows the sequence of activities during a period in the model.

![Figure 1 here.]

3 Symmetric Stationary Equilibria with Global Punishments

First, we will analyze equilibria that are symmetric and stationary, in that each buyer and each seller receive the same allocation, and consume the same amount in each period. Further, these equilibria will be supported by off-equilibrium-path global punishments, in which all agents are punished for the bad behavior of any agent. We will first examine equilibria without government debt, and then introduce government debt to show what difference this makes.

3.1 No Government Debt

In a $DM$ meeting, we will assume that the buyer makes a take-it-or-leave-it offer to the seller. Let $v$ denote the continuation value (constant for all $t$) for a buyer at the end of the $CM$, with $\hat{v}$ denoting the punishment continuation value. Then, $v$ is determined by

$$v = \max_x [u(x) - x + \beta v]$$

subject to

$$x \leq \beta (v - \hat{v}).$$

Here, $x$ is the quantity of goods received by the buyer from the seller (i.e. the loan quantity) during the $DM$ and the buyer promises to repay $\frac{x}{\hat{v}}$ goods in the following $CM$ so as to make the seller indifferent to accepting the contract offer. Inequality (4) is an incentive constraint which states that, given limited commitment, the buyer must have the incentive to repay the loan during the $CM$ rather than facing punishment, represented by the continuation value $\hat{v}$.

We assume that no one can be forced to work, so that the worst possible punishment is $\hat{v} = 0$, i.e. perpetual autarky. Recall that $\hat{v} = 0$ is accomplished off-equilibrium with global punishments. If any buyer defaults then this triggers global autarky. Note that global autarky is also an equilibrium, since if $\hat{v} = 0$ then $v = x = 0$ solves the problem (3) subject to (4).
3.1.1 Incentive Constraint Does Not Bind

First, suppose that the incentive constraint (4) does not bind, which implies, from (3), that \( x = x^* \), where \( x^* \) solves

\[
u'(x^*) = 1.
\]

Then, from (3), we have \( v = \frac{u(x^*) - x^*}{1 - \beta} \), and checking the incentive constraint (4) with \( \hat{v} = 0 \), this equilibrium exists if and only if

\[
\beta \geq \frac{x^*}{u(x^*)},
\]

i.e. buyers have to be sufficiently patient for this equilibrium to exist, in that they need to suffer sufficiently from the off-equilibrium punishment.

3.1.2 Incentive Constraint Binds

Next, suppose that the incentive constraint (4) binds. Then, \( x = \beta v \), and from (3), \( x \) solves

\[
x = \beta u(x),
\]

and (6) has two solutions, one with \( x = 0 \), and one with \( x > 0 \). This first equilibrium always exists, and is inefficient, while the equilibrium with \( x > 0 \) is efficient if it exists, and exists if and only if (checking that \( x < x^* \))

\[
\beta < \frac{x^*}{u(x^*)}.
\]

Thus, from (5) and (7), two equilibria exist, one with \( x = 0 \) and one with \( x > 0 \), and in the latter either the incentive constraint binds or it does not. Note that, in the equilibrium where \( x > 0 \), efficient trade is supported in spite of the fact that the seller does not observe the buyer’s history in a limited-information meeting during the DM. If a buyer defaults on any loan contract, whether the loan was received in a limited-information or full-information meeting, this will trigger global autarky, so that no loans are made on the off-equilibrium path.

3.2 Government Debt

Now, suppose that the government issues \( B \) units of government bonds each period in the CM. Each bond is a promise to pay one unit of goods in the next CM, and these promises sell at the price \( q \). Further, each buyer incurs a tax \( \tau \) during the CM to pay the net interest on the government’s debt. Then, the continuation value \( v \) is determined by

\[
v = \max_{l,b,b'} \{-qb + u(l + \beta b - \beta b') - l + \beta b' - \beta \tau + \beta v\}
\]

subject to

\[
l + \beta \tau - \beta b' \leq \beta(v - \hat{v}),
\]
\[ b' \leq b \]  

where \( l \) is the quantity borrowed by the buyer during the \( DM \), \( b \) denotes the quantity of bonds acquired by the buyer in the \( CM \), and \( b' \) is the quantity of bonds that are not sold by the buyer in the subsequent \( DM \) but are held to be redeemed in the next \( CM \). Note that, in the incentive constraint (9), the right-hand side represents how much the buyer has to work to pay off his or her debts, where these debts include tax liabilities. The government is assumed to be no better or worse than private sector lenders at collecting on its debts. In particular, the government relies on the off-equilibrium punishments of private sector agents in order to enforce payment of taxes (see for example Sanches and Williamson 2010 or Andolfatto 2011).

In equilibrium, the demand for government debt is equal to the supply,

\[ b = B, \]  

and the government budget constraint holds, or

\[ \tau = B(1 - q). \]  

### 3.2.1 Incentive Constraint Does Not Bind

First, if the constraint (9) does not bind, then in equilibrium \( l + \beta b - \beta b' = x^* \), and \( q = \beta \), so from (8), (11), and (12), we have

\[ v = \frac{u(x^*) - x^* - \beta B(1 - \beta)}{1 - \beta}, \]  

and the incentive constraint (9) in equilibrium then reduces to

\[ x \leq \beta u(x) \]

so the equilibrium exists if and only if (5) holds, which is the same condition we obtained without government debt, and the equilibrium allocation is identical, so \( B \) is irrelevant if (5) holds.

### 3.2.2 Incentive Constraint Binds

Next, consider the case where the incentive constraint (9) binds. Then, from (8)-(12), the quantity of consumption in the quantity of consumption in the DM, \( x \), solves (6), and

\[ q = \beta u'(x). \]  

If the incentive constraint (9) binds, then from (8)-(12) and (14), we can write (8) as

\[ x = \beta u(x), \]  

and so an equilibrium with \( x > 0 \) and a binding incentive constraint exists if and only if (7) holds. There also always exists an equilibrium with \( x = 0 \). Thus, in this case \( B \) is irrelevant, just as when the incentive constraint does not bind.
Thus, in these equilibria with global punishments, the issue of government
debt accomplishes nothing. In these cases the economy is Ricardian. In spite
of the limited commitment friction, government debt is irrelevant as the gov-
ernment is no better at collecting on its debts than are private sector lenders.
If the incentive constraint binds in the absence of government debt, then is-
suing government debt does not relax the incentive constraint, as taxation is
required to support tradeable government debt, and buyers can default on their
tax liabilities to the same extent they can default on their private debts.

4 Symmetric Equilibria with Individual Punish-
ment

Studying equilibria with global punishments, as in the previous section, serves
as a useful baseline, but global punishments are obviously unrealistic. In this
section, we construct equilibria with individual punishments, in that only the in-
dividual who defaults, off-equilibrium, is punished for his or her behavior. These
equilibria are also symmetric, in that all buyers behave in the same manner.

An interesting feature of individual punishment equilibria is that a buyer who
defaults will be able to get away with something. Sellers in limited information
meetings cannot observe histories, and are therefore unable to detect a previous
default. As a result, as long as a buyer who has defaulted mimics the equilibrium
behavior of other buyers on meeting a seller in a limited information match, he or
she may be able to receive a loan. Thus, it is possible that the continuation value
in the event of default, \( \hat{v} \), is greater than zero. Mimicking equilibrium behavior
requires that the defaulter offer the same quantity of bonds, \( \beta \), and negotiate the
same loan \( \lambda \), in exchange for goods with a limited-information seller as do buyers
in equilibrium. A defaulting buyer will not be able to trade in full information
meetings in the DM.

Though a defaulting buyer may be able to obtain loans by mimicing equi-
lbrium behavior, it may be optimal for a defaulting buyer to choose autarky.
Therefore, if in equilibrium the loan quantity and quantity of bonds traded in
limited information meetings are \( \lambda \) and \( b \), respectively, then the continuation
value of a buyer on default is given by

\[
\hat{v} = \max \left[ 0, \frac{-qb + \rho u(l + \beta b) + (1 - \rho)\beta b}{1 - \beta} \right].
\]  

(16)

Thus, in equation (16), \( \hat{v} \) is the maximum of the continuation value in autarky
(zero) and the continuation value if, each period, the defaulting buyer acquires
\( b \) bonds in the CM at price \( q \), trades those bonds for goods with a seller and
receives a loan \( \lambda \) in limited information meetings, and exchanges the bonds for
goods in the next CM if a full information meeting occurs, as the seller will not
trade in that case.
4.1 Equilibria with no Government Debt

As in the previous section, we first consider the case where there is no government debt. Then, $v$ is determined by (3) subject to (4), and from (16),

$$\hat{v} = \frac{\mu(l)}{1 - \beta} \geq 0,$$

so it is optimal when there is no government debt for defaulting buyers to mimic equilibrium behavior.

4.1.1 Incentive Constraint Does not Bind

If there is efficient exchange in all $DM$ meetings, with $x = x^*$, then from (3) and (17) we obtain

$$v - \hat{v} = \frac{(1 - \rho)u(x^*) - x^*}{1 - \beta},$$

and checking the incentive constraint (4), this equilibrium exists if and only if

$$\beta \geq \frac{x^*}{(1 - \rho)u(x^*)}.$$

(18)

4.1.2 Incentive Constraint Binds

Now, suppose that trade is not efficient in the $DM$, i.e. the incentive constraint (4) binds. Then from (3), (4), and (17), we can solve for the quantity of goods $x$ consumed by the buyer in the $DM$, i.e. $x$ solves

$$x = \beta(1 - \rho)u(x),$$

(19)

and we require that the solution satisfy $x < x^*$, in order that the incentive constraint not bind. There always exists an equilibrium with $x = 0$, and an equilibrium with $x > 0$ exists if and only if

$$\beta < \frac{x^*}{(1 - \rho)u(x^*)}.$$

(20)

4.1.3 Individual Punishments vs. Global Punishments

Not surprisingly, with weaker punishments relative to the global punishment case, the quantity of exchange and welfare is in general reduced. Let the welfare measure be the sum of period utilities, so that welfare is denoted by

$$W = u(x) - x.$$

Then, welfare is increasing in $x$ for $x \leq x^*$. Let $x_G$ denote the quantity of goods exchanged in $DM$ meetings with global punishments, and $x_I$ the quantity with individual punishments. Then, from (5), (6), (7), (18), (19), and (20),

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1. If $\beta \geq \frac{\beta^*}{(1-\rho)u(x^*)}$, then $x_G = x_I = x^*$ and welfare is the same whether there are global or individual punishments.

2. If $\frac{\beta^*}{u(x^*)} \leq \beta < \frac{\beta^*}{(1-\rho)u(x^*)}$, then $x_G = x^* > x_I$, and welfare is higher with global punishments.

3. If $\beta < \frac{\beta^*}{u(x^*)}$, then $x_I < x_G < x^*$, and welfare is higher with global punishments.1

4.2 Equilibria with Government Debt

Now, suppose that $B > 0$. To focus on whether or not the issue of government debt can improve matters, we will construct equilibria in which government debt is just sufficiently large to crowd out private credit, and all bonds acquired by buyers in the $CM$ are sold in the $DM$ in exchange for goods. Thus, $x = \beta b = \beta B$. Then, from (8), (9), (11), (12), and (16),

$$v = -qB + u(\beta B) - \beta B(1-q) + \beta v,$$

$$\hat{v} = \max \left[ 0, \frac{-qB + \rho u(\beta B) + (1-\rho)\beta B}{1-\beta} \right].$$

The bond price $q$ is determined by

$$q = \beta u'(\beta B).$$

Then, our approach will be to solve for $B$ from (21)-(23), and then study the characteristics of the equilibrium.

4.2.1 Incentive Constraint Does Not Bind

In this case $x = \beta B = x^*$, and from (23) we have $q = \beta$. Then, from (21) and (22) we get

$$v = \frac{-x^* + u(x^*) - x^*(1-\beta)}{1-\beta},$$

$$\hat{v} = \rho \frac{u(x^*) - x^*}{1-\beta}.$$ 

Checking the incentive constraint (9), this equilibrium exists if and only if

$$\beta \geq \frac{x^*}{(1-\rho)u(x^*) + \rho x^*}$$

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1When the incentive constraint binds with individual punishments, $x_I$ is determined by $x_I = \beta(1-\rho)u(x_I)$, whereas when the incentive constraint binds with global punishments $x_G$ is determined by $x_G = \beta(1-\rho)u(x_G)$. Since $\rho < 1$, therefore $x_G > x_I$. 
4.2.2 Incentive Constraint Binds and $\hat{\beta} > 0$

If the incentive constraint binds, we need to consider two possibilities. First, it may be the case that $\hat{\beta} > 0$, so that a defaulting buyer prefers to mimic equilibrium behavior. Second, we could have $\hat{\beta} = 0$, in which case a defaulting buyer prefers autarky. In this subsection, consider the first case.

Then, from (21)-(23), and the binding incentive constraint (9) we have $x [1 - \beta u'(x)] = \beta (1 - \rho) [u(x) - x]$. (25)

**Proposition 1** For $x \in (0, \infty)$, equation (25) has a unique solution $x_E$.

**Proof.** Rewrite equation (25) as

$$1 - \beta u'(x) = \beta (1 - \rho) \left[ \frac{u(x)}{x} - 1 \right].$$

The right-hand side of (26) is monotonically decreasing in $x$, since $u(\cdot)$ is strictly concave. The right-hand side of (26) tends to $\infty$ as $x \to 0$, and to 0 as $x \to \infty$. The left-hand side of (26) is monotonically increasing in $x$. The left-hand side of (26) tends to $-\infty$ as $x \to 0$ and to 1 as $x \to \infty$. Therefore, by the intermediate value theorem, and given monotonicity, there exists a unique $x_E \in (0, \infty)$ that solves (26) and (25).

An equilibrium of this type must involve a solution $x_E$ to (25), and we have shown that a unique solution always exists. However, for this to be an equilibrium of the type we are looking for, $x_E$ must satisfy two other properties. First, the incentive constraint binds if and only if $x_E < x^*$. Second, it must be the case that $\hat{\beta} > 0$ in equilibrium, or $\phi(x_E) > 0$, where

$$\phi(x) \equiv -xu'(x) + \rho u(x) + (1 - \rho)x.$$ (27)

and let $\hat{x}$ denote the solution to

$$\phi(\hat{x}) = 0.$$

**Proposition 2** Assume that $-\frac{xu''(x)}{u'(x)}$ is constant. If $-\frac{xu''(x)}{u'(x)} \geq 1 - \rho$, then

$$\beta < \frac{x^*}{(1 - \rho)u(x^*) + \rho x^*}$$ (28)

is necessary and sufficient for existence of an asymmetric equilibrium with individual punishments and a binding incentive constraint with $\hat{\beta} > 0$. If $-\frac{xu''(x)}{u'(x)} < 1 - \rho$, then (28) and

$$\beta > \frac{\hat{x}}{(1 - \rho)[u(\hat{x}) - \hat{x}] + \hat{x}u'(\hat{x})}$$ (29)

are necessary and sufficient for existence, where $\hat{x}$ solves $\phi(\hat{x}) = 0$. 

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Therefore, if \( x_E < x^* \). If we rewrite (25) as (26), then the right-hand side of (26) is monotonically decreasing and the left-hand side of (26) is monotonically increasing. The previous proposition shows that the solution \( x_E \) is unique, so it follows that \( x_E < x^* \) if and only if (28) holds. Second, we want to find necessary and sufficient conditions for \( \phi(x_E) > 0 \). Differentiating (27), we obtain

\[
\phi'(x) = u'(x) [-1 + \rho + \eta] + 1 - \rho \tag{30}
\]

Therefore, if \( \eta \geq 1 - \rho \), then \( \phi'(x) > 0 \) for \( 0 \leq x \leq x^* \), and since \( \phi(0) = 0 \) therefore \( \phi(x) > 0 \) for \( x \in [0, x^*] \). Thus, if \( \eta \geq 1 - \rho \) then (28) is necessary and sufficient for existence of the equilibrium. Alternatively, suppose \( \eta < 1 - \rho \), then \( \phi'(0) = -\infty, \phi''(x) > 0 \) for \( x \in [0, x^*] \), \( \phi(0) = 0, \phi(x^*) = \rho[u(x^*) - x^*] > 0 \), so there exists a unique \( \tilde{x} \in (0, x^*) \) which solves \( \phi(\tilde{x}) = 0 \). Further, \( \phi(x) > 0 \) for \( x \in (\tilde{x}, x^*] \), and \( \phi(x) \leq 0 \) for \( x \in (0, \tilde{x}] \). It is then necessary and sufficient for \( \phi(x_E) > 0 \) that \( x_E \in (\tilde{x}, x^*) \). Then, once more using (26), \( x_E \in (\tilde{x}, x^*) \) if and only if (28) and (29) hold.

**4.2.3 Incentive Constraint Binds and \( \hat{v} = 0 \)**

In this case, from (9), (21), and (22), the quantity of goods traded in DM meetings, \( x \), solves (15).

**Proposition 3** Assume that \( -\frac{xu''(x)}{u'(x)} \) is constant. A necessary condition for existence of a symmetric equilibrium with individual punishments and a binding incentive constraint with \( \hat{v} = 0 \) is (7). If \( -\frac{xu''(x)}{u'(x)} \geq 1 - \rho \), then this equilibrium does not exist. If \( -\frac{xu''(x)}{u'(x)} < 1 - \rho \), then (7) and

\[
\beta \leq \frac{\tilde{x}}{u(\tilde{x})} \tag{31}
\]

are necessary and sufficient conditions for existence.

**Proof.** Checking that \( x < x^* \), from (15) a necessary condition for the equilibrium to exist is (7). As well, \( \hat{v} = 0 \) requires that a defaulting buyer not wish to mimic equilibrium behavior, which requires \( \phi(x) \leq 0 \). Therefore, this equilibrium does not exist if \( -\frac{xu''(x)}{u'(x)} \geq 1 - \rho \), as this implies \( \phi(x) > 0 \) for \( x \in (0, x^*) \).

If \( -\frac{xu''(x)}{u'(x)} < 1 - \rho \), then \( \phi(x) \leq 0 \) for \( x \in [0, \tilde{x}] \), and so from (15), a necessary and sufficient condition for existence in this case is (31).

**4.2.4 Effects of Government Debt with Individual Punishments**

Government debt clearly matters in equilibria with individual punishments, as the equilibrium allocation with government debt is in general different from the one without government debt. As we have been doing, take welfare to be the sum of expected utilities across agents, which implies that welfare is increasing.
in the quantity of goods $x$ exchanged in each meeting in the $DM$, for $x \leq x^*$, which will always hold in equilibrium. The following sequence of propositions demonstrates that government debt at worst has no effect on welfare and at best increases it by increasing the volume of exchange in the $DM$. Let $x_N$ denote the quantity of goods exchanged in each meeting in the $DM$ in the absence of government debt, and $x_D$ the quantity exchanged when there is government debt.

**Proposition 4** If $\beta \geq \frac{x^*}{(1-\rho)u(x^*)}$, then $x_D = x_N = x^*$, and welfare is the same in the equilibrium without government debt and the one with government debt.

**Proof.** From (18) and (24), equilibria with a nonbinding incentive constraint exist if $\beta \geq \frac{x^*}{(1-\rho)u(x^*)}$, both with and without government debt. ■

**Proposition 5** If $\frac{x^*}{(1-\rho)u(x^*)+px^*} \leq \beta < \frac{x^*}{(1-\rho)u(x^*)}$, then $x^* = x_D > x_N$, and welfare is greater with government debt than without.

**Proof.** From (24), the incentive constraint does not bind with government debt if $\frac{x^*}{(1-\rho)u(x^*)+px^*} \leq \beta < \frac{x^*}{(1-\rho)u(x^*)}$, so $x_D = x^*$. However, since (20) holds, the incentive constraint binds without government debt, so $x_N < x^*$. ■

**Proposition 6** If $-\frac{u''(x)}{u'(x)}$ is constant, if $-\frac{u''(x)}{u'(x)} \geq 1 - \rho$ or if $-\frac{xu''(x)}{u'(x)} < 1 - \rho$ and $\beta > \frac{\bar{x}}{(1-\rho)u(x^*)+px^*}$, and $\beta < \frac{x^*}{(1-\rho)u(x^*)+px^*}$, then $x_N < x_D < x^*$.

**Proof.** From (20), (28), and (29), the incentive constraint binds in both equilibria, so $x_N < x^*$ and $x_D < x^*$. Further, we can write equation (??) as

$$x[1 - \beta(1 - \rho)] = \beta(1 - \rho)[u(x) - x].$$

(32)

Then, since the right-hand sides of equations (32) and (25) are identical, and $u'(x) > 1 - \rho$ for $x < x^*$, therefore $x_D > x_N$. ■

**Proposition 7** If $-\frac{u''(x)}{u'(x)}$ is constant, if $-\frac{xu''(x)}{u'(x)} > 1 - \rho$ and $\beta \leq \frac{\bar{x}}{u'(x)}$, then $x_N < x_D < x^*$.

**Proof.** From (20) and (31), the incentive constraint binds in both equilibria, so $x_N < x^*$ and $x_D < x^*$. In the equilibrium with no government debt, $x$ is determined by (19), and in the equilibrium with government debt, $x$ is determined by (15). It is immediate from (19) and (15) that $x_N < x_D$. ■

Note that, since

$$-\ddot{x}u'(\ddot{x}) + \rho u(\ddot{x}) + (1 - \rho)\ddot{x} = 0$$

determines $\ddot{x}$, therefore

$$\frac{\ddot{x}}{u'(\ddot{x})} = \frac{\ddot{x}}{(1-\rho)[u(\ddot{x}) - \ddot{x}] + \ddot{x}u''(\ddot{x})}.$$
so the right-hand sides of inequalities (29) and (31) are equal. As a result, the
above four propositions exhaust the parameter space. With government debt,
the equilibrium is unique among the class of equilibria we are examining in
this section, and an equilibrium always exists. The same is the case without
government debt.

Figure 2 shows how the parameter space is subdivided, with the parameter
\( \rho \) on the horizontal axis, and \( \beta \) on the vertical axis, and with \( \eta = -\frac{xu'(x)}{u(x)} \).
In region 1, Proposition 1 applies. In this case, the incentive constraint does
not bind with or without government debt, and there is efficient exchange in
either case, or \( x_N = x_D = x^* \). In region 2, Proposition 2 applies, in which case
the incentive constraint binds without government debt, but does not bind with
government debt, or \( x_N < x_D = x^* \). In region 2, the introduction of government
debt is welfare-improving, as it increases exchange in the \( DM \). In region 3,
Proposition 3 applies, with incentive constraints binding in equilibrium with or
without government debt. With government debt, defaulting buyers choose to
mimic equilibrium behavior, or \( \hat{v} > 0 \). In region 3, exchange is greater in the
\( DM \) with government debt, and welfare is higher. Finally, in region 4 of the
parameter space, in Figure 3, Proposition 4 applies. In this case, the incentive
constraint binds with or without government debt, but there is more exchange
in the \( DM \), and higher welfare, with government debt. In the equilibrium with
government debt in region 4, defaulting buyers choose autarky, or \( \hat{v} = 0 \).

We can conclude from the above four propositions that introducing gov-
ernment debt in symmetric equilibria with individual punishments is welfare
improving as, in general, it increases the quantity of exchange in decentralized
meetings. The introduction of government debt acts to relax incentive con-
straints, as it tends to make default less desirable. At best, the introduction of
government debt can make it so costly to mimic equilibrium behavior for a de-
faulting buyer, that a buyer will choose autarky if default occurs, off-equilibrium.
In the absence of government debt, a buyer who defaults can get something for
nothing. By simply posing as a buyer who has not defaulted, a defaulting buyer
can receive a loan with probability \( \rho \). However, when government debt is traded,
a defaulting buyer has to work to acquire the government debt in order to pose
as a buyer who has not defaulted.

Effectively, government debt plays the role of collateral, in that the govern-
ment supplies a safe asset, supported by taxation, that is used to support credit
market activity. An interpretation of the exchange that takes place involving
government debt in the \( DM \), is that a buyer and seller write a credit contract,
with government debt posted as collateral. Then, in the subsequent \( CM \), the
buyer sells the collateral to pay off the debt. With a ready supply of govern-
ment debt, the collateralized credit arrangements arise endogenously, and they
increase welfare by promoting more exchange. This role for government debt
as collateral in the credit market is consistent with empirical observations (see
Garbade 2006), in that activity in the repurchase agreement market appears to
have grown partly in response to growth in the quantity of government debt outstanding in the United States.

Intuition might tell us that the provision of government debt might be bad for incentives in this context, since it allows for anonymous exchange. The possibility of defecting from the credit system might look more desirable if there is another mode of exchange available. Indeed, this intuition captures the forces at work in Aiyagari and Williamson (2000). In Aiyagari/Williamson, exchange of government liabilities (fiat money, in that case) provides an alternative to participation in a credit system, and the better this system of government-liability exchange works, the worse that is for incentives in the credit system. In the Aiyagari/Williamson model, government liabilities are effectively a substitute for credit, while in this model (at least in the symmetric equilibria we have studied thus far) government liabilities are complementary to credit, as government debt essentially serves as collateral.

5 Asymmetric Equilibria with Individual Punishment and Equilibrium Default

We will now consider equilibria where agents behave asymmetrically, with some buyers defaulting in equilibrium. These are equilibria where a fraction $\alpha$ of buyers (the good buyers) never defaults, but a fraction $1 - \alpha$ (bad buyers) will default on their debts if anyone chooses to lend to them. Here, $\alpha$ is endogenous. In an asymmetric equilibrium, good buyers never default because they would be punished by being treated in the same way as bad buyers, off equilibrium, losing access to full-information lending. Bad buyers always default as they have nothing to lose – sellers who know their type will not lend to them.

In typical limited commitment models of credit, such as Kehoe and Levine (1993) and Kocherlakota (1996), there is no default in equilibrium, just as in the symmetric equilibria in our model. While the threat of default can be important in constraining credit contracts in those models, the fact that default does not occur in equilibrium limits the ability of such models to explain observations. In models of bankruptcy with incomplete markets, such as Athreya (2002) and Corbae et al. (2007), it is typical to assume that contracts take the form of non-contingent debt, and there is then equilibrium default. Of course, much is imposed in such models, including the absence of contingent claims markets, and restrictions on contracts. In our model, the contracts are optimal given the economic environment, but default can occur in equilibrium.

5.1 Asymmetric Equilibria Without Government Debt

In the absence of government debt, the continuation value $v$ for a good buyer is given by

$$v = \max_{i_1, i_2} \left\{ \rho u(x_L) + (1 - \rho)u(x_F) - \rho \frac{x_L}{\alpha} - (1 - \rho)x_F + \beta v \right\}$$

(33)
subject to

\[
\frac{x_L}{\alpha} \leq \beta (v - \hat{v}), \quad (34)
\]

\[
x_F \leq \beta (v - \hat{v}) \quad (35)
\]

In (33)-(35), the good buyer takes out a loan \( x_L \) in a limited information meeting, and a loan \( x_F \) in a full information meeting. In order to receive consumption goods from the seller in a limited-information meeting, a bad buyer must mimic the behavior of a good buyer. Thus, there is a pooling equilibrium in which the good borrower promises to make a payment \( \frac{x}{\alpha} \) on a loan from a seller so as to compensate the seller for defaults by bad buyers. Thus, limited-information loans carry a default premium. In full information meetings, bad buyers will not receive loans, as sellers in these meetings know that bad buyers always default. Constraints (34) and (35) are the incentive constraints that must hold for a good buyer following a limited-information meeting and a full-information meeting, respectively.

A bad buyer can consume the same quantity as a good buyer in a limited information meeting in the decentralized market if he or she mimics the behavior of a good buyer. Without government debt, he or she will always wish to mimic equilibrium behavior, so

\[
\hat{v} = \frac{\rho u(x_L)}{1 - \beta}. \quad (36)
\]

If neither incentive constraint, (34) or (35), binds, then \( x_L = \hat{x} \) and \( x_F = x^* \), where \( \hat{x} \) solves

\[
u'(\hat{x}) = \frac{1}{\alpha}.
\]

Clearly \( \hat{x} < x^* \) for \( \alpha < 1 \), so a smaller quantity of goods is exchanged in limited information meetings, even if incentive constraints do not bind. Further, since \( -\frac{u'(x)}{u''(x)} < 1 \), \( \hat{x} < x^* \) for \( \alpha < 1 \), which implies that, in equilibrium, incentive constraint (35) is always tighter than (34). Thus, the only cases we need to consider, in the absence of government debt, are equilibria where neither incentive constraint binds, where constraint (35) binds and (34) does not, and where both incentive constraints bind. We will consider each of these three cases in turn.

\section{5.1.1 Neither Incentive Constraint Binds}

If (34) and (35) do not bind, then from (33) - (36) we get

\[
v - \hat{v} = \frac{(1 - \rho)u(x^*) - \rho \hat{x} - (1 - \rho)x^*}{1 - \beta}. \quad (37)
\]

To check incentive constraints, it is sufficient to check (35), as this is always the tighter constraint. This tells us that this equilibrium exists if and only if

\[
\beta \geq \frac{x^*}{(1 - \rho)u(x^*) + \rho (x^* - \hat{x})} \quad (38)
\]
5.1.2 Limited Information Incentive Constraint Does Not Bind, Full-Information Incentive Constraint Does

Next, we analyze the case where (34) does not bind, but (35) does. Then, from (33) - (36) we get

\[ v = \rho u(\hat{x}) + (1 - \rho)u[\beta(v - \hat{v})] - \rho \frac{\hat{\beta}}{\alpha} + \beta \rho v + \beta(1 - \rho)\hat{v}, \quad (39) \]

\[ \hat{v} = \rho u(\hat{x}) + \beta \hat{v}, \quad (40) \]

and then (39), (40), and (35) with equality gives

\[ \beta(1 - \rho)u(x_F) - x_F + \rho \beta \left( x_F - \frac{\hat{x}}{\alpha} \right) = 0, \quad (41) \]

which solves for \( x_F \). There are potentially two solutions to (41), one solution with \( x_F < \frac{\hat{x}}{\alpha} \), and one with \( x_F > \frac{\hat{x}}{\alpha} \). Only the latter can be an equilibrium as, if \( x_F < \frac{\hat{x}}{\alpha} \), then incentive constraint (34) must bind, but we are attempting to construct an equilibrium in which this constraint does not bind. For an equilibrium, we first require that there be a solution to (41). A necessary and sufficient condition for that is

\[ \beta \geq \frac{\frac{\hat{x}}{\alpha}}{(1 - \rho)u(\frac{\hat{x}}{\alpha})}. \quad (42) \]

As well, we require that the solution satisfy \( x_F < x^* \), so that the full-information incentive constraint binds or, from (41),

\[ \beta < \frac{x^*}{(1 - \rho)u(x^*) + \rho \left( x^* - \frac{\hat{x}}{\alpha} \right)}. \quad (43) \]

Then, (42) and (43) are necessary and sufficient conditions for the existence of this equilibrium to exist.

5.1.3 Both Incentive Constraints Bind

If (34) and (35) bind, then \( x_L = \alpha x_F \), and from (33) - (36), \( x_F \) solves

\[ x_F = \beta(1 - \rho)u(x_F). \quad (44) \]

To determine necessary and sufficient conditions for existence, it is sufficient to check that the solution to (44) implies that the incentive constraint (34) binds, since (35) is always the tighter constraint. Thus, we require that \( x_F < \frac{\hat{x}}{\alpha} \), which from (44) gives

\[ \beta < \frac{\frac{\hat{x}}{\alpha}}{(1 - \rho)u(\frac{\hat{x}}{\alpha})}. \quad (45) \]

Then, inequality (45) is necessary and sufficient for the existence of this equilibrium.
5.1.4  Asymmetric Equilibria vs. Symmetric Equilibria

The presence of defaulting borrowers in equilibrium has two sets of effects. First, there are direct effects on good borrowers in limited-information credit arrangements. There is an adverse selection problem, with a fraction $1-\alpha$ of buyers who borrow from sellers in limited-information meetings defaulting. Good buyers in these meetings will be able to borrow less than if there were full information. Consumption will then be lower in limited-information meetings. The indirect effects on good borrowers are incentive effects. Following a limited information meeting in the $DM$, a good borrower will have less debt to pay off, in spite of having to pay a default premium to the seller (this depends on curvature in the utility function; if the coefficient of relative risk aversion is greater than one, this effect goes the other way). Perhaps counterintuitively, the presence of defaulting borrowers can make default less likely for good borrowers, in the sense that incentive constraints are relaxed.

More explicitly, note first that

\[
\frac{x^*}{(1-\rho)u(x^*) + \rho \left( x^* - \frac{2}{\alpha} \right)} < \frac{x^*}{(1-\rho)u(x^*)} \tag{46}
\]

and

\[
\frac{x^*}{(1-\rho)u \left( \frac{x^*}{\alpha} \right)} < \frac{x^*}{(1-\rho)u(x^*)} \tag{47}
\]

for $0 < \alpha < 1$. Therefore, from (18), (20), (38) and (45), if $\alpha$ falls, so that there are fewer good buyers in the population, this acts to relax incentive constraints, expand the region of the parameter space in which incentive constraints do not bind, and shrink the region where incentive constraints bind. For this result, it is important that $-x^* \frac{u''(x^*)}{u'(x^*)} < 1$, which implies that $\hat{x}$ is increasing in $\alpha$.

Therefore, if \[\beta \geq \frac{x^*}{(1-\rho)u(x^*)},\]

then from (18), (38), and (46), incentive constraints do not bind in a symmetric equilibrium or in any asymmetric equilibrium with $0 < \alpha < 1$. But, since $\hat{x} < x^*$, consumption is smaller in limited information meetings in the $DM$ in asymmetric equilibria than in the symmetric equilibrium, and consumption is the same in full information meetings. Therefore, welfare is higher in the symmetric equilibrium in this case.

If \[\beta < \frac{x^*}{(1-\rho)u \left( \frac{x^*}{\alpha} \right)},\]

then given (47), the incentive constraint binds in the symmetric equilibrium, and both incentive constraints bind in an asymmetric equilibrium. But from \[2\] The result goes the other way if $-x^* \frac{u''(x^*)}{u'(x^*)} > 1$.
(44) and (19), consumption is the same in full information meetings in the symmetric and asymmetric equilibria, but is smaller in limited information meetings in the asymmetric equilibrium. Therefore, welfare is higher in the symmetric equilibrium.

In the case where

\[
\beta \in \left[ \frac{\hat{\alpha}}{\alpha}, \frac{x^*}{(1 - \rho)u(\frac{\hat{\alpha}}{\alpha})}, \frac{x^*}{(1 - \rho)u(x^*)} \right],
\]

it is difficult to make comparisons between symmetric and asymmetric equilibria.

**Multiplicity of Asymmetric Equilibria** One feature of asymmetric equilibria that we did not see with symmetric equilibria is that, for given \( \rho \), there can be multiple equilibria. From (38), (42), (43), and (45), if

\[
0 < \rho < \frac{u(\hat{x}) - u(x^*)}{u(\hat{x}) - u(x^*) + 1 - \frac{\hat{x}}{x}},
\]

then for high \( \beta \), neither incentive constraint binds, for middle levels of \( \beta \) the limited-information incentive constraint does not bind while the other incentive constraint does, and for low levels of \( \beta \) both incentive constraints bind. However, if

\[
\frac{u(\hat{x}) - u(x^*)}{u(\hat{x}) - u(x^*) + 1 - \frac{\hat{x}}{x}} < \rho \leq \frac{u(x^*) - x^*}{u(x^*) - x^* + \frac{\hat{x}}{x}}, \tag{48}
\]

then for middle levels of \( \beta \) two equilibria exist – the equilibrium where neither incentive constraint binds and the one where both incentive constraints bind, as the following proposition summarizes.

**Proposition 8** Suppose (48) holds and

\[
\beta \in \left[ \frac{x^*}{(1 - \rho)u(x^*) + \rho (x^* - \frac{\hat{x}}{x})}, \frac{\hat{x}}{\alpha} \right]. \tag{49}
\]

Then, for given \( \alpha \), there exist two equilibria, one where neither incentive constraint binds, and one where both incentive constraints bind.

**Proof.** Consider the necessary and sufficient conditions for existence of equilibria with neither incentive constraint binding, (38), and both incentive constraints binding, (45). The set defined by (48) and (49) is the set satisfying (38) and (45). Then, any \((\rho, \beta)\) satisfying (48) and (49) is such that both (38) and (45) are satisfied, so both equilibria exist.
5.2 Asymmetric Equilibria with Government Debt

In this section, we take the same approach as for symmetric equilibria, in constructing equilibria in which the quantity of bonds issued by the government is just sufficient to drive out private credit. In this case however, we are constructing equilibria for given \( \alpha \), and then characterizing such equilibria and determining under what conditions they exist. In these equilibria, there is a mass \( \alpha \) of good buyers who trade government bonds for goods in limited-information and full-information meetings in the \( DM \), and who always pay their taxes in the \( CM \). There is also a mass of \( 1 - \alpha \) bad buyers, who are able to trade in limited-information meetings in the \( DM \) if they mimic the behavior of good buyers, but cannot trade in full-information \( DM \) meetings. These bad buyers always default on their tax liabilities.

The continuation values \( v \) and \( \hat{v} \) are determined by (8), (9), and (16). Further, taxes on good buyers finance the net interest on government bonds, or

\[
\tau = \frac{B(1-q)}{\alpha},
\]

and the bond market clears, which implies

\[
b = B,
\]

if bad buyers mimic good buyers in limited information meetings in the \( DM \), or

\[
\alpha b = B,
\]

if bad buyers choose autarky. The price of bonds is determined by

\[
q = \beta u'(\beta B).
\]

5.2.1 Incentive Constraint Does Not Bind

In this equilibrium \( \beta B = x^* \), and from (53), \( q = \beta \). From (16),

\[
\hat{v} = \frac{\rho [u(x^*) - x^*]}{1 - \beta} > 0,
\]

so bad buyers will mimic the behavior of good buyers in limited information meetings in the \( DM \). Then, from (8) and (54),

\[
v - \hat{v} = \frac{(1 - \rho)u(x^*) - \frac{x^*(1-\beta)}{\alpha} - (1 - \rho)x^*}{1 - \beta}.
\]

To determine existence of this equilibrium, it is sufficient to check the incentive constraint (9), so from (9) and (55) we obtain

\[
\beta \geq \frac{x^*}{\alpha(1-\rho)u(x^*) + [1 - \alpha(1-\rho)]x^*},
\]

Inequality (56) is then a sufficient condition for an equilibrium of this type to exist for given \( \alpha \).
5.2.2 Incentive Constraint Binds and $\hat{v} > 0$

Next, consider the case where the asymmetric equilibrium with government debt involves a binding incentive constraint, and $\hat{v} > 0$, so that bad buyers strictly prefer to mimic good buyers in limited information meetings in the DM. The quantity of consumption in DM meetings is $x = \beta B$, and the price of government bonds in the CM is $q = \beta u'(x)$. Then, from (8), (9), and (16), $x$ solves

$$x[1 - \beta u'(x)] = \alpha \beta (1 - \rho)[u(x) - x].$$  \hfill (57)

A unique solution to (57) exists, and we require that the incentive constraint bind, or $x < x^*$. Thus, from (57), a necessary condition for an equilibrium of this type to exist is

$$\beta < \frac{x^*}{\alpha (1 - \rho)u(x^*) + [1 - \alpha (1 - \rho)]x^*}.$$  \hfill (58)

Further, we require $\hat{v} > 0$. As in our analysis in the symmetric equilibrium case, suppose that $-\frac{xu''(x)}{u'(x)}$ is constant. Then, as we showed above, if $-\frac{xu''(x)}{u'(x)} \geq 1 - \rho$, then $\hat{v} > 0$, and if $-\frac{xu''(x)}{u'(x)} < 1 - \rho$ then $\hat{v} > 0$ if and only if $x > \tilde{x}$ where $\tilde{x}$ solves

$$-\tilde{x} u'(\tilde{x}) + pu(\tilde{x}) + (1 - \rho)\tilde{x} = 0.$$  \hfill (59)

Then, since the left-hand side of (57) is increasing in $x$ and the right-hand side is decreasing in $x$, (57) and (59) imply $x > \tilde{x}$ if and only if

$$\beta > \frac{\tilde{x}}{\alpha (1 - \rho)[u(\tilde{x}) - \tilde{x}] + \tilde{x} u'(\tilde{x})}.$$  \hfill (60)

5.2.3 Incentive Constraint Binds and $\hat{v} = 0$

In this case, $x = \beta \tilde{x}$, since bad buyers do not hold bonds, and $q = \beta u'(x)$. Then, from (8), (9), and (16), $x$ solves (6). A necessary condition for this equilibrium to exist is that $x < x^*$, which from (6) gives (7). As well, bad buyers must not prefer to mimic the behavior of good buyers in limited information meetings in the DM, i.e.

$$-x u'(x) + pu(x) + (1 - \rho)x \leq 0.$$  \hfill (61)

But (61) holds if and only if $-\frac{xu''(x)}{u'(x)} < 1 - \rho$ and $x \leq \tilde{x}$, where $\tilde{x}$ is defined by (59). But from (6) and (59), $x \leq \tilde{x}$ is equivalent to

$$\beta \leq \frac{\tilde{x}}{u(\tilde{x})}.$$  \hfill (62)

Therefore (7), $-\frac{xu''(x)}{u'(x)} < 1 - \rho$, and (62) are necessary and sufficient for this equilibrium to exist.
5.3 Effects of Government Debt in Asymmetric Equilibria

In an asymmetric equilibrium, if we measure welfare as the sum of expected utilities across agents, then we need to take account of bad buyers as well as good ones, and we obtain

\[ W = \rho [u(x_L) - x_L] + \alpha (1 - \rho) [u(x_F) - x_F], \]

where \( x_L \) and \( x_F \) denote, respectively, consumption in limited information and full information meetings in the \( DM \), and \( W \) is strictly increasing in \( x_L \) and \( x_F \) for \( x_L \leq \hat{x} \) and \( x_F < x^* \).

We have two goals in this subsection. The first is to get some idea how the trading of government debt affects exchange and welfare for given \( \alpha \). In other words, given an economy with a particular fraction of defaulting buyers in the population, how will the existence of government debt affect what is consumed in limited-information and full-information \( DM \) meetings? Our second goal is to determine the effects of trading in government debt on the existence of particular asymmetric equilibria. Does government debt encourage or discourage default?

First, for given \( \alpha \), compare the properties of the equilibrium (equilibria) that exists (exist) without government debt, and with government debt, in alternative regions of the parameter space. Let \( x_{LN} \) and \( x_{FN} \) denote consumption in limited-information and full-information meetings, respectively, in an asymmetric equilibrium without government debt. Similarly, \( x_{LD} \) and \( x_{FD} \) are consumption quantities in the \( DM \) when government debt is traded.

**Proposition 9** If \( \beta \geq \frac{x^*}{\alpha (1 - \rho) u(x^*) + [1 - \alpha (1 - \rho)] x^*} \), then \( x_{LN} < x_{LD} = x^* \) and \( x_{FN} \leq x_{FD} \). Welfare is higher with government debt than without it.

**Proof.** From (56), an asymmetric equilibrium with government debt exists in which the incentive constraint does not bind, so \( x_{LN} = x_{FD} = x^* \). If (38) holds, then an asymmetric equilibrium without government debt exists in which \( x_{LN} = \hat{x} \) and \( x_{FN} = x^* \), so \( x_{LN} < x_{LD} \) and \( x_{FN} = x_{FD} \), and welfare is higher in the equilibrium with government debt. If (42) and (43) hold, then an asymmetric equilibrium without government debt exists in which \( x_{LN} = \hat{x} \) and \( x_{FN} < x^* \). Therefore, \( x_{LN} < x_{LD} \) and \( x_{FN} < x_{FD} \) in this case, and welfare is higher in the equilibrium with government debt. Finally, if (45) holds, then an equilibrium without government debt exists in which \( x_{LN} < \hat{x} \) and \( x_{FN} < x^* \). Therefore \( x_{LN} < x_{LD} \) and \( x_{FN} < x_{FD} \) in this case, and welfare is higher in the equilibrium with government debt. \( \blacksquare \)

**Proposition 10** If \( \beta \leq \frac{x}{u(x)} \) and \( \beta < \frac{x}{(1 - \rho) u(x)} \), then \( x_{LN} < x_{LD}, x_{FN} = x_{FD} \), and welfare is higher with government debt than without it.

**Proof.** From (62), an equilibrium exists with government debt in which there is a binding incentive constraint, where \( x = x_{LD} = x_{FD} \) solves (6). From (45), an equilibrium also exists without government debt where \( x = x_{FN} \) also solves (45), so \( x_{FD} = x_{FN} \). But in the equilibrium without government debt,
Figure 3 shows the subdivision of the parameter space, according to which asymmetric equilibria exist, with and without government debt. First, Proposition 5 deals with regions 1, 2, and 3 in Figure 3, where an asymmetric equilibrium with government debt exists in which the incentive constraint does not bind. In region 1, there exists an equilibrium without government debt in which neither incentive constraint binds. In region 2, there exist two equilibria without government debt; in one of these equilibria neither incentive constraint binds, and in the other both incentive constraints are binding. In region 3, there exists an equilibrium with no government debt where both incentive constraints bind. In regions 1, 2, and 3, the equilibrium with government debt dominates any equilibrium without government debt that exists.

Second, Proposition 6 deals with region 12 in Figure 3. In that subset of the parameter space, an equilibrium without government debt exists in which the incentive constraint binds, and bad buyers choose autarky rather than defaulting. This equilibrium dominates the equilibrium without government debt in which both incentive constraints bind, which also exists in region 12.

In Propositions 5 and 6, we have shown that there exist subsets of the parameter space for which issuing government debt will improve matters in asymmetric equilibria, for given $\alpha$. First (Proposition 5), for any $\alpha$ with $0 < \alpha < 1$, there is a subset of the parameter space in which there exists an equilibrium with government bonds for which the incentive constraint does not bind, and in which there is more exchange in the $DM$, and higher welfare, than in any asymmetric equilibrium without government bonds that exists in that subset. Second (Proposition 6), for any $\alpha$ with $0 < \alpha < 1$, there is a subset of the parameter space in which there exists an equilibrium with government debt for which the incentive constraint binds, in which there exists an equilibrium without government debt in which both incentive constraints bind, and in which there is less exchange in the $DM$ and therefore lower welfare in the equilibrium without government debt.

In regions 4 through 11 in Figure 3, it is more difficult to draw conclusions, though we know that, by continuity, the introduction of government debt must be welfare improving for sufficiently large $\alpha$. This follows from the fact that government debt always improves matters in symmetric equilibria – essentially the special case where $\alpha = 1$.

In the equilibrium with government bonds in which the incentive constraint does not bind (regions 1, 2, and 3 of Figure 3), the welfare improvement from trade in government bonds does not result from effects on default behavior. Indeed, if such an equilibrium exists for given $\alpha$, then there exists an equilibrium without government bonds for the same $\alpha$. The key welfare-improving effect is that introducing government bonds solves the adverse selection problem in limited-information meetings in the $DM$. With government bonds, good buyers
are no longer charged a default premium in limited-information exchange, and
more goods are traded. However, in the equilibrium with government bonds,
good buyers suffer because bad buyers always default on their taxes. Thus,
good buyers have to bear the entire burden of paying the net interest on the
government debt.

In an asymmetric equilibrium with government debt in which the incentive
constraint binds and bad buyers choose autarky ($\hat{v} = 0$), in region 12 of Figure 3,
government debt improves the allocation for two reasons. First, as we outlined
for the case where the incentive constraint does not bind, government debt
solves the adverse selection problem in limited information meetings in the $DM$.
Second, in region 12 of Figure 3, bad buyers do not default in equilibrium, but
instead choose autarky. Therefore, the behavior of bad buyers has no effect on
good buyers.

Next, government debt can act to mitigate default in that it can kill some
undesirable asymmetric equilibria. Note in particular that, without government
bonds, an asymmetric equilibrium exists for any $\alpha \in (0, 1)$ and $(\beta, \rho) \in (0, 1) \times
(0, 1)$. However, from (56), (58), (60), and (62), with government bonds, there
is a subset of the parameter space where an asymmetric equilibrium does not
exist, for given $\alpha$, where parameters satisfy

$$\rho < 1 + \frac{xu''(x)}{u'(x)}$$

and

$$\frac{\hat{x}}{u(\hat{x})} < \beta \leq \frac{\hat{x}}{\alpha(1 - \rho)|u(\hat{x}) - \hat{x}| + \hat{x}u'(\hat{x})}$$

This subset comprises regions 8, 9, and 10 in Figure 3. Note further that this
subset expands as $\rho$ decreases, so government debt more successfully does away
with default the more severe the default problem is.

Note that we are not saying that government debt is useful because it leads
to nonexistence of equilibrium. Indeed, we showed that, with government debt,
a symmetric equilibrium always exists (i.e. $\alpha = 1$). Another way to state the
idea in the previous paragraph is that there are some regions of the parameter
space for which the introduction of government debt implies that an asymmet-
ric equilibrium exists only if $\alpha$ is sufficiently large. Thus, government debt
eliminates some equilibria with large numbers of defaulters.

6 Discussion

The problem in this model that government debt mitigates or solves is an en-
dogenous one. There always exists an equilibrium under global punishments
in which the allocation is efficient. In such an equilibrium, government debt is
irrelevant – the economy is Ricardian. However, there are many equilibria, and
we have focused here on ones that we think are particularly interesting.

First, in symmetric equilibria with individual punishments in which default
does not occur in equilibrium, the existence of government debt matters for the
equilibrium allocation, and the optimal provision of such debt will in general increase the quantity of exchange and economic welfare. This can be interpreted as a role for government debt as collateral. If government debt is provided, then it will be used as collateral in decentralized credit arrangements, as it acts to relax incentive constraints. Without government debt, credit is limited by the potential for a borrower to default on a private credit arrangement. But when there is enough government debt to displace private credit, secured credit (or, what is equivalent here, the use of government debt in exchange) is limited by the potential for agents to default on their tax liabilities, where taxes are necessary to finance the net interest on the government debt. Effectively, the potential burden of default is transferred from the private sector to the government. But, in spite of the fact that the government is no better at collecting on its debts than are private sector lenders, this transfer increases welfare. This is because government debt acts to alter the incentives of borrowers who default — effectively the off-equilibrium punishment from defaulting is higher with government debt than without it.

Second, in asymmetric equilibria with individual punishments, some borrowers always default on their debts because they have little to lose from defaulting — their behavior is bad because they are treated badly. In such equilibria, there are additional welfare-enhancing effects of government debt. Government debt can solve an adverse selection problem in credit market arrangements subject to limited information. Effectively government debt permits all credit to be secured, which can permit equilibria with more exchange and higher welfare. As well, government debt can eliminate equilibria with high default rates. Thus, government debt can mitigate default, and can enhance the ability of private sector agents to defend themselves against default.

The model illustrates two roles for collateral, which are somewhat related. The first role is related to a borrower’s incentive to default. When government debt is available to post as collateral in credit contracts, default becomes more costly for borrowers, which makes lending more attractive. The second role is related to the terms on which a lender is willing to extend credit when it is difficult to screen borrowers. When some borrowers always default, and the credit is secured by government debt, the borrower’s type is irrelevant to the seller, and the existence of collateral allows lending to proceed on better terms, with a lower interest rate and larger loan quantity. The government debt solves the adverse selection problem by making the loan information-insensitive\textsuperscript{3} and thus enhancing the credit arrangement.

The collateral-role of government debt in our model is consistent with how markets in repurchase agreements (repos) work in practice, and with some features of the recent financial crisis. A repo is a bilateral agreement between a short-term lender and a dealer (borrower) involving the sale and repurchase of securities. This is essentially a loan where the securities serve as collateral.\textsuperscript{4}

\textsuperscript{3}See Gorton and Metrick (2010).
\textsuperscript{4}See Garbade (2006).
is an important collateral asset in the repo market. During the recent financial
crisis, the quality of collateral, as measured by repo haircuts ⁵, differed sub-
stantially among Treasury securities, agency securities, asset-backed securities
issued by government-sponsored enterprises (GSEs), other kinds of asset-backed
securities, and corporate bonds (see for example Martin et al. 2012). Treasury
securities, Agency securities, and mortgage-backed securities issued by GSEs
traded with very low haircuts during the period 2008-2010.

Thus, consistent with our model, in times of credit market dysfunction such
as the recent financial crisis, government-related securities are relatively more
valuable as collateral. In our model government debt acts to free up the credit
arrangements between buyers and sellers, so that welfare increases. In the
model, liquid government securities work on the intensive margin; the loan that
the seller grants the buyer is larger when it is secured by good collateral. A
larger loan is (weakly) welfare improving⁶.

We have shown here that it is not the limited commitment friction alone
that is the ultimate source of a role for government debt in credit markets,
since efficient equilibria exist in which government debt is irrelevant. What
makes government debt useful is limited commitment in conjunction with a poor
endogenous choice of punishment for default, and the existence of defaulting
debtors.

7 Conclusion

In this model, government debt can act to discourage default, and to reduce
the quantity of equilibrium default. Effectively, government debt plays a role
as collateral in credit contracts subject to a limited commitment friction. The
model uses ideas from the monetary theory literature, in which memory and
recordkeeping play a critical role in giving government-issued liabilities a role in
exchange. Government debt matters in this model because of limited memory, in
conjunction with weak punishments for default. This creates the possibility that
agents can borrow even if they have defaulted in the past. This can tighten the
incentive constraint, even though default is only an off-equilibrium phenomenon,
or it permits the existence of equilibrium default.

Typical limited commitment models (e.g. Kehoe and Levine 1993 or Kocher-
lakota 1996) do not exhibit equilibrium default. The only explicit contracting
model with default as an outcome is the static costly state verification model
of Townsend (1979). Limited commitment has some distinct advantages as we
can study credit in complex dynamic environments in a tractable way. This
framework, which permits default with limited commitment, opens up interest-
ing possibilities for future work. In particular, it is possible to study endogenous
fluctuations in which the aggregate default rate varies over time. Such a model
would be useful for studying financial panics.

⁵The haircut is the difference between the value of an asset and the value of the loan that
is secured by the asset as collateral. See Martin et al. (2010).
⁶It is strictly welfare improving when the incentive constraint is binding.
8 References


Figure 1: Sequence of Events in a Period

- Debts from period $DM_{t-1}$ settled and taxes paid
- Bonds from $CM_t$ redeemed
- Govt. bonds issued
- Random matching. Exchange of personal IOUs and government debt for goods.
Figure 2: Symmetric Equilibria with Individual Punishments

\[ \rho = \text{fraction of limited information trades} \]

\[ \beta = \text{Discount Factor} \]

\[ 1 - x^*/u(x^*) \]

\[ x^*/u(x^*) \]

\[ 1 - \eta \]

\[ (0,0) \]

\[ (0,1) \]

\[ (1,0) \]

\[ (1,1) \]
$\rho = \text{fraction of limited information trades}$

Figure 3: Asymmetric Equilibria with Individual Punishments