Asset Pricing and the Welfare Effects of Monetary Policy

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Abstract

In this study I examine the welfare implications of monetary policy by constructing a novel New Keynesian model that properly accounts for asset pricing facts. Focusing on a simple interest rate rule, I find that substantially more (less) weight should be given to output (inflation) stabilization than in previous studies. In the model, a high average equity premium is associated with sizable welfare costs of recessions, rendering output stabilization strongly beneficial. Simultaneously, a low risk-free rate suggests that agents are very patient and forward looking, implying that monetary policy can more effectively and persistently reduce the average markup with higher inflation volatility.

Keywords: Asset Pricing, Long-Run Risk, Monetary Policy

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1 Introduction

As we continue to recover from the Great Recession, monetary policy makers are confronted with the decision of when to start raising interest rates and by how much. Underlying these choices is a concern about the trade-off between inflation and output. On one side there are “inflation hawks” who suggest that the primary goal of monetary policy should be price stability, with overwhelming concern for the stabilization of inflation—potentially at the expense of output. This course of action has historically been followed by the European Central Bank. On the other side, “inflation doves” suggest that monetary policy should place a greater emphasis on stabilizing output to decrease the severity of recessions, potentially at the expense of increased inflation. This more closely aligns with the approach of the Federal Reserve, which has a dual mandate to promote both maximum employment and stable prices. For policy makers, determining the proper emphasis to place on inflation versus output is a high-stakes decision that impacts everyone in the economy.

Existing studies of monetary policy predominantly find that stabilizing inflation is strongly preferred to stabilizing output. However, these studies suffer from the risk-free rate puzzle (Weil, 1989) and the equity premium puzzle (Mehra and Prescott, 1985), and thus they ignore key characteristics of financial data. The present study addresses this issue by incorporating financial data while evaluating the welfare implications of simple monetary policy rules that are functions of inflation and output. Specifically, I construct a model that is consistent with the historical risk-free rate, the historical equity premium, and the presence of long-run risk in productivity. Each of these features is important in capturing key aspects of the macroeconomy. The equity premium captures the welfare costs of recessions; the risk-free rate dictates the extent to which households and firms are patient and forward looking; and long-run productivity risk is crucial because of its major impact on the pricing decisions of forward-looking firms. Taken together, these three characteristics lead to policy recommendations that place significantly greater weight on output and less weight on inflation than is the case in the existing literature.

To explain this result, I investigate the trade-offs among three sources of welfare losses in the New Keynesian model: inflation volatility, output volatility, and the average markup. Output volatility and

1See Canzoneri et al. (2011) and Woodford (2010) for extensive overviews.
the average markup have greater impact within my model relative to previous studies because households and firms are assumed to have recursive preferences. Unlike constant relative risk aversion (CRRA) preferences, recursive preferences break the inverse relationship between risk aversion and the intertemporal elasticity of substitution (IES). This allows my model to replicate both the high equity premium and low risk-free rate observed in financial data. High levels of risk aversion combined with a high IES causes households to strongly dislike recessions while being much more patient and forward looking.

The combination of greater patience with long-run productivity shocks dramatically alters the influence of monetary policy on firms’ price setting and the average markup. The reduction in the average markup, a key determinant of welfare, provides substantial welfare benefits. To fully understand the mechanism that drives the importance of the markup, it is first necessary to recognize that in the context of my fully specified nonlinear model, risk-adjusted measures are what matter for pricing and consumer decisions. The likelihood of positive shocks is down-weighted, while negative shocks receive greater weight; this creates an asymmetry that is present in any standard macro model that does not linearize first-order conditions. This second-order-based asymmetry is crucial for risk characterization, the determination of patience, and ultimately the determination of optimal price setting.

This asymmetry also generates important implications for monetary policy when bad news for long-run growth is realized. Specifically, negative long-run news shocks to productivity lead patient (i.e., very forward-looking) firms to choose substantially higher prices, which pushes inflation persistently higher. Higher inflation in turn mechanically lowers relative prices set by firms in previous periods and erodes the real value of the average markup. The persistent reduction of the average markup works like a hedge when the household receives negative long-run news shocks. By allowing inflation to rise, monetary policy provides good long-run news thanks to the reduction in the average markup.

Across a wide range of values for the coefficient on inflation in the monetary policy rule, my model yields an interior solution of 3.0, where the benefits and costs to stabilizing inflation perfectly balance each other. With higher values on the coefficient of inflation in the monetary policy rule, inflation does not rise as much in response to a negative shock, which implies a smaller reduction in the markup. As a result of this smaller reduction, the steady-state markup rises with the inflation coefficient, placing a greater implicit tax on labor and capital as monetary policy increasingly stabilizes inflation. At values
greater than 3.0 for the inflation coefficient, I find that the costs of the higher markup outweigh the benefits associated with lower inflation volatility.

This result stands in contrast to the conclusions of a number of previous studies, which suggest that the reduction of inflation volatility should be the primary focus of monetary policy. I replicate this finding in my model with CRRA utility, in which the weight placed on fluctuations of inflation in the monetary policy rule is set to infinity. Results show that the markup channel is insignificant in the standard (second-order CRRA) setting because its movements are not as asymmetric and persistent as in the recursive preferences setting. In the CRRA utility setting, both the representative agent and firms are relatively impatient, and monetary policy is unable to persistently influence the price setting and average markup. Furthermore, not as much weight is placed on negative shocks, which reduces the asymmetry compared to the setting with recursive preferences.

Another justification for the low value of the inflation coefficient is the trade-off between real and nominal uncertainty. In my model, monetary policy can lower nominal uncertainty by placing a greater weight on inflation fluctuations, but this comes at the expense of the greater volatility of real variables such as output. This makes intuitive sense, because greater stabilization of inflation is achieved only through greater changes in real interest rates. The higher IES by definition makes households more willing to substitute consumption intertemporally due to changes in the real interest rates, which implies that monetary policy is more effective in altering real quantities. This channel is also present in a standard model but is much smaller (due to the restricted, lower IES) and is dominated in terms of welfare by the price dispersion channel. Holding all else constant, higher output and consumption volatility reduce welfare. Thus, this channel also contributes to the finding of an interior solution of 3.0 for the coefficient on inflation.

In addition to the lower value for the coefficient on inflation, my proposed model also yields a high value on the coefficient for output. Unlike most previous studies, I focus on output growth rather than the typical output gap (the deviation between the actual level and the flexible-price output level). I include output growth because it is easier to observe in real time and does not require policy makers to make decisions based on the unobservable flexible-price level of output. As pointed out in Sims (2013), focusing on output growth allows monetary policy to respond to recessions while also anchoring inflation.
expectations. These expectations are better anchored because monetary policy is implicitly promising to raise rates as the economy recovers and growth rates rise. Anchoring inflation expectations is imperative because current inflation depends on expectations of future inflation, and in the proposed model, firms are very patient and forward looking.

Placing a higher weight on the output growth benefits welfare because it reduces fluctuations in consumption. Moreover, it also pushes up inflation during negative long-run productivity shocks, which effectively lowers the markup. Matching the high equity premium in the data implies that households strongly dislike the recessions associated with output fluctuations, so that the costs and benefits equal at a coefficient value of 1.5 for output growth in the interest rate rule. This weight is three times greater than the optimal weight on the output growth of Sims (2013), who does not include long-run productivity shocks and instead uses habits. Results from my model with CRRA utility show that zero weight should be placed on the output growth; this is because the welfare costs of recessions are significantly lower and the costs coming from inflation volatility dominate.

1.1 Related Literature

This paper contributes to the sizable literature on monetary policy as well as a growing body of work on production-based asset pricing. A benchmark result in the monetary policy literature is that attention should be completely focused on inflation stabilization (Goodfriend and King, 1997; Rotemberg and Woodford, 1997; Woodford, 2001; King and Wolman, 1999; Benigno and Woodford, 2005; Khan et al., 2003; Siu, 2004; Schmitt-Grohé and Uribe, 2007; Kollmann, 2008). However, my asset pricing–driven approach suggests that strict inflation stabilization may be suboptimal. In contrast to Erceg et al. (2000); Giannoni and Woodford (2004); and Schmitt-Grohé and Uribe (2005), my results do not hinge on wage stickiness. To better isolate the welfare effects of asset pricing data, I do not include sticky wages or pure cost push shocks. Rather, I simply use a second-order approximation around a distorted steady state and

\(^{2}\)For instance, the Ramsey solution for Schmitt-Grohé and Uribe (2007) yields an optimal inflation volatility of 0.01\%. Moreover, in a similar study Kollmann (2008) finds the optimal value on the inflation coefficient in the interest rate rule is 8,660.
assume Epstein and Zin (1989) preferences for households. I resolve both the equity premium puzzle (Mehra and Prescott, 1985) and the risk-free rate puzzle (Weil, 1989) by introducing productivity long-run risk, in the spirit of Croce (2014) and more broadly of Bansal and Yaron (2004). This setting has not been explored to date in the monetary policy literature.

Long-run risk is a key driver of my qualitative results. Croce (2014), Beaudry and Portier (2004), Schmitt-Grohé and Uribe (2012), Kurmann and Otrok (2010), and Barsky and Sims (2011) have all found evidence of long-run news shocks to productivity, which explain a large fraction of business cycle fluctuations. Endogenous growth models have also been used to generate long-run consumption risk, as in Kung and Schmid (2011). The connection between endogenous growth, monetary policy and the term structure of interest rates has also been explored by Kung (2014). None of these studies has explored the welfare implications of long-run news for the trade-off between inflation and output stabilization.

To be clear, my study is not the first to incorporate recursive preferences when evaluating the welfare effects of monetary policy. Levin et al. (2008) use a very stylized, one-shock New Keynesian model with no capital in order to study the linearized Ramsey planner problem. They find that the planner is more risk averse and permits fewer fluctuations in output with more volatile inflation. In contrast to my study, there is no long-run risk and no quantification of optimal inflation volatility or the optimal coefficients on an interest rate rule.

An (2010) shows how to use perturbation methods to solve models with recursive preferences and simple monetary policy rules. Long-run risk is not incorporated, and his optimal policy coefficients and inflation volatility are low and almost identical to those in the setting of Schmitt-Grohé and Uribe (2007) with CRRA preferences.

Darracq Paries and Loublier (2010) come closest to my study by moving beyond a highly stylized setting to look at the effects of recursive preferences. Their study differs from mine in a number of ways: (1) habits are incorporated, (2) the IES is less than one, (3) wages are rigid, and (4) there is no growth and no long-run risk. They find that the optimal inflation volatility is less than 0.05 percent after including Epstein-Zin preferences. Finally, Benigno and Paciello (2014) incorporate doubts and ambiguity aversion and find that as doubts rise, monetary policy finds it optimal to further deviate from strict inflation targeting. However, while they target a high equity premium, they do not match the low risk-free rate.
None of the studies that incorporate recursive preference when evaluating the effects of monetary policy includes long-run risk shocks or attempts to match both the equity premium and the risk-free rate.  

Certain studies, such as Gavin et al. (2009), do look at the effects of monetary policy with permanent changes in the growth rate of productivity. However, they use log utility and do not match financial data. In their model, the central bank’s optimal policy is to fully stabilize the inflation rate at its steady state in order to completely eliminate the sticky price distortion.

The rest of the paper is organized as follows. Section 2 discusses the model and empirical motivation. Section 3 shows the optimal interest rate rules and explains the welfare channels driving the results. Section 4 explores the underlying dynamics of the welfare channels and Section 5 concludes.

2 Model and Empirical Motivation

In the discussion below I focus on two key differences of my model with respect to existing monetary policy analysis: the preferences and the productivity process. The economy consists of a continuum of identical households, a continuum of intermediate-goods firms, and a government that conducts monetary and fiscal policy. The structure of the model is a standard neoclassical growth model augmented with real and nominal frictions. The nominal friction is sticky prices. The real friction is monopolistic competition, which results in a markup of price over marginal costs. Monetary policy assumes full commitment to an interest rate rule that is a function of inflation, output growth, and the previous period’s interest rate. Fiscal policy raises lump-sum taxes to pay for exogenous expenditures.

Preferences. The households have Epstein-Zin preferences defined over consumption goods, \( c_t \), and leisure, \( 1-h_t \). These preferences exhibit a CES aggregate of current and future utility certainty equivalent weighted by \((1-\beta)\) and \(\beta\), respectively.

\[
v_t = \left\{ (1-\beta)(c_t(1-h_t)^{1-\gamma})^{1-\frac{1}{\psi}} + \beta (E_t[v_{t+1}^{1-\gamma}])^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}
\]

s.t.

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3The full model is described in appendix A.
\[ b_t + c_t + i_t + \tau_t = R_{t-1} \frac{b_{t-1}}{\pi_t} + w_t h_t + u_t k_t + \ddot{\phi}_t \]

The real value of debt is \( b_t \); \( c_t \) is consumption; \( i_t \) is investment; \( R_{t-1} \) is the risk-free rate; \( \pi_t \) is the inflation rate \( \frac{P_t}{P_{t-1}} \); \( \tau_t \) is the lump-sum tax; \( w_t \) is the real wage; \( h_t \) is labor hours; \( u_t \) is the rental rate of capital; \( k_t \) is capital; and \( \ddot{\phi}_t \) is profits.

Unlike CRRA preferences, Epstein-Zin preferences allow for the disentanglement of \( \gamma \), the coefficient of relative risk aversion, and \( \psi \), the elasticity of intertemporal substitution. When \( \frac{1}{\psi} = \gamma \), the utility collapses to CRRA preferences, with additively separable expected utility both in time and state. When \( \gamma > \frac{1}{\psi} \), the agent prefers early resolution of uncertainty, so the agent dislikes shocks to long-run expected growth rates.

**Stochastic Discount Factor.** The stochastic discount factor (SDF) represents the intertemporal marginal rate of substitution for consumption. It is the major focal point in the forward-looking New Keynesian model, as it translates the value of future income/profits to the present. Given the fact that monetary policy relies on long-term expectations to influence the economy, the functional form is very relevant for all of the household’s and firm’s intertemporal maximization decisions:

\[
M_{t,t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{(1-\frac{1}{\psi})-1} \left( \frac{1 - h_{t+1}}{1 - h_t} \right)^{(1-i)(1-\frac{1}{\psi})} \frac{P_t}{P_{t+1}} \left[ \frac{v_{t+1}}{E_t[v_{t+1}]^{1-\gamma}} \right]^{\frac{1}{\psi}-\gamma} \tag{1}
\]

The last part of equation 1 is unique to recursive preferences. This factor captures news regarding the continuation value of the representative agent. Future utility, as represented by the continuation value, is very sensitive to long-run news and this allows for greater variation of the stochastic discount factor without the need for excessive levels of risk aversion. Compared to CRRA preferences, the last factor implies a significantly higher weight on negative outcomes as marginal utility rises. This results in endogenous asymmetric responses to shocks because agents are more concerned with negative long-run news.
Productivity. The law of motion of the productivity process captures both short-run and long-run productivity risks:

\[
\frac{\log A_{t+1}}{A_t} \equiv \Delta a_{t+1} = \mu + x_t + \sigma_a \epsilon_{a,t+1},
\]

\[
x_{t+1} = \rho x_t + \sigma_x \epsilon_{x,t+1},
\]

\[
\begin{bmatrix}
\epsilon_{a,t+1} \\
\epsilon_{x,t+1}
\end{bmatrix} \sim i.i.d. N\left(\begin{bmatrix}
0 \\
0
\end{bmatrix}, \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}\right), \quad t = 0, 1, 2, \ldots.
\]

According to the above specification, short-run productivity shocks, \(\epsilon_{a,t+1}\), affect contemporaneous output directly, but they have no effect on future productivity growth. Shocks to long-run productivity, represented by \(\epsilon_{x,t+1}\), carry news about future productivity growth rates but do not affect current output.

2.1 Welfare Analysis: Three Major Channels

In this section I describe the three key sources of welfare losses in the New Keynesian model: inflation volatility, the average markup, and output volatility. In short, inflation volatility causes relative price dispersion and inefficient production; the average markup acts as an implicit tax on factors of production; and output volatility reduces concave utility. The average markup and output volatility will have a greater impact within my model relative to previous studies. The reasons for this will be discussed in the following section.

2.1.1 Inflation Volatility: Price Dispersion

From a welfare perspective, price dispersion is a crucial characteristic of the Calvo-based time-dependent sticky price models. Each period, a fraction \(\alpha \in [0, 1]\) of randomly picked firms is not allowed to optimally set the nominal price of the good they produce. The remaining \(1 - \alpha\) firms choose \(P_{t,t}\) to maximize the expected present discounted value of profits.

Price dispersion arises when a subset of firms finds it optimal to choose a different price relative to the remaining firms who are unable to update. Differences in pricing across firms are important because
it is assumed that monopolistically competitive firms choose a price and agree to supply the quantity demanded. The quantity demanded for each firm \((Y_{i,t})\) follows a downward-sloping demand schedule based on the firm’s price \((P_{i,t})\) and the elasticity of substitution across goods, \(\eta\):

\[
Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} Y_t
\]

Given that supply must equal demand at the firm level,

\[
K_{i,t}^\theta(A_{t}N_{i,t})^{1-\theta} = \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} Y_t
\]

One can then aggregate across all firms to arrive at

\[
K_t^\theta(A_tN_t)^{1-\theta} = \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} di \cdot Y_t.
\]

Defining the resource cost of price dispersion as \(s_t = \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} di\), one can see that its effect is similar to a negative aggregate productivity shock, as it increases the amount of labor and capital required to produce a given level of output.

\[
\frac{1}{s_t} K_t^\theta(A_tN_t)^{1-\theta} = Y_t
\]

This happens because firms with relatively low prices produce an inefficiently high quantity to meet relatively high demand, while the opposite occurs for the high-price firms. As shown by Schmitt-Grohé and Uribe (2007), \(s_t \geq 1\), and \(s_t\) is equal to one when there is no price dispersion. Furthermore, price dispersion can be recursively decomposed into the following equation:

\[
s_t = (1 - \alpha)(p_t^*)^{-\eta} + \alpha \pi_t^n s_{t-1}
\]

where \(p_t^* = \frac{P^*}{P_t}\) represents the relative price of the optimizing firm at time \(t\). Price dispersion is an increasing function of \(\alpha\), the probability that firms are unable to update their prices; \(\eta\), the elasticity of substitution across goods; and \(\pi\), the steady-state rate of inflation.\(^4\)

\(^4\)If prices were flexible \((\alpha = 0)\), all firms would be able re-optimize every period, and there would be no price
The main driver of price dispersion within the model is inflation volatility. Given the elasticity of substitution \((\eta \geq 1)\), more volatile inflation will lead to higher price dispersion on average due to the Jensens Inequality \((\pi_t^\eta)\) in equation 2. This effect is absent in a first-order approximation.

**Price Dispersion and the Stochastic Discount Factor.** The stochastic discount factor affects price dispersion through the relative price of the optimizing firm. The forward-looking firm chooses a price to maximize expected profits while knowing that their price may become stuck for multiple periods. The price is a markup over a ratio of the present discounted value of future marginal costs and the present discounted value of future marginal revenues:

\[
p_t^* = \frac{\eta}{\eta - 1} \frac{E_t \sum_{j=0}^{\infty} \alpha^j M_{t,t+j} \pi_{t,t+j}^\eta y_{t+j} m_{t+j}}{E_t \sum_{j=0}^{\infty} \alpha^j M_{t,t+j} \pi_{t,t+j}^{\eta-1} y_{t+j}}
\]

where \(\frac{\eta}{\eta - 1}\) is the nonstochastic steady-state markup, \(\alpha\) is the probability of not being able to update next quarter, \(M_{t,t+j}\) is the nominal stochastic discount factor between \(t\) and \(t+j\), and \(m_{t+j}\) is the marginal costs.

The stochastic discount factor is an important component of equation 3, the pricing equilibrium condition. The use of financial data to discipline the dynamics of the pricing equilibrium condition can have a significant impact on the predictions of the model. Specifically, matching the low risk-free rate in the data makes the firms more patient and forward looking, placing more weight on future economic conditions as compared to the present. This is especially important in a world with highly persistent shocks, such as the long-run risk shocks to productivity. Although these shocks are quantitatively small, their persistence combined with the inability of firms to update each period tends to magnify firms’ pricing decisions. Firms also use expected inflation rates to discount future marginal costs and marginal revenues, because inflation erodes the markup over time. Higher expected inflation places greater weight on future marginal costs, which again is important in a setting with small but highly persistent productivity shocks.

dispersion \((s = 1)\). A higher elasticity of substitution across intermediate goods moves the economy closer to perfect competition (where \(\eta = \infty\)). This results in greater costs of price dispersion because households are more willing to switch from the high-price good to the low-price good, causing aggregate production to become more inefficient because the low-price firm must meet the higher demand. Lastly, price dispersion increases with steady-state inflation because firms that are resetting their prices optimally choose a higher price than the existing price level.
The greater weight on future marginal costs (versus future marginal revenues) can be seen by the different exponents on expected inflation ($\eta$ vs. $\eta - 1$) in the numerator and denominator.

2.1.2 The Markup: Implicit Tax

Monopolistic competition allows firms to charge a price that is higher than their marginal costs. The difference between the price and the marginal costs is known as the average markup. From a welfare perspective, the average markup is very important because it acts as a wedge between factor prices and marginal products, which causes inefficiently low levels of labor, capital, and output. Hence, the average markup is akin to an implicit tax on capital and labor.

The implicit tax can be seen in the firm’s demand for labor and capital:

$$MPL_t = \mu_t w_t \quad MPK_t = \mu_t u_t$$

where $\mu_t$ is the markup. Higher markups imply lower real wages and rental rates of capital, and this affects both labor and capital supply. For example, combining the labor supply equation with labor demand yields

$$\frac{1 - \iota}{\iota} \frac{c_t}{1 - h_t} = w_t = \frac{MPL_t}{\mu_t}$$

where $\iota$ is the share of consumption in the consumption-leisure bundle. This shows that a higher markup reduces the amount of labor supplied. Moreover, given that the slope of the labor supply is both positive and increasing, increases in the markup result in greater and greater decreases in the quantity of labor supplied. This fact is important in explaining the reasons that the costs of higher markups dominate the benefits of reduced price dispersion as monetary policy increasingly stabilizes inflation.

**Markup decomposition.** As outlined in King and Wolman (1996), the average markup of price over marginal cost can be decomposed into two components:

$$\mu_t = \left( \frac{P_t}{P_t^*} \right) \left( \frac{P_t^*}{MC_t} \right)$$
where \( \frac{P^*_t}{P_t} \) is defined as the price adjustment gap and \( \frac{P_t}{MC_t} \) is defined as the marginal markup. The price adjustment gap is just the inverse of the relative price of the optimizing firm at time \( t \) and the marginal markup is the ratio of price to marginal cost for firms allowed to adjust their price in period \( t \).

If inflation increases, it must be that \( P^*_t \) is greater than \( P_t \), and the price adjustment gap falls. The price adjustment gap captures the notion that higher inflation automatically decreases relative prices set by firms in previous periods and decreases the real value of the average markup. King and Wolman (1996) find that if the average markup only consisted of the price adjustment gap, an increase in inflation from 5 to 10 percent would raise output permanently by 7 percent.

**Marginal markup.** The marginal markup captures the markup that re-optimizing firms are able to charge. This can be shown to depend positively on expected future inflation as firms choose a higher price when they are allowed to update their prices. Firms will choose a higher price because they are concerned that they will get “stuck” (i.e., be unable to re-optimize) and that inflation will erode their relative price. Erosion of a firm’s relative price causes households to substitute toward their good as it becomes relatively less expensive, and this can be problematic as the firm is required to meet demand by securing more labor at higher costs. Inflation will also erode the real value of any markup established at time \( t \), so that per-unit profits will decline for as long as the firm is stuck.

Ascari and Sbordone (2013) show that as trend inflation increases, the increase in the marginal markup dominates so that the overall average markup also rises. As the average markup rises, output declines along with welfare. This raises the question of which channel dominates in a setting with zero trend inflation or a zero inflation target. In a comparison across different policy rules, I find that the price adjustment gap channel dominates the marginal markup for negative productivity shocks, so that the average markup falls as inflation volatility increases. This is not surprising because under the benchmark calibration, most firms are unable to update their prices each period.\(^5\)

\(^5\)Under the benchmark calibration, 75% of the firms are unable to update in any given period, which is equivalent to firms getting stuck on average for 12 months. This duration is in the middle of empirical estimates and will be further discussed in the calibration section.
The Markup and the Stochastic Discount Factor. The stochastic discount factor significantly impacts both the price adjustment gap and the marginal markup. Matching the low risk-free rate in the data makes the firms more patient and forward looking, raising concerns about the erosion of future markups. This suggests that more patient firms will choose higher marginal markups in response to persistently higher expected inflation. At the same time, the price adjustment gap will also decline as firms choose prices further away from the existing price level. Results show that the price adjustment gap dominates in the benchmark model.  

2.1.3 The Output Gap: Consumption-Leisure Volatility

The output gap is defined as the deviation between the actual level of output and its natural level (the level of output in the absence of nominal rigidities). In the typical New Keynesian setup, a second-order approximation to household welfare gives rise to a loss function in the variances of the output gap and inflation. The loss function is then used to evaluate various monetary policy rules. However, the reason the output gap enters the loss function is due to the simplifying assumption that consumption is equal to output.

In my model, output of final goods and services goes toward not only consumption but also investment and government expenditures. In addition, agents care not about consumption but rather a consumption-leisure bundle. Therefore, I focus on the bundle rather than output when evaluating the effects of policy on welfare. With regard to the welfare costs of recessions, the magnitude is much greater in the asset pricing–oriented New Keynesian model, as indicated by the higher equity premium. The higher welfare costs suggest that greater weight should be placed on stabilizing output and the consumption-leisure bundle, in contrast to what is the case in existing studies that do not match the equity premium.

\[ \chi_{c} = 1 - \left( \frac{v_{i}}{v_{j}} \right)^{\iota} \] where \( \iota \) is the share of consumption in the consumption-leisure bundle and \( v_{k}^{i} \) represents the lifetime welfare based on policy \( k = i, j \).
2.2 Empirical Motivation for Welfare Channels

The existing literature on optimal monetary policy finds that movement toward a hawk policy has no effect on the steady-state markup. However, I show below that the theoretical prediction of a nil effect is not supported by the data. In other words, CRRA preferences are not only inconsistent with asset pricing facts but also with the empirical evidence on the average markup when monetary policy increasingly stabilizes inflation. In addition, CRRA models do not predict a meaningful trade-off between reducing nominal uncertainty at the expense of higher real uncertainty. This lack of a trade-off is also inconsistent with empirical evidence discussed in more detail below.

Compared to empirical estimates of the Taylor rule, the optimal coefficients on inflation in the interest rate rule as determined by the existing literature are often orders of magnitude higher. This is because the CRRA models are not fully taking into account the explicit costs of the higher markup and greater consumption volatility implied by the data, so the benefits of reducing inflation volatility dominate all other channels. In contrast, my novel asset pricing-oriented New Keynesian model is consistent with the empirical evidence on the average markup and the real-nominal trade-off. As this model endogenously captures these channels as reflected in the data, the optimal policy coefficients are much closer to empirical estimates for the interest rate rule.

In the sections that follow, I provide a discussion along with empirical evidence on the markup and the trade-off involved in reducing nominal volatility at the expense of increased consumption volatility. I then estimate interest rate rules over the pre-Volcker and post-Volcker periods to connect the observed behavior of the Federal Reserve with the markup channel and real-nominal trade-off.

The Markup. The empirical evidence of the association of a higher markup with greater inflation stabilization was first found by Benabou (1992). Using US data on the retail trade sector, he finds that inflation has significantly negative effects on the markup. Other studies have focused on the inverse of the labor share, as this can be shown to be theoretically proportional to the markup if the production function is Cobb-Douglas. Nekarda and Ramey (2013) show that an upward trend of the markup began in the early 1980s. The rise in the markup coincides with the more aggressive role of monetary policy in stabilizing inflation. This can be seen in figure 1, which plots data on the average markup according to
Fig. 1: Historical Average Markup: Higher Markup Since 1980

The data depicted in this figure are based on the inverse of the labor share of income, as computed by the Bureau of Labor Statistics. The inverse of the labor share of income can be shown to be theoretically equal to the average markup when the production function is Cobb-Douglas. Splitting the time series into 1950–1980 and 1980–2007, the solid red line reflects the increase in the mean of the markup for the post-Volcker period (1980-2007), a period in which inflation was stabilized to a much greater extent. This figure provides empirical evidence for the theoretical result that increasingly stabilizing inflation is associated with a higher markup, which is relevant for my welfare analysis.

the Bureau of Labor Statistics records dating back to 1950. During the post-Volcker period (1980–2007), the mean of the markup increased as inflation was stabilized to a greater extent. Other studies such as Alcala and Sancho (2000) and Raurich et al. (2012) have shown using various measures that the markup has risen since 1980.\(^8\)

The rise in the markup due to greater inflation stabilization is consistent with the theoretical dynamics of my asset pricing-oriented New Keynesian model. Specifically, figure 1 shows that the mean of the markup rises between 2.5–3% when moving from pre-Volcker to post-Volcker. Similarly, my general

\(^8\)The mechanism for the relationship between inflation and labor share is simple. As pointed out by Alcala and Sancho (2000), accelerated inflation is correlated with higher employment, higher employment leads to greater bargaining power, and greater bargaining power is correlated with lower markups. In my New Keynesian model inflation is largely due to output being pushed above the natural level, which reduces the real average markup because labor costs rise but most firms are unable to update their prices.
equilibrium model predicts that the average markup will also rise 3%, from 12% to 15%, as monetary policy increasingly stabilizes inflation.\(^9\) Therefore, the theoretical effect of monetary policy on the markup lines up with the empirical evidence. Moreover, the volatility of the markup is 3% under the optimal rule within my model, while in the data, the markup volatility is historically between 5% and 6%. This suggests that my model is conservative with respect to the observed volatility of the markup and that the relative importance of the markup channel for welfare is not driven by exaggerated or implausible dynamics.

**Real-Nominal Trade-Off.** The real-nominal trade-off suggests that as monetary policy increasingly stabilizes inflation, nominal uncertainty falls while real uncertainty increases. This trade-off arises as an endogenous outcome of my general equilibrium model. The intuition is that as the central bank increasingly targets inflation, this induces greater changes in both nominal rates and real rates (due to sticky prices), which causes higher real consumption volatility. This channel is also present in the endogenous growth model of Kung (2014). Bansal and Shaliastovich (2013) provide empirical evidence that real uncertainty is higher relative to nominal uncertainty for the post-Volcker period.

**Empirical Interest Rate Rules.** A number of studies suggest that simple interest rate rules can characterize the behavior of the Federal Reserve over various time periods. Taylor (1993) proposes a simple rule of the Federal Funds Rate as a function of inflation and the output gap around a trend. Weights of 1.5 and 0.5 are assumed on inflation and output, respectively, and seem to roughly capture the behavior of monetary policy. Other papers have since evaluated the empirical fit of simple interest rate rules, including Judd and Rudebusch (1998), Taylor (1999), Clarida et al. (1998) and Orphanides (2003). In contrast to many of these studies, I choose to focus on output growth rather than the output gap because there is greater consensus in terms of its measurement.\(^{10}\)

I conduct my own estimation of the interest rate reaction function using data from 1983Q1 to 2002Q4. The starting and ending dates are chosen to match Coibion and Gorodnichenko (2011). I regress the

\(^9\)The rise in the markup within the asset pricing-oriented New Keynesian model is further demonstrated in section 3, figure 2.

\(^{10}\)Other studies have estimated interest rate rules that include output growth, including Ireland (2004), Carlstrom and Fuerst (2012), and Coibion and Gorodnichenko (2011).
Table 1: Estimated Interest Rate Reaction Function

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_r$</th>
<th>$\alpha_\pi$</th>
<th>$\alpha_{\Delta y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parsimonious Regression</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1955:1–1982:4</td>
<td>0.90</td>
<td>1.60</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>(0.41)</td>
<td>(0.55)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>1983:1–2002:4</td>
<td>0.92</td>
<td>2.46</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>(0.35)</td>
<td>(0.82)</td>
<td>(0.26)</td>
</tr>
<tr>
<td><strong>Coibion &amp; Gorodnichenko (2012)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1983:1–2002:4</td>
<td>0.93</td>
<td>2.20</td>
<td>1.56</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.40)</td>
<td>(0.39)</td>
</tr>
</tbody>
</table>

This table shows results of regressions of the Federal Funds Rate on the lagged rate ($\alpha_r$), inflation based on the GDP price deflator ($\alpha_\pi$), and the growth of real GDP ($\alpha_{\Delta y}$). Results from Coibion and Gorodnichenko (2011) over a similar time period are shown for sake of comparison. Their method uses real-time measures of expected inflation and expected output growth based on Greenbook forecasts from the Federal Open Market Committee (FOMC). Standard errors are listed in parentheses.

Federal Funds Rate on its lagged term, the demeaned inflation rate according to the GDP price deflator, and the demeaned output growth rate of real GDP:

$$R_t = \alpha_r \cdot R_{t-1} + (1 - \alpha_r) (\alpha_\pi \cdot (\pi_t - \pi^*) + \alpha_{\Delta y} \cdot (\Delta y_t - \Delta y^*)) + \epsilon_t$$

where $\alpha_r$ captures partial adjustment or inertia, $\alpha_\pi$ is the weight on inflation, $\alpha_{\Delta y}$ is the weight on output growth, and the starred variables indicate means that act as proxies for the targets.

The results of my regression for post-1983 are in line with Coibion and Gorodnichenko (2011), as shown in table 1. Coibion and Gorodnichenko (2011) solely focus on characterizing Federal Reserve policy, which leads them to use a more richly parameterized empirical model. Here, I characterize this behavior in a more parsimonious and straightforward manner. The small difference in results can be attributed to these modeling differences, along with Coibion and Gorodnichenko (2011)’s use of expected inflation and expected output growth based on the Greenbook forecasts from the Federal Open Market Committee (FOMC).

In both instances, it is clear that the Federal Reserve did not target inflation as much for the pre-Volcker period. With less weight placed on inflation, higher inflation volatility was associated with lower
average markups, which is an important channel for my welfare analysis. Most importantly, the values on the inflation coefficient for the post-Volcker regressions are much lower than the optimal monetary policy literature frequently suggests. For example, the model of Schmitt-Grohé and Uribe (2007) in a similar setting with no long-run risk and CRRA preferences yields an optimal inflation coefficient of 332 with no weight on output. Likewise, Kollmann (2008) finds an optimal inflation coefficient of 8,660.

The optimal policy in my asset pricing-oriented New Keynesian model places a weight on inflation that is within the standard error of the point estimate based on my post-Volcker regression. Also, the optimal weight placed on the output growth is higher than that estimated in both of my regressions. Thus, this model, with its high welfare costs of recessions, suggests that it would have been optimal for the Federal Reserve to respond more to output growth over the estimated time period.

Overall, CRRA preferences are inconsistent not only with asset pricing facts but also with the empirical evidence described in this section. Once Epstein-Zin preferences are introduced, the empirical evidence on the markup and consumption volatility is reproduced in a general equilibrium setting and the implications for monetary policy change dramatically.

2.3 Calibration

In the proposed model, I calibrate the time period to a quarterly frequency. I then annualize the moments and focus on matching the behavior of macroeconomic variables over the long sample of US data from 1929–2008. Data on consumption and investment are from the Bureau of Economic Analysis (BEA). Fiscal policy variables such as government spending and steady-state debt are taken from Schmitt-Grohé and Uribe (2007). The parameters described below are listed in table 2.

For the New Keynesian parameters, the markup due to monopolistic competition is set to 15%, which is in line with previous studies (Bils and Klenow, 2002). Firms are assumed to re-optimize their prices every 12 months, which is in the middle of empirical estimates that range from 6 to 18 months (see Altig et al. (2011)). With regard to fiscal policy, steady-state government purchases make up 17% of GDP, and the steady-state debt-GDP ratio is set to the historical average of 44%, following Schmitt-Grohé and Uribe (2007). Taxes are collected by lump sum to pay for an exogenous expenditure stream. The
### Table 2: Model Features and Parameter Values

<table>
<thead>
<tr>
<th></th>
<th>Recursive</th>
<th>CRRA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High IES</td>
<td>Low IES</td>
</tr>
<tr>
<td></td>
<td>High RA</td>
<td>High RA</td>
</tr>
<tr>
<td><strong>Preference parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.9955</td>
</tr>
<tr>
<td>Effective risk aversion</td>
<td>$\gamma$</td>
<td>10</td>
</tr>
<tr>
<td>Intertemporal elasticity of substitution</td>
<td>$\psi$</td>
<td>2.0</td>
</tr>
<tr>
<td>Leisure weight</td>
<td>$\phi$</td>
<td>0.35</td>
</tr>
<tr>
<td><strong>Technology parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital share</td>
<td>$\theta$</td>
<td>0.25</td>
</tr>
<tr>
<td>Quarterly depreciation rate</td>
<td>$\delta$</td>
<td>1.725%</td>
</tr>
<tr>
<td><strong>Productivity parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk exposure of new investment</td>
<td>$\phi_0$</td>
<td>0</td>
</tr>
<tr>
<td>Average quarterly growth rate</td>
<td>$\mu$</td>
<td>0.005</td>
</tr>
<tr>
<td>Volatility of short-run risk</td>
<td>$\sigma_a$</td>
<td>0.0055</td>
</tr>
<tr>
<td>Volatility of long-run risk</td>
<td>$\sigma_x$</td>
<td>0.15•$\sigma_a$</td>
</tr>
<tr>
<td>AR(1) of expected growth</td>
<td>$\rho$</td>
<td>0.98</td>
</tr>
<tr>
<td><strong>New Keynesian parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price elasticity of demand</td>
<td>$\eta$</td>
<td>7</td>
</tr>
<tr>
<td>Probability firm cannot change price</td>
<td>$\alpha$</td>
<td>75%</td>
</tr>
<tr>
<td><strong>Policy parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steady-state debt to GDP</td>
<td>$S_B$</td>
<td>0.44</td>
</tr>
<tr>
<td>Steady-state $\frac{G}{Y}$</td>
<td>$S_G$</td>
<td>0.17</td>
</tr>
<tr>
<td>Monetary policy inflation coefficient</td>
<td>$\alpha_\pi$</td>
<td>1.5</td>
</tr>
<tr>
<td>Output growth gap coefficient</td>
<td>$\alpha_{\Delta y}$</td>
<td>0.75</td>
</tr>
<tr>
<td>Inertia coefficient</td>
<td>$\alpha_r$</td>
<td>0.8</td>
</tr>
</tbody>
</table>

The four models were calibrated to minimize the distance between the moments implied by the model and the moments taken from the data. The moments to match were the volatilities of consumption and investment growth, the level of the risk-free rate, and if possible the equity premium.

Persistence and standard deviation of the government spending shocks follow Croce et al. (2012), but given the assumed nondistortionary nature of taxes, they essentially have a nil effect on the results.

For monetary policy, the default interest rate rule used for calibration includes an inertia coefficient of 0.8 and an output growth coefficient of 0.75. The parameters chosen for these coefficients are set to match inflation dynamics and the interest rate serial correlation observed in the data. An inertia
coefficient of 0.8 is consistent with empirical quarterly estimates, which set it as high as 0.9. Without the output growth, inflation in the model is positively correlated with permanent productivity shocks, which is counterfactual. The incorporation of the output growth causes inflation to fall with good productivity shocks and rise with bad productivity shocks. The inflation coefficient is set to 1.5 to match the original work of Taylor (1993).

Production and Preference Parameters. The parameters for effective risk aversion ($\gamma = 10$) and intertemporal elasticity of substitution ($\psi = 2$) are consistent with the estimates of Bansal et al. (2007), Bansal et al. (2010), and Colacito and Croce (2011). The capital share ($\alpha = 0.25$) and the quarterly depreciation rate of physical capital ($\delta = 0.01725\%$) are consistent with the labor share of income in the data and with the quarterly depreciation rate of 1.5 to 4% observed in the real business cycle (RBC) literature. I calibrate $\mu$ at 2% per year, consistent with the average annual real growth rate of the US economy. I set the persistence parameter on long run risk at $\rho = 0.98$, which is close to the point estimate of Croce (2014).

The subjective discount factor is set to be consistent with the low risk-free rate observed in the data. Labor is endogenous and set to match the level of hours in the data, $\iota = 0.35$. I set $\sigma_a$ to match the standard deviation of consumption. The smaller long-run shock is set to $0.15 \cdot \sigma_a$ as estimated by Croce (2014).

Free Parameters and Moments Matched. The free parameters above are the subjective discount factor ($\beta$), the depreciation rate for capital ($\delta$), the volatility of short-run risk ($\sigma_a$), and the persistence parameter on long-run risk ($\rho$). These parameters are chosen within a grid of values accepted by the literature in order to maximize the model’s ability to reproduce the moments of interest, which are the mean of the real risk-free rate, the levered excess return, and the volatilities of consumption and investment growth. I minimize the distance between the moments implied by the model and the moments

\[^{11}\text{See Swanson (2012) for a discussion of risk aversion when leisure is present.}\]
The four models are calibrated to minimize the distance between the moments implied by the model and the moments taken from the data. The minimization is done by searching over a grid of values: $\beta = [0.995, 1]$, $\delta = [0.15, 0.4]$, $\sigma_a = [0.005, 0.02]$, and $\rho = [0.92, 0.98]$. The moments to match were the volatilities of consumption and investment growth, the level of the risk-free rate, and the equity premium, if possible. For recursive preferences, settings tested were High RA ($\gamma = 10$), High IES ($\psi = 2$), and Low IES ($\psi = 0.2$). For CRRA preferences, settings tested were High RA ($\gamma = 10$) and High IES ($\psi = 2$). All entries for the models are obtained from repetitions of small samples. Data refer to the US and include pre-World War II observations (1930–2012). Quarterly calibrations are reported in table 2. Excess returns are levered by a factor of three, consistent with García-Feijóo and Jorgensen (2010). Note that with CRRA preferences, a low risk free rate and high equity premium are not possible given the inverse link of RA and IES.

The four models are calibrated to minimize the distance between the moments implied by the model and the moments taken from the data. The minimization is done by searching over a grid of values: $\beta = [0.995, 1]$, $\delta = [0.15, 0.4]$, $\sigma_a = [0.005, 0.02]$, and $\rho = [0.92, 0.98]$. The moments to match were the volatilities of consumption and investment growth, the level of the risk-free rate, and the equity premium, if possible. For recursive preferences, settings tested were High RA ($\gamma = 10$), High IES ($\psi = 2$), and Low IES ($\psi = 0.2$). For CRRA preferences, settings tested were High RA ($\gamma = 10$) and High IES ($\psi = 2$). All entries for the models are obtained from repetitions of small samples. Data refer to the US and include pre-World War II observations (1930–2012). Quarterly calibrations are reported in table 2. Excess returns are levered by a factor of three, consistent with García-Feijóo and Jorgensen (2010). Note that with CRRA preferences, a low risk free rate and high equity premium are not possible given the inverse link of RA and IES.

The distance is measured by

$$\min_a \zeta(a) = [\hat{F}_T - f(a)]'[\hat{F}_T - f(a)]$$

where $f(a)$ is the vector of moments generated by the model, and $\hat{F}_T$ is the moments in the data. The minimization is done by searching over a grid of values for $a$: $\beta = [0.995, 1]$, $\delta = [0.15, 0.4]$, $\sigma_a = [0.005, 0.02]$, and $\rho = [0.92, 0.98]$. Note that in the data, excess returns are levered. Hence, I use the following excess return: $R_{x,t}^{LEV} = \chi_{LEV}(R_t^K - R_t^f)$. The calibration of $\chi_{LEV}$ is set to 3 to match the debt-equity ratio in the data along with the degree of operating leverage as estimated in García-Feijóo and Jorgensen (2010).

**Simulation.** The policy rules are numerically computed using second-order approximations from Dynare++. The simulations consist of random draws of the two productivity shocks (short-run and
long-run) and a fiscal shock (government spending). The number of periods is 200 (the first 100 are discarded) and the number of simulations is 100. The moments for the models are listed in table 3. The preferred High IES ($\psi = 2$), High RA ($\gamma = 10$) model comes closest to matching the low risk-free rate, the high equity premium, the smooth volatility of consumption growth, and volatile investment growth. Matching data on both macroeconomic aggregates and asset pricing facts imposes joint structural restrictions on both the quantity and price of risk in the data.

3 Optimal Interest Rate Rules

In the following section, I describe the characteristics of the interest rate rules that yield the highest welfare. I then restrict the model to a setting with only long-run shocks and a setting with only short-run shocks to provide a deeper understanding of the underlying dynamics. Following this, I examine the effect of the inflation coefficient and output growth coefficient on the three major channels of welfare.

To obtain the simple monetary policy rule that yields the highest welfare, I search across a grid for the inflation coefficient, $\alpha_\pi$, the output growth coefficient, $\alpha_{\Delta y}$, and the inertia coefficient, $\alpha_r$. Their respective grids are $(1, \infty)$, $(-\infty, \infty)$, and $(-\infty, \infty)$. I constrain $\alpha_\pi > 1$ in order to be consistent with the Taylor principle. With $\alpha_\pi > 1$, the real interest rate rises with inflation, and this ensures determinacy. The rule is formulated as follows:

$$\hat{R}_t = \alpha_r \cdot \hat{R}_{t-1} + (1 - \alpha_r)(\alpha_\pi \cdot \hat{\pi}_t + \alpha_{\Delta y} \cdot \Delta \hat{y}_t)$$

(4)

The variables denoted with a hat are log deviations from steady state. The first two panels of table 4 list the optimal inflation volatility and inflation coefficient for each model, and the third panel shows the optimal output growth coefficient. The bottom panel shows the relative welfare gain (in the model that matches asset prices) for the optimal policy compared to the policy that completely stabilizes inflation. The columns are split into settings based on the sources of the shocks in order to capture the role of long-run risk.
Table 4: Optimal Coefficients, Volatility, and Welfare

<table>
<thead>
<tr>
<th>Preferences</th>
<th>RA</th>
<th>IES</th>
<th>All Shocks</th>
<th>LRR only</th>
<th>SRR only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recursive</td>
<td>High</td>
<td>High</td>
<td>0.42%</td>
<td>0.39%</td>
<td>0.05%</td>
</tr>
<tr>
<td>Recursive</td>
<td>High</td>
<td>Low</td>
<td>0.16%</td>
<td>0.18%</td>
<td>0.06%</td>
</tr>
<tr>
<td>CRRA</td>
<td>High</td>
<td>Low</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>CRRA</td>
<td>Low</td>
<td>High</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Preferences</th>
<th>RA</th>
<th>IES</th>
<th>All Shocks</th>
<th>LRR only</th>
<th>SRR only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recursive</td>
<td>High</td>
<td>High</td>
<td>3</td>
<td>3</td>
<td>1.75</td>
</tr>
<tr>
<td>Recursive</td>
<td>High</td>
<td>Low</td>
<td>16</td>
<td>11</td>
<td>3.25</td>
</tr>
<tr>
<td>CRRA</td>
<td>High</td>
<td>Low</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
</tr>
<tr>
<td>CRRA</td>
<td>Low</td>
<td>High</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Preferences</th>
<th>RA</th>
<th>IES</th>
<th>All Shocks</th>
<th>LRR only</th>
<th>SRR only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recursive</td>
<td>High</td>
<td>High</td>
<td>1.5</td>
<td>1.625</td>
<td>0.25</td>
</tr>
<tr>
<td>Recursive</td>
<td>High</td>
<td>Low</td>
<td>3</td>
<td>3.25</td>
<td>0.25</td>
</tr>
<tr>
<td>CRRA</td>
<td>High</td>
<td>Low</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>CRRA</td>
<td>Low</td>
<td>High</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Rec. Pref., High RA, High IES:

<table>
<thead>
<tr>
<th></th>
<th>All Shocks</th>
<th>LRR only</th>
<th>SRR only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Welfare Gain</td>
<td>0.07%</td>
<td>0.07%</td>
<td>0.00037%</td>
</tr>
<tr>
<td>Consumption Equivalent Units</td>
<td>0.19%</td>
<td>0.20%</td>
<td>0.001%</td>
</tr>
<tr>
<td>Perpetuity of Welfare Gain</td>
<td>$3,255</td>
<td>$3,427</td>
<td>$17.13</td>
</tr>
</tbody>
</table>

This table shows the optimal inflation volatility, optimal inflation coefficient, and optimal output growth coefficient across different shocks and different utility specifications. RA is risk aversion, IES is intertemporal elasticity of substitution, and CRRA is constant relative risk aversion. For recursive preferences, settings tested were High RA ($\gamma = 10$), High IES ($\psi = 2$), and Low IES ($\psi = 0.2$). For CRRA preferences, settings tested were High RA ($\gamma = 10$) and High IES ($\psi = 2$). Quarterly calibrations are reported in table 2. The bottom panel shows the relative welfare gain compared to the policy that places infinite weight on inflation. The consumption equivalent units are computed as in An (2010). The perpetuity of the welfare gain is computed by assuming an interest rate of 2% and real consumption expenditures per capita of $34,270 per year. The consumption equivalent unit percentage is multiplied by $34,270 and divided by the interest rate to arrive at the value of the perpetuity.
Inflation Volatility. The setting with recursive preferences, high risk aversion ($\gamma = 10$), and a high IES ($\psi = 2$) yields the highest optimal inflation volatility of 0.42% with all shocks. Compared to results in extant studies with similar model features, this value is relatively high and represents close to 20% of the observed historical inflation volatility.\(^{12}\) Most of the inflation volatility can be attributed to the long-run shocks. The higher optimal inflation volatility is due to the rise in the average markup as monetary policy increasingly stabilizes inflation. These dynamics are explained in greater detail in section 3.1.

Output Growth. Under recursive preferences, I find that the output growth coefficient is dramatically larger for the setting with long-run shocks compared to that with only short-run shocks. Increasing the weight on the output growth reduces the markup and lowers the volatility of the consumption-leisure bundle. For CRRA preferences, I find it optimal to completely eliminate price dispersion and place an infinite coefficient on inflation with no weight on output growth. The average markup and the volatility of the consumption-leisure bundle stay relatively constant as monetary policy increasingly stabilizes inflation, which is counterfactual to empirical evidence.

Welfare Gains. For the benchmark setting with a low risk free rate and high equity premium (table 4, last panel), I find the optimal policy provides a welfare gain of 0.07% compared to the policy that places an infinite weight on inflation. This translates into about 0.20 additional percentage of consumption units every quarter. While this may be small compared to the gains found in the policy literature that incorporates endogenous growth, a perpetuity that pays this amount each year from now on is worth over $3,000, assuming an interest rate of 2%. In other words, a greater response to output growth and less to inflation (relative to that advocated in the existing literature) would be worth a one-time payment of over $3,000 to each individual.\(^{13}\) This is in contrast to results from the setting with only short-run shocks, in which an equivalent calculation would yield a perpetuity worth only $17. This small magnitude is in line with the existing literature (see Schmitt-Grohé and Uribe (2007)).

To explain the low inflation coefficient and high weight on the output growth for the asset pricing-

\(^{12}\)Inflation is defined as the annualized percentage change in the quarterly GDP price deflator dating back to 1947.

\(^{13}\)Base year 2009 dollars.
oriented model, I focus on the setting in which there are only long-run shocks. Although the long-run shock is one-seventh the size of the short-run shock, its persistence (combined with the very forward-looking nature of agents) dominates the other shocks.

3.1 Long-Run Productivity Shocks Only

The small but persistent news shock to productivity growth has been empirically documented by Croce (2014), Beaudry and Portier (2004), Schmitt-Grohé and Uribe (2007), Barsky and Sims (2011), and Kurmann and Otrok (2010). Specifically, Croce (2014) finds that the conditional mean of productivity growth is extremely persistent and time-varying. In a setting with forward-looking monopolistically competitive firms, a low-frequency predictable fluctuation in productivity can have significant ramifications for the prices chosen by firms, which reverberates through the demand-driven New Keynesian model. Below, I examine the channels that drive the finding of a low optimal inflation coefficient and relatively high weight on output growth.\footnote{Discussion of the optimal inertia coefficient, $\alpha_r$, is reserved for appendix B.}

Optimal Inflation Coefficient and Three Welfare Channels. In this section I examine the reasons the optimal inflation coefficient is low for the recursive preferences asset pricing-oriented model and high for the CRRA preferences model. In figures 2–3, I compare three different models: (1) recursive preferences with high risk aversion ($\gamma = 10$) and high IES ($\psi = 2$); (2) recursive preferences with high risk aversion ($\gamma = 10$) and low IES ($\psi = 0.2$); and (3) CRRA preferences with high risk aversion ($\gamma = 10$) and low IES ($\psi = 1/10$). Figure 2 depicts the effects of increasing the inflation coefficient and figure 3 shows the welfare. The increase in $\alpha_\pi$ can be thought of as moving from a dove regime to a hawk regime. The level of price dispersion decreases in all settings due to the lower inflation volatility (far right panel of figure 2). Monetary policy, by increasingly stabilizing inflation, reduces nominal uncertainty.

Major differences across the three settings can be seen in the left two panels of figure 2. The far left panel shows the volatility of the consumption-leisure bundle. While volatility is increasing with the degree of inflation stabilization for all three settings, the increase is lowest for the CRRA preferences. This makes sense because of the inverse nature of CRRA preferences, in which the IES is lower due to the
This figure shows the three key channels for welfare (volatility of consumption-leisure bundle, average markup, price dispersion) and the effects of greater inflation stabilization. The setting is a world with only long-run news shocks to productivity. The other coefficients are set to match the rule that yields the highest welfare for the high IES ($\psi = 2$) and high risk aversion ($\gamma = 10$) case, as in equation 4 and table 2. Both of the recursive preference settings exhibit strong increases in the volatility of the consumption-leisure bundle compared to the CRRA preferences. The average markup rises the most for the high IES and high risk aversion setting. All of the settings see a reduction in price dispersion as the inflation coefficient rises.

High risk aversion. With a lower IES, by definition agents will react less to monetary policy and so the volatility does not rise as much for a given change in policy. This is also true for the recursive preferences as one moves from a low IES to a high IES. The increase in volatility of the consumption-leisure bundle is greatest for the high IES case.

The average markup rises the most for the model with the high IES. The markup falls more in response to negative productivity shocks, and the more that monetary policy stabilizes inflation, the less the markup will be allowed to decline, resulting in a higher average markup. Note that with less asymmetry, the low IES markup does not rise as much with the inflation coefficient. All three models converge to the average markup that would occur in the nonstochastic steady-state, in which the variance of the markup is zero. In other words, they all converge to the level of the markup that would occur under a first-order approximation.

In terms of overall welfare, the optimal inflation coefficient is lowest for the model with high IES ($\psi = 2$) and high risk aversion ($\gamma = 10$), as shown in figure 3. Beyond the value of 3 for the inflation coefficient, the benefits of reducing price dispersion are outweighed by the costs of the higher markup.

\[15\text{This is perfectly consistent with the asymmetric responses discussed in appendix C and figure A1}\]
**Fig. 3: Effects of Increasing the Inflation Coefficient: Welfare ($\alpha_\pi$)**

This figure shows the effects on welfare of increasing the inflation coefficient in a world with only long-run productivity shocks. The other coefficients are set to match the rule that yields the highest welfare for the high IES ($\psi = 2$) and high risk aversion ($\gamma = 10$) case as in equation 4 and table 2. The greatest fall in welfare comes with the High IES, High Risk Aversion on the far left due to the higher markup from stabilizing inflation and the increased volatility of the consumption-leisure bundle. This is in contrast to the CRRA preferences setting in which completely stabilizing inflation is optimal.

and greater volatility of the consumption-leisure bundle. For the low IES case, the optimal inflation coefficient is higher with lesser increases in the markup. Finally, the CRRA preferences setting has a value on the inflation coefficient of infinity as the reduction in price dispersion dominates the markup channel. Monetary policy has very little effect on the volatility of the consumption-leisure bundle and the markup, which means the costs of stabilizing inflation are essentially nil.

**Output Growth Coefficient and Three Welfare Channels.** In this section I examine the reasons the optimal output growth coefficient is high for the asset pricing-oriented model. As shown in figure 4, placing weight on the output growth reduces the variability of the consumption-leisure bundle and also provides a greater anchor for inflation expectations. This is because when output falls below potential, monetary policy lowers interest rates with the implicit assurance that interest rates will rise as output increases back to potential. Anchoring inflation expectations is imperative because current inflation depends on expectations of future inflation, and in the model firms are very patient and forward looking.\(^{16}\)

In addition, the markup monotonically decreases with the output growth coefficient. Greater weight

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\(^{16}\)This is in contrast to reacting to simply the level of output, which destabilizes inflation expectations. I find that a zero weight on the level of output is optimal as the negative effects of price dispersion dominate.
Fig. 4: Effects of Increasing the Output Growth Coefficient ($\alpha\Delta y$)

This figure shows the effect of placing higher weight on the output growth in a world with only long-run productivity shocks. The other coefficients are set to match the rule that yields the highest welfare for the high IES ($\psi = 2$) and high risk aversion ($\gamma = 10$) case, as in equation 4 and table 2. The volatility of the consumption-leisure bundle and the average markup decline, while price dispersion is increasing due to higher inflation volatility, which causes a hump-shape for welfare.

on the output growth means relatively less weight on inflation, so that inflation volatility and price dispersion increase. Thus, after the negative long-run productivity shock shown in Figure 5, the average markup is persistently lower in the medium to long term, while inflation is higher. Furthermore, the decline in the response of the consumption-leisure bundle is also beneficial to welfare. Eventually, the consumption-leisure bundle volatility increases due to monetary policy overcompensation. This, combined with the higher price dispersion, leads to the decline in welfare when moving beyond an output growth coefficient of 1.5.

3.2 CRRA Preferences

This section shows how agents and firms with CRRA preferences react to negative long-run productivity shocks. Figure 6 shows the real stochastic discount factor in the upper left panel. Note the rise for recursive preferences is eight times greater in magnitude due to the continuation value that captures news. The higher real SDF combined with the higher IES makes firms more forward looking, which causes them to raise prices to a greater extent because they are concerned with higher future marginal costs.

The higher inflation directly translates into a lower real markup, which acts as a hedge and provides good long-run news. The low IES with CRRA preferences reduces the effectiveness of monetary policy
Fig. 5: Higher Output Growth Response: Negative Long-Run Shock ($\alpha \Delta y$)

This figure shows the response to a negative long-run productivity shock. The parameterization is based on the calibration in table 2 for the high IES ($\psi = 2$) and high risk aversion ($\gamma = 10$) case and focuses on the policy that yields the highest welfare in equation 4. The volatility of the consumption-leisure bundle declines and the average markup falls more over the medium to long term due to the higher inflation.

so that the consumption-leisure bundle and average markup quickly converge to the steady state. This analysis concurs with the evidence in figure 2, which shows that for CRRA preferences, monetary policy is less effective in reducing the average markup and the volatility of the consumption-leisure bundle.

### 3.3 Short-Run Shocks Only

In a setting with only short-run shocks, I find that the differences in welfare across policies are negligible, as shown in figure 7. Under CRRA preferences, the policy that yields the highest welfare places an infinite value on the inflation coefficient. Under recursive preferences, the following rule yields the highest welfare:

$$\hat{R}_t = 0.9 \cdot \hat{R}_{t-1} + 1.75 \cdot (1 - 0.9) \cdot \hat{\pi}_t + \frac{1}{4} \cdot (1 - 0.9) \cdot \Delta \hat{y}_t.$$

However, in contrast to the policy that places an infinite weight on inflation, the welfare benefit is only 0.000367%. This is 200 times smaller than a similar comparison with a setting comprising only long-run shocks, where the welfare benefit of the best policy is 0.07% compared to the one that places an infinite weight on inflation.

The differences in welfare across policies are small due to the largely symmetric responses of the
average markup and inflation to short-run shocks. Moreover, these variables revert to the steady state very quickly compared to those in the setting with long-run shocks. In summary, the lack of persistence and asymmetry in response to short-run shocks combined with the near-zero equity premium makes all three welfare channels insignificant. Note that the small differences in welfare across policies in this setting are consistent with the findings of Schmitt-Grohé and Uribe (2007), who also reach this conclusion in a setting without long-run risk.

4 Further Inspection of Dynamics

In this section I directly compare the low IES ($\psi = 0.2$) and high IES ($\psi = 2$) settings to further emphasize the importance of making firms and agents more patient. Following that, I show the reasons the average markup rises as monetary policy increasingly stabilizes inflation. Further sensitivity analysis
Fig. 7: Effects of Increasing the Inflation Coefficient ($\alpha_{\pi}$), Short-Run Shocks Only

This figure shows the three key channels for welfare (volatility of consumption-leisure bundle, average markup, price dispersion) and the effects of greater inflation stabilization. The setting is a world with only short-run shocks to productivity. The other coefficients are set to match the rule that yields the highest welfare for the high IES ($\psi = 2$) and high risk aversion ($\gamma = 10$) case, as in table 2. The effect on the average markup and price dispersion is essentially flat for all three models. The effects on the volatility of the consumption-leisure bundle are muted compared to the setting with long-run shocks only. Overall, I find the welfare differences across policies are negligible.

with respect to changes in the risk aversion and intertemporal elasticity of substitution can be found in appendix D.\textsuperscript{17}

High vs. Low IES. The high IES model better captures the low risk-free rate in the data and makes forward-looking agents and firms more patient, as shown in the left panel of figure 8. With greater precautionary savings, the consumption-leisure bundle growth declines more, and this results in the doubling of the stochastic discount factor when moving from the low to high IES model. In this context, forward-looking firms place greater weight on future marginal costs, so that the decrease in inflation in the initial period is five times less in the high IES model than the low IES model. The higher inflation pushes down the relative prices set by firms in previous periods and decreases the real value of the average markup. All of the above results suggest that the high IES model has a greater negative effect on the average markup.

\textsuperscript{17}In appendix D, I also show why the markup channel is largely not present in other studies—namely, due to the absence of both a high IES and high risk aversion. When the IES is relatively low (which is typically the case for CRRA preferences), moving from a first-order to second-order approximation barely changes the dynamics.
Fig. 8: Negative Long-Run Shock

This figure shows the effects of a negative long-run productivity shock on the three key channels for welfare: inflation volatility, markup, and consumption-leisure volatility. The parameterization is based on the calibration in table 2 and focuses on the policy that yields the highest welfare in equation 4. The left panel shows that moving from a low IES to a high IES leads to significantly different implications for each channel. The right panel shows that moving from the low inflation coefficient of 2.25 to the high inflation coefficient of 5, there is a decrease in inflation volatility which means the markup does not fall by as much in recessions. This suggests the average markup will be higher as monetary policy increasingly stabilizes inflation.
**High IES: Low vs. High $\alpha_\pi$ (Dove vs. Hawk).** As shown in the right panel of figure 8, a higher inflation coefficient combined with a high coefficient on the lagged interest rate imply a higher real interest rate for many periods in response to inflation. Forward-looking firms take this into account when setting prices, knowing that monetary policy is actively attempting to stabilize inflation. Therefore, firms choose lower prices due to the lower expected inflation, and this causes the initial inflation to be lower.

However, the decrease of inflation volatility lowers the inefficient allocation coming from price dispersion. Therefore, a trade-off exists between increasing the average markup and decreasing price dispersion as monetary policy increasingly stabilizes inflation. Note that this markup channel is nonexistent for first-order approximations and is not as large for the low IES model. Since the decrease in inflation volatility means the markup does not decrease by as much in recessions, this implies the average markup will be higher as monetary policy increasingly stabilizes inflation.

5 Conclusion

Asset pricing is important for monetary policy analysis because it reveals how much agents dislike recessions and the extent to which they are forward looking and patient. I have shown in this study that the combination of these two characteristics, along with the presence of long-run risk, leads to policy recommendations that are very different from those of prior studies. Specifically, in my asset pricing-oriented New Keynesian model, much greater weight is placed on output growth, and a much smaller weight is placed on inflation. Price dispersion is no longer the dominant channel in this setting. In addition, the effects of monetary policy on the volatility of the consumption-leisure bundle and the average markup become first-order concerns.

I find that the welfare gain of moving away from a policy that completely stabilizes inflation is two hundred times greater for settings with long-run shocks relative to those with short-run shocks. This translates into the equivalent of a one-time benefit of over $3,000 for every individual. Moreover, the optimal inflation volatility is forty times greater in this setting than in similar settings without long-run risk. In my model, the weight on the output growth is over 14 times greater than the weight that is placed on the output level in the original study by Taylor (1993). All of these findings would likely be
magnified in a setting with endogenous rather than exogenous growth. In the current economic setting, monetary policy is unable to increase or decrease long term growth, and I leave this as a topic for future research.
References


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Appendix

A Model

The economy consists of a continuum of identical households, a continuum of intermediate-goods firms, and a government that conducts monetary and fiscal policy. The structure of the model is the standard neoclassical growth model augmented with real and nominal frictions. The nominal friction is sticky prices. The real friction is monopolistic competition, which results in a markup of price over marginal costs. Monetary policy assumes full commitment to an interest rate rule that is a function of inflation and output growth. Fiscal policy raises lump-sum taxes to pay for exogenous expenditures.

Preferences. The households have Epstein-Zin preferences defined over consumption goods, $c_t$, and leisure, $1-h_t$. These preferences exhibit a CES aggregate of current and future utility certainty equivalent weighted by $(1-\beta)$ and $\beta$, respectively.

$$v_t = \max_{\{c_j,h_j,i_j,b_j,k_{j+1}\}_{j=t}} \left\{ (1 - \beta)(c^\beta (1 - h_t)^{1-\beta})^{1 - \frac{1}{\psi}} + \beta(E_t[v_{t+1}^{1-\gamma}])^{1 - \frac{1}{\psi}} \right\}^{\frac{1}{1 - \frac{1}{\psi}}}$$

s.t.

$$b_t + c_t + i_t + \tau_t = R_{t-1} \frac{b_{t-1}}{\pi_t} + w_t h_t + u_t k_t + \tilde{\phi}_t$$

The real value of debt is $b_t$; $c_t$ is consumption; $i_t$ is investment; $R_{t-1}$ is the risk-free rate; $\pi_t$ is the inflation rate $\frac{P_t}{P_{t-1}}$; $\tau_t$ is the lump-sum tax; $w_t$ is the real wage; $h_t$ is labor hours; $u_t$ is the rental rate of capital; $k_t$ is capital; and $\tilde{\phi}_t$ is profits.

Unlike standard preferences, Epstein-Zin preferences allow for the disentanglement of $\gamma$, the coefficient of relative risk aversion, and $\psi$, the elasticity of intertemporal substitution. When $\frac{1}{\psi} = \gamma$, the utility collapses to standard preferences with additively separable expected utility both in time and state. When $\gamma > \frac{1}{\psi}$, the agent prefers early resolution of uncertainty, so the agent dislikes shocks to long-run expected growth rates.
Intermediate good bundling. The consumption good is assumed to be a composite made of a continuum of differentiated goods $c_{it}$ indexed by $i \in [0,1]$ via the aggregator:

$$c_t = \left[ \int_{0}^{1} c_{it}^{1 - \frac{i}{\eta}} \, di \right]^{\frac{1}{1 - \frac{i}{\eta}}}$$

The elasticity of substitution across different varieties of consumption goods is $\eta > 1$ (also the price elasticity of demand for good $j$). As $\eta \to \infty$, the goods become closer and closer substitutes, so that individual firms have less market power.

The household minimizes total expenditures subject to an aggregation constraint, where $P_{jt}$ is price of intermediate good $j$:

$$\min_{c_{jt}} \int_{0}^{1} P_{jt} c_{jt} \, dj$$

s.t.

$$\left[ \int_{0}^{1} c_{it}^{1 - \frac{i}{\eta}} \, di \right]^{\frac{1}{1 - \frac{i}{\eta}}} \geq c_t$$

The optimal demand for the level of intermediate consumption good $c_{jt}$ is given by

$$c_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\eta} c_t$$

where $P_t$ is the nominal price index

$$P_t \equiv \left[ \int_{0}^{1} P_{jt}^{1 - \eta} \, dj \right]^{\frac{1}{1 - \eta}}$$

Productivity. The law of motion of the productivity process captures both short-run and long-run productivity risks:

$$\log \frac{A_{t+1}}{A_t} \equiv \Delta a_{t+1} = \mu + x_t + \sigma_a \varepsilon_{a,t+1}, \quad (5)$$

$$x_{t+1} = \rho x_t + \sigma_x \varepsilon_{x,t+1}, \quad (6)$$

$$\begin{bmatrix} \varepsilon_{a,t+1} \\ \varepsilon_{x,t+1} \end{bmatrix} \sim i.i.d. N \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad t = 0, 1, 2, \cdots \quad (7)$$
According to the above specification, short-run productivity shocks, $\varepsilon_{a,t+1}$, affect contemporaneous output directly but have no effect on future productivity growth. Shocks to long-run productivity, represented by $\varepsilon_{x,t+1}$, carry news about future productivity growth rates but do not affect current output.

**Capital Accumulation Technology.** Assume that investments in different vintages of capital have heterogeneous exposure to aggregate productivity shocks. In other words, there will be vintage-specific productivity growth that is going to depend on the age $j = 0, 1, ..., t - 1$ of the vintage of capital

$$\frac{A_{t+1}^{t-j}}{A_{t}^{t-j}} = e^{\mu + \phi_j(\Delta a_{t+1} - \mu)}$$

Under the above specification, production units of all generations have the same unconditional expected growth rate. Also, $A_{t}^{t-0} = A_{t}^{0}$ is set to ensure that new production units are on average as productive as older ones. The log growth rate of the productivity process for the initial generation of production units, $\Delta a_{t+1}$, is given by equation (5). Heterogeneity is driven solely by differences in aggregate productivity risk exposure, $\phi_j$.

The empirical findings in Ai et al. (2012) suggest that older production units are more exposed to aggregate productivity shocks than younger ones, i.e., the exposure $\phi_j$ is increasing in $j$. To capture this fact, a parsimonious specification for $\phi_j$ is adopted:

$$\phi_j = \begin{cases} 
0 & j = 0 \\
1 & j = 1, \ldots
\end{cases}$$

New production units have zero exposure to aggregate productivity shocks in the first period of life. Every period thereafter, they have 100% exposure to aggregate productivity shocks as do all other existing vintages.

\[\begin{align*}
\frac{A_{t+1}^{t-0}}{A_{t}^{t-0}} &= e^\mu \\
\frac{A_{t+1}^{t-j}}{A_{t}^{t-j}} &= e^{\mu + x_t + \sigma a \varepsilon_{a,t+1}} \\
\end{align*}\]

\(8\)

18. Multiple frictions for capital accumulation have been tested, and this friction was chosen because standard capital adjustment costs result in counterfactually low investment growth volatility. The friction in this paper does not suffer from this issue.
Let $K_t$ denote the productivity-adjusted physical capital stock. Despite the heterogeneity in productivity, aggregate production can be represented as a function of $K_t$ and $N_t$. The law of motion of the productivity-adjusted physical capital stock $K_t$, takes the following form:

$$K_1 = I_0, \quad K_{t+1} = (1 - \delta)K_t + \omega_{t+1}I_t$$

$$\omega_{t+1} = \left( \frac{A_{t+1}^{t-1}}{A_{t+1}^0} \right)^{\frac{1-\alpha}{\alpha}} = e^{-\frac{1-\alpha}{\alpha}(x_t+\sigma_a\epsilon_{a,t+1})(1-\phi_0)}$$

where $I_t$ is the total mass of new vintage capital produced at time $t$, and $\omega_{t+1}$ is an endogenous process that accounts for the productivity gap between the newest vintage of capital and all older vintages. Note that when $\phi_0 = 1$, the new capital vintage has the same exposure to aggregate productivity shocks as older ones. In this case, $\omega_{t+1} = 1$ for all $t$ and capital of all generations are identical.

**The Government.** The government issues one-period nominal risk-free bonds, $b_t$, collects taxes in the amount of $\tau_t$, and faces an exogenous expenditure and transfers stream, $g_t$ and $tr_t$. Its period by period budget constraint is given by

$$b_t = \frac{R_t}{\pi_t}b_{t-1} + g_t - \tau_t + tr_t$$

The exogenous expenditure streams are formulated as in Croce et al. (2012)

$$\frac{G}{Y} = \frac{1}{1 + e^{-gy}}$$

$$gy_t = (1 - \rho_g)\bar{gy} + \rho_ggy_{t-1} + \epsilon_{G,t}, \quad \epsilon_{G,t} \sim N(0, \sigma_{gy}^2)$$

$$tr_y = (1 - \rho_{gt})\bar{tr_y} + \rho_{gt}tr_{y,t-1} + \epsilon_{tr,t}, \quad \epsilon_{tr,t} \sim N(0, \sigma_{tr_y}^2)$$

Total tax revenues, $\tau_t$, consist of lump-sum tax revenues.

Tax smoothing by the government consists of tax revenues rising whenever the previous period’s debt rises

$$\tau_t - \tau^* = \gamma_1(b_{t-1} - b^*)$$

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where $\gamma_1 > 0$ ensures that the debt-GDP ratio is bounded and there is a unique solution.

The monetary authority sets short-term nominal interest rate according to a simple Taylor rule

$$
\ln(R_t/R^*) = \alpha_r \ln(R_{t-1}/R^*) + \alpha_\pi \ln(\pi_t/\pi^*) + \alpha_\Delta y \ln(\Delta y_t/\Delta y^*)
$$

where $\alpha_r$ is the inertia coefficient, $\alpha_\pi$ is the inflation coefficient, $\alpha_\Delta y$ is the output growth gap coefficient, $\pi^*$ is the inflation target, and $\Delta y^*$ is the output growth target.

**Firms.** Each variety $i \in [0, 1]$ is produced by a single firm in a monopolistically competitive environment. Each firm $i$ produces output using as factor inputs capital services, $k_{it}$, and labor services, $h_{it}$. It is assumed that the firm must satisfy demand at the posted price. Formally,

$$
h^{\theta}_{it}(A_t h_{it})^{1-\theta} \geq \left(\frac{P_{it}}{P_t}\right)^{-\eta} y_t
$$

The objective of the firm is to choose $P_{it}$, $h_{it}$, $k_{it}$ to maximize the present discounted value of profits, given by

$$
E_t \sum_{s=t}^{\infty} m_{t,s} P_s \phi_{is}
$$

where real profits of firm $i$ are

$$
\phi_{it} = \frac{P_{it}}{P_t} y_{it} - u_t k_{it} - w_t h_{it}
$$

Prices are assumed to be sticky as in Calvo (1983). Each period, a fraction $\alpha \in [0, 1)$ of randomly picked firms is not allowed to optimally set the nominal price of the good they produce. Instead, these firms index their prices to past inflation according to the equation

$$
P_{it} = P_{it-1} \pi_{t-1}^\chi.
$$

Note, that in all settings $\chi = 0$, which implies there is no price indexation. The remaining $1 - \alpha$ firms choose $\hat{P}_t$ to maximize the expected present discounted value of profits:
The firm’s first-order conditions for labor, capital, and optimal price are

\[ mc_t (1 - \theta) \left( \frac{\tilde{k}_t}{\tilde{h}_t} \right)^{\theta} = \tilde{\omega}_t \]  
(9)

\[ mc_t \theta \left( \frac{\tilde{k}_t}{\tilde{h}_t} \right)^{\theta - 1} = u_t \]  
(10)

The firm’s optimal price is set such that marginal revenues are equal to some markup over marginal costs

\[ \frac{\eta}{\eta - 1} x^1_t = x^2_t \]  
(11)

where

\[ \tilde{x}^1_t = p^*_t \left( \frac{\tilde{y}_t}{mc_t} \right) \]  
(12)

\[ \tilde{x}^2_t = p^*_t \left( \frac{\tilde{y}_t}{mc_t} \right) + \alpha E_t D_{t,t+1} \frac{\pi_{t+1}^{\chi} \chi^{-\eta}}{\pi_{t+1}^{\chi}} \left( \frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right) \]  
(13)

**Aggregation and Equilibrium.** This period’s price level is a weighted average of the firm’s optimal price and the previous period’s price level:

\[ 1 = \alpha \pi_{t-1}^{1+\eta} \pi_{t-1}^{(1-\eta)} + (1 - \alpha) p^*(1-\eta) \]  
(14)

It can be shown that the resource costs of inefficient price dispersion are characterized as follows

\[ s_t = (1 - \alpha) p^*_t \left( \frac{\pi_t}{\pi_{t-1}} \right) + \alpha \left( \frac{\pi_t}{\pi_{t-1}} \right) s_{t-1} \]  
(15)
Given the price dispersion, output is described by

\[ \tilde{y}_t = \frac{1}{s_t} \left[ \hat{k}_t^\theta (A_t h_t)^{1-\theta} \right] \] (16)

and aggregate demand is the following sum

\[ \tilde{y}_t = \tilde{c}_t + \tilde{i}_t + \tilde{g}_t \] (17)

### B Optimal Inertia Coefficient

In the New Keynesian model, forward-looking expectations are a crucial component through which monetary policy can influence the economy. This is because consumption and investment (i.e., aggregate demand) are dependent upon not just the current real interest rate, but also all future expected real interest rates. Given that the policymaker in my model is committing to a rule in which it has complete credibility, monetary policy is fully capable of altering the expectations of the households and firms.

Since firms are forward looking when resetting their prices, inflation (which is a function of the firms optimal prices) will not rise by as much if the firms are promised by monetary policy that future output gaps and marginal costs will be lower (which is what commitment allows). This improves the short-run trade-off between inflation and output and is the reason that policies under commitment welfare-dominate discretionary policies. It is in this spirit that a positive inertia coefficient could potentially improve welfare.

With a positive inertia coefficient, dependence on a lagged term permits the policy maker to manipulate long-term interest rates with more modest movements in the short-term rate than would otherwise be necessary. One way to influence the future path of short-term rates is to maintain a higher level of interest rates for a period of time after they have been raised. Hence, given the agents’ forward-looking expectations, monetary policy can impose significant effects on aggregate demand without using extremely volatile movements in the short-term interest rate. In my model, a high coefficient on the lagged interest rate reduces the variance of both inflation and consumption growth, and the optimal value is 0.9.
Fig. A1: Asymmetric Responses to Positive and Negative Long-Run Shock

This figure shows the asymmetric endogenous responses to a negative long-run productivity shock on inflation and the markup. The parameterization is based on the calibration in table 2 and focuses on the policy that yields the highest welfare in equation 4. The colored line shows the difference between the positive and negative shocks. This asymmetry is greatest for the high IES setting and decreases with a low IES or high inflation coefficient. The implication is that the steady state markup will be close to the markup that occurs in the linear, symmetric setting. The negative shock dominating for the high IES implies that the average markup would be lower in that setting.

C Asymmetric, persistent responses to long-run shocks

If all of the above dynamics were symmetric with respect to positive and negative shocks, policy would have no impact on the stochastic steady state of any of the endogenous variables. The second-order approximation is crucial because it captures nonlinearities that are inherent to the data. Nonlinearities imply asymmetric responses of endogenous variables. This potential asymmetry means that a greater variance can impact the steady-state values of variables such as the average markup, which is important for my welfare analysis.

Figure A1 shows the response of inflation and the average markup to both a positive and negative shock. The differences in responses have been magnified for pedagogical reasons. The red line is the difference in magnitude between the positive and negative long-run shock for the model with a high IES. It is clear that the red line follows the negative shock, which implies that the negative shock dominates.
The magnitude of the difference is greater for the high IES model than for the low IES model. In the high IES model, the more forward-looking firms are choosing a higher optimal price in response to the long-run negative productivity shock, and this erodes the average markup to a greater extent. The responses are practically symmetric with respect to the setting with a high inflation coefficient. This implies that the steady-state markup will be close to the markup that would occur in a linear, symmetric setting. Although not shown, the first-order approximations were perfectly symmetrical so the difference was precisely zero for every period.

D Sensitivity Analysis

In terms of robustness, I test how sensitive the optimal inflation volatility and optimal inflation coefficients are to changes in the risk aversion coefficient and the intertemporal elasticity of substitution. Figure A2 shows that both a high IES ($\psi = 2$) and high risk aversion ($\gamma = 10$) are necessary for monetary policy to find it optimal to place less weight on stabilizing inflation. As these two parameters rise, monetary policy finds it optimal to stabilize consumption to a greater extent to lessen the severity of recessions and also to more effectively reduce the markup in bad times.

First- vs. Second-Order Approximation, Low IES

The first studies of New Keynesian monetary policy (e.g. Goodfriend and King (1997), Woodford (2001)) used linear approximations. Studies thereafter used second-order approximations (e.g. Kollmann (2008), Schmitt-Grohé and Uribe (2007)) and they came to the same conclusions: it is optimal to focus solely on stabilizing inflation. All of the studies mentioned use standard preferences, where the IES is low and typically below one because it represents the inverse of the risk aversion coefficient.

To understand the negligible difference, note that the second-order approximation captures the effect of the variance of future shocks. Given the relatively low patience of the agent, it stands to reason that future effects would play a minor role in determining the optimal allocations, and this is confirmed by the impulse response functions in Figure A3. Overall, my findings suggest that one should expect to see very little difference in dynamics and outcomes when moving from first order to second order for a model.
with a low IES and relatively high risk free rate. This finding is roughly consistent with the notion that
the typical macroeconomic model that abstracts from financial data can be well approximated with a
first-order approximation.

**First- vs. Second-Order Approximation, High IES**

There are sizable differences between the first- and second-order approximations for the high IES ($\psi = 2$)
case, as shown in Figure A3. The growth of the consumption-leisure bundle reacts more and does not
revert to steady state as quickly as the first-order approximation. This is intuitive, because the second-
order approximation yields a greater motive for precautionary savings, so that the agent works more and
decreases leisure more at the onset of bad news. Firms also become more forward looking and are choose
much higher prices as the negative productivity shock continues well into the future. For the second-order
approximation, the average markup no longer quickly reverts to steady state but instead stays low for an
extended period of time.

By taking into account the future shocks to the variance along with a greater precautionary savings
motive, the second-order approximation implies much lower average markups are associated with negative
long-run productivity shocks. Recall that the average markup acts as an implicit tax on factors of
production, so that a decrease in its value is positive for welfare. This substantial drop in the average
markup is missing from the previous section that focuses on the low IES and is also missing in the
literature. By allowing inflation to rise so that the average markup falls, monetary policy is providing
good long-run news to counter the bad productivity shock.
Fig. A2: Effects of Increasing IES and Risk Aversion with Long-Run Shocks Only

This figure shows the effects of increasing the intertemporal elasticity of substitution and risk aversion. The setting is a world with only long-run shocks to productivity. The other coefficients are set to match the rule that yields the highest welfare for the high IES ($\psi = 2$) and high risk aversion ($\gamma = 10$) case as in table 2. The higher IES increases the patience of households and lowers the risk-free rate. This also makes firms more forward looking and reduces the average markup. The lower markup combined with the greater sensitivity of consumption to changes in real rates leads to higher optimal inflation volatility and a lower inflation coefficient as the IES rises, as shown in the bottom two panels. The higher risk aversion increases the equity premium, which coincides with higher welfare costs of recessions.
This figure shows the effects of a negative long-run productivity shock on the three key channels for welfare: inflation volatility, markup, and consumption-leisure volatility. The parameterization is based on the calibration in table 2 and around the policy that yields the highest welfare in equation 4. When moving from a first- to second-order approximation, if the IES is low, the differences are negligible. If the IES is high, the effect on the average markup is significant.